Bandwidth-Efficient OFDM Cooperative Protocol with Applications to UWB Communications

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Abstract— **In this paper, we propose an OFDM cooperative protocol that not only achieves full diversity but also efficiently utilizes available bandwidth. The proposed protocol exploits limited feedback from the destination terminal (central node) such that each relay is able to help forward information of multiple sources in one OFDM symbol. To specify how relay-source pairs should be assigned, we propose two practical relay-assignment schemes, including fixed-location scheme in which the relays are fixed at optimum locations, and centralized-control scheme in which the relays are assigned by the central node. We provide outage probability analysis of the proposed protocol in wireless indoor environment. Moreover, a lower bound on the outage probability of any relay-assignment schemes is established, and the performance of the proposed relay-assignment schemes is analyzed. We also investigate the application of the proposed protocol to enhance the performance of UWB systems. In UWB wireless indoor scenarios, both theoretical and simulation results show that the proposed cooperative protocol can achieve 75% power saving and 200% coverage extension compared to the non-cooperative UWB system proposed in the IEEE 802.15.3a standard.**

I. INTRODUCTION

Cooperative diversity has recently emerged as a promising alternative to combat fading in wireless channels. The basic idea is that users or nodes in a wireless network share their information and transmit cooperatively as a virtual antenna array, thus providing diversity without the requirement of additional antennas at each node. In [1], the authors proposed various cooperative strategies including fixed relaying (e.g. amplify-and-forward and decode-and-forward), selection relaying, and incremental relaying schemes. In [2], a similar concept, called user cooperation diversity, was proposed for CDMA systems in which orthogonal codes are used to mitigate multiple access interference.

In broadband communications, orthogonal frequency division multiplexing (OFDM) is an effective means to capture multipath energy, mitigate the intersymbol interferences, and offer high spectral efficiency. OFDM is used in many communications systems, e.g., WLANs as specified by the IEEE 802.11a/g and ultra wideband (UWB) networks. Recently, UWB multiband OFDM (MB-OFDM) [3] that utilizes OFDM together with time-frequency interleaving across subbands has been proposed for the IEEE 802.15.3a WPAN standard [4]. To improve the performance of OFDM systems, the fundamental concept of cooperative diversity can be applied. Nevertheless, special modulations/cooperation strategies are needed to efficiently exploit the available multiple carriers. In [5], an oversampling technique is used in combination with the intrinsic properties of OFDM symbols to provide efficient resource utilization. An application of space-time cooperation in OFDM systems was investigated in [6]. In [7], pairing of users and level of cooperation are jointly determined to minimize overall transmitted power of OFDM system. Most of the existing works are based on fixed relaying protocols, in which the relays always repeat the source information. Moreover, these works rely on an assumption of fixed channel variances which implies a fixed network topology and fixed source-relay pairs.

In this paper, we propose an OFDM cooperative protocol that improves spectral efficiency over those based on fixed relaying protocols while achieving the same performance of full diversity. By exploiting limited feedback from the destination node, the proposed protocol allows each relay to help forward information of multiple sources in one OFDM symbol. We also propose two practical relay-assignment schemes, namely fixed-location scheme and centralized-control scheme, for implementing the proposed cooperative protocol in OFDM networks considering the random users' spatial distribution. Outage probability is provided as a performance measure of the proposed protocol. A lower bound on the outage probability of any relay-assignment schemes is established, and the performance of the proposed relay-assignment schemes is analyzed. Furthermore, we investigate the application of the proposed protocol to enhance the performance of UWB communications. Simulation results are shown to validate our proposed schemes and support our theoretical analysis.

II. SYSTEM MODEL

We consider an OFDM wireless network such as a WLAN or a WPAN with a circular cell of radius ρ. The cell contains one central node and multiple users, each communicating with the central node. The central node can be a base station or an access point in case of the WLAN, and it can be a piconet coordinator in case of the WPAN. Suppose the central node is located at the center of the cell, and K users are uniformly located within the cell. Then, the user's distance D from the central node has the probability density function (PDF)

$$p_D(D) = \frac{2D}{\rho^2}, \quad 0 \leq D \leq \rho, \tag{1}$$

and the user's angle is uniformly distributed over $[0, 2\pi)$. We assume that each node is equipped with single antenna, and its transmission is constrained to half-duplex mode, i.e., any node cannot transmit and receive simultaneously [1]. We consider an uplink scenario where all users transmit their information to the central node. Similar to that specified in the IEEE 802.11a/g standard and the IEEE 802.15.3a standard proposal [3], the data packet of each user consists of preamble, header, and frame payload which carries several OFDM data symbols. The header includes the pilot symbols which allow channel estimation to be performed at the central node. Channel access within the cell is based on orthogonal multiple access mechanism as used in many current OFDM wireless networks.

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A. Channel Model

For subsequent analysis, we utilize a channel model based on the Saleh-Valenzuela (S-V) model [8], which has been widely used for indoor environments and has recently been specified in the IEEE 802.15.3a standard as UWB channel model [9]. In the S-V model, the channel impulse response is modeled by

$$h(t) = \sum_{c=0}^{C} \sum_{l=0}^{L} \alpha(c,l)\delta(t - T(c) - \tau(c,l)), \tag{2}$$

where $\alpha(c,l)$ is the gain of the l^{th} multipath component in the c^{th} cluster, $T(c)$ is the delay of the c^{th} cluster, and $\tau(c,l)$ is the delay of the l^{th} path in the c^{th} cluster relative to the cluster arrival time. The cluster arrivals and the path arrivals within each cluster are modeled as Poisson distribution with rate Λ and rate λ (where $\lambda > \Lambda$), respectively. The gain $\alpha(c,l)$ is modeled as zero-mean, complex Gaussian random variable with variance [8] $\Omega(c,l) = \mathrm{E}\left[|\alpha(c,l)|^2\right] = \Omega(0,0)\exp\left(-\frac{T(c)}{\Gamma} - \frac{\tau(c,l)}{\gamma}\right)$, where $\mathrm{E}[\cdot]$ is the expectation operation, $\Omega(0,0)$ is the mean energy of the first path of the first cluster, Γ is the cluster decay factor, and γ is the ray decay factor. The total energy of the multipath components is normalized to one, i.e., $\sum_{c=0}^{C}\sum_{l=0}^{L}\Omega(c,l) = 1$. The channel fading for each transmit-receive link is assumed to stay constant during the transmission of each packet. This assumption is reasonable for slow fading scenarios including UWB environments [9].

B. Signal Model

With the choice of cyclic prefix length greater than the duration of the channel impulse response, OFDM allows the frequency-band to be divided into a set of orthogonal narrowband subcarriers. Accordingly, the received signal at subcarrier n of destination d (central node) from source user s can be modeled as

$$y_{s,d}(n) = \sqrt{P_{NC} k D_{s,d}^{-\nu}} H_{s,d}(n) x_s(n) + z_{s,d}(n), \tag{3}$$

where P_{NC} is the transmitted power at the source in non-cooperative mode, $x_s(n)$ denotes an information symbol to be transmitted from the source s at subcarrier n, $H_{s,d}(n)$ represents the frequency response at the n^{th} subcarrier of the channel from the source to the destination, and $z_{s,d}(n)$ is an additive noise. The power P_{NC} is assumed equal for all subcarriers, i.e., no bit loading is performed, as in the current MB-OFDM standard proposal [3]. In (3), k is a constant whose value depends on the propagation environment and antenna design, ν is the propagation loss factor, and $D_{s,d}$ represents the distance between node s and node d. The noise term $z_{s,d}(n)$ is modeled as a complex Gaussian random variable with zero mean and variance N_0. Since different users transmit via orthogonal channels, no multiple access interference is considered in the signal model. From (2), the channel frequency response $H_{s,d}(n)$ is given by

$$H_{s,d}(n) = \sum_{c=0}^{C} \sum_{l=0}^{L} \alpha_{s,d}(c,l) e^{-\mathbf{j}2\pi n \Delta f [T_{s,d}(c) + \tau_{s,d}(c,l)]}, \tag{4}$$

where $\mathbf{j} \triangleq \sqrt{-1}$, $\Delta f = 1/T$ is the frequency separation between two adjacent subcarriers, T is the OFDM symbol period, and the subscript $\{s,d\}$ indicates the channel link from the source to the destination. We assume that the nodes are spatially well separated such that the channel fades for for different propagation links are statistically mutually independent, i.e., $H_{i,j}(n)$ are independent for different transmit-receive links.

Note that the information can be jointly encoded across time or frequency to achieve diversity. For instance, in the MB-OFDM approach [3], the frequency-domain spreading is obtained by choosing conjugate symmetric inputs to the IFFT, while the time-domain spreading is achieved by repeating the same information in an OFDM symbol on two different subbands [3]. When the frequency spreading is performed, the same information can be transmitted in more than one subcarrier. For subsequent performance evaluation, we denote Φ_n as a set of subcarriers that carry the information $x_s(n)$. The case when time spreading is performed is not considered here due to space limitation.

At the destination, the same information transmitted via different subcarriers is combined using the maximum ratio combining (MRC). Assume that each transmitted symbol has unit energy, then the signal-to-noise ratio (SNR) of the MRC output is [10]

$$\zeta_{s,d} = \frac{P_{NC} k D_{s,d}^{-\nu}}{N_0} \sum_{n \in \Phi_n} |H_{s,d}(n)|^2. \tag{5}$$

In this paper, we characterize the system performance in terms of outage probability [10], which is defined as the probability that the combined SNR, ζ, falls below a specified threshold, ζ_o:

$$\mathrm{P_{out}} = \mathrm{P}\left(\zeta \le \zeta_o\right). \tag{6}$$

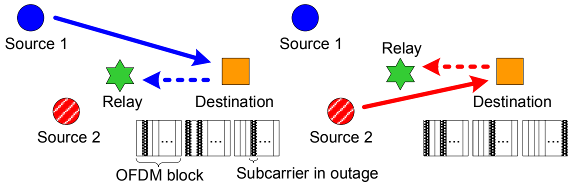
If the combined SNR of any subcarrier symbol is larger than the given threshold ζ_o, the symbol is assumed to be decoded correctly. Otherwise, an outage occurs, and the symbol is considered lost.

III. PROPOSED COOPERATIVE PROTOCOL AND RELAY-ASSIGNMENT SCHEMES

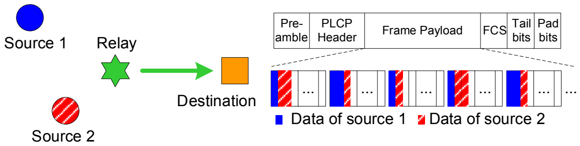
In this section, we first describe the proposed cooperative protocol, and then provide two relay-assignment schemes, namely fixed-location and centralized-control relay-assignment schemes.

A. Proposed Cooperative Protocol

Consider a cooperation scenario where a source can employ another node (relay) to forward its information to the destination. The proposed cooperative protocol is based on the incremental relaying protocols [1], which exploit a bit feedback from the destination that indicates the success or failure of the direct transmission. The proposed protocol consists of two phases. In Phase 1, each user transmits its packet to the destination (central node) and the packets are also received at the relay. After receiving the user's packet, the destination performs channel estimation using the OFDM pilot symbols in the packet header. Based on the estimated channel coefficients, the destination is able to specify which subcarrier symbols are not received successfully (i.e. those in the subcarriers of which the combined SNRs fall below the SNR threshold), and then broadcasts the indices of the subcarriers carrying those symbols. In Phase 2, the relay forwards the source symbols that are unsuccessfully transmitted in Phase 1 to the destination. Since it is unlikely that all subcarrier symbols are sent unsuccessfully, the proposed protocol makes efficient use of the available bandwidth by allowing the relay to help forward



(a) Phase 1



(b) Phase 2

Fig. 1: Illustrations of the proposed cooperative protocol for UWB MB-OFDM system with 2 users and 1 relay.

the information of multiple users in one OFDM block. The users' data to be forwarded by the relay can be arranged such that the destination can specify which subcarriers carry information of which users. For instance, if ω_i subcarriers of user i are in outage, then in Phase 2, the relay can use the first ω_1 subcarriers to transmit the data of user 1, the next ω_2 subcarriers to transmit the data of user 2, and so on. Before transmission, the relay can also perform subcarrier permutation (see [11] and references therein) to alleviate the effect of burst error. As an example, Fig. 1 illustrates the proposed protocol for a UWB MB-OFDM system with 2 source users and 1 relay. The multiple access is based on TDMA, and the first three subbands are used [3]. Figs. 1(a) and 1(b) depict transmission in Phase 1 and Phase 2, respectively.

In Phase 1, the received signals at the destination and the relay are

$$y_{s,d}(n) = \sqrt{P_{CO} k D_{s,d}^{-\nu}} H_{s,d}(n) x_s(n) + z_{s,d}(n); \tag{7}$$

$$y_{s,r}(n) = \sqrt{P_{CO} k D_{s,r}^{-\nu}} H_{s,r}(n) x_s(n) + z_{s,r}(n), \tag{8}$$

where P_{CO} is the transmitted power in the cooperative mode. As we will show in Section V, P_{CO} can be determined rigourously to ensure the same average transmitted power of both non-cooperative and cooperative protocols. In Phase 2, the signal received at the destination from the relay is given by

$$y_{r,d}(n) = \sqrt{P_{CO} k D_{r,d}^{-\nu}} H_{r,d}(n) \tilde{x}_s(n) + z_{r,d}(n), \tag{9}$$

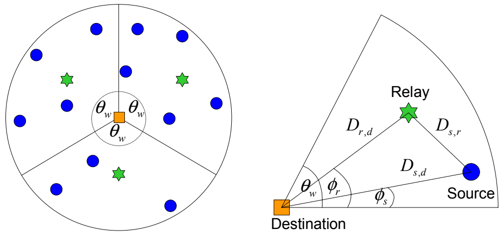
where $\tilde{x}_s(n)$ denotes the source symbols that are not captured by the destination in Phase 1.

B. Relay Assignment Schemes

We propose in this subsection two practical relay assignment schemes for cooperative OFDM networks. In both schemes, the cell is equally divided into w sectors, each with central angle $2\pi/w$; one relay is assigned to help users within each sector, as illustrated in Fig. 2(a) for a cell with $w = 3$ sectors. We describe two practical relay-assignment schemes as follows.

1) Fixed relay location: In each sector, one relay is placed at an optimum relay location which minimizes the outage probability for all possible source-destination pairs within the sector.

2) Centralized control: The central node assigns a user in each sector to be the relay for that sector. Since users are randomly located in the cell, the users may not be located in the optimum location. The central node selects the user whose location is nearest to the optimum location as the relay. This scheme requires that the locations of all users in the cell is known at the central node. This can be done via network aid position techniques (see [12] and references therein). Once the relay is assigned, it continues helping the users. The relay assignment can be updated when the network topology changes considerably.



(a) A cell with 3 sectors (b) One sector

Fig. 2: An example of relay assignment for a multiuser OFDM system.

IV. Performance Analysis

In this section, we first derive outage probability of the non-cooperative and the proposed cooperative protocols. Next, we provide a lower bound on the outage performance. Finally, we analyze performance of the proposed relay-assignment schemes.

A. Non-Cooperative and Cooperative Protocols

Given a distance $D_{i,j}$ of a transmit-receive link (i,j), the probability that the link (i,j) is in outage can be obtained from (5) and (6) as

$$\mathrm{P_{out}}(D_{i,j}) = \mathrm{P}\left(\sum_{n \in \Phi_n} |H_{i,j}(n)|^2 \leq \frac{N_0 \zeta_o D_{i,j}^{\nu}}{kP} \right), \tag{10}$$

where P is the transmitted power at node i. The outage probability in (10) can be determined from the PDF of $\xi_{i,j} \triangleq \sum_{n \in \Phi_n} |H_{i,j}(n)|^2$, which in turn can be obtained from the moment generating function (MGF) of $\xi_{i,j}$ (denoted by $\mathcal{M}_{\xi_{i,j}}(s)$). If the data is jointly encoded across multiple subcarriers, it is difficult, if not impossible, to obtain closed-form formulations of the MGF $\mathcal{M}_{\xi_{i,j}}(s)$. In the sequel, we exploit an approximation approach in [13] which allows us to approximate $\mathcal{M}_{\xi_{i,j}}(s)$ as

$$\mathcal{M}_{\xi_{i,j}}(s) \approx \prod_{n=1}^{M} \frac{1}{1 - s\beta_{i,j}(n)} = \sum_{n=1}^{M} \frac{A_{i,j}(n)}{1 - s\beta_{i,j}(n)}, \tag{11}$$

where M is the cardinality of the set Φ_n, and $A_{i,j}(n)$ is [13]

$$A_{i,j}(n) = \prod_{n'=1, n' \neq n}^{M} \frac{\beta_{i,j}(n)}{\beta_{i,j}(n) - \beta_{i,j}(n')}. \tag{12}$$

Here, $\beta_{i,j}(n)$ denote the eigenvalues of an $M \times M$ correlation matrix $\mathbf{R}_{i,j}$ whose diagonal component is one and the $(p,q)^{th}$ $(p \neq q)$ component is given by

$$R_{p,q} = \Omega(0,0) g_{p,q}(\Lambda, \Gamma^{-1}) g_{p,q}(\lambda, \gamma^{-1}), \tag{13}$$

where $g_{p,q}(a,b) \triangleq (a + b + \mathbf{j}2\pi(n_p - n_q)\Delta f)/(b + \mathbf{j}2\pi(n_p - n_q)\Delta f)$ in which n_p denotes the p^{th} element in the set Φ_n. By applying the inverse Laplace transform to the MGF in (11), and then substituting the obtained PDF into (10), we have

$$\mathrm{P_{out}}(D_{i,j}) \approx \sum_{n=1}^{M} A_{i,j}(n) \Big(1 - \exp\big(- \frac{N_0 \zeta_o D_{i,j}^{\nu}}{kP\beta_{i,j}(n)} \big) \Big). \tag{14}$$

Note that the outage probability in (14) is exact in case of no jointly encoding across subcarriers.

The conditional outage probability of the non-cooperative protocol can be obtained from (5) and (14) as

$$\mathrm{P}_{\text{out}}^{NC}(D_{s,d}) \approx \sum_{n=1}^{M} A_{s,d}(n)\Big(1 - \exp\big(-\frac{N_0\zeta_o D_{s,d}^{\nu}}{kP_{NC}\beta_{s,d}(n)}\big)\Big). \quad (15)$$

The eigenvalues $\beta_{s,d}(n)$ depend on the channel model parameters of the source-destination link. For mathematical tractability, we assume that the channel parameters of all source-destination links are the same. By averaging (15) over the user distribution in (1), we obtain the average outage probability

$$\begin{aligned}\mathrm{P}_{\text{out}}^{NC} &= \int_0^{\rho} \mathrm{P}_{\text{out}}^{NC}(D_{s,d})p_{D_{s,d}}(D_{s,d})dD_{s,d}\\ &\approx \sum_{n=1}^{M} A_{s,d}(n)\left(1 - \frac{2\Upsilon(2/\nu, B_{s,d}(n)\rho^{\nu})}{\nu\rho^2 B_{s,d}^{2/\nu}(n)}\right), \quad (16)\end{aligned}$$

where $B_{s,d}(n) = N_0\zeta_o/(kP_{NC}\beta_{s,d}(n))$ and $\Upsilon(a,x) \triangleq \int_0^x e^{-t}t^{a-1}dt$ is the incomplete Gamma function.

Under the proposed cooperative protocol, the destination broadcasts the indices of the subcarriers of which the combined SNR falls below the SNR threshold, and the assigned relay re-transmits the information conveyed in those subcarriers. Given locations of the source user and the relay, the conditional outage probability can be calculated as

$$\begin{aligned}\mathrm{P}_{\text{out}}^{CO}(D_{s,d}) = {}&\mathrm{P}\big((\zeta_{s,d} \le \zeta_o) \cap (\zeta_{r,d} \le \zeta_o) \cap (\zeta_{s,r} > \zeta_o)\big)\\ &+ \mathrm{P}\big((\zeta_{s,d} \le \zeta_o) \cap (\zeta_{s,r} \le \zeta_o)\big), \quad (17)\end{aligned}$$

where the first term corresponds to the event that both the source-destination link and relay-destination link are in outage while the source-relay link is not, and the the second term corresponds to the event that both the source-destination link and source-relay link are in outage. Using the signal models in (7)-(9), the outage probability in (14), and the assumption of independent channel links among all nodes, the conditional outage probability in (17) can be calculated as

$$\mathrm{P}_{\text{out}}^{CO}(D_{s,d}) = \big(1 - F_{s,d}(D_{s,d})\big)\big(1 - F_{s,r}(D_{s,r})F_{r,d}(D_{r,d})\big); \quad (18)$$

$$F_{i,j}(D_{i,j}) = \sum_{n=1}^{M} A_{i,j}(n)e^{-\frac{N_0\zeta_o D_{i,j}^{\nu}}{kP_{CO}\ \beta_{i,j}(n)}}. \quad (19)$$

Finally, given specific relay locations, the average outage probability of the proposed cooperative protocol can be obtained as

$$\mathrm{P}_{\text{out}}^{CO} = \frac{2}{\rho^2}\int_0^{\rho} D_{s,d}\mathrm{P}_{\text{out}}^{CO}(D_{s,d})dD_{s,d}, \quad (20)$$

where $\mathrm{P}_{\text{out}}^{CO}(D_{s,d})$ is given in (18). From (20), we can clearly see that the performance of the proposed cooperative protocol depends on how the relays are assigned to help the source users. To get more insights of the cooperation systems, we provide the performance lower bound and the performance of the proposed relay-assignment schemes in the following subsections.

B. Performance Lower Bound

To obtain a lower bound on the outage probability of the proposed cooperative protocol, we first determine an optimum relay location that minimizes the outage probability for a fixed source-destination pair. Then, the lower bound can be determined as the outage performance of a network in which the assigned relay for every source is located in the optimum location [14].

It is obvious that if the relay can be placed anywhere in the cell, the optimum relay location must be on the line joining the source and the destination. In this case, the distance between the source and the relay can be written as $D_{s,r} = D_{s,d} - D_{r,d}$. Consequently, from the conditional outage probability in (18), the optimum relay location for a source-destination pair can be obtained by solving $\hat{D}_{r,d} = \arg\min_{D_{r,d}} \mathrm{P}_{\text{out}}^{CO}(D_{s,d})$, which is equivalent to

$$\begin{aligned}\hat{D}_{r,d} = \arg\max_{D_{r,d}}\ & F_{s,r}(D_{s,d} - D_{r,d})F_{r,d}(D_{r,d}) \quad (21)\\ \text{subject to } & 0 \le D_{r,d} \le D_{s,d}.\end{aligned}$$

For simplicity, we resort to the scenario that the channel model parameters of the source-relay links and relay-destination link are the same. In this case, (21) can be written as

$$\hat{D}_{r,d} = \arg\max_{D_{r,d}} \sum_{n=1}^{M} A_n e^{-B_n(D_{s,d}-D_{r,d})^{\nu}} \sum_{n=1}^{M} A_n e^{-B_n D_{r,d}^{\nu}}, \quad (22)$$

where $A_n = A_{s,r}(n) = A_{r,d}(n)$, and $B_n = N_0\zeta_o/(kP_{CO}\beta_n)$ in which $\beta_n = \beta_{s,r}(n) = \beta_{r,d}(n)$. By taking the derivatives of the right hand side of (22) with respect to $D_{r,d}$, we can show that the optimum relay location is $\hat{D}_{r,d} = D_{s,d}/2$. Finally, replacing $D_{r,d}$ in (20) with $\hat{D}_{r,d} = D_{s,d}/2$, we have

$$\begin{aligned}\mathrm{P}_{\text{out}}^{LB} = \frac{2}{\rho^2}\int_0^{\rho} & D_{s,d}\big(1 - \sum_{n=1}^{M}\sum_{n'=1}^{M} A_n A_{n'} e^{-\frac{N_0\zeta_o D_{s,d}^{\nu}}{2^{\nu}kP_{CO}}(\frac{1}{\beta_n}+\frac{1}{\beta_{n'}})}\big)\\ &\times\big(1 - \sum_{n=1}^{M} A_{s,d}(n)e^{-\frac{N_0\zeta_o D_{s,d}^{\nu}}{kP_{CO}\ \beta_{s,d}(n)}}\big)dD_{s,d}. \quad (23)\end{aligned}$$

The outage probability in (23) serves as a lower bound on the outage probability of the proposed cooperative protocol. The performance of the proposed protocol employing any practical relay-assignment schemes can be lower bounded as

$$\mathrm{P}_{\text{out}}^{CO} \ge \mathrm{P}_{\text{out}}^{LB}. \quad (24)$$

C. Proposed Relay-Assignment Schemes

In this subsection, we derive an outage probability of the proposed cooperative protocol with fixed location and centralized control relay-assignment schemes. In both schemes, the cell is divided into w sectors, each containing one relay which is assigned to help all users in the sector. Without loss of generality, we consider the sector as shown in Fig. 2(b), in which the relay is located at $D_{r,d}e^{\mathbf{j}\phi_r}$ and a source user is located at $D_{s,d}e^{\mathbf{j}\phi_s}$ $(0 \le \phi_r, \phi_s \le \theta_w)$. The distance between the source and the relay can be expressed as

$$D_{s,r} = [D_{s,d}^2 + D_{r,d}^2 - 2D_{s,d}D_{r,d}\cos(\phi_r - \phi_s)]^{\frac{1}{2}} \triangleq f(\phi_s, \phi_r).$$

Assuming that users are uniformly distributed within the cell, the PDF of the user's distance D from the destination conditioned that the user is located in the sector can be given by

$$p_D(D \mid 0 \le \phi_s \le \theta_w) = 2D/(w\rho^2), \quad 0 \le D \le \rho. \quad (25)$$

Given a fixed relay location within each sector, the average outage probability of the proposed relay-assignment schemes can be determined by averaging (18) over the user distribution in (25) as

$$\mathrm{P}_{\text{out}}^{CO} = \frac{2}{w\rho^2}\int_0^{\rho} D_{sd}[1 - F_{s,d}(D_{s,d})][1 - G(D_{s,d})]dD_{s,d} \quad (26)$$

where $G(D_{s,d}) = w/(2\pi)\int_0^{\theta_w} F_{s,r}(f(\phi_s, \phi_r))F_{r,d}(D_{r,d})d\phi_s$.

Based on the average outage probability in (26), we can determine the optimum relay location as follows. Since the users are uniformly located in the cell, one can show that the optimum relay angle is $\hat{\phi}_r = \theta_w/2$. Substitute $\hat{\phi}_r$ into (26) and take the first derivative of $\mathrm{P}_{\text{out}}^{CO}$ with respect to $D_{r,d}$, then the optimum relay distance $\hat{D}_{r,d}$ can be obtained by solving

$$\int_0^{\rho} D_{s,d}\big(1 - F_{s,d}(D_{s,d})\big)\int_0^{\theta_w} \mathcal{G}(D_{s,d})d\phi_s dD_{s,d} = 0; \quad (27)$$

$$\begin{aligned}\mathcal{G}(D_{s,d}) = {} & CF_{r,d}(D_{r,d})\tilde{D}_{s,r}^{\nu-1}\sum A_{s,r}(n)B_{s,r}(n)e^{-B_{s,r}(n)\tilde{D}_{s,r}^{\nu}} \\ & + F_{s,r}(\tilde{D}_{s,r})D_{r,d}^{\nu-1}\sum A_{r,d}(n)B_{r,d}(n)e^{-B_{r,d}(n)D_{r,d}^{\nu}},\end{aligned}$$

in which $\tilde{D}_{s,r} = f(\phi_s, \pi/w)$, $C = (D_{r,d} - D_{s,d}\cos(\pi/w - \phi_s))$ and $B_{i,j}(n) = N_0\zeta_o/(kP_{CO}\beta_{i,j}(n))$.

To get more insightful understanding, we also provide here an explicit relay location that achieves close performance to that of optimum relay location. First, we calculate the average value of the user location as

$$\bar{D}_{s,d} = \int_0^{\rho} D_{s,d}p_{D_{s,d}}(D_{s,d})dD_{s,d} = 2\rho/3. \quad (28)$$

Then, an approximate relay location can be determined as

$$\bar{D}_{r,d} = \arg\min_{0\le D_{r,d}\le \bar{D}_{s,d}} \mathrm{P}_{\text{out}}^{CO}(\bar{D}_{s,d}|D_{s,r} = \bar{D}_{s,d} - D_{r,d}), \quad (29)$$

where $\mathrm{P}_{\text{out}}^{CO}(D_{s,d})$ is evaluated in (18). Using the results from Section IV-B, we can approximate the relay location by

$$\bar{D}_{r,d} = \bar{D}_{s,d}/2 = \rho/3. \quad (30)$$

As will be shown in the next section, the relay location obtained from this approximation leads to almost the same performance as that of optimum relay location.

V. SIMULATION RESULTS

We perform computer simulations to compare performance of the proposed relay-assignment schemes and to validate the above theoretical analysis. All simulations are based on UWB MB-OFDM systems with 128 subcarriers and the subband bandwidth of 528 MHz. The channel model parameters of every link follow those for channel model 4 [9], the path loss exponent is $\nu = 2$, and the number of users in the cell is set at 10 users. Unless stated otherwise, the cell radius is fixed at 10 meters.

In Figs. 3 and 4, we show the outage probability of the two proposed relay-assignment schemes. Fig. 3 depicts the outage performance versus the SNR per subcarrier symbol (E_s/N_0) in case of $w = 2$ relays. For both relay-assignment schemes, the approximate relay location $\bar{D}_{r,d}$ results in very close performance to that of the optimum relay location. Moreover, both fixed-location and central-controlled relay-assignment schemes yield almost the same performance. This is due to the fact that the cell in UWB systems is of small size, so there is high chance that a user is located close to the optimum relay location. In Fig. 4, the outage probability is plotted as a function of the number of relays. Notice that the outage probability slightly decreases with the number of relays. In this case, less than two relays are necessary for practical implementation of UWB system.

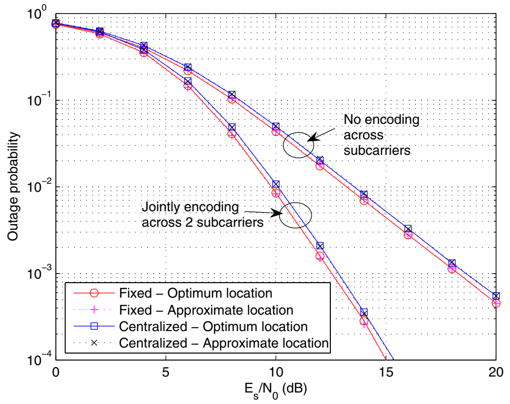


Fig. 3: Outage probability of two proposed relay assignment schemes.

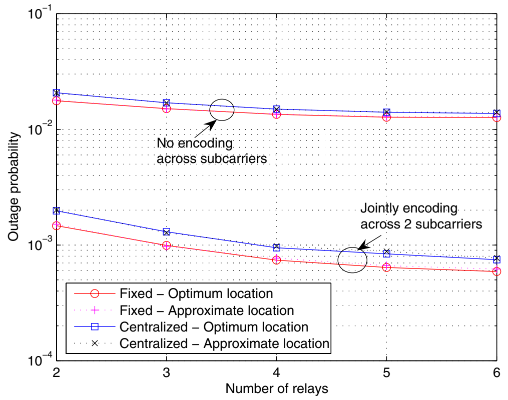


Fig. 4: Outage probability of two proposed relay assignment schemes versus the number of relays.

In Figs. 5-7, we compare the performance of the proposed cooperative protocol with that of non-cooperative protocol and the lower bound. Since the two relay-assignment schemes yield very close performance, we show only the performance of the fixed relay location scheme. Along with the simulation curves, we also plot the theoretical outage performance that is derived in the previous sections. For fair comparison between the non-cooperative and cooperative protocols, we use the same average transmitted power in both protocols [14]. The average transmitted power of cooperative protocol is $\bar{P}_{CO} = P_{CO}\mathrm{P}(\text{Source transmits only}) + 2P_{CO}\mathrm{P}(\text{Source and relay transmit})$, which can be determined as

$$\bar{P}_{CO} = P_{CO}(1 + \mathrm{P}_{\text{out}}^{s,d}(P_{CO}) - \mathrm{P}_{\text{out}}^{s,d}(P_{CO})\mathrm{P}_{\text{out}}^{s,r}(P_{CO})), \quad (31)$$

where $\mathrm{P}_{\text{out}}^{i,j}(P_{CO})$ denotes the outage probability of the direct transmission for the link $i - j$ when transmitted power P_{CO} is used. We set $P_{NC} = P_{CO}(1 + \mathrm{P}_{\text{out}}^{s,d}(P_{CO}))$ which is in favor of the non-cooperative protocol. With the power in (31), the bandwidth efficiency of the proposed cooperative protocol is approximately the same as that of non-cooperative protocol. Figs. 5 and 6 depicts the outage probability versus E_s/N_0 for the case of no coding and jointly coding across 2 subcarriers, respectively. Clearly, the theoretical results match with the simulation results in

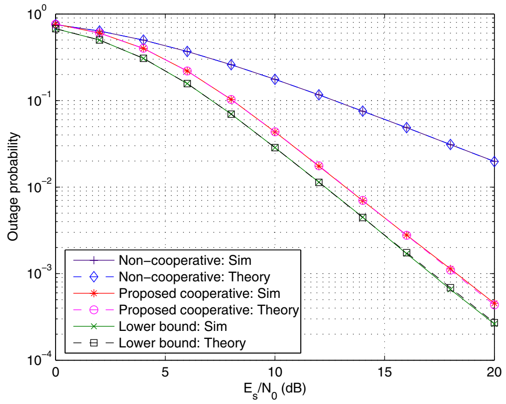


Fig. 5: Outage probability versus E_s/N_0 in case of no encoding across subcarriers.

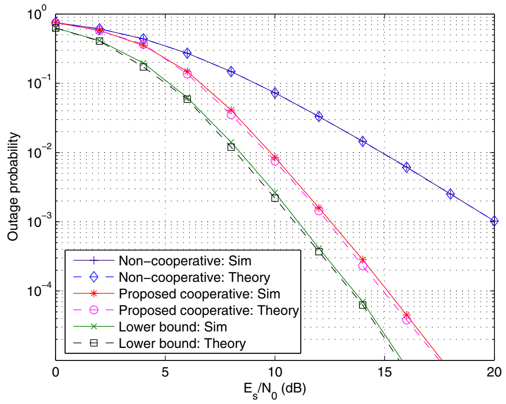


Fig. 6: Outage probability versus E_s/N_0 in case of jointly encoding across two subcarriers.

all cases. In case of no coding, the proposed cooperative protocol achieves 6dB performance improvement compared to the non-cooperative protocol at an outage probability of 0.05; in other words, 75% power saving is achieved. Also, there is only about 1dB performance gap between the proposed scheme and the lower bound. The same tendencies of the performance curves can be observed in case of jointly coding across subcarriers.

Fig. 7 depicts the outage probability as a function of the cell radius. The average SNR per symbol is fixed at $E_s/N_0 = 10$ dB. Again, the theoretical results closely match with the simulation results. If the outage probability is required to be at most 0.01, then the cell radius can be at most 3 meters. By employing the proposed cooperative protocol with 2 relays, the cell radius can be improved to 9m, i.e., 200% increase. Also, the cell radius of the proposed scheme is only 1m less than that of the lower bound.

VI. CONCLUSIONS

We propose in this paper a bandwidth-efficient cooperative protocol for OFDM systems. In the proposed protocol, the destination broadcasts subcarriers indices of which the received SNR falls below a specific SNR threshold, and the relay forwards only the source symbols carried in those subcarriers. In this way, the relay can help forward the data of multiple sources in one OFDM symbol, and the proposed protocol greatly improves the spectral efficiency, while still achieving full diversity. For practical implementation of the proposed cooperative protocol in OFDM networks, we proposed two relay-assignment schemes: fixed-location scheme in which each relay is placed in the optimum location, and centralized-control scheme in which the central node assigns the user that is nearest to the optimum location to be the relay. Performance analysis in terms of outage probability is provided. Furthermore, we investigate the application of the proposed protocol to enhance the performance of UWB communications. Both analytical and theoretical results show that the proposed cooperative protocol can achieve 75% power saving and 200% coverage extension compared to the non-cooperative UWB multiband OFDM at the same data rate.

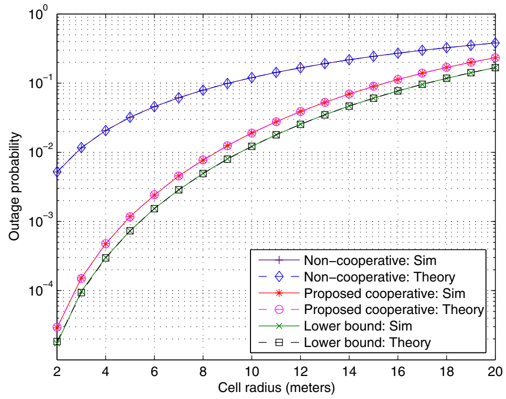


Fig. 7: Outage probability versus cell radius.

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