Joint Power Control and Beamforming for Peer-to-Peer MIMO Relay Systems

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Abstract—**In this paper, we consider an interference multiple-input multiple-output (MIMO) relay system where multiple source nodes communicate with their desired destination nodes with the aid of distributed relay nodes all equipped with multiple antennas. We aim at minimizing the total source and relay transmit power such that a minimum signal-to-interference-plus-noise ratio (SINR) threshold is maintained at each receiver. An iterative joint power control and beamforming algorithm is developed to achieve this goal. The proposed algorithm exploits transmit-relay-receive beamforming technique to mitigate the interferences from the unintended sources in conjunction with transmit power control. Numerical simulations are performed to demonstrate the effectiveness of the proposed iterative algorithm.**

I. Introduction

In a large wireless network with many nodes, multiple source-destination links must share a common frequency band concurrently to ensure a high spectral efficiency of the whole network. In such network, cochannel interference (CCI) is one of the main impairments that degrades the system performance. Developing schemes that mitigate the CCI is therefore important.

By exploiting the spatial diversity, multi-antenna technique provides an efficient approach to CCI minimization. A joint power control and receiver beamforming scheme is developed in [1] to meet the signal-to-interference-plus-noise ratio (SINR) threshold with the minimal transmission power. A joint transmit-receive beamforming and power control algorithm is proposed in [2], when the source nodes also have multiple antennas.

In addition to the transmit and/or receive beamforming considered in [1], [2], distributed/network beamforming technique [3] can further increase the reliability of the communication link especially for long-distance communication. In [4], a wireless ad hoc network consisting of d source-destination pairs and R relaying nodes, each having a single antenna, is considered, where the network beamforming scheme is used to meet the SINR threshold at all links with the minimal total transmission power consumed by all relay nodes. Relay beamformers are designed in [5] for multiple-antenna relay nodes with single-antenna source-destination pairs. Multiple-input multiple-output (MIMO) relay technique has been applied to multi-cellular (interference) systems in [6] where transceiver beamformers are designed using partial zero-forcing (PZF) technique.

However, [4]-[6] assume that each source node uses its maximum available transmit power. Such assumption not only raises the system transmit power consumption, but also increases the interference from one user to all other users. So the beamforming and the power control problem should be considered jointly as in [1], [2]. In this paper, we consider a two-hop interference MIMO relay system consisting of L source-destination pairs communicating with the aid of K relay nodes. Each of the source, relay and destination nodes is equipped with multiple antennas. The amplify-and-forward scheme is used at each relay node due to its practical implementation simplicity.

We aim at developing a joint power control and beamforming algorithm such that the total transmission power consumed by all source nodes and relay nodes are minimized while maintaining the SINR at each receiver above a minimum threshold. Compared with [4], [5], we not only use the network beamforming technique at the relay nodes, but also apply the joint transmit-receive beamforming technique for multiple-antenna users to mitigate the CCI. In contrast to [6], we develop an iterative technique to solve the total power minimization problem rather than using the so called PZF approach. Moreover, transmit power control is used in our algorithm to minimize the total transmit power and the interference to other users, which is not considered in [4]-[6]. Numerical simulations are carried out to evaluate the performance of the proposed algorithm.

II. System Model

We consider a two-hop interference MIMO relay system with L source-destination pairs as illustrated in Fig. 1. Each source node communicates with its corresponding destination node with the aid of a network of K distributed relays. Moreover, the direct links between the source nodes and the destination nodes are not considered since we assume that these direct links undergo relatively larger path attenuations compared with the links via relays. The source and destination nodes of the lth link are equipped with $N_{\mathrm{s},l}$ and $N_{\mathrm{d},l}$ antennas, respectively, whereas the kth relay node is mounted with $N_{\mathrm{r},k}$ antennas.

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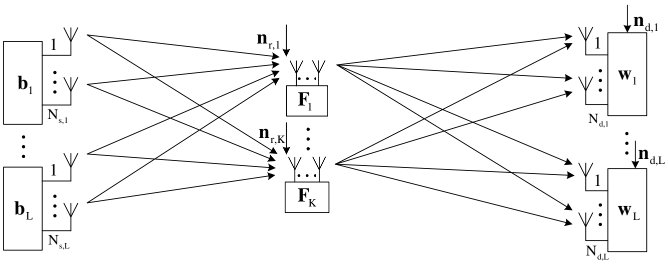

Fig. 1. Block diagram of an interference MIMO relay system.

We assume that all relay nodes work in half-duplex mode, thus the communication between the source-destination pairs is completed in two time slots. In the first time slot, the lth source node transmits an $N_{\mathrm{s},l} \times 1$ signal vector $\mathbf{b}_l s_l$, where s_l is the information-carrying symbol and $\mathbf{b}_l$ is the transmit beamforming vector. The received signal vector at the kth relay node is given by

$$\mathbf{y}_{\mathrm{r},k} = \sum_{l=1}^{L} \mathbf{H}_{k,l}\mathbf{b}_l s_l + \mathbf{n}_{\mathrm{r},k}$$

where $\mathbf{H}_{k,l}$ is the $N_{\mathrm{r},k} \times N_{\mathrm{s},l}$ MIMO channel matrix between the lth transmitting node and the kth relay node and $\mathbf{n}_{\mathrm{r},k}$ is the $N_{\mathrm{r},k} \times 1$ additive Gaussian noise vector at the kth relay node.

The kth relay multiplies its received signal vector by an $N_{\mathrm{r},k} \times N_{\mathrm{r},k}$ matrix $\mathbf{F}_k$. Thus the signal vector transmitted by the kth relay node is given by

$$\mathbf{x}_{\mathrm{r},k} = \mathbf{F}_k \mathbf{y}_{\mathrm{r},k}. \tag{1}$$

The received signal at the lth destination node is obtained as the weighted sum of the received signals at each antenna element of that node, and is given by

$$y_{\mathrm{d},l} = \mathbf{w}_l^H \left(\sum_{k=1}^{K} \mathbf{G}_{l,k}\mathbf{F}_k \left(\sum_{m=1}^{L} \mathbf{H}_{k,m}\mathbf{b}_m s_m + \mathbf{n}_{\mathrm{r},k} \right) + \mathbf{n}_{\mathrm{d},l} \right) \tag{2}$$

where $\mathbf{G}_{l,k}$ is the $N_{\mathrm{d},l} \times N_{\mathrm{r},k}$ MIMO channel matrix between the kth relay node and the lth destination node, $\mathbf{w}_l$ and $\mathbf{n}_{\mathrm{d},l}$ are the $N_{\mathrm{d},l} \times 1$ beamforming weight vector and the additive Gaussian noise vector at the lth destination node, respectively, and $(\cdot)^H$ denotes matrix (or vector) Hermitian transpose. We assume that all noises are complex circularly symmetric with zero mean and variance σ_{n}^2.

Let us now introduce the following definitions

$$\begin{aligned}
\tilde{\mathbf{h}}_l &\triangleq \left[(\mathbf{H}_{1,l}\mathbf{b}_l)^T, \cdots, (\mathbf{H}_{K,l}\mathbf{b}_l)^T \right]^T \in \mathcal{C}^{\bar{N}_{\mathrm{r}} \times 1} \\
\tilde{\mathbf{G}}_l &\triangleq [\mathbf{G}_{l,1}, \cdots, \mathbf{G}_{l,K}] \in \mathcal{C}^{N_{\mathrm{d},l} \times \bar{N}_{\mathrm{r}}} \\
\mathbf{F} &\triangleq \mathrm{blkdiag}(\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_K) \in \mathcal{C}^{\bar{N}_{\mathrm{r}} \times \bar{N}_{\mathrm{r}}} \\
\tilde{\mathbf{n}}_{\mathrm{r}} &\triangleq \left[\mathbf{n}_{\mathrm{r},1}^T, \cdots, \mathbf{n}_{\mathrm{r},K}^T \right]^T \in \mathcal{C}^{\bar{N}_{\mathrm{r}} \times 1}
\end{aligned}$$

where $\bar{N}_{\mathrm{r}} \triangleq \sum_{k=1}^{K} N_{\mathrm{r},k}$, $(\cdot)^T$ denotes matrix (or vector) transpose, and $\mathrm{blkdiag}(\cdot)$ stands for a block-diagonal matrix. Using these definitions, (2) can be rewritten as

$$y_{\mathrm{d},l} = \mathbf{w}_l^H \tilde{\mathbf{G}}_l \mathbf{F} \left(\sum_{m=1}^{L} \tilde{\mathbf{h}}_m s_m + \tilde{\mathbf{n}}_r \right) + \mathbf{w}_l^H \mathbf{n}_{\mathrm{d},l}. \tag{3}$$

From (3), the total power of the received signal at the lth link destination node is given by

$$\mathrm{E}\{y_{\mathrm{d},l} y_{\mathrm{d},l}^*\} = \sum_{m=1}^{L} p_m \mathbf{w}_l^H \boldsymbol{\psi}_{ml} \boldsymbol{\psi}_{ml}^H \mathbf{w}_l + \mathbf{w}_l^H \mathbf{C}_l \mathbf{w}_l \tag{4}$$

where $(\cdot)^*$ denotes complex conjugate, $\boldsymbol{\psi}_{ml} \triangleq \tilde{\mathbf{G}}_l \mathbf{F} \tilde{\mathbf{h}}_m$ is the equivalent vector channel response between the mth source node and the lth destination node, $\mathbf{C}_l \triangleq \sigma_{\mathrm{n}}^2 \tilde{\mathbf{G}}_l \mathbf{F}\mathbf{F}^H \tilde{\mathbf{G}}_l^H + \sigma_{\mathrm{n}}^2 \mathbf{I}_{N_{\mathrm{d},l}}$ is the covariance matrix of the equivalent noise at the lth receiver, and $\mathbf{I}_n$ is an $n \times n$ identity matrix. Here we assume that $\mathrm{E}\{|s_l|^2\} = p_l$ is the transmit power of the lth information-carrying symbol. Using (4), the SINR at the lth destination node is given by

$$\Gamma_l = \frac{p_l \mathbf{w}_l^H \boldsymbol{\psi}_{ll} \boldsymbol{\psi}_{ll}^H \mathbf{w}_l}{\sum_{m \neq l}^{L} p_m \mathbf{w}_l^H \boldsymbol{\psi}_{ml} \boldsymbol{\psi}_{ml}^H \mathbf{w}_l + \mathbf{w}_l^H \mathbf{C}_l \mathbf{w}_l}. \tag{5}$$

Using (1), the transmit power at the kth relay node can be expressed as

$$P_{\mathrm{r},k} = \mathrm{tr}\big(\mathbf{F}_k \mathbf{R}_{\mathrm{y},k} \mathbf{F}_k^H \big), \; k = 1, \cdots, K \tag{6}$$

where $\mathbf{R}_{\mathrm{y},k} = \sum_{l=1}^{L} p_l \mathbf{H}_{k,l} \mathbf{b}_l \mathbf{b}_l^H \mathbf{H}_{k,l}^H + \sigma_{\mathrm{n}}^2 \mathbf{I}_{N_{r,k}}$ is the covariance matrix of the received signal at the kth relay node, and $\mathrm{tr}(\cdot)$ denotes matrix trace. Thus the total transmit power consumed by the whole network can be expressed as

$$P_{\mathrm{T}} = \sum_{k=1}^{K} P_{\mathrm{r},k} + \sum_{l=1}^{L} p_l \mathbf{b}_l^H \mathbf{b}_l. \tag{7}$$

III. Joint Power Control and Beamforming

Let us define the relay beamforming vector $\mathbf{f}$ from the relay amplifying matrices $\mathbf{F}_1, \cdots, \mathbf{F}_K$ as

$$\mathbf{f} = \left[\mathbf{f}_1^T, \cdots, \mathbf{f}_K^T \right]^T \in \mathcal{C}^{\tilde{N}_{\mathrm{r}} \times 1} \tag{8}$$

where $\mathbf{f}_k \triangleq \mathrm{vec}(\mathbf{F}_k)$, $k = 1, \cdots, K$, $\tilde{N}_{\mathrm{r}} \triangleq \sum_{k=1}^{K} N_{\mathrm{r},k}^2$, and $\mathrm{vec}(\cdot)$ stands for a vector obtained by stacking all column vectors of a matrix on top of each other. We design the optimal source transmit power vector $\mathbf{p} \triangleq [p_1, p_2, \cdots, p_L]^T$, the relay beamforming vector $\mathbf{f}$, transmit beamforming vectors $\{\mathbf{b}_l\} \triangleq \{\mathbf{b}_l, l = 1, \cdots, L\}$, and receive beamforming vectors $\{\mathbf{w}_l\} \triangleq \{\mathbf{w}_l, l = 1, \cdots, L\}$, such that a target SINR threshold $\gamma_l, l = 1, \cdots, L$, is maintained at the lth destination node with the minimal P_T. The optimization problem can be written as

$$\min_{\mathbf{p},\mathbf{f},\{\mathbf{b}_l\},\{\mathbf{w}_l\}} \quad P_{\mathrm{T}} \tag{9}$$

$$\text{s.t.} \quad \Gamma_l \geq \gamma_l, \qquad l = 1, \cdots, L. \tag{10}$$

The problem (9)-(10) is nonconvex due to the constraints in (10). In the following, we solve corresponding subproblems to optimize each variable.

A. Receive Beamforming

The optimal receiver beamforming vectors $\{\mathbf{w}_l\}$ for fixed $\mathbf{p}$, $\mathbf{f}$, and $\{\mathbf{b}_l\}$ can be obtained such that it minimizes the noise-plus-interference power at the receiver under the condition of unity gain for the signal of interest, which can be written as

$$\min_{\mathbf{w}_l} \quad \sum_{m\neq l}^{L} p_m \mathbf{w}_l^H \boldsymbol{\psi}_{ml}\boldsymbol{\psi}_{ml}^H \mathbf{w}_l + \mathbf{w}_l^H \mathbf{C}_l \mathbf{w}_l \tag{11}$$

$$\text{s.t.} \quad \mathbf{w}_l^H \boldsymbol{\psi}_{ll} = 1. \tag{12}$$

The unity gain condition ensures that the desired signal is unaffected by beamforming. Using the Lagrangian multiplier method, the solution to the problem (11)-(12) is given by

$$\mathbf{w}_l = \frac{\boldsymbol{\Phi}_l^{-1}\boldsymbol{\psi}_{ll}}{\boldsymbol{\psi}_{ll}^H \boldsymbol{\Phi}_l^{-1}\boldsymbol{\psi}_{ll}} \tag{13}$$

where $\boldsymbol{\Phi}_l \triangleq \sum_{m\neq l}^{L} p_m \boldsymbol{\psi}_{ml}\boldsymbol{\psi}_{ml}^H + \mathbf{C}_l$ is the interference-plus-noise covariance matrix at the lth receiver, and $(\cdot)^{-1}$ denotes matrix inversion.

B. Transmit Power Allocation

To obtain optimal $\mathbf{p}$ with given beamforming vectors $\mathbf{f}$, $\{\mathbf{b}_l\}$, and $\{\mathbf{w}_l\}$, we reformulate the problem (9)-(10) as

$$\min_{\mathbf{p}} \quad P_{\mathrm{T}} \tag{14}$$

$$\text{s.t.} \quad \frac{p_l[\bar{\mathbf{H}}]_{l,l}}{\sum_{m\neq l}^{L} p_m[\bar{\mathbf{H}}]_{m,l} + \bar{n}_l} \geq \gamma_l, \quad l = 1,\cdots,L \tag{15}$$

where $\bar{\mathbf{H}}$ is an $L\times L$ covariance matrix such that $[\bar{\mathbf{H}}]_{m,l} = \mathbf{w}_l^H \boldsymbol{\psi}_{ml}\boldsymbol{\psi}_{ml}^H \mathbf{w}_l$ and $\bar{n}_l \triangleq \mathbf{w}_l^H \mathbf{C}_l \mathbf{w}_l$. Here for a matrix $\mathbf{A}$, $[\mathbf{A}]_{i,j}$ indicates the (i,j)th element of $\mathbf{A}$. In an optimal power allocation, the transmit power of each user is set to the minimum required level such that the target SINR is just met. Thus the optimal power solution to the problem (14)-(15) is given by

$$\mathbf{p} = (\mathbf{I}_L - \tilde{\mathbf{H}})^{-1}\mathbf{u} \tag{16}$$

where $[\tilde{\mathbf{H}}]_{l,m} = \begin{cases} 0, & m = l \\ \gamma_l[\bar{\mathbf{H}}]_{m,l}/[\bar{\mathbf{H}}]_{l,l}, & m\neq l \end{cases}$, and $\mathbf{u}$ is an $L\times 1$ vector whose lth element is given by $\gamma_l\bar{n}_l/[\bar{\mathbf{H}}]_{l,l}$, $l = 1,\cdots,L$.

C. Transmit Beamforming

With given $\mathbf{p}, \mathbf{f}$ and $\{\mathbf{w}_l\}$, the optimal $\{\mathbf{b}_l\}$ can be obtained simply by swapping the roles of the transmitters and the receivers as in [2]. The received SINR in the lth virtual link can be expressed similar to (5), and is given by

$$\tilde{\Gamma}_l = \frac{\tilde{p}_l \mathbf{b}_l^T \boldsymbol{\xi}_{ll}\boldsymbol{\xi}_{ll}^H \mathbf{b}_l^*}{\sum_{m\neq l}^{L} \tilde{p}_m \mathbf{b}_l^T \boldsymbol{\xi}_{ml}\boldsymbol{\xi}_{ml}^H \mathbf{b}_l^* + \tilde{N}_l \mathbf{b}_l^T \mathbf{b}_l^*}. \tag{17}$$

Here $\boldsymbol{\xi}_{ml} \triangleq \mathbf{H}_l^T \mathbf{F}^T \tilde{\mathbf{G}}_m^T \mathbf{w}_m^*$, $\mathbf{H}_l \triangleq \left[\mathbf{H}_{1,l}^T,\cdots,\mathbf{H}_{K,l}^T\right]^T$ is the equivalent MIMO channel between the lth user and all relay nodes, $\tilde{p}_l$ is the transmit power and $\tilde{N}_l$ is the noise power in the lth virtual link. Thus the optimal transmit beamformers $\{\mathbf{b}_l\}$ which are the receive beamformers in the virtual links, can be obtained by solving the following problem for each l

$$\min_{\mathbf{b}_l} \quad \sum_{m\neq l}^{L} \tilde{p}_m \mathbf{b}_l^T \boldsymbol{\xi}_{ml}\boldsymbol{\xi}_{ml}^H \mathbf{b}_l^* + \tilde{N}_l \mathbf{b}_l^T \mathbf{b}_l^* \tag{18}$$

$$\text{s.t.} \quad \mathbf{b}_l^T \boldsymbol{\xi}_{ll} = 1. \tag{19}$$

The solution to this problem is given by

$$\mathbf{b}_l^* = \frac{\boldsymbol{\Theta}_l^{-1}\boldsymbol{\xi}_{ll}}{\boldsymbol{\xi}_{ll}^H \boldsymbol{\Theta}_l^{-1}\boldsymbol{\xi}_{ll}} \tag{20}$$

where $\boldsymbol{\Theta}_l \triangleq \sum_{m\neq l}^{L} \tilde{p}_m \boldsymbol{\xi}_{ml}\boldsymbol{\xi}_{ml}^H + \tilde{N}_l \mathbf{I}_{N_l}$ is the noise-plus-interference covariance matrix at the lth receiver of the virtual link. The virtual link transmit power can be obtained as

$$\tilde{\mathbf{p}} \triangleq (\mathbf{I}_L - \tilde{\mathbf{H}}^T)^{-1}\tilde{\mathbf{u}} \tag{21}$$

where $[\tilde{\mathbf{u}}]_l \triangleq \frac{\gamma_l \tilde{N}_l \mathbf{b}_l^T \mathbf{b}_l^*}{\mathbf{b}_l^T \boldsymbol{\xi}_{ll}^H \boldsymbol{\xi}_{ll} \mathbf{b}_l^*}$, $l = 1,\cdots,L$. Here for a vector $\mathbf{v}$, $[\mathbf{v}]_l$ stands for the lth element of $\mathbf{v}$.

D. Relay Beamforming

To optimize the relay amplifying matrices we reformulate problem (9)-(10). Now the SINR of the lth link in (5) can be expressed as

$$\Gamma_l = \frac{\mathrm{tr}\big(\mathbf{R}_{\mathrm{g},l}\mathbf{F}\mathbf{R}_{\mathrm{h},l}\mathbf{F}^H\big)}{\mathrm{tr}\big(\mathbf{R}_{\mathrm{g},l}\mathbf{F}\big(\sum_{m\neq l}^{L}\mathbf{R}_{\mathrm{h},m} + \mathbf{I}_{\tilde{N}_{\mathrm{r}}}\big)\mathbf{F}^H\big) + \sigma_{\mathrm{n}}^2 \mathbf{w}_l^H \mathbf{w}_l} \tag{22}$$

where $\mathbf{R}_{\mathrm{g},l} \triangleq \tilde{\mathbf{G}}_l^H \mathbf{w}_l \mathbf{w}_l^H \tilde{\mathbf{G}}_l$ and $\mathbf{R}_{\mathrm{h},m} \triangleq p_m \tilde{\mathbf{h}}_m \tilde{\mathbf{h}}_m^H$. Applying the fact that $\mathrm{tr}(\mathbf{A}^H\mathbf{B}\mathbf{A}\mathbf{C}) = \mathrm{vec}(\mathbf{A})^H(\mathbf{C}^T \bigotimes \mathbf{B})\mathrm{vec}(\mathbf{A})$ [7], where $\bigotimes$ denotes the matrix Kronecker product, the SINR in (22) can be expressed as

$$\Gamma_l = \frac{\mathrm{vec}(\mathbf{F})^H\big(\mathbf{R}_{\mathrm{h},l}^T \bigotimes \mathbf{R}_{\mathrm{g},l}\big)\mathrm{vec}(\mathbf{F})}{\mathrm{vec}(\mathbf{F})^H\big(\tilde{\mathbf{R}}_{\mathrm{h},l}^T \bigotimes \mathbf{R}_{\mathrm{g},l}\big)\mathrm{vec}(\mathbf{F}) + \sigma_{\mathrm{n}}^2 \mathbf{w}_l^H \mathbf{w}_l} \tag{23}$$

where $\tilde{\mathbf{R}}_{\mathrm{h},l} \triangleq \sum_{m\neq l}^{L}\mathbf{R}_{\mathrm{h},m} + \mathbf{I}_{\tilde{N}_{\mathrm{r}}}$. Let us now introduce the link between $\mathbf{f}$ in (8) and $\mathrm{vec}(\mathbf{F})$ as $\mathrm{vec}(\mathbf{F}) = \mathbf{D}_{\mathrm{F}}\mathbf{f}$, where $\mathbf{D}_{\mathrm{F}} \in \mathcal{R}^{\tilde{N}_{\mathrm{r}}^2\times\tilde{N}_{\mathrm{r}}}$ is a matrix of ones and zeros and is constructed by observing the nonzero entries of $\mathrm{vec}(\mathbf{F})$. Then (23) can be rewritten as

$$\Gamma_l = \frac{\mathbf{f}^H \mathbf{D}_{\mathrm{F}}^T\big(\mathbf{R}_{\mathrm{h},l}^T \bigotimes \mathbf{R}_{\mathrm{g},l}\big)\mathbf{D}_{\mathrm{F}}\mathbf{f}}{\mathbf{f}^H \mathbf{D}_{\mathrm{F}}^T\big(\tilde{\mathbf{R}}_{\mathrm{h},l}^T \bigotimes \mathbf{R}_{\mathrm{g},l}\big)\mathbf{D}_{\mathrm{F}}\mathbf{f} + \sigma_{\mathrm{n}}^2 \mathbf{w}_l^H \mathbf{w}_l}. \tag{24}$$

From (8) we have $\mathbf{f}_k = \mathbf{D}_k\mathbf{f}, k = 1,\cdots,K$, with $\mathbf{D}_k \in \mathcal{R}^{N_{\mathrm{r},k}^2\times\tilde{N}_{\mathrm{r}}}$ defined as $\mathbf{D}_k = [\mathbf{D}_{k,1},\cdots,\mathbf{D}_{k,K}]$, where $\mathbf{D}_{k,k} = \mathbf{I}_{N_{\mathrm{r},k}^2\times N_{\mathrm{r},k}^2}$ and $\mathbf{D}_{k,j} = \mathbf{0}_{N_{\mathrm{r},k}^2\times N_{\mathrm{r},j}^2}$, $j = 1,\cdots,K$, $j\neq k$. Using the identity of $\mathrm{tr}(\mathbf{A}^H\mathbf{A}\mathbf{B}) = \mathrm{vec}(\mathbf{A})^H\big(\mathbf{B}^T \bigotimes \mathbf{I}_n\big)\mathrm{vec}(\mathbf{A})$ for $\mathbf{A},\mathbf{B} \in \mathcal{C}^{n\times n}$ [7], the transmit power of the kth relay in (6) can be expressed alternatively as

$$P_{\mathrm{r},k} = \mathbf{f}_k^H\big(\mathbf{R}_{\mathrm{y},k}^T \bigotimes \mathbf{I}_{N_{\mathrm{r},k}}\big)\mathbf{f}_k = \mathbf{f}^H\mathbf{D}_k^T\big(\mathbf{R}_{\mathrm{y},k}^T \bigotimes \mathbf{I}_{N_{\mathrm{r},k}}\big)\mathbf{D}_k\mathbf{f}. \tag{25}$$

Using (24) and (25), with given $\mathbf{p}$, $\{\mathbf{b}_l\}$ and $\{\mathbf{w}\}$, the problem (9)-(10) can be reformulated as

$$\min_{\mathbf{f}} \quad \mathbf{f}^H\mathbf{A}\mathbf{f} \tag{26}$$

$$\text{s.t.} \quad \mathbf{f}^H\mathbf{B}_l\mathbf{f} \geq \gamma_l\sigma_{\mathrm{n}}^2\mathbf{w}_l^H\mathbf{w}_l, \qquad l = 1,\cdots,L \tag{27}$$

where $\mathbf{A} \triangleq \sum_{k=1}^{K} \mathbf{D}_k^T \big(\mathbf{R}_{\mathrm{y},k}^T \bigotimes \mathbf{I}_{N_{\mathrm{r},k}}\big) \mathbf{D}_k$ and

$$\mathbf{B}_l \triangleq \mathbf{D}_{\mathrm{F}}^T \big(\mathbf{R}_{\mathrm{h},l}^T \bigotimes \mathbf{R}_{\mathrm{g},l} - \gamma_l \tilde{\mathbf{R}}_{\mathrm{h},l}^T \bigotimes \mathbf{R}_{\mathrm{g},l}\big) \mathbf{D}_{\mathrm{F}}. \tag{28}$$

The problem (26)-(27) is non-convex, since $\mathbf{B}_l$ in (28) can be indefinite. By introducing $\mathbf{X} = \mathbf{f}\,\mathbf{f}^H$, the problem (26)-(27) can be equivalently rewritten as

$$\min_{\mathbf{X}} \quad \mathrm{tr}(\mathbf{A}\mathbf{X}) \tag{29}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{B}_l \mathbf{X}) \geq \gamma_l \sigma_{\mathrm{n}}^2 \mathbf{w}_l^H \mathbf{w}_l, \quad l = 1, \cdots, L \tag{30}$$

$$\mathbf{X} \succeq 0 \tag{31}$$

$$\mathrm{rank}(\mathbf{X}) = 1 \tag{32}$$

where $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is a positive semidefinite (PSD) matrix, and $\mathrm{rank}(\cdot)$ denotes the rank of a matrix. Note that in the problem (29)-(32), the rank constraint on $\mathbf{X}$ is not convex. Interestingly, the problem (29)-(32) can be solved by the semidefinite relaxation technique [8] as explained in the following.

First we drop the rank constraint (32) to obtain a relaxed SDP problem which is convex in $\mathbf{X}$. The relaxed problem can be conveniently solved by using interior point methods at a complexity order that is at most $O((L + \tilde{N}_{\mathrm{r}}^2)^{3.5})$ [9]. We use CVX MATLAB toolbox for disciplined convex programming [10] to obtain the optimal $\mathbf{X}$. Due to the relaxation, $\mathbf{X}$ obtained by solving the relaxed problem is not necessarily rank one in general. If it is, then its principal eigenvector will be the optimal solution to the original problem. Otherwise, we have to use alternative techniques such as randomization [8] to obtain a (suboptimal) rank-one solution from $\mathbf{X}$. Different randomization techniques have been studied [8], [9]. Note that when $\mathrm{rank}(\mathbf{X}) > 1$, at least one of the constraints in (10) will be violated after the randomization operation. However, a feasible relay beamforming vector can be obtained by simply scaling $\mathbf{f}$ so that all the constraints are satisfied.

Now the original total transmit power minimization problem (9)-(10) can be solved by an iterative algorithm as shown in Table I. Here ε is a small positive number close to zero up to which convergence is acceptable, $\max$ stands for the maximum element of a vector, and the superscript (n) denotes the number of iterations. It can be easily shown as in [1], that starting with random $\mathbf{p}^{(0)}$, $\{\mathbf{b}_l^{(0)}\}$, and $\mathbf{f}^{(0)}$, the algorithm in Table I converges to (at least) a locally optimal solution. Note that updating the relay beamforming vector $\mathbf{f}$ at step-4 in Table I involves solving an SDP problem whereas updating other variables involves simpler matrix operations. Therefore, most of the computational time of the algorithm is required to update $\mathbf{f}$ at a complexity order that is at most $O((L + \tilde{N}_{\mathrm{r}}^2)^{3.5})$. The computational time to update any other optimization variable is negligible compared to that of $\mathbf{f}$.

TABLE I
PROCEDURE OF SOLVING THE PROBLEM (9)-(10) BY THE PROPOSED ITERATIVE ALGORITHM

1) Initialize the algorithm with an arbitrary forward link power vector $\mathbf{p}^{(0)}$, virtual link power vector $\tilde{\mathbf{p}}^{(0)}$, and randomly generated transmit beamforming vectors $\{\mathbf{b}_l^{(0)}\}$ and relay beamforming vector $\mathbf{f}^{(0)}$; Set $n = 0$.
2) Solve the subproblem (11)-(12) using given $\mathbf{p}^{(n)}$, $\{\mathbf{b}_l^{(n)}\}$, and $\mathbf{f}^{(n)}$ to obtain $\{\mathbf{w}_l^{(n)}\}$ as in (13).
3) Solve the subproblem (14)-(15) with fixed $\mathbf{f}^{(n)}$, $\{\mathbf{b}_l^{(n)}\}$, and $\{\mathbf{w}_l^{(n)}\}$ to obtain power vector $\mathbf{p}^{(n+1)}$ as in (16).
4) Solve the relaxed problem of the subproblem (29)-(32) using known $\{\mathbf{b}_l^{(n)}\}$, $\{\mathbf{w}_l^{(n)}\}$, and $\mathbf{p}^{(n+1)}$ to obtain $\mathbf{X}$.
   a) Use the randomization technique to obtain $\mathbf{f}$.
   b) Find the most violated constraint in the original problem (9)-(10) using such $\mathbf{f}$.
   c) Scale $\mathbf{f}$ so that the most violated constraint is satisfied with equality to obtain $\mathbf{f}^{(n+1)}$.
5) Update the transmit beamforming vectors $\{\mathbf{b}_l^{(n+1)}\}$ as in (20) by solving subproblem (18)-(19) with given $\mathbf{f}^{(n+1)}$, $\{\mathbf{w}_l^{(n)}\}$, and $\tilde{\mathbf{p}}^{(n)}$.
6) Update the virtual link transmit power $\tilde{\mathbf{p}}^{(n+1)}$ with fixed $\{\mathbf{b}_l^{(n+1)}\}$, $\{\mathbf{w}_l^{(n)}\}$, and $\mathbf{f}^{(n+1)}$ as in (21).
7) If $\max |\mathbf{p}^{(n+1)} - \mathbf{p}^{(n)}| \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to step 2.

IV. NUMERICAL EXAMPLES

In this section, we study the performance of the proposed algorithm through numerical simulations. For simplicity, we assume $\gamma_l = \gamma, N_{\mathrm{s},l} = N_{\mathrm{s}}$, $N_{\mathrm{d},l} = N_{\mathrm{d}}$, $l = 1, \cdots, L$, and $N_{\mathrm{r},k} = N_{\mathrm{r}}$, $k = 1, \cdots, K$, in all simulations. All noises are complex circularly symmetric Gaussian random variables with zero mean and unit variance (i.e. $\sigma_n^2 = 1$). In each simulation, the channel matrices have entries generated as i.i.d. complex Gaussian random variables with zero mean and variances σ_{h}^2 and σ_{g}^2 for $\mathbf{H}_{k,l}$ and $\mathbf{G}_{l,k}$, $l = 1, \cdots, L, k = 1, \cdots, K$, respectively. All simulation results are averaged over 200 independent channel realizations.

For the proposed algorithm, the procedure in Table I is carried out in each simulation to obtain the optimal power vector $\mathbf{p}$, transmit beamforming vectors $\{\mathbf{b}_l\}$, relay beamforming vector $\mathbf{f}$, and receive beamforming vectors $\{\mathbf{w}_l\}$. To initialize the algorithm in Table I, we randomly generate $\{\mathbf{b}_l\}$ and $\mathbf{f}$, along with arbitrary transmit power vector $\mathbf{p}$ and virtual power vector $\tilde{\mathbf{p}}$. We observed from our simulations that in a typical run with $L = 2$, $K = 20$, $N_{\mathrm{s}} = N_{\mathrm{r}} = 2$, $N_{\mathrm{d}} = 4$, $\sigma_{\mathrm{h}}^2 = 15$, and $\sigma_{\mathrm{g}}^2 = 10$, the algorithm converges within 3 to 5 iterations. Also, the algorithm requires less iterations for lower target SINR thresholds.

In the first example, we compare the performance of the proposed joint optimal power control and beamforming algorithm (Proposed TxRxBF) with the conventional SVD-based transmit beamforming approach (SVD-based TxBF). For the latter scheme, we iteratively update $\mathbf{p}$, $\mathbf{f}$, and $\{\mathbf{w}_l\}$, based on the proposed structure, and pickup the strongest SVD-based transmit beamformers $\mathbf{b}_l, l = 1, \cdots, L$, from the source-relay channels, i.e., $\mathbf{b}_l = \mathbf{q}_l$, where $\mathbf{q}_l$ is the strongest right singular vector of $\mathbf{H}_l$. We plot the total power consumed by all source nodes and relay nodes versus the target SINR threshold γ (dB). Fig. 2 shows the performance of both algorithms for $L = 2$, $K = 20$, $N_{\mathrm{s}} = N_{\mathrm{r}} = 2$, $N_{\mathrm{d}} = 4$, $\sigma_{\mathrm{h}}^2 = 15$, and $\sigma_{\mathrm{g}}^2 = 10$. It can be seen from Fig. 2 that the proposed jointly optimal algorithm requires less total power compared with the SVD-based transmit beamforming scheme.

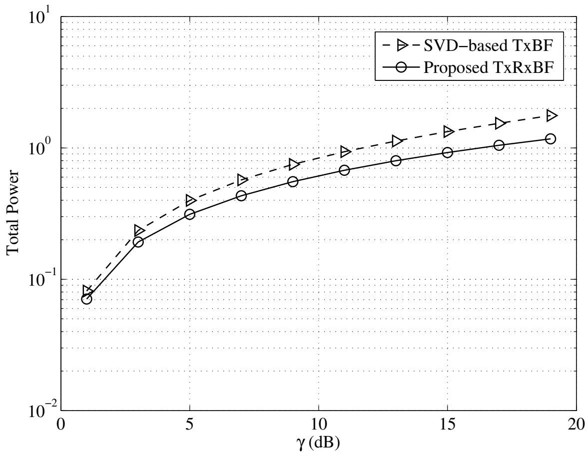

Fig. 2. Total power versus target SINR. $L = 2$, $K = 20$, $N_{\mathrm{s}} = N_{\mathrm{r}} = 2$, $N_{\mathrm{d}} = 4$, $\sigma_{\mathrm{h}}^2 = 15$, and $\sigma_{\mathrm{g}}^2 = 10$.

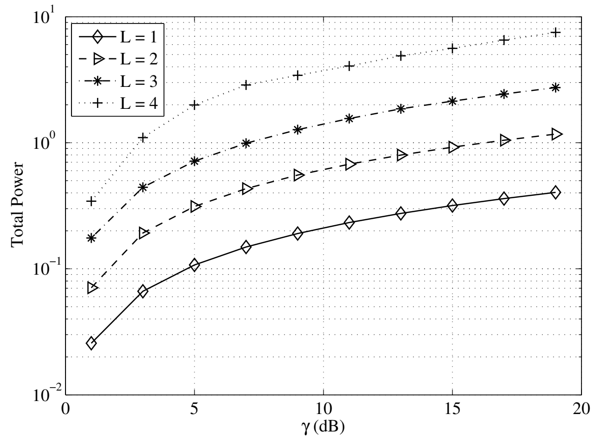

Fig. 4. Total power versus target SINR for different number of users. $K = 20$, $N_{\mathrm{s}} = N_{\mathrm{r}} = 2$, $N_{\mathrm{d}} = 4$, $\sigma_{\mathrm{h}}^2 = 15$, and $\sigma_{\mathrm{g}}^2 = 10$.

In the next example, we study the performance of the proposed algorithm for different number of relay antennas N_{r} with $L = 2$, $K = 15$, $N_{\mathrm{s}} = 2$, $N_{\mathrm{d}} = 4$, $\sigma_{\mathrm{h}}^2 = 15$, and $\sigma_{\mathrm{g}}^2 = 10$. The impact of the number of relay antennas is displayed in Fig. 3. As expected, if we increase the number of relay antennas the proposed algorithm requires less power since more antennas provide more spatial diversity. But at the same time, the complexity of the algorithm significantly increases. An important observation from Fig. 3 is that after a reasonably higher value of N_{r}, the performance improvement is not noticeable. The reason is that increasing the number of relay antennas not only strengthen the desired signals but also the interferences. So one should make a tradeoff between the performance and complexity based on the system requirements and the available resources.

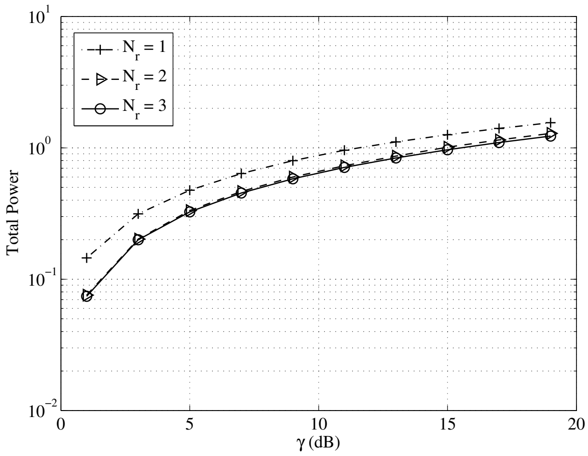

Fig. 3. Total power versus target SINR for different number of relay antennas. $L = 2$, $K = 15$, $N_{\mathrm{s}} = 2$, $N_{\mathrm{d}} = 4$, $\sigma_{\mathrm{h}}^2 = 15$, and $\sigma_{\mathrm{g}}^2 = 10$.

In the last example, we study the effect of channel interferences on the proposed algorithm. By increasing the number of source-destination pairs L, the interfering signal received at each destination node is also increased. The performance of the algorithm for different L is illustrated in Fig. 4. From this figure it is clear that if there are more active users communicating simultaneously in the system, we need more power to achieve the same target SINR threshold γ.

V. Conclusions

We considered a two-hop interference MIMO relay system with distributed relay nodes and developed an iterative technique to minimize the total transmit power consumed by all source and relay nodes such that a minimum SINR threshold is maintained at each receiver. The proposed algorithm exploits beamforming techniques at the source, relay and destination nodes in conjunction with transmit power control. Simulation results demonstrate that the jointly optimal power control and beamforming algorithm outperforms the existing techniques.

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