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Libration Control of Electrodynamic Tethers Using the Extended Time-Delayed Autosynchronization Method

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Electrodynamic tethers working in inclined orbit are affected by dynamic instability due to continuous pumping of energy from electromagnetic forces into the tether attitude motion. This paper proposes a new control scheme to remove this instability. The procedure is based on an extended delay feedback control method that has been used successfully in problems with one degree of freedom. When simple dynamic models are used, unstable periodic orbits appear in the attitude dynamics. By adding appropriate forces to the system, the unstable periodic orbits become asymptotically stable. Such a stabilized periodic orbit can be taken as the starting point for the operation of the electrodynamic tether. This analysis assumes a rigid tether, with two end masses orbiting along a circular inclined orbit, and a constant tether current, which does not depend on the attitude and orbital position of the tether. The Earth's magnetic field is modeled as a dipole aligned with the Earth's rotation axis.

Nomenclature

a	=	radius of the circular orbit, m
$\boldsymbol{B}$	=	Earth magnetic field, T
$Ex_1y_1z_1$	=	inertial geocentric frame, with origin at Earth's mass center
F_A, F_B	=	components of the control force, N
$\boldsymbol{F}_C$	=	control force, N
G	=	system center of mass
$Gxyz$	=	orbital frame with origin at G
$Gx_1y_1z_1$	=	moving frame with origin at G and axes parallel to those of $Ex_1y_1z_1$
$\boldsymbol{H}_G$	=	central angular momentum of the system, $\mathrm{kg \cdot m^2/s}$
h	=	distance from the upper end, m
h_G	=	distance between m_2 and G, m
$\mathcal{I}$	=	interval $[\phi_{\min}, \phi_{\max}]$
$I_e(h)$	=	tether current profile, A
I_s	=	central moment of inertia about a line perpendicular to the tether, $\mathrm{kg \cdot m^2}$
$\bar{\bar{\mathbf{I}}}_G$	=	central inertia tensor of the system
i	=	orbital inclination, rad
$(\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$	=	basis of the orbital frame, $Gxyz$
$(\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{k}_1)$	=	basis of the inertial frame, $Ex_1y_1z_1$
J_1	=	$\int_0^L (h_G - h) I_e(h)\, \mathrm{d}h$, $\mathrm{A \cdot m^2}$
k, k_1, k_2	=	feedback control gains
L	=	length of tether, m
$\boldsymbol{M}_C$	=	gravitational torque about G, $\mathrm{N \cdot m}$
$\boldsymbol{M}_E$	=	Lorentz torque about G, $\mathrm{N \cdot m}$
$\boldsymbol{M}_G$	=	control torque about G, $\mathrm{N \cdot m}$
m	=	total mass of the system $(m_1 + m_2 + m_t)$, kg
m_t	=	tether mass, kg
m_1	=	lower-end mass, kg
m_2	=	upper-end mass, kg
R, R_1, R_2	=	parameters of the extended control method
t	=	time, s
$\boldsymbol{u}$	=	unit vector along the tether
$\boldsymbol{u}_2$	=	$-\sin\theta \boldsymbol{i} + \cos\theta \boldsymbol{k}$, unit vector in orbital plane, perpendicular to tether line
$\boldsymbol{\alpha}_{\mathcal{R}}$	=	angular acceleration of the orbital frame, $\mathrm{rad/s^2}$
ε	=	nondimensional parameter defined in Eq. (17)
θ	=	in-plane angle formed by Gx axis and the projection of the tether on the orbital plane, rad
Λ_t	=	m_t/m, nondimensional tether mass
μ_E	=	Earth gravitational constant, $\mathrm{m^3/s^2}$
μ_m	=	strength of the magnetic dipole, $\mathrm{T \cdot m^3}$
ν	=	true anomaly measured from the line of nodes, rad
τ	=	feedback control delay
ϕ	=	mass angle [see Eqs. (1) and (2)], rad
φ	=	out-of-plane angle formed by the tether and the orbital plane, rad
$\boldsymbol{\omega}$	=	tether angular velocity, rad/s
$\boldsymbol{\omega}_{\mathcal{R}}$	=	angular velocity of the orbital frame relative to the inertial frame, rad/s
$\dot{(*)}$	=	$\mathrm{d}(*)/\mathrm{d}\nu$

I. Introduction

IN RECENT years, new control techniques have been developed to be applied to nonlinear dynamical systems in order to transform chaotic or unstable behavior into regular or periodic motions [1]. These new techniques have as their goal the possibility of bringing order to chaos. Some research has been undertaken using control schemes with and without feedback. However, feedback control methods form a distinguished and important group among the available control techniques. The reason is probably found in the advantage that they offer: in general, they need smaller forces than nonfeedback schemes to gain control of the system [2].

Pyragas [3] proposed a feedback control scheme designed to synchronize the current state of a system and a time-delayed version of itself. Taking this delayed time as the period of an unstable periodic orbit (UPO), such a control scheme can be used to stabilize the orbit. This method of control is usually called time-delayed autosynchronization, or TDAS. Two important advantages of this method are related to the feedback used: it requires neither rapid

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switching or sampling, nor a reference signal corresponding to the desired orbit. Socolar et al. [4] and Bleich and Socolar [5] improved on this technique, using a more elaborate feedback called the extended time-delayed autosynchronization, or ETDAS, in which TDAS appears as a limiting case.

Frequently, dynamical systems exhibit UPOs, which usually appear embedded in chaotic attractors. Substantial research effort has been devoted to controlling the chaotic behavior found in many of these unstable orbits, a constant feature in low-dimensional dynamical systems. Such orbits can be controlled with small forces that decrease over time as the system approaches the stabilized periodic orbit, where they vanish. The contemplated instabilities come from different sources, depending on the case analyzed.

Control schemes using delayed feedback have been used in orbital and attitude dynamics of spacecraft. One example of this kind of analysis can be found in [6] for a system with one degree of freedom. In that paper, TDAS control is used to stabilize the libration of a gravity–gradient satellite in an elliptical orbit; this kind of satellite exhibits self-excited dynamics, which is strongly chaotic in some regions of the parameter space.

There are studies that try to extend these techniques to tethered spacecraft, especially when electrodynamic tethers are involved. Basically, an electrodynamic tether is made of a thin conductive wire. When flying in a circular orbit with zero current, the electrodynamic tether has a stable equilibrium position on the local vertical. In the absence of damping or control, however, this gravity–gradient-stabilized equilibrium position disappears when the current begins to flow through the wire, and the tether becomes unstable in inclined orbits. The source of the instability is the Lorentz torque produced by the electromagnetic forces that act on the tether. Under some assumptions, the mathematical treatment of this problem simplifies, and some characteristics of the instability can be determined, as it is shown in previous analysis with different dynamic models [7–11]. These studies indicate that the source of instability of the electrodynamic tether is a nonlinear resonance mechanism that continually pumps energy into the system's attitude dynamics. After several orbits, the attitude motion eventually becomes unstable. In the previously mentioned studies, the tether current was assumed constant along the orbit and, in particular, independent of the actual tether position. This assumption, which is also adopted in this paper, leads to general results that are valid for electrodynamic tether systems, regardless of the particular device used to collect electrons from the surrounding plasma. It should be noted that, although the constant current assumption was relaxed in many simulations carried out in our group, the instability continued to appear with similar dynamics.

For the case of constant tether current, the governing equations exhibit periodic solutions instead of yielding equilibrium positions, with the period of the circular orbit followed by the system center of mass. In the absence of damping or control, these periodic orbits are unstable; the eigenvalues of the monodromy matrix of these periodic solutions provide a measure of the strength of such instability. The dynamic instability increases with the tether current and the inclination of the orbit. Consequently, it is quite natural to research the possibility of stabilizing such periodic orbits using the previously-mentioned techniques, which have been specially designed to stabilize chaotic periodic orbits. The analysis carried out by Peláez and Lorenzini [12] is an attempt to apply these techniques to the stabilization of the attitude motion of electrodynamic tethers working in inclined orbits. They explore some control laws, obtaining the families of periodic solutions that appear in the analysis and their stability properties, by using a numeric algorithm described in [13]. An alternative procedure can be found in the work of Williams [14]. Therefore, the subject has interest in and of itself.

To reduce the complexity of the analysis, some simplifying assumptions are introduced here: the Earth's magnetic field is modeled as a dipole aligned with the Earth's rotational axis, and a rigid tether with two end masses orbiting along a circular and inclined orbit is assumed. Consequently, the analysis does not include the tether lateral dynamics, which are also affected by the instability. Usually, the tether mass is small when compared with the end masses; therefore, the energy involved in the lateral modes is small when compared with the energy associated with the librational modes. Thus, control of the librations is a necessary condition for the reliable operation of the tether and, although it is the most important condition, unfortunately, it is not sufficient. Because the coupling between the librations and the lateral modes is complex due to electrodynamic forces (see [15] for a linear approximation), further analysis would be required to asses the behavior of the lateral modes. In this sense, the hypothesis here is that the destabilization of the lateral modes is also due to the input of energy to the attitude motion. The energy input is distributed between all modes in a complex way. If the most important input, the one associated with the librational modes, is removed, destabilization of the lateral modes will take longer; thus, control of the lateral modes becomes easier.

Peláez and Lorenzini [12], in exploring the possibility of using the TDAS method for this kind of problem, stated that the TDAS scheme does not work well with electrodynamic tethers; although this control law delays the onset of instability, it does not stabilize the UPO for reasonable values of the control parameters. They suggested the use of the ETDAS method, because it has been used with success in some cases for which TDAS failed. This paper is an attempt to extend the analysis of [12] by checking the value of the ETDAS method for some of the cases studied there. The results obtained here are preliminary, and they need to be confirmed by more detailed analysis. However, they are interesting and open the door to other control laws that allow for the stabilization of electrodynamic tethers that are not capable of self-balancing.

II. Attitude Dynamics and the Basic Periodic Solutions

A detailed derivation of the tether equations of motion, using the dumbbell model, can be found in [16] for the general elliptic case. Here, the salient points in the derivation are summarized for the case of circular orbit. The approach described is interesting, because it allows for the controlling forces that will be used in the control process to be easily introduced.

Consider a system formed by two end masses connected by a rigid rod of length L and a mass m_t and aligned with the unit vector $\boldsymbol{u}$ (see Fig. 1). The upper mass is m_2 and the lower one is m_1. The motion of the system, relative to the geocentric inertial frame $Ex_1y_1z_1$, is to be examined. Instead of (m_1, m_2, m_t), the parameters (m, ϕ, Λ_t) are introduced, where the mass angle ϕ is defined by

$$\cos^2\phi = \frac{1}{m}\left\{m_1 + \frac{1}{2}m_t\right\} \Rightarrow m_1 = m\left(\cos^2\phi - \frac{1}{2}\Lambda_t\right) \tag{1}$$

$$\sin^2\phi = \frac{1}{m}\left\{m_2 + \frac{1}{2}m_t\right\} \Rightarrow m_2 = m\left(\sin^2\phi - \frac{1}{2}\Lambda_t\right) \tag{2}$$

and ϕ belongs to the interval $\mathcal{I} = [\phi_{\min}, \phi_{\max}]$, with

$$\phi_{\min} = \arcsin\left(\sqrt{\frac{\Lambda_t}{2}}\right); \qquad (m_2 = 0)$$

$$\phi_{\max} = \arccos\left(\sqrt{\frac{\Lambda_t}{2}}\right); \qquad (m_1 = 0)$$

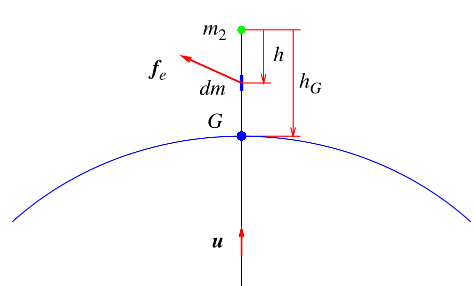


Fig. 1 System mass geometry.

If $\Lambda_t = 0, \mathcal{I} = [0,\ \pi/2]$. If both end masses are equal, $\phi = \pi/4$. The distance h_G and the inertia I_s are given, respectively, by

$$h_G = L\cos^2\phi \tag{3}$$

$$I_s = \frac{1}{12} mL^2(3\sin^2 2\phi - 2\Lambda_t) \tag{4}$$

Following Peláez et al. [7], the electrodynamic tethers in inclined orbit are unstable. Such instability is associated with the attitude dynamics. Therefore, the analysis is focused on the attitude motion. The center of mass G is assumed to follow a circular orbit of radius a and an inclination i, which is frozen; that is, the orbit of G is not perturbed by the electrodynamic drag (this is equivalent to assuming a very large total mass m).

A. Attitude Dynamics

The attitude dynamics of the tether system are governed by the angular momentum equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{H}_G) = \boldsymbol{M}_G + \boldsymbol{M}_E + \boldsymbol{M}_C \tag{5}$$

where $\boldsymbol{H}_G = \bar{\bar{\mathbf{I}}} \circ \boldsymbol{\omega}$ and the tether angular velocity[‡] $\boldsymbol{\omega}$ is given by

$$\boldsymbol{\omega} = \boldsymbol{u} \times \dot{\boldsymbol{u}} + \alpha \boldsymbol{u} \tag{6}$$

In Eq. (6), $\dot{\boldsymbol{u}}$ is the time derivative of $\boldsymbol{u}$ in the inertial frame $Ex_1y_1z_1$. On the right-hand side of Eq. (5),

$$\boldsymbol{M}_G \approx \frac{3\mu_E}{a^3} \boldsymbol{i} \times (\bar{\bar{\mathbf{I}}}_G \circ \boldsymbol{i}) = \frac{3\mu_E}{a^3} I_s (\boldsymbol{u} \times \boldsymbol{i})(\boldsymbol{u} \cdot \boldsymbol{i})$$

This approximated expression (terms of order L/a, and higher, have been neglected when compared with unity) assumes a perfectly spherical gravitational field. The unit vector $\boldsymbol{i}$ is along the local vertical pointing to the zenith.

The Lorentz torque $\boldsymbol{M}_E$ produced by the current tether profile $I_e(h)$ is

$$\boldsymbol{M}_E = \boldsymbol{u} \times (\boldsymbol{u} \times \boldsymbol{B}) J_1 \tag{7}$$

where $\boldsymbol{B}$ is the Earth's magnetic field at G (the variation of $\boldsymbol{B}$ along the tether is negligible). To model the field $\boldsymbol{B}$, a nontilted dipole is used. Its components in the orbital frame are [7]

$$B_x = -\frac{2\mu_m}{a^3} \sin i \sin \nu; \qquad B_y = -\frac{\mu_m}{a^3} \cos i;$$
$$B_z = +\frac{\mu_m}{a^3} \sin i \cos \nu$$

The control torque $\boldsymbol{M}_C$ must be modeled in an appropriate way. Let $Gxyz$ be the orbital frame (right-handed) with the origin at G, the Gx axis along the local vertical pointing to the zenith, and the Gy axis normal to the orbital plane (see Fig. 2). It is possible to produce a torque $\boldsymbol{M}_C$ with the help of a force, normal to the tether, and placed at one of the tether ends. For instance, consider a control force,

$$\boldsymbol{F}_C = F_A(\boldsymbol{u} \times \boldsymbol{u}_2) + F_B \boldsymbol{u}_2 \tag{8}$$

at the lower end of the tether. The control torque produced by $\boldsymbol{F}_C$ is

$$\boldsymbol{M}_C = L\sin^2\phi\{F_A \boldsymbol{u}_2 - F_B(\boldsymbol{u} \times \boldsymbol{u}_2)\}$$

B. Governing Equations

The angular momentum $\boldsymbol{H}_G$ can be written as

$$\boldsymbol{H}_G = I_s(\boldsymbol{u} \times \dot{\boldsymbol{u}})$$

[‡]In Eq. (6), the particular value of α is irrelevant, because the moment of inertia relative to the tether line vanishes.

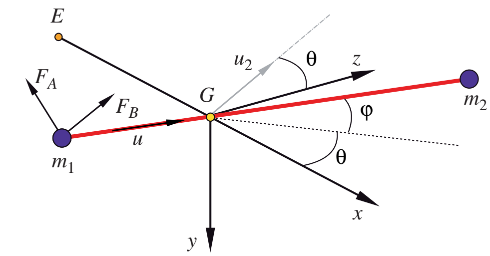


Fig. 2 Orbital frame and tether position.

because $(\boldsymbol{u} \times \dot{\boldsymbol{u}})$ is parallel to a central principal direction for the system. Equation (5) then takes the form:

$$\boldsymbol{u} \times \ddot{\boldsymbol{u}} = \frac{1}{I_s}(\boldsymbol{M}_G + \boldsymbol{M}_E + \boldsymbol{M}_C) \tag{9}$$

A study of the system's attitude dynamics reduces to the analysis of the motion relative to the orbital frame $Gxyz$. The angles φ and θ will be taken as generalized coordinates in the study of the attitude dynamics of the system. The unit vector $\boldsymbol{u}$ takes the form:

$$\boldsymbol{u} = \cos\varphi \cos\theta \boldsymbol{i} - \sin\varphi \boldsymbol{j} + \cos\varphi \sin\theta \boldsymbol{k}$$

Let $\boldsymbol{\omega}_{\mathcal{R}} = -\omega_{\mathcal{R}} \boldsymbol{j}$. For a circular orbit, $\boldsymbol{\omega}_{\mathcal{R}}$ is constant ($\omega_{\mathcal{R}} = \sqrt{\mu_E/a^3}$), and so $\boldsymbol{\alpha}_{\mathcal{R}} = 0$. It is straightforward to show that

$$\ddot{\boldsymbol{u}} = \boldsymbol{u}'' + 2\boldsymbol{\omega}_{\mathcal{R}} \times \boldsymbol{u}' + \boldsymbol{\omega}_{\mathcal{R}} \times (\boldsymbol{\omega}_{\mathcal{R}} \times \boldsymbol{u}) \tag{10}$$

where $\boldsymbol{u}'$ and $\boldsymbol{u}''$ are the time derivatives of $\boldsymbol{u}$ in the orbital frame. Equation (9) then takes the form

$$\boldsymbol{u} \times \boldsymbol{u}'' = \frac{1}{I_s}(\boldsymbol{M}_G + \boldsymbol{M}_E + \boldsymbol{M}_C) - \boldsymbol{P} \tag{11}$$

where

$$\boldsymbol{P} = 2(\boldsymbol{\omega}_{\mathcal{R}} \cdot \boldsymbol{u})\boldsymbol{u}' + (\boldsymbol{u} \times \boldsymbol{\omega}_{\mathcal{R}})(\boldsymbol{u} \cdot \boldsymbol{\omega}_{\mathcal{R}})$$

provides two scalar relations when projected onto two independent directions different from the tether line. Projection onto the vectors $\boldsymbol{j}$ and $\boldsymbol{u}_2$ yields

$$\boldsymbol{j} \cdot (\boldsymbol{u} \times \boldsymbol{u}'') = \boldsymbol{j} \cdot \left\{\frac{1}{I_s}(\boldsymbol{M}_G + \boldsymbol{M}_E + \boldsymbol{M}_C) - \boldsymbol{P}\right\} \tag{12}$$

$$\boldsymbol{u}_2 \cdot (\boldsymbol{u} \times \boldsymbol{u}'') = \boldsymbol{u}_2 \cdot \left\{\frac{1}{I_s}(\boldsymbol{M}_G + \boldsymbol{M}_E + \boldsymbol{M}_C) - \boldsymbol{P}\right\} \tag{13}$$

Introducing the libration angles $(\theta,\ \varphi)$, and after some algebra, these equations take the form

$$\begin{cases} \ddot{\theta} = 2(1+\dot{\theta})\dot{\varphi}\tan\varphi - \frac{3}{2}\sin(2\theta) - \varepsilon[\sin i \tan\varphi h_1(z,\ \theta) + \cos i] \\ \quad - \dfrac{LF_B \sin^2\phi}{\omega^2 I_s \cos\varphi} \\ \ddot{\varphi} = -\frac{1}{2}\sin(2\varphi)[(1+\dot{\theta})^2 + 3\cos^2\theta] + \varepsilon \sin i h_2(z,\ \theta) \\ \quad - \dfrac{LF_A \sin^2\phi}{\omega^2 I_s} \\ \dot{z} = 1 \end{cases} \tag{14}$$

where

$$h_1(z,\ \theta) = 2\sin z \cos\theta - \cos z \sin\theta;$$
$$h_2(z,\ \theta) = 2\sin z \sin\theta + \cos z \cos\theta$$

Here, and throughout the paper, the dot is used to symbolize the derivative with respect to ν, $\nu = \nu_0 + \omega t$. The variable z has been introduced to make the system of differential equations autonomous. It is defined in one orbital period $[z_0,\ z_0 + 2\pi]$, and it is equal to the true anomaly ν but for a constant. The tether current is on at the initial time ($t = 0$), and the equations must be integrated, starting from the appropriate initial conditions, at

$$\nu = \nu_0(t = 0); \quad \theta = \theta_0; \quad \varphi = \varphi_0; \quad \dot{\theta} = \dot{\theta}_0; \quad \dot{\varphi} = \dot{\varphi}_0 \tag{15}$$

Similar equations have been obtained using classical methods of analytical mechanics by Peláez and Lorenzini [12] for the uncontrolled case ($F_A = F_B = 0$).

C. Uncontrolled Tether

Now, consider the case in which the tether is uncontrolled ($F_A = F_B = 0$). The governing equations become:

$$\begin{cases} \ddot{\theta} = 2(1 + \dot{\theta})\dot{\varphi}\tan\varphi - \frac{3}{2}\sin(2\theta) - \varepsilon[\sin i \tan\varphi h_1(z,\ \theta) + \cos i] \\ \ddot{\varphi} = -\frac{1}{2}\sin(2\varphi)[(1 + \dot{\theta})^2 + 3\cos^2\theta] + \varepsilon \sin i h_2(z,\ \theta) \\ \dot{z} = 1 \end{cases} \tag{16}$$

These equations involve two free parameters i and ε. This last parameter, defined by

$$\varepsilon = \frac{J_1}{I_s}\frac{\mu_m}{\mu_E} \tag{17}$$

compares the Lorentz torque against the torque produced by the gravity and inertia forces, and it is an indication of the strength of the electrodynamic interaction. It vanishes for zero tether current and also for self-balanced electrodynamic tethers, regardless of the value of the tether current [17]. In summary, the terms in Eqs. (16), affected by the parameter ε, come from the Lorentz torque; the rest of the terms arise from the gravity gradient, the inertial Coriolis forces, and the control torque $\boldsymbol{M}_C$.

For an inert tether, that is $\varepsilon = 0$, Eqs. (16) produce steady solutions. In one of the singular points, the tether is aligned along the local vertical ($\theta = \varphi = 0$); this equilibrium position is stable. However, when $\varepsilon \neq 0$ (that is, when the current is flowing through the tether), the steady solutions disappear. If ε is assumed constant, instead of the equilibrium positions, Eqs. (16) then yield periodic solutions with the orbital period (2π in the nondimensional time ν). These basic periodic solutions collapse to the stable equilibrium position along the local vertical when $\varepsilon \to 0$.

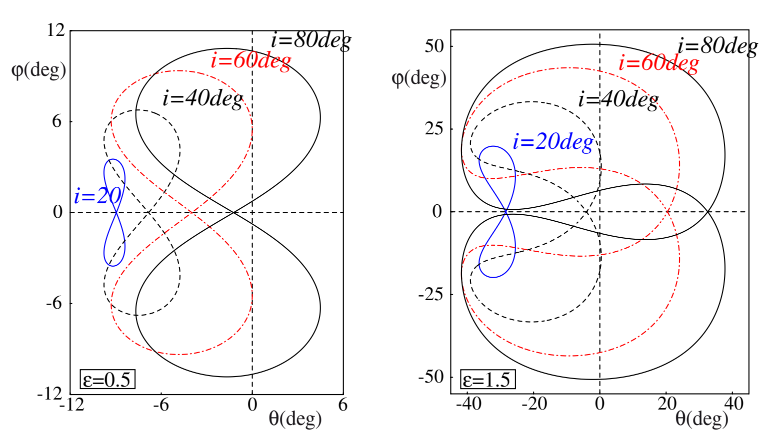
The basic periodic solutions depend on the two free parameters ε and i, and they have been described in [12]. However, to increase the readability of this paper, a brief summary is presented here. Figure 3 shows the form of these periodic solutions for different values of ε and i. In Fig. 3a, the basic periodic solutions are for a small value of $\varepsilon(=0.5)$, and several values of i are depicted. For small values of i, the periodic solution is an oscillation in φ, with θ almost constant. For increasing values of i, the amplitudes of both angles grow noticeably. Figure 3b shows the basic periodic solutions for a greater value of $\varepsilon(=1.5)$ and for the same values of the orbital inclination i. It is worth noting that, when $\varepsilon = 0.5$, the amplitudes of the oscillations are quite small. In fact, they are smaller than 15 deg. On the other hand, for $\varepsilon = 1.5$, the amplitudes become significant, and they can even reach values close to 50 deg. From these pictures, it is clear that the amplitudes of both oscillations increase with ε.

Apart from these basic periodic motions, it is important to note that the uncontrolled electrodynamic tether also exhibits other periodic solutions for Eqs. (16). These other secondary periodic solutions have the same period (2π) and appear in pairs that are approximately symmetric with respect to the orbital plane. Moreover, there are other periodic solutions for which the periods are multiples of 2π. For more details about these secondary periodic libration motions, see Peláez and Lara [9].

The stability properties of the basic periodic solutions depend on the two free parameters ε and i. When the system is not controlled, all the basic periodic solutions are unstable for any values of ε and i [7]. The secondary periodic libration motions are also unstable; in fact, they are more unstable than the basic periodic solutions [9].

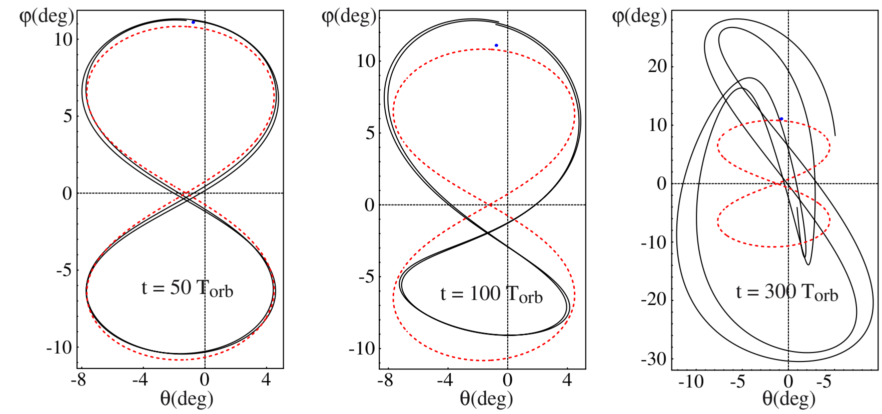
Figures 4 and 5 graphically represent two examples of the unstable character of the basic periodic motions of the uncontrolled tether. The dashed line represents the basic periodic solution, and the continuous line represents a libration motion starting from initial conditions very close to that periodic solution. Figure 4 corresponds to the case $i = 80$ deg and a small value of the electrodynamic parameter $\varepsilon = 0.5$. The plot represents the librational motion followed by the tether after 50, 100, and 300 orbital periods (only the last two periods are shown in the figure). From these graphs, it is clear that, although the motions start with initial conditions close to the periodic solution, after 300 orbital periods, the libration motion of the tether is very far away from the periodic trajectory. Figure 5 shows another example of the instability of the basic periodic solutions for a larger value of the electrodynamic parameters $\varepsilon = 1.5$ and $i = 40$ deg. In this case, the corresponding basic periodic motion is much more unstable. As can be seen in Fig. 5b, after only 6.5 orbital periods, the libration motion of the tether is very far away from the periodic solution; in fact, the motion has undergone a transition from libration to rotation.

These two examples graphically demonstrate the fact that the instability of the basic periodic solutions strongly increases with the parameter ε. Peláez and Lara [9] have done an extensive analysis of the eigenvalues of the monodromy matrix of the periodic motions of the uncontrolled electrodynamic tether. They studied the dependence of the eigenvalues with the parameter ε and the inclination i, and they showed that the instability of the periodic solutions increases with the electrodynamic parameter ε.



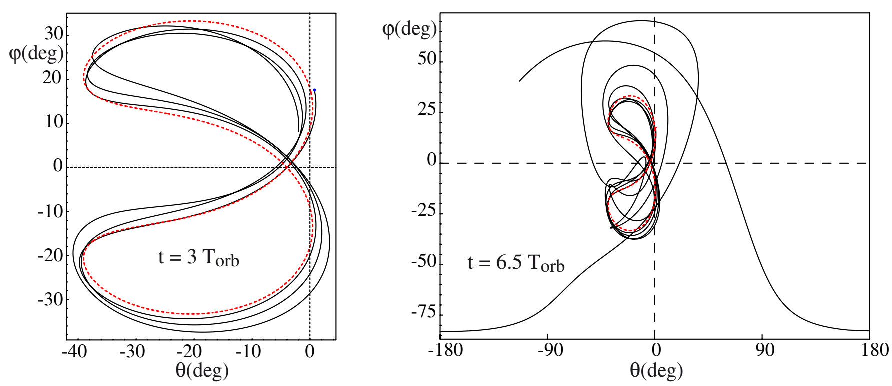
a) b)

Fig. 3 Basic periodic solutions for two values of the orbital inclination: a) $\varepsilon = 0.5$and b) $\varepsilon = 1.5$.



a) b) c)

Fig. 4 Unstable behavior of a libration motion, with initial conditions very close to the basic periodic solution (dashed line), in the case of $\varepsilon = 0.5$ and $i = 80$ deg after different orbital periods. Only the libration during the two last orbital periods is represented.



a) b)

Fig. 5 Unstable behavior of a libration motion, with initial conditions very close to the basic periodic solution (dashed line), in the case of $\varepsilon = 1.5$ and $i = 40$ deg during different orbital periods.

III. Libration Control with the Time-Delayed Autosynchronization Method

The particular feedback control method that is used in this section is the so-called TDAS [3]. This technique has two important advantages: it does not require fast switching or sampling, nor does it need a reference signal corresponding to the desired regular motion. It only requires the knowledge of the period of the desired periodic orbit.

The basic block diagram describing the TDAS control technique is shown in Fig. 6. The control variable y of the system is delayed at the output by some amount of time τ, and then it is reintroduced into the system through the feedback control signal $F(t) = k\{y(t-\tau) - y(t)\}$. When considering periodic motions, the delay time τ usually coincides with the period of the orbit. This control perturbation can be adjusted through the parameter k in order to achieve the stabilization of the desired periodic orbit; that is, k is a free parameter of the problem. It should be pointed out that, for any value of k, when the controlled system follows a periodic orbit of period τ, the control signal $F(t)$ vanishes, because in that case, $y(t-\tau) = y(t)$.

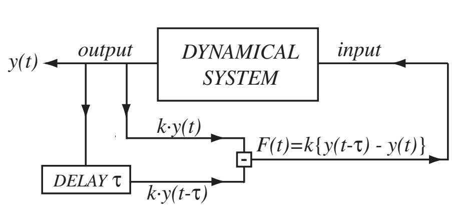


Fig. 6 Block diagram of the TDAS control method.

In the case of the electrodynamic tether, Peláez and Lorenzini [12] used the TDAS control method in order to convert unstable periodic libration motions into stable periodic ones. However, the instability to be controlled in this case is associated with a nonlinear resonance that continually pumps energy into the system. Hence, the system is a two-degree-of-freedom system that has a destabilizing mechanism different from the one usually found in the literature on control methods for nonlinear systems. In the study cited previously, Peláez and Lorenzini assumed that the tether was acted upon by additional forces, which introduces new terms in the governing equations in order to effectively control the tether dynamics. This way, the TDAS control method they applied leads to the following governing equations for the controlled attitude motion:

$$\begin{cases} \ddot{\theta} = 2(1+\dot{\theta})\dot{\varphi}\tan\varphi - \frac{3}{2}\sin(2\theta) - \varepsilon[\sin i \tan\varphi h_1(z,\theta) \\ \quad + \cos i] + F_1(z) \\ \ddot{\varphi} = -\frac{1}{2}\sin(2\varphi)[(1+\dot{\theta})^2 + 3\cos^2\theta] + \varepsilon \sin i h_2(z,\theta) + F_2(z) \\ \dot{z} = 1 \end{cases} \tag{18}$$

where the two control signals $F_i(z)$ are given by

$$F_1(z) = k_1[\dot{\theta}(z) - \dot{\theta}(z-\tau)]; \qquad F_2(z) = k_2[\dot{\varphi}(z) - \dot{\varphi}(z-\tau)]$$

The control variables they chose were the angular velocities $\dot{\theta}$ and $\dot{\varphi}$. The delay time τ must precisely be the period of the unstable periodic motions in the nondimensional time ν; that is, $\tau = 2\pi$. In this way, there are two parameters, k_1 and k_2, in the added control terms to achieve the stabilization of the basic periodic librational motions of the tether. Notice that, if these control signals are to be produced by the control force $\boldsymbol{F}_C$, defined in Eq. (8), their components should take the values:

$$F_A = -\frac{I_s\omega^2}{L\sin^2\phi}F_2(z); \qquad F_B = -\frac{I_s\omega^2}{L\sin^2\phi}\cos\varphi F_1(z) \tag{19}$$

Notice that, when the controlled tether follows a 2π-periodic orbit, both control signals $(F_1,\ F_2)$ vanish; that is, any 2π-periodic motion of the uncontrolled tether [Eq. (16)] is also a 2π-periodic orbit of the controlled one [Eq. (18)]. As a consequence, when the system moves in the neighborhood of the periodic solution, the controlling signals $(F_1,\ F_2)$, as well as the controlling forces $(F_A,\ F_B)$ should take small values.

Let us assume, for a moment, that this control method is successful. The unstable basic periodic motion of the uncontrolled system [Eq. (16)] would become asymptotically stable when considered as a periodic libration of the controlled tether [Eq. (18)]. Therefore, any motion of the controlled system (starting in the attraction basin of that stabilized periodic libration) would approach it over time, and (after a while) the control terms would become very small, because they would tend to zero when $\nu \to \infty$. Thus, if from the very beginning, the tether is operated close to the basic periodic solution, it can be controlled with small controlling forces. This is an attractive feature of this control method. The growth of the libration angles θ and φ, due to the instability of the uncontrolled system, could be removed by the control terms added to the governing equations. Unfortunately, the numerous tests carried out by Peláez and Lorenzini in [12] showed that the TDAS control technique fails to stabilize the basic periodic motion of the tether. Thus, this control method is not able to convert the unstable periodic motions of the uncontrolled tether into asymptotically stable ones.

Figure 7 shows the typical behavior of the TDAS control technique when it tries to stabilize one basic periodic orbit of the electrodynamic tether. In this example the free and the control parameters take the values $\varepsilon = 1.5$, $i = 40$ deg, $k_1 = 0.2$, and $k_2 = 1.2$. Figure 7 shows the evolution with time of the libration, starting from initial conditions very close to the periodic orbit in which several increasing multiples of the orbital period have been considered. The controlled equations of motion [Eq (18)] have been integrated for different combinations of the control parameters k_1 and k_2, always starting from initial conditions very close to the periodic orbit. Unfortunately, in all cases, the trajectory always moved away from the periodic motion after several orbital periods.

IV. Libration Control with the Extended Time-Delayed Autosynchronization Method

As Peláez and Lorenzini [12] pointed out, the TDAS control scheme fails to stabilize the periodic motion because of the energy flow to the system coming from the electrodynamic interaction with the Earth's magnetic field. Similar behavior has been observed, for example, in a forced pendulum in which the TDAS control method also fails [5].

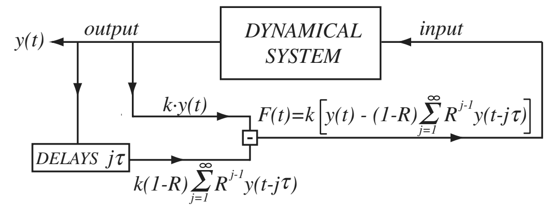


Fig. 8 Block diagram of the ETDAS control method.

The failure of the TDAS method led us to try a natural extension of this control technique: the so-called ETDAS. This extension was first proposed by Socolar et al. [4] to overcome the limitations of the TDAS technique in stabilizing periodic orbits, and it has been successfully applied in several systems in which TDAS had previously failed [2,4,5,18]. The use of the ETDAS with the electrodynamic tether was already mentioned by Peláez and Lorenzini in [12] as a natural continuation of their work.

The basic block diagram of the ETDAS control method is shown in Fig. 8. The control variable y is progressively delayed at the output by multiples of some amount of time τ. Then, all these delayed control values $y(t - j\tau)$ are reintroduced into the system through the feedback control signal

$$F(t) = k\left[y(t) - (1-R)\sum_{j=1}^{\infty} R^{j-1}y(t-j\tau)\right]$$

where $0 \leq R < 1$ and k are the two adjustable parameters of this control signal.

When applied to periodic motion, the delay time τ coincides with the period of the motion. So, the ETDAS method uses information about many previous states of the system in order to stabilize the periodic orbit with period τ. It is worth emphasizing that, for any values of the control parameters R and k, when the system follows a τ-periodic orbit, the control signal $F(t)$ vanishes because, in that case, $y(t - j\tau) = y(t)$ for all j (the identity,

$$\frac{1}{1-R} = \sum_{k=0}^{\infty} R^k$$

has to be taken into account). Note also that, in the limit $R \to 0$, the ETDAS method coincides with TDAS.

To stabilize the basic periodic librations of the electrodynamic tether, the ETDAS method has been applied in such a way that the governing equations of motion of the controlled tether take the same form as Eqs. (18), where now the two control terms $F_i(z)$ are given by

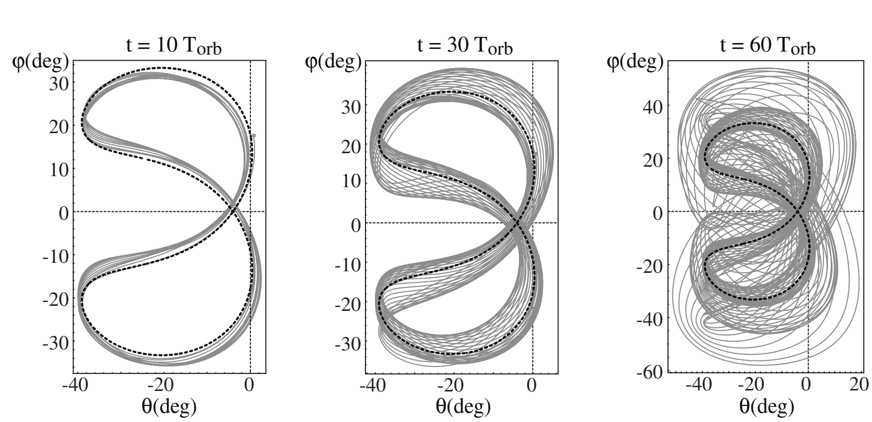


Fig. 7 Failure of the TDAS control method for the case of $\varepsilon = 1.5$ and $i = 40$ deg. Libration motion after 10, 30, and 60 orbital periods. Control parameters are $k_1 = 0.2$ and $k_2 = 1.2$. The dashed line represents the corresponding periodic motion.

$$F_1(z) = k_1\left[\dot{\theta}(z) - (1-R_1)\sum_{j=1}^{\infty} R_1^{j-1}\dot{\theta}(z-j\tau)\right]$$

$$F_2(z) = k_2\left[\dot{\varphi}(z) - (1-R_2)\sum_{j=1}^{\infty} R_2^{j-1}\dot{\varphi}(z-j\tau)\right]$$

The angular velocities $\dot{\theta}$ and $\dot{\varphi}$ have been chosen as control variables, and there are four different adjustable control parameters of k_1, k_2, and R_1, R_2, with $0 \le R_i < 1$. Notice that the control forces (F_A, F_B) at the lower end of the tether are also given by Eq. (19) but now with the previously stated values of (F_1, F_2).

Figure 9 shows an example of a test carried out by numerically integrating the equations of motion controlled by the ETDAS method. The example corresponds to the same case shown in Fig. 7, with identical values for the parameters $\varepsilon = 1.5$ and $i = 40$ deg and the same initial conditions close to the corresponding periodic motion. The values of the control parameters have been taken as $k_1 = k_2 = -0.2$ and $R_1 = R_2 = 0.9$. For the sake of simplicity, the number of free parameters is reduced by taking $k_1 = k_2$ and $R_1 = R_2$. Hence, analysis of the control domains of the ETDAS method, which is carried out in the next section, becomes manageable. The values of the control parameters $k_1 = k_2 = -0.2$ and $R_1 = R_2 = 0.9$, selected in this example, lie in the stable domain of the method.

Figure 9 shows the evolution of the controlled librational motion for increasing multiples of the orbital period. In Figs. 9b and 9c, the libration is represented only for the two last orbital periods. It is clear from the figure that the ETDAS technique is successful in stabilizing the periodic orbit. After 30 orbital periods, the tether libration practically coincides with the basic periodic motion, so that it is almost impossible to distinguish one from the other. Therefore, the ETDAS control method is able to change the dynamical character of the basic periodic motion, which has become asymptotically stable. Similar qualitative behavior was found for different values of ε and i.

V. Control Domains of the Extended Time-Delayed Autosynchronization Method

After checking, with several numerical tests, that the ETDAS method is able to stabilize the electrodynamic tether, a stability analysis of the basic periodic motions of the tether, controlled by means of the ETDAS method, is carried out. The analysis uses the technique proposed by Bleich and Socolar in [5], which is briefly described in the following paragraphs.

A. Summary of the Extended Time-Delayed Autosynchronization Method

Consider an uncontrolled dynamical system with equations of motion

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y},\ t)$$

where $\mathbf{y}$ is the n-dimensional vector that describes the dynamical state of the system. A UPO, $\mathbf{y}_p(t)$, of the uncontrolled system with period τ is known, and a nearby orbit $\mathbf{y}(t)$ is also considered. The interest is in controlling the system, so that the UPO becomes a stable periodic orbit in such a way that the nearby orbit $\mathbf{y}(t)$ tends asymptotically to the periodic orbit $\mathbf{y}_p(t)$. This implies that the difference between both orbits, $\mathbf{x}(t) = \mathbf{y}(t) - \mathbf{y}_p(t)$, must satisfy the condition $\lim_{t\to\infty}\mathbf{x}(t) = 0$. To achieve this goal, the dynamical system is modified by the addition of an extended TDAS control signal to the equations of motion, which then take the form

$$\dot{\mathbf{y}}(t) = \mathbf{f}(\mathbf{y},\ t) + k\mathbf{M}\left[\mathbf{y}(t) - (1-R)\sum_{j=1}^{\infty} R^{j-1}\mathbf{y}(t-j\tau)\right] \tag{20}$$

where $\mathbf{M}$ is an $n\times n$ matrix that contains the information about the specific way the feedback control signal is applied to the system. Note that the periodic orbit $\mathbf{y}_p(t)$ is also a periodic solution of the controlled system [Eq. (20)]. To study the stability properties of the UPO $\mathbf{y}_p(t)$ in the new controlled system, the time derivative of the deviation $\mathbf{x}(t)$ is written to first order as

$$\dot{\mathbf{x}}(t) = \mathbf{J}(t)\mathbf{x}(t) + k\mathbf{M}\left[\mathbf{x}(t) - (1-R)\sum_{j=1}^{\infty} R^{j-1}\mathbf{x}(t-j\tau)\right] \tag{21}$$

where $\mathbf{J}(t)$ is the Jacobian matrix of the uncontrolled dynamical system.

The goal of the control method is to transform the UPO into an asymptotically stable orbit. Therefore, a suitable form for the solutions $\mathbf{x}(t)$ of Eq. (21) is $\mathbf{x}(t) = \mathbf{p}_\lambda(t)e^{\lambda t/\tau}$, where $\mathbf{p}_\lambda(t)$ is a τ-periodic function $\mathbf{p}_\lambda(t+\tau) = \mathbf{p}_\lambda(t)$, and λ is a complex number with $\Re(\lambda) < 0$. Inserting this solution into Eq. (21) gives

$$\dot{\mathbf{p}}_\lambda(t) = \left[\mathbf{J}(t) - \frac{\lambda}{\tau}\mathbf{I} + k\mathbf{M} - k\mathbf{M}(1-R)\sum_{j=1}^{\infty} R^{j-1}e^{-j\lambda}\right]\mathbf{p}_\lambda(t)$$

where $\mathbf{I}$ is the identity matrix. The solution of this differential equation for a given initial condition can be written as

$$\mathbf{p}_\lambda(t) = e^{-\lambda t/\tau}\mathbf{U}_\lambda(t)\mathbf{p}_\lambda(0) \tag{22}$$

where $\mathbf{U}_\lambda(t)$ is a matrix, which is the solution of the following problem:

$$\dot{\mathbf{U}}_\lambda(t) = \left[\mathbf{J}(t) + k\mathbf{M} - k\mathbf{M}(1-R)\sum_{j=1}^{\infty} R^{j-1}e^{-j\lambda}\right]\mathbf{U}_\lambda(t)$$

$$\mathbf{U}_\lambda(0) = \mathbf{I}$$

Performing the geometric sum included in the right-hand side, this equation takes the form:

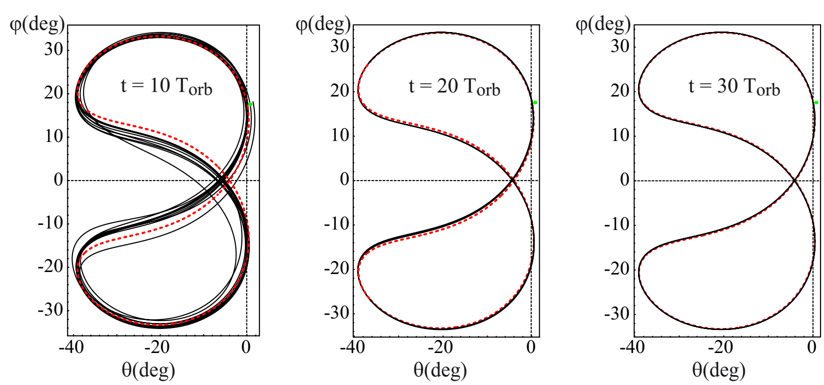


Fig. 9 Success of the ETDAS control method for the case of $\varepsilon = 1.5$ and $i = 40$ deg. Controlled libration motion after 10, 20, and 30 orbital periods. Control parameters are $k_1 = k_2 = -0.2$ and $R_1 = R_2 = 0.9$. The dashed line stands for the corresponding periodic motion. In b) and c), only the libration during the two last orbital periods is represented.

$$\dot{\mathbf{U}}_\lambda(t) = \left[\mathbf{J}(t) + k\mathbf{M}\frac{1-e^{-\lambda}}{1-Re^{-\lambda}}\right]\mathbf{U}_\lambda(t); \qquad \mathbf{U}_\lambda(0) = \mathbf{I} \tag{23}$$

Because $\mathbf{p}_\lambda(t)$ is a τ-periodic function, the periodicity condition $\mathbf{p}_\lambda(0) = \mathbf{p}_\lambda(\tau)$ can be written as

$$[e^{-\lambda}\mathbf{U}_\lambda(\tau) - \mathbf{I}]\mathbf{p}_\lambda(0) = 0$$

in which the relation [Eq. (23)] has been taken into account. As a consequence, the following determinant vanishes:

$$g(\mu^{-1}) \equiv \det[\mu^{-1}\mathbf{U}_\lambda(\tau) - \mathbf{I}] = 0 \tag{24}$$

where $\mu \equiv e^{\lambda}$ is the Floquet multiplier.

The control method will be effective if, for any solution of Eq. (23), the corresponding deviation $\mathbf{x}(t) = \mathbf{p}_\lambda(t)e^{\lambda t/\tau}$ goes asymptotically to zero, which means that all solutions of Eq. (24) must satisfy $\Re(\lambda) < 0$. Therefore, the asymptotic stability of the periodic orbit $\mathbf{y}_p$ in the controlled system requires that all zeros of $g(\mu^{-1})$ lie outside the unit circle as $\|\mu^{-1}\| > 1 \Leftrightarrow \Re(\lambda) < 0$. For $R < 1$, the determinant $g(\mu^{-1})$ has no zeros inside the unit circle. Thus, by a well-known theorem in complex analysis, the number of roots of $g(\mu^{-1})$ inside the unit circle is equal to the number of times the path traced by $g(\mu^{-1})$ winds around the origin as μ^{-1} runs completely over the unit circle. Hence, the periodic orbit in the controlled system is stable if, and only if, this number of encirclements vanishes.

B. Domain of Stability of the Extended Time-Delayed Autosynchronization Method

Calculation of the domains of stability with the ETDAS method involves four different tasks for given values of the parameters k, R, and the matrix $\mathbf{M}$:

1) The first task is the determination of the particular UPO in the uncontrolled system.

2) The second task is the computation of the matrix $\mathbf{U}_\lambda(\tau)$. Because this cannot be done analytically, except for very special cases, this computation must be done numerically by integrating Eq. (23) between 0 and τ.

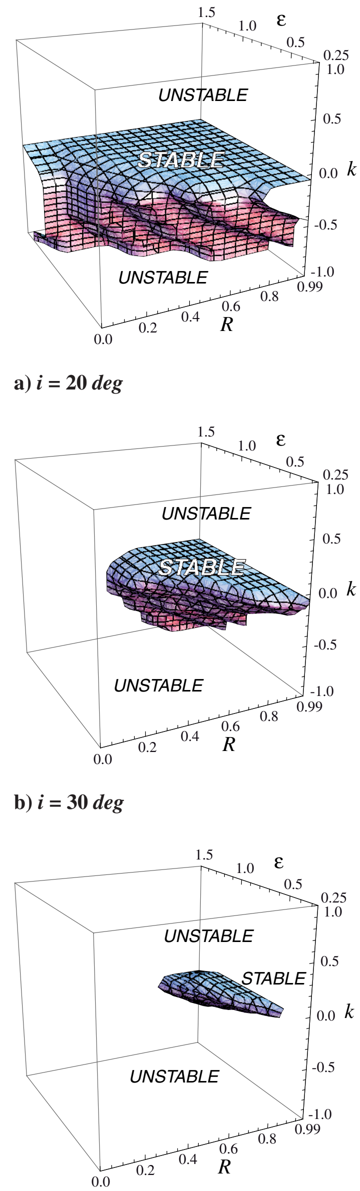
3) The third task is the calculation of the determinant $g(\mu^{-1})$ for a sequence of sufficiently closely-spaced values of μ over the unit circle.

4) The fourth task is the determination of the number of encirclements of the origin corresponding to the path traced by $g(\mu^{-1})$.

Although this method of stability analysis involves some cumbersome calculations, it has several important advantages. The method avoids integrating the equations of motion of the controlled system with time delay. This integration would be a very delicate matter due to two nontrivial difficulties: first, the accuracy of the numerical integrator over long times and, second, the choice of the initial conditions in the corresponding basin of attraction. The alternative method of stability analysis proposed by Bleich and Socolar [5] only requires integrating the equations of motion without the time-delay control terms F_i. Moreover, this integration must be carried out over only one period of the corresponding periodic motion. Basically, the method reduces to the calculation of the number of encirclements of the origin of a curve in the complex plane.

Using this technique, the stability domains of the electrodynamic tether have been calculated, taking the ETDAS method as controller, as functions of the free parameters of the problem. For simplicity, the study on the domains of control was limited to the case in which $k_1 = k_2 = k$ and $R_1 = R_2 = R$. Therefore, the stability domains have been calculated as functions of the control parameters R, k, and the electrodynamic parameter ε for different values of the orbital inclination i. This analysis will be extended in the future to study the general case $k_1 \neq k_2$ and $R_1 \neq R_2$ as a natural continuation of this line of this work.

Figure 10 shows the stability domains provided by the ETDAS method in the three-dimensional parametric space $(R,\ k,\ \varepsilon)$ for three increasing values of the orbital inclination i. The gray regions stand



a) $i = 20$ *deg*

b) $i = 30$ *deg*

c) $i = 40$ *deg*

Fig. 10 Evolution of the stability domains of the ETDAS method in space ($R,\ k,\ \varepsilon$) for three increasing orbital inclinations (i).

for the domains at which the ETDAS method succeeds in stabilizing the periodic motion, whereas in the rest of the (R, k, ε) volume, this control technique fails.

Figure 10 shows that, as the orbital inclination i increases, the stability domains shrink, and the efficacy of the ETDAS method decreases. This happens not only globally, but also for any particular value of the electrodynamic parameter ε. For orbital inclinations $i > 50$ deg, the ETDAS method fails for any value set of the parameters $(R,\ k,\ \varepsilon)$. However, for a fixed inclination i, as the parameter ε increases, the stability domains enlarge. This behavior seems quite paradoxical, as it means that the more unstable the uncontrolled tether is (large values of ε), the more effective the ETDAS method seems to be for the control parameters used.

Note that Fig. 10 for the ETDAS method also includes the stability domains of the TDAS method, because both methods coincide in the limit $R \to 0$. Indeed, the control domains of the TDAS method are represented in the left vertical plane $(k,\ \varepsilon)$ with $R = 0$ of each parametric cube $(R,\ k,\ \varepsilon)$ for $k_1 = k_2 = k$. Thus, Fig. 10 shows that the TDAS method is much less powerful than the ETDAS method. As can be seen in Fig. 10a, the TDAS technique only succeeds in the case $i = 20$ deg for $k_1 = k_2 = k < 0$ and high enough values of the electrodynamic parameter ε.

To check the validity of the calculated stability domains, several numerical tests were carried out by integrating the controlled equations of motion for values of the parameters (R, k, ε) belonging to each one of control domains. Figure 11 shows an example of these tests for the particular case $\varepsilon = 1$ and $i = 25$ deg. Figure 11b shows the corresponding stability domains in the parametric plane (R, k). Chosen representative examples are the values of the control parameters of points $A(0.1, -0.25)$, placed at the unstable domain and $B(0.6, -0.25)$, situated at the stable one. Figures 11a and 11c share the UPO corresponding to $\varepsilon = 1$ and $i = 25$ deg and also the initial conditions of both simulations (close to the periodic orbit).

The stable region, shown in Fig. 11c, shows the periodic orbit (dashed line) and the controlled tether libration after 90 orbital periods, during only the last one. The values of the control parameters correspond to point B, included in the stable region. In this case, the success of the ETDAS method is evident, because the controlled libration is almost indistinguishable from the basic periodic motion.

The unstable region, shown in Fig. 11a, shows the periodic orbit (dashed line) and the controlled libration after 600 orbital periods, during only the last 100. The values of the control parameters correspond to point A inside the unstable control region. Now, the ETDAS method fails, because the controlled libration moves away from the periodic orbit. Note that, in spite of a long integration time, the characteristic transition from libration to rotation in the attitude motion of the uncontrolled tether is not observable in this case. On the contrary, for even longer times of integration, the trajectory of the tether libration completely fills the same region shown in Fig. 11a without escaping from it.

The reason for the curious behavior described previously is not clear. The existence of other secondary periodic librational motions in the dynamics of the uncontrolled tether [9] could shed some light on this. Indeed, some of these secondary periodic motions have the same period 2π as the basic periodic ones, and they appear almost symmetrically with respect to the orbital plane ($\varphi = 0$). Thus, although the ETDAS method fails in the asymptotic stabilization of the basic periodic motion, it seems that it stabilizes one pair of those secondary periodic motions in such a way that the trajectory of the controlled libration moves away from the basic periodic orbit. But, it ends up oscillating indefinitely between both symmetric secondary periodic motions, filling up the region delimited by them. However, there are alternatives to this explanation; perhaps, there is some invariant manifold stabilized by the controlling terms of the ETDAS method that prevents the typical transition from libration to rotation, which characterizes the long-term behavior of the electrodynamic tether instability. In any case, this behavior should be studied in detail in the future in order to clarify the underlying reasons for it.

C. Comments on the Model

The analysis carried out in this paper should be considered a first step toward exploring the performance of the ETDAS in stabilizing electrodynamic tethers in inclined orbit. The analysis assumes a constant value for the parameter ε. Such a constant value does not occur naturally in the electrodynamic tether, because the tether current depends on two distinct parameters: the driving electric field $E_m = \boldsymbol{u} \cdot (\boldsymbol{v}_G \times \boldsymbol{B})$ and the electronic plasma density of the ionosphere n_∞. Both parameters change along any inclined orbit. Moreover, the tether librations affect the value of E_m, which decreases when the tether deviates from the local vertical. Consequently, ε is not constant unless an additional control on the tether current forces the achievement of such a condition. To do that, a variable resistor could be introduced in series with the tether. This is not practical, however, because the constant value of ε would be very close to the minimum tether current (along the orbit), and the thruster performance of the tether would be seriously diminished.

The assumption of constant value for ε can be relaxed. In such a case, the analysis must opt for a particular tether configuration (bare tether, for example) and a particular tether regime (generator or thruster), because the electron collection depends closely on them. Once the tether current is modeled, the next point is to determine periodic orbits in the librational motion. Some work in this vein can be found in [19,20], in which the bare tether has been analyzed in the long-tether regime of the generator mode. Both papers show how to reformulate the governing equations in order to account for the dependence of the tether current on the actual tether position in the orbital frame and the variations along the orbit. There are also 2π-periodic solutions that are unstable without damping or control.

For an actual tether in Earth's orbit, the geomagnetic field is more complex than the field given by an aligned dipole model. A more accurate description of this field introduces forcing terms in the governing equations, with a new period associated with the daily Earth rotation. As a consequence, the basic periodic solutions disappear for most orbits, and they only exist in some resonant cases. Moreover, real tethers exhibit flexibility and elasticity, which introduce other temporal scales in the problem that contribute to destroying the periodicity. Finally, the orbit of the system center of

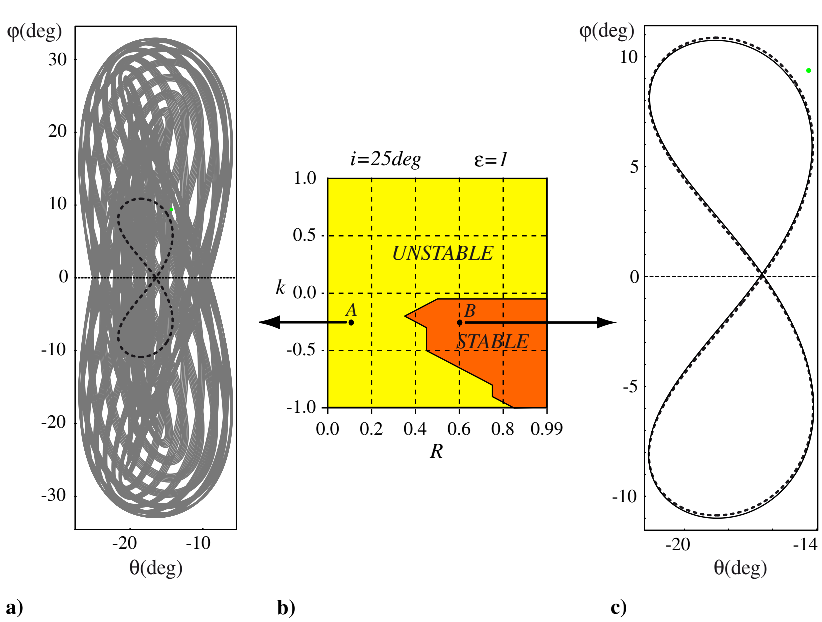


Fig. 11 Two examples of the different efficacies of the ETDAS method in both regions of the stability domains when $\varepsilon = 1$ and $i = 25$ deg: a) unstable control for $R = 0.1$ and $k = -0.25$, b) control domains in the parametric plane (R, k), and c) stable control for $R = 0.6$ and $k = -0.25$.

mass is not frozen; it evolves and introduces additional perturbations into the problem. Any eccentricity, for instance, would induce self-excited librations that reinforce the tether instability. Thus, in general, in a real tether, the periodic solutions considered in this paper do not exist. However, the effects that prevent the existence of periodic solutions can be considered as perturbations of the model that is presented here. It would then become necessary to assess if those perturbations remove the asymptotic stable character from the periodic solution; such a stable character is the main consequence of the ETDAS method analyzed in this paper.

Another point to be considered is the transition between the inert tether, usually at rest along the local vertical, and the live tether that will closely follow the basic periodic orbit with the help of the controlling forces. The stabilization carried out by this control scheme requires starting the procedure not too far away from the periodic orbit; otherwise, the controlling forces would grow excessively large. It would be necessary to start from the natural equilibrium position of the tether along the local vertical and to increase the tether current progressively to the desired value of ε. Probably, the increase of the current would require several steps. In the first step, the tether would be stabilized around a value $\varepsilon_1 < \varepsilon$; once the trajectory is close enough to the basic periodic motion corresponding to ε_1, the current would be increased until it reaches a larger value ε_2, which satisfies $\varepsilon_1 < \varepsilon_2 < \varepsilon$ and so on. The advantage of the ETDAS method is that it works better for larger values of ε; such a feature likely makes this initial strategy easier to implement. It is suggested that all these important points be analyzed and checked in future work.

VI. Conclusions

In this study, the ETDAS method is applied to the control of the libration motion of an electrodynamic tether in inclined circular orbit. The study was conducted for a set of reasonable assumptions that allow for a tractable mathematical treatment of the problem. The main idea is to use one of the basic periodic motions that appear in the model as the starting point for the operation of the electrodynamic tether. This periodic motion, which is unstable, can be stabilized with the help of the ETDAS method.

Numerical simulations of the libration motion of the controlled tether show that the proposed extended control scheme succeeds in many cases in converting the basic unstable periodic motion of the uncontrolled system into an asymptotically stable orbit of the controlled tether. Note that the scheme used in the analysis only involves two control parameters (R and k) instead of the four control parameters (k_1, R_1, k_2, and R_2), which appear in a natural way in the method. This fact opens the door to some kind of optimization of the control scheme, through the relaxation of some of the simplifying assumptions.

A stability analysis of the basic periodic motions of the tether controlled by the extended method was performed. This analysis provides the stability domains of the extended control scheme as functions of its control parameters R and k for several values of the free parameters i and ε. The analysis confirms that the extended method is much more efficient than the reduced one in stabilizing the basic periodic motions. The study of the control domains shows that, paradoxically, the more unstable the uncontrolled tether is (large values of ε), the more effective the extended method seems to be. However, for values of inclination larger than 45 deg, the extended scheme also fails to stabilize the electrodynamic tether.

The stability domains calculated for the extended method ($R \neq 0$) also include the stability domains for the reduced one ($R = 0$). In this regard, a small region of stability unknown from previous studies was found for small values of the inclination i and large values of ε.

It was also found that, in the cases in which the extended scheme fails to stabilize a basic unstable periodic solution, the unstable trajectory prevents the transition from libration to rotation in the attitude motion of the tether, as usually happens in this kind of instability. This control method seems to be able to stabilize a pair of secondary unstable periodic solutions that are symmetric with respect to the orbital plane and have the same period as the basic one.

Finally, the success of the extended method in stabilizing the electrodynamic tether, in the particular case studied, opens the door to other different control schemes (also based on the ETDAS theory) but using the tether current as control parameter.

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