Using a Fuzzy Object-Relational Database for Colour Image Retrieval

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Abstract. The paper presents a fuzzy database management system, and a fuzzy method for dominant colour description of images, on which an image retrieval system is built. The paper shows the suitability of the fuzzy database management system for this kind of applications when the images are characterized by fuzzy data. The synergy of these two introduced components, improves traditional image retrieval systems in three aspects: natural and automatic image description, a natural and easy query language, and high performance in query resolution.

1 Introduction

Nowadays, large image collections offered to non expert users in near to real time environments (i.e. through web sites) are becoming more common. This kind of systems manage a huge amount of data, are concurrently accessed by many users, and require fast query resolution. In our opinion, three key issues are decisive in this kind of systems, natural and automatic image descriptors, a natural and easy query definition mechanism and high performance in query resolution. This paper proposes the usage of a combination of an automatic image description method and a Fuzzy Object-Relational Database Management System (FORDBMS) to solve these issues.

The initial approaches to image retrieval systems were based in textual descriptions of the images. Although it is an useful mechanism, the descriptions must be made by humans. Currently, image retrieval systems are based in automatic extracted image features, such as colour, texture and shape. Usually, these systems uses sample images or sketches to define queries, which can be inappropriate for non expert users.

In this paper we use a novel method for describing images based on its dominant colours [1]. This method takes advantage of fuzzy set theory for dealing with vagueness in colour descriptions. It describes the dominant colours of an image as fuzzy colours, and assign to each one a dominance degree. This fuzzy

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approach lets define query conditions on the basis of linguistic terms, which seems to be more natural for non expert users.

The large amount of data requires a database management system (DBMS) to manage efficiently its storage and retrieval. The DBMS has to deal with the other two key issues: easy query definition and query processing performance.

To ensure high query processing performance, the employed DBMS must implement advanced query processing techniques, such as indexing and query optimization, and high availability, scalability and distribution degrees. Actual market leader DBMSs offer seamlessly all the previously required features. Despite of that, most actual DBMSs implements the relational data model, and some of them including object-relational features, which is not suitable to seamlessly manage fuzzy data required by the kind of image description algorithm considered in this paper.

On the other hand, a great research effort has been spend in making possible to store and manage fuzzy data in databases [2, 3, 4, 5, 6, 7]. Nevertheless, the existing fuzzy DBMSs (FDBMS) are research prototypes. These FDBMS, due to their early stage, are not able to ensure the previously required features for applications for the storage and retrieval of such large image databases.

In order to solve the drawbacks of both, actual DBMSs and the prototypical FDBMSs, we propose a Fuzzy Object-Relational model [8]. This model defines a group of types and operators to store, manage and query fuzzy data, which can be used to extend an Object-Relational DBMS (ORDBMS) to implement a FORDBMS. This extension consist in adding a group of user-defined types and operators to the ORDBMS taking advantage of its extension mechanisms. This approach combines the desired features of actual DBMSs and the ability of fuzzy data storage and handling of prototypical FDBMS. The resulting FORDBMS is very suitable for supporting flexible content based retrieval of images for the discussed type of systems.

This usage of extended datatypes and operators for including fuzzy data management in classical databases, matches with the SQL:1999 standard. This standard compliance lets express a fuzzy query in a fully SQL compliant sentence, which lets implement a SQL compatible fuzzy query definition language.

The rest of the paper is organized as follows. Section 2 introduces the mechanism for describing images based on its dominant colours. Section 3 describes the proposed FDBMS and depicts how it can be used to model the image descriptors. Section 4 describes the proposed operators for defining conditions used in image queries. Section 5 shows some query examples and their results. Finally, Section 6 highlight the concluding remarks and future works.

2 Dominant Fuzzy Colour Descriptor

To achieve a good performance in the retrieval process, efficient methods for describing images are needed. The current retrieval systems face this issue by means of features, such as colour, texture or shape [9]. In this context, dominant colours arise as a powerful tool for describing the representative colours in an image.

In this paper we propose to describe images by means of dominant fuzzy colours. Concretely, we face the colour description in two stages: firstly, a set of crisp dominant colours is extracted (sect. 2.1); then, each colour calculated in the previous stage is used to obtain the set of dominant fuzzy colours (sect. 2.2).

2.1 Dominant Colours

In this section a methodology to extract dominant crisp colours from images is presented using the HSI colour space. In this perceptual space, the hue component (H) represents the colour tone, saturation (S) is the amount of colour and the third component (I) is the amount of light. Let us remark that, although the red-green-blue (RGB) is the most used model to acquire digital images, it is well known that it is not adequate for colour image analysis. Furthermore, the colour components of this space do not have an intuitive interpretation according to the human perception of colour [9].

Dominant Crisp Colour Extraction. In the literature there are many crisp approaches to dominant colour extraction, for example those based on histogram analysis or clustering techniques. In this paper we will perform a clustering approach using the Batchelor&Wilking algorithm [10], where the number of clusters is unknown a priori. This method is initialized with one cluster consisting of all pixels and then an iterative split procedure is performed until a stopping criterion is met. This stopping criterion is based on a parameter $\theta \in [0,1]$ related to the maximum distance to be achieved between points within each cluster (in this paper we have fixed empirically $\theta = 0.3$). To measure the distance between points, the distance between HSI colours proposed in [11] will be used. As a result, we obtain a set of N clusters where the centroid of each cluster, calculated as the mean value, defines a dominant colour. In the following, the set of dominant colours will be noted as DCS, with

$$DCS = \{\mathbf{dc}_1, \mathbf{dc}_2, \ldots, \mathbf{dc}_N\} \ . \tag{1}$$

and $\mathbf{dc}_k = [h_k, s_k, i_k]$ being a dominant crisp colour represented in the HSI colour space.

Degree of Dominance. Intuitively, a colour is dominant to the extent it appears frequently in a given image. It seems natural to model the idea of frequent apparition by means of a fuzzy set over the percentages, i.e. a fuzzy subset of the real interval $[0,1]$. Hence, we define the fuzzy subset *Dominant* of colours as follows:

$$Dom(\mathbf{c}) = \begin{cases} 0 & fr(\mathbf{c}) \leq u_1 \\ \frac{fr(\mathbf{c})-u_1}{u_2-u_1} & u_1 \leq fr(\mathbf{c}) \leq u_2 \\ 1 & fr(\mathbf{c}) \geq u_2 \end{cases} \ . \tag{2}$$

where $fr(c)$ is the percentage of pixels with colour $\mathbf{c}$ in the image under consideration, and u_1 and u_2 are two parameters such that $0 \leq u_1 < u_2 \leq 1$. We have empirically fixed them to be $u_1 = 0.05$ and $u_2 = 0.2$.

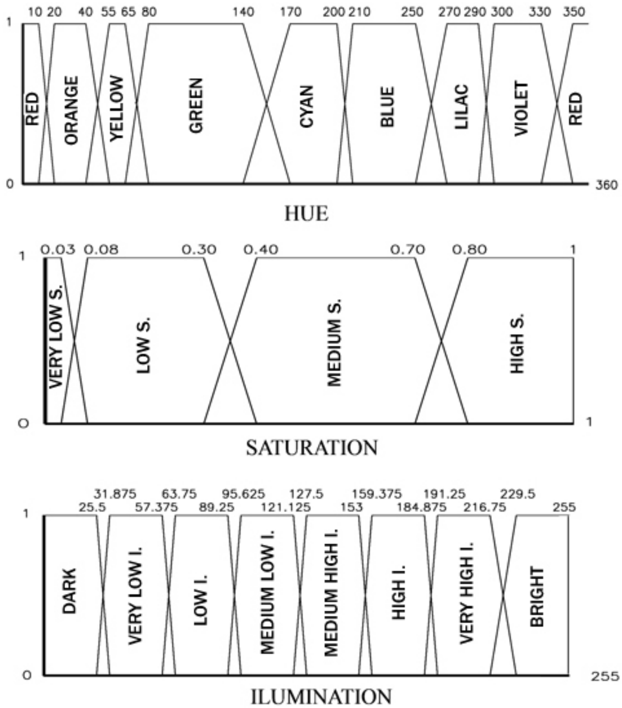

Fig. 1. Fuzzy HSI colour space

2.2 Dominant Fuzzy Colours

In this section, a set of dominant fuzzy colours is obtained taking as starting point the set of dominant crisp colours extracted in the previous one.

For this purpose, the fuzzy HSI colour space presented in [1] will be used. In this proposal, a fuzzy HSI colour $\widetilde{C}$ is defined as a linguistic label whose semantics is represented by a fuzzy subset of $[0, 2\pi] \times [0, 1] \times \{0, \ldots, 255\}$. Then, a fuzzy HSI colour space $\widetilde{HSI}$ is defined as a set of fuzzy HSI colours that define a partition of $[0, 2\pi] \times [0, 1] \times \{0, \ldots, 255\}$.

To define and represent a fuzzy HSI colour space, [1] propose to employ a fuzzy hue space, a fuzzy saturation space and a fuzzy intensity space, consisting of fuzzy hues, fuzzy saturations and fuzzy intensities, respectively (see Fig. 1). Then, a fuzzy HSI colour $\widetilde{C}$ can be defined and represented in practice by a triple $[\widetilde{C}_H, \widetilde{C}_S, \widetilde{C}_I]$, where $\widetilde{C}_H$, $\widetilde{C}_S$, and $\widetilde{C}_I$ are a fuzzy hue, a fuzzy saturation and a fuzzy intensity, respectively (see [1] for more details)

On the basis of the fuzzy HSI colour space defined in [1], we introduce the concept of dominant fuzzy colour in an image as follows:

Definition 1. *A dominant fuzzy colour is a fuzzy HSI colour that appears frequently in the image.*

As in the case of dominant crisp colours, this definition is imprecise in nature, i.e., "dominant" is an imprecise concept defined on the set of fuzzy colours.

Many approaches are possible to calculate how dominant a fuzzy colour is in an image. One possible approach is to calculate the frequency with which each fuzzy colour appears in the image, by using some fuzzy cardinality measure. This will be dealt with in future papers.

One alternative approach we adopt in this paper is to obtain the fuzzy subset of dominant fuzzy colours from the set of crisp dominant colours. We shall consider that a fuzzy colour is dominant to the extent that it matches a dominant crisp colour. This leads to the following definition:

Definition 2. *Let* $DCS = \{\boldsymbol{dc}_1, \ldots, \boldsymbol{dc}_N\}$ *be the set of dominant crisp colours where* $\boldsymbol{dc}_k = [h_k, s_k, i_k]$. *The fuzzy subset of dominant fuzzy colours for an image will be*

$$\widetilde{DCS} = \bigcup_{k \in \{1,\ldots,N\}} \widetilde{DCS}_k \ . \tag{3}$$

where

$$\widetilde{DCS}_k = \sum_{\widetilde{C} \in \widetilde{HSI}} \left(\widetilde{C}(\boldsymbol{dc}_k) \otimes Dom(\boldsymbol{dc}_k) \right) / \widetilde{C} \ . \tag{4}$$

with $\otimes$ *being a t-norm (we use the minimum in this paper) and where* $\widetilde{C}$ *is a fuzzy colour of the fuzzy HSI colour space* $\widetilde{HSI}$, *and the union is performed using the maximum.*

Hence, for each dominant crisp colour $\mathbf{dc}_k$, we obtain the possibility distribution given by equation (3), where the degree of dominance associated to each $\widetilde{C}$ is calculated as the minimum between the membership degree of $\mathbf{dc}_k$ to $\widetilde{C}$ and the dominant degree of $\mathbf{dc}_k$. If a fuzzy colour $\widetilde{C}$ is compatible with several dominant crisp colours, then different degree of dominance will be obtained for $\widetilde{C}$ corresponding to each crisp colour compatible with it; in this case, the maximum of these degrees will be selected as the final degree of dominance of $\widetilde{C}$ as (4) shows.

3 Fuzzy Object-Relational Database Management System

This section briefly introduces our FORDBMS datatypes to show its modeling capabilities. Afterwards, the usage of this capabilities to model image descriptors for an image retrieval system is detailed.

3.1 User-Defined Types for Fuzzy Data

A wide variety of fuzzy data can be handled and represented by our FORDBMS through its user-defined datatypes. The available datatypes for fuzzy data management are the following:

- Atomic fuzzy types (AFT): This type groups subtypes for representing fuzzy data as a possibility distribution defined on crisp domains.
  - Ordered AFT (OAFT): This type handles fuzzy data defined on ordered domains by means of trapezoidal possibility distributions.
  - Non ordered AFT (NOAFT): The type lets define possibility distributions on non ordered finite scalar domains with a similarity relation defined between the domain elements.
- Fuzzy collections (FC): This type groups subtypes for representing fuzzy sets of objects of a particular class. The elements can be fuzzy or crisp data.
  - Conjunctive FC (CFC): The semantics of the set is inclusive, the set represents every element within it.
  - Disjunctive FC (DFC): The semantics of the set is exclusive, the set can only represent one of its elements.
- Fuzzy objects (FO): This type is used to group user-defined datatypes whose attribute types, or some of them, are fuzzy types. Every attribute of the datatypes extending FO datatype is associated with a degree to measure its *importance* in the fuzzy object comparison algorithm.

3.2 Modeling the Flexible Dominant Colour Image Descriptor

The previously introduced method for image description generates a data set representing the dominant colours of an image. In an image retrieval system supported by a DBMS, a user-defined database datatype, for this data set, would ease storage, handling and querying these image descriptors. The datatypes for representing fuzzy data of our FORDBMS are a convenient base to build the image descriptor datatype because the image description data set includes fuzzy data representing flexible colour descriptions. Additionally, the FORDBMS operators for fuzzy data querying let the definition of flexible conditions on the image descriptor.

The datatype *DominantColorSet* models the proposed image description dataset. This datatype is modeled as show in Fig. 2. The classes in this figure follow a colour code where, the datatypes provided by our FORDBMS to represent fuzzy data have a dark grey background, a black background for the abstraction of the basic types of the host ORDBMS on which the FORDBMS is built, and a white background for the datatypes created to built de image descriptor datatype. Let us describe the definition of *DominantColorSet* datatype by a *bottom to top* approach.

The basic components of a fuzzy colour are the linguistics labels representing fuzzy hue, saturation and intensity values. These linguistics labels represents trapezoidal possibility distributions defined on crisp hue, saturation and intensity domains, as shown previously in Fig. 1. As all the underlying domains are ordered, an OAFT derived datatype could be defined for each HSI colour component. Hence, the datatypes *FHue*, *FSaturation* and *FIntensity* are defined for each colour component, representing the fuzzy hue ($\widetilde{H}$), saturation ($\widetilde{S}$) and intensity ($\widetilde{I}$) spaces respectively. On these datatypes the linguistic labels in Fig. 1 are defined, in order to use these linguistic labels as value for the datatypes.

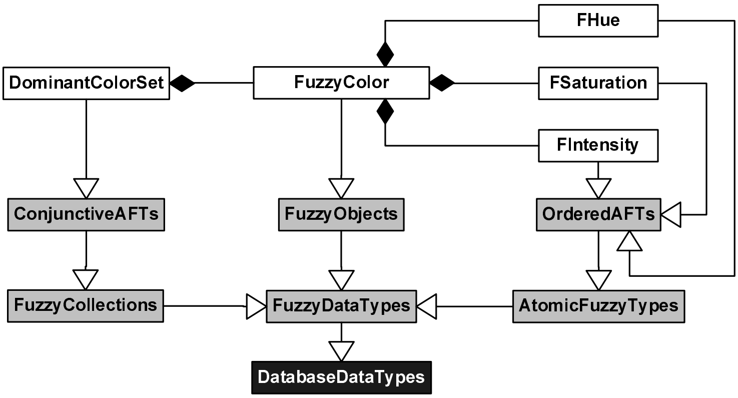

Fig. 2. UML diagram for DominantColorSet datatype

According to the previous definitions, a fuzzy colour is represented as a triple $[\widetilde{H}, \widetilde{S}, \widetilde{I}]$. In fact, a fuzzy colour can be viewed as a group of three linguistic labels, each one representing a value on fuzzy hue, saturation and intensity spaces respectively. In the database, a fuzzy colour is represented by the datatype *FuzzyColor*. This datatype is defined by composing three values of *FHue*, *FSaturation* and *FIntensity*, the same way that a fuzzy colour is defined previously. The *FuzzyColor* datatype is modeled as a FO derived datatype with three attributes of *FHue*, *Fsaturation* and *FIntensity* datatypes respectively.

Finally, the *DominantColorSet* datatype is defined. The data of this datatype represents a fuzzy subset of fuzzy colours $\widetilde{DCS}$. The membership degree of each fuzzy colour in the fuzzy set corresponds to its degree of dominance, which is calculated following (4). As $\widetilde{DCS}$ is a fuzzy set of complex elements, which are also fuzzy, the *DominantColorSet* datatype must be derived from FC. Taking into account the conjunctive semantics of $\widetilde{DCS}$, the *DominantColorSet* datatype is defined as a CFC derivative whose members are of *FuzzyColor* datatype.

4 Fuzzy Operators for Colour Based Image Retrieval

This section describes the most significant operators for the fuzzy datatypes described earlier. These operators will be used later for defining flexible selection conditions in image retrieval queries.

4.1 Fuzzy Inclusion Operator

The fuzzy inclusion operator, `FInclusion(A,B)`, returns the degree of which $A \subseteq B$, where A and B are instances of CFC. This degree is calculated in our approach by a adaptation of the *Resemblance Driven Inclusion Degree* introduced in [12]. This proposal is peculiar because it computes the inclusion degree of two fuzzy sets whose elements are also imprecise.

Definition 3. (Resemblance Driven Inclusion Degree). *Let A and B be two fuzzy sets defined over a finite reference universe $\mathcal{U}$, μ_A and μ_B the membership functions of these fuzzy sets, S the resemblance relation defined over the elements of $\mathcal{U}$, $\otimes$ be a t-norm, and I an implication operator. The inclusion degree of A in B driven by the resemblance relation S is calculated as follows:*

$$\Theta_S(B|A) = \min_{x \in \mathcal{U}} \max_{y \in \mathcal{U}} \theta_{A,B,S}(x,y) \ . \tag{5}$$

where

$$\theta_{A,B,S}(x,y) = \otimes(I(\mu_A(x), \mu_B(y)), \mu_S(x,y)) \ . \tag{6}$$

The resemblance driven inclusion degree does not allow partial inclusion, which means that $Theta_S(B|A) > 0$ even though some of the least important elements of A are not members of B. It can be very interesting for colour based retrieval for decrease the importance of less dominant colours with respect to very dominant colours. For this reason, we propose, inspired by [13], to modify (5) substituting the minimum aggregation by a weighted mean aggregation, whose weight values are the membership degrees in A of the elements of $\mathcal{U}$. This proposal allow partial inclusion, and takes into account the *importance* of each included element in terms of relative membership. The *Modified Resemblance Inclusion Degree* is defined in (7), where $|A| = \sum_{x \in \mathcal{U}} \mu_A(x)$.

$$\Theta_S(B|A) = \sum_{x \in \mathcal{U}} \frac{\mu_A(x)}{|A|} \cdot \max_{y \in \mathcal{U}} \theta_{A,B,S}(x,y) \ . \tag{7}$$

The implementation of `FInclusion(A,B)`, used to calculate the results shown in this paper, takes the minimum as t-norm, and as implication operator the one defined in (8).

$$I(x,y) = \begin{cases} 1 & \text{if } x \leq y \\ y/x & \text{otherwise} \end{cases} \ . \tag{8}$$

4.2 Fuzzy Equality Operator

The fuzzy equality operator, `FEQ(A,B)`, calculates the resemblance degree between two values of a fuzzy datatype. The way this calculus is done depends on the fuzzy datatype of the operands.

Fuzzy Equality Operator for Conjunctive Fuzzy Collections. If `A` and `B` are two instances of a CFC derived datatype, the resemblance degree is calculated according to the *Generalized Resemblance between Fuzzy Sets* proposed in [12]. This method is based on the *Resemblance Driven Inclusion Degree* by applying the concept of double inclusion, shown in (9).

$$A = B \text{ if, and only if, } (A \subseteq B) \wedge (B \subseteq A) \ . \tag{9}$$

Definition 4. *(Generalized resemblance between fuzzy sets). Let A and B be two fuzzy sets defined over a finite reference universe $\mathscr{U}$, over which a resemblance relation S is defined, and $\otimes$ be a t-norm. The generalized resemblance degree between A and B restricted by $\otimes$ is calculated by means of the following formulation:*

$$\sqsupset_{S,\otimes}(A,B) = \otimes(\Theta_S(B|A), \Theta_S(A|B)) \ . \tag{10}$$

The previous definition uses the modifications proposed in (7), therefore this operator aggregates the results of `FInclusion(A,B)` and `FInclusion(B,A)`. In order to increase the flexibility of the equality operator, a modification for (10) consisting in substituting the t-norm by a arithmetic mean aggregation is proposed. This modification makes the fuzzy equality operator to not penalize so much asymmetric inclusions degrees as minimum t-norm does, which leads to get more flexible comparisons.

Fuzzy Equality Operator for Fuzzy Colour Instances. When the operator `FEQ(A,B)` is applied on instances of a class derived from FO, in this paper on instances of the datatype *FuzzyColor*, the resemblance degree of these objects is calculated using the following method.

Definition 5. *(Object Resemblance Degree). Let o_1 and o_2 be two objects of the class C, $obj.a_i$ the value of the i-th attribute of the object obj, n the number of attributes defined in the class C, and FEQ the resemblance operator.*

$$OR(o_1, o_2) = \frac{1}{n} \sum_{i \in \mathcal{A}}^{n} FEQ(o1.a_i, o2.a_i) \ . \tag{11}$$

5 Retrieving Images by Dominant Colour Criteria

The previous set of fuzzy datatypes and operators makes our FORDBMS able to answer queries including conditions defined on the set of dominant colours which describes images in a database. The proposed approach to create a FORDBMS based on a commercial ORDBMS by extending it with user defined datatypes and operators, let to express these queries as SQL, following the latest standard.

A dominant colour based condition is, actually, a fuzzy condition defined on the fuzzy set of fuzzy colours which describes each image in a database. This kind of conditions are based on the usage of the previously defined *fuzzy inclusion operator* or the *fuzzy equality operator for CFC*. Through these operators, a condition requiring that a fuzzy set of fuzzy colours is included, or resembles to, the dominant colour descriptor of each image in a database can be defined.

Each user defined fuzzy colour for a condition can be defined by using the linguistic labels previously defined for each HSI colour component. This way, a query is defined with natural colour descriptors, which means an advantage in contrast to numerical definition. Additionally, this condition definition way makes possible to define different conditions on each HSI colour component, and to omit requirements for a colour component by using the linguistic label *unknown* defined by the membership function $\mu(a) = 1, \forall a \in D(A)$, where $D(A)$ is the underlying domain for the attribute A.

5.1 Query Examples

This section shows some query examples, based on the fuzzy inclusion and equality of fuzzy sets of dominant colours, and its results.

Dominant Colour Inclusion Query. An example of a condition using the fuzzy inclusion operator could be "*Retrieve all the images including the fuzzy colour bright very high saturated red*". This condition is defined using the fuzzy inclusion operator on the image descriptor and the user defined fuzzy set of dominant colours, which in the example includes only the fuzzy colour [*red*, *veryhighsat*,*bright*]. This query is expressed in SQL using the previously defined datatypes and operators as the following sentence:

```
SELECT image,cdeg(1) FROM images WHERE FCond( FInclusion(
  ColorDescriptor,
  DominantColorSet(1.0, FuzzyColor(
        FHue('red'),FSaturation('veryhighsat'),Intensity('bright')
        ) ) ) , 1 ) > 0 ORDER BY 2 DESC;
```

Figure 3 shows the results of the previous query and a more complex example of this type of queries applied on a database of 160 flag images. In this figure the first column shows the fuzzy colours which must be included in the resulting images, and for each fuzzy colour a sample crisp colour which fits it. The second column shows the query results ordered by relevance.

Another example of this kind of condition is the requirement "*Retrieve all the images including bright colours*". In this case, the inclusion operator must ensure to include the fuzzy colour [*unknown*, *unknown*, *bright*]. Note the usage of the label *unknown* to avoid to define the condition for the hue and saturation components. The previously defined query, applied on a database of about 700 colour images, obtains the results show in Fig. 4 which are ordered by relevance.

Dominant Colour Resemblance Query. Another interesting kind of queries is the one which includes a condition to retrieve the set of images of a database

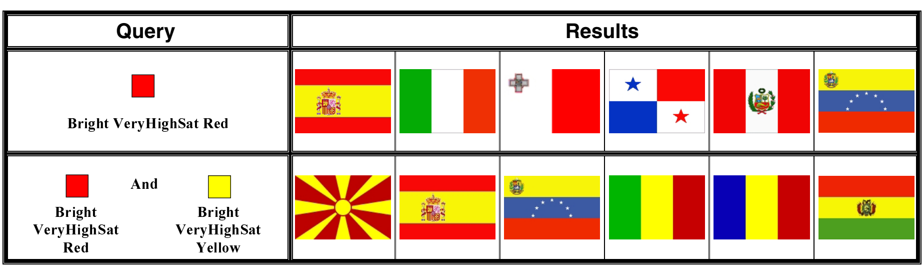

Fig. 3. Colour inclusion query results using all the colour components

Fig. 4. Colour inclusion query results using only the intensity component

with a dominant colour pattern similar to the one associated with a sample image. This kind of conditions are defined by using the *fuzzy equality operator for CFC*, which computes the resemblance degree between the sets of dominant colours of an image in the database and the sample image. A sample query of this kind is:

```
SELECT a.image,cdeg(1) FROM images a, images b WHERE b.id=# AND
FCond( FEQ( a.ColorDescriptor, b.ColorDescriptor ),1 ) > 0
ORDER BY 2 DESC;
```

Several examples of this kind of queries, applied on a database with about 700 colour images, are shown in Fig. 5. In each example, the fist column shows the sample image, and the second column shows the set of images ordered by relevance with a dominant colour pattern similar to the image sample.

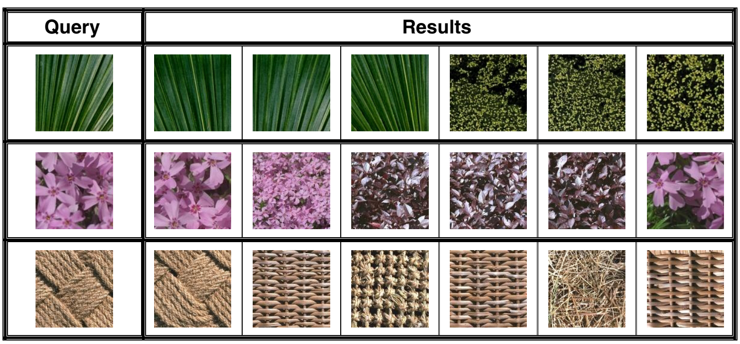

Fig. 5. Image resemblance query results

6 Concluding Remarks and Future Works

This paper has shown the suitability of a FORDBMS for flexible image retrieval systems. The synergy of a fuzzy approach for dominant colour description extraction, and the ability of the proposed FORDBMS to represent and handle complex fuzzy data, results in a powerful system for content based image retrieval.

Future works will focus on new methods for indexing fuzzy data in order to boost query processing performance. These indexing methods will be based on traditional indexing methods for an easy integration in the host ORDBMS.

References

1. J.Chamorro-Martínez and J.M. Medina and C. Barranco and E. Galán-Perales and J.M. Soto-Hidalgo, "An Approach to Image Retrieval on Fuzzy Object-Relational Database using Dominant Color Descriptors", in Proc. of 4th Conference of the European Society for Fuzzy Logic and Technology, pp. 676–684, 2005
2. R. De Caluwe, Fuzzy and Uncertain Object-Oriented Databases: Concepts and Models, Advances in Fuzzy Systems-Applications and Theory. Vol 13. World Scientific, 1997.
3. S. Fukami, M. Umano, M. Muzimoto, H. Tanaka, "Fuzzy Database Retrieval and Manipulation Language", IEICE Technical Reports, Vol. 78, N. 233, pp. 65–72, AL-78–85 (Automata and Language) 1979.
4. J. Galindo, J.M. Medina, O.Pons, J.C. Cubero, "A Server for Fuzzy SQL Queries", Flexible Query Answering Systems, eds. T. Andreasen, H. Christiansen and H.L. Larsen, Lecture Notes in Artificial Intelligence (LNAI) 1495, pp. 164–174. Ed. Springer, 1998.
5. H. Prade, C. Testemale, "Generalizing Database Relational Algebra for the Treatment of Incomplete or Uncertain Information and Vague Queries", Information Sciences Vol. 34, 1984, pp. 115–143.
6. M. Umano, "Freedom-O: A Fuzzy Database System", Fuzzy Information and Decision Processes. Gupta-Sanchez edit. North-Holand Pub. Comp. 1982.
7. M. Zemankova-Leech, A. Kandel, "Fuzzy Relational Databases – A Key to Expert Systems", Köln, Germany, TÜV Rheinland, 1984.
8. J.C. Cubero, N. Marín, J.M. Medina, O. Pons, M.A. Vila, "Fuzzy object Management in an Object-Relational Framework", X Intl. Conf. of information processing and management of uncertainty in knowledge-based systems, pp.1767–1774. 2004. Perugia (Italy).
9. A. Del Bimbo, "Visual Information Retrieval", Morgan Kaufmann Publishers, 2001
10. A.K. Jain and R.C. Dubes, "Algorithms for Clustering Data", Prentice Hall, 1988
11. J. Chamorro-Martínez, D. Sánchez, B. Prados-Suarez and E. Galán-Perales, "Fuzzy Connectivity Measures for Path-based Image Segmentation", in Proc. of IEEE International Conference on Fuzzy Systems, pp. 218–223, 2005
12. Marín, M., Medina, J.M., Pons, O., Sánchez, D. and Vila, M.A. Complex object comparison in a fuzzy context. Inf. and Software Technology, 45(7):431–444. 2003
13. J. A. Goguen, "The logic of inexact concepts", Synthese, 9(3–4), 1969, pp. 325–373.