Multi-Mode Precoding for MIMO Wireless Systems

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***Abstract*— Multiple-input multiple-output (MIMO) wireless systems obtain large diversity and capacity gains by employing multi-element antenna arrays at both the transmitter and receiver. The theoretical performance benefits of MIMO systems, however, are irrelevant unless low error rate, spectrally efficient signaling techniques are found. This paper proposes a new method for designing high data-rate spatial signals with low error rates. The basic idea is to use transmitter channel information to adaptively vary the transmission scheme for a fixed data-rate. This adaptation is done by varying the number of substreams and the rate of each substream in a precoded spatial multiplexing system. We show that these substreams can be designed to obtain full diversity and full rate gain using feedback from the receiver to transmitter. We model the feedback using a limited feedback scenario where only finite sets, or codebooks, of possible precoding configurations are known to both the transmitter and receiver. Monte Carlo simulations show substantial performance gains over beamforming and spatial multiplexing.**

Index Terms-Diversity methods, MIMO systems, Quantized precoding, Rayleigh channels, Spatial multiplexing.

I. Introduction

Wireless systems employing multi-element antenna arrays at both the transmitter and receiver, known as multiple-input multiple-output (MIMO) systems, promise large gains in capacity and quality compared with single antenna links [1]. Spatial multiplexing is a simple MIMO signaling approach that achieves large spectral efficiencies with only moderate transmitter complexity. Receivers for spatial multiplexing range from the high complexity, low error rate maximum likelihood decoding to the low complexity, moderate error rate linear receivers [2], [3]. Unfortunately, rank deficiency of the matrix channel can cause all spatial multiplexing receivers to suffer increases in the probability of error [2]–[4].

Linear transmit precoding, where the transmitted data vector is premultiplied by a precoding matrix that is adapted to some form of channel information, adds resiliency against channel ill-conditioning. Linear precoded spatial multiplexing has been proposed for transmitter's with full channel state information (CSI) [5]–[7], channel first-order statistics [8]–[10], channel second order statistics [11]–[15], partial subspace knowledge [16], or limited feedback from the receiver (which includes antenna subset selection) [4], [17]–[26]. Optimization techniques for choosing the precoding matrix include minimizing the mean squared error (MSE) [5], [6], maximizing the receive minimum distance (with receive distance defined as the two-norm of the difference between two unique symbol vectors after multiplication by the channel) [7], maximizing the minimum singular value [22], and maximizing the mutual information assuming Gaussian signaling [4], [17], [20], [26]. These precoding methods provide probability of error improvements compared with unprecoded spatial multiplexing [2], but *linear precoding does not guarantee full diversity order and full rate growth*. This follows from the fact that linear precoders usually limit the number of substreams transmitted in order to obtain improved error rate performance [2].

The full rate and full diversity problem was first discussed in space-time code design in [27], [28]. These open-loop (i.e. no channel knowledge at the transmitter) space-time codes use spatial and temporal redundancy combined with sphere decoding at the receiver. Unfortunately, these codes are difficult to decode even using the low-complexity sphere decoder. Given these results, the natural question is whether full rate and full diversity gain transmission can be obtained in a more practical solution at the expense of feedback from the receiver to transmitter.

This problem was also addressed for the special case of antenna subset selection precoding in [29], [30]. These papers studied systems where the size of the antenna subset, along with the spatial multiplexing constellation, could be varied in order to guarantee full diversity performance for a fixed data-rate. Various selection criteria were proposed for both dual-mode (i.e. selecting between spatial multiplexing and selection diversity) and multi-mode antenna selection. These methods provided substantial performance improvements compared with traditional spatial multiplexing. Antenna subset selection, however, is quite limited because the precoding matrices are restricted to columns of the identity matrix.

In this paper, we propose a modified version of linear precoding called *multi-mode precoding*. Multi-mode precoding varies the number of substreams contained in the precoded spatial multiplexing vector assuming that the *transmitted data-rate is independent of the number of substreams chosen for transmission*. This allows the transmitter to adapt the signal using a combination of linear precoding and adaptive modulation. This substream selection has been studied independently in related work [31], [32]. We present methods for choosing the multi-mode precoder based on minimizing the probability of error and maximizing the mutual information assuming independent and identically distributed (i.i.d.) Gaussian signaling. Because it is often impractical to assume perfect channel information at the transmitter, we present a limited feedback multi-mode precoder using limited feedback approaches developed in [23]–[25], [33], [34]. Limited feedback multi-mode precoders use precoder codebooks, finite sets

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of precoder matrices, for each of the supported substream numbers. These codebooks are designed offline and stored at both the transmitter and receiver. The chosen multi-mode precoder is then conveyed from the receiver to transmitter over a limited capacity feedback channel using a small number of bits.

Multi-mode precoding represents a new approach to diversity-multiplexing tradeoff in MIMO wireless systems. Previous work in diversity-multiplexing tradeoff, see for example [27], [28], [35]–[37], emphasized that every spatio-temporal signaling method has an accompanying diversity-multiplexing gain tradeoff curve. Diversity systems, such as orthogonal space-time block codes or transmit beamforming, maximize the diversity gain for a fixed rate, while spatial multiplexing maximizes the multiplexing gain. Our system actually "adapts" the transmission scheme between different diversity-multiplexing curves by varying the number of substreams for transmission. This allows multi-mode precoding to maximize the multiplexing gain if the rate is allowed to vary with SNR and maximize the diversity gain if the rate is held constant. This adaptation is in the spirit of that studied in [36] but is more general because we are not restricted to only two transmission types (such as Alamouti coding and spatial multiplexing as in [36]). Unlike [36], we do not switch between space-time coding and spatial multiplexing but rather vary the number of spatial multiplexing substreams and the precoder.

Limited feedback multi-mode precoding is also one solution to the important problem of covariance optimization for transmitters without any form of CSI besides feedback. This problem has previously been studied in [38]–[41]. Unlike [38], we are concerned with maximizing the average mutual information rather than obtaining a better quantized estimate of the optimal waterfilling covariance matrix. Our approach also uses fixed codebooks known to both the transmitter and receiver as opposed to the random vector quantization (RVQ) technique described in [39], [40]. Most importantly, we do not require that the codebook be redesigned when the SNR changes as is assumed in [38], [41]. We give a technique that can generate codebooks off-line. These codebooks can then be used regardless of the operating SNR. Practically, this is a large savings compared to the algorithms in [38], [41] that require vector quantization algorithms that generate thousands of channel realizations and then perform iterative optimizations to be run given an SNR value.

This paper is organized as follows. Section II introduces the multi-mode precoded spatial multiplexing system model. Criteria for choosing the optimal matrix from the codebook are presented in Section III. Multi-mode precoding using limited feedback from the receiver to the transmitter is considered in Section IV. The relationship between limited feedback multi-mode precoding and covariance quantization is explored in Section V. Section VI illustrates the performance improvements over previously proposed techniques using Monte Carlo simulations of the symbol error rate and mutual information. Conclusions are presented in Section VII.

II. System Overview

The M_t transmit and M_r (with $M_t \leq M_r$) receive antenna MIMO wireless system studied in this paper is shown in Fig. 1. For each channel use, R bits are demultiplexed into M different bit streams. Each bit stream is modulated using the same constellation $\mathcal{S}$, producing a vector $\mathbf{s}_k$ at the k^{th} channel use. This means that *each substream carries R/M bits of information.* The spatial multiplexing symbol vector[1] $\mathbf{s}_k = [s_{k,1}\ s_{k,2}\ \ldots\ s_{k,M}]^T$ is assumed to have power constraints so that $E_{\mathbf{s}_k}[\mathbf{s}_k\mathbf{s}_k^*] = \frac{\mathcal{E}_s}{M}\mathbf{I}_M$. Note that this means the *average of the total transmitted power* at any channel use is *independent of the number of substreams M*.

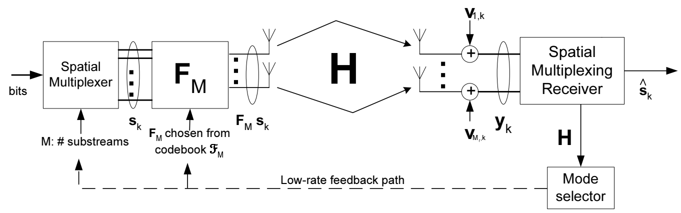

Fig. 1. Block diagram of a limited feedback precoding MIMO system.

An $M_t \times M$ linear precoding matrix $\mathbf{F}_M$ maps $\mathbf{s}_k$ to an M_t-dimensional spatial signal that is transmitted on M_t transmit antennas. The transmitted signal vector encounters an $M_r \times M_t$ matrix channel $\mathbf{H}$ before being added with an M_r-dimensional white Gaussian noise vector $\mathbf{v}_k$. Assuming perfect pulse-shaping, sampling, and timing, this formulation yields an input-output relationship

$$\mathbf{y} = \mathbf{H}\mathbf{F}_M\mathbf{s} + \mathbf{v} \tag{1}$$

where the channel use index k has been suppressed because we are interested in vector-by-vector detection of $\mathbf{s}_k$. We assume that $\mathbf{H}$ has i.i.d. entries with each distributed according to $\mathcal{CN}(0,1)$. We employ a block-fading model where the channel is constant over multiple frames before independently taking a new realization. As well, the noise vector $\mathbf{v}$ is assumed to have i.i.d. entries distributed according to $\mathcal{CN}(0, N_0)$.

We assume that the receiver has perfect knowledge of $\mathbf{H}$ and $\mathbf{F}_M$. The matrix $\mathbf{H}\mathbf{F}_M$ can be thought of as an *effective channel*, and the receiver decodes $\mathbf{y}$ using this effective channel and a spatial multiplexing decoder. Spatial multiplexing decoders include ML, sub-optimal ML, linear, and V-BLAST receivers. An ML receiver decodes to an estimated signal vector $\widehat{\mathbf{s}}$ using

$$\widehat{\mathbf{s}} = \underset{\mathbf{s}' \in \mathcal{S}^M}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{F}_M\mathbf{s}'\|_2^2. \tag{2}$$

Sub-optimal ML receivers, such as the sphere decoder [42], [43], solve the optimization in (2) by performing reduced complexity ML decoding over a reduced set of candidate vectors.

[1]We use $\mathcal{CN}(0,\sigma^2)$ to denote the complex Gaussian distribution with independent real and imaginary parts distributed according to $\mathcal{N}(0,\sigma^2/2)$, * for conjugate, ^{T} for transpose, * for conjugate transpose, $^+$ for the matrix pseudo-inverse, $\mathbf{I}_M$ for the $M \times M$ identity matrix, $\log_2$ is the base two logarithm, ln is the natural logarithm, $\lambda_j\{\mathbf{H}\}$ to denote the j^{th} largest singular value of a matrix $\mathbf{H}$, $tr(\ )$ for the trace operator that gives the sum of the diagonal elements of a matrix, $\|\cdot\|_2$ for the matrix two-norm, $\|\cdot\|_F$ to denote the matrix Frobenius norm, $|a|$ to denote the absolute value of a complex number a, $|\mathcal{M}|$ to denote the cardinality of a set $\mathcal{M}$, $\mathcal{A}\backslash\mathcal{B}$ denotes the elements in the set $\mathcal{A}$ that are not in $\mathcal{B}$, and $E_s[\cdot]$ to denote expectation with respect to random variable s.

A linear receiver applies an $M \times M_r$ matrix transformation $\mathbf{G}$ to $\mathbf{y}$ and then independently detects each entry of $\mathbf{Gy}$. If a zero-forcing (ZF) linear receiver is used, $\mathbf{G} = (\mathbf{HF}_M)^+$. Minimum mean squared error (MMSE) decoding uses $\mathbf{G} = [\mathbf{F}_M^*\mathbf{H}^*\mathbf{HF}_M + (N_0 M/\mathcal{E}_s)\mathbf{I}_M]^{-1}\mathbf{F}_M^*\mathbf{H}^*$. V-BLAST decoding uses successive cancellation with ordering using ZF or MMSE receivers. An overview of the V-BLAST algorithm can be found in [44].

Note that the total instantaneous transmitted power for this system is given by $\mathbf{s}^*\mathbf{F}_M^*\mathbf{F}_M\mathbf{s}$. The precoder matrix $\mathbf{F}_M$ must therefore be constrained in order to limit the transmitted power. We will restrict $\lambda_1^2\{\mathbf{F}_M\} \le 1$ (i.e. the largest squared singular value of $\mathbf{F}_M$) in order to limit the peak-to-average ratio. This means that $E_{\mathbf{s}}[\mathbf{s}^*\mathbf{F}_M^*\mathbf{F}_M\mathbf{s}] \le \mathcal{E}_s$ regardless of the modulation scheme or the value of M. It was shown in [5], [24] that matrices that optimize MSE, capacity, the minimum effective channel singular value (i.e. maximize $\lambda_{min}^2\{\mathbf{HF}_M\}$), and total effective channel power (i.e. maximize $\|\mathbf{HF}_M\|_F$) over the set of all matrices

$$\mathcal{L}(M_t, M) = \{\mathbf{L} \in \mathbb{C}^{M_t, M} \;\; | \;\; \lambda_1^2\{\mathbf{L}\} \le 1\} \tag{3}$$

are all members of the set

$$\mathcal{U}(M_t, M) = \{\mathbf{U} \in \mathbb{C}^{M_t, M} \;\; | \;\; \mathbf{U}^*\mathbf{U} = \mathbf{I}_M\}.$$

For this reason, we will further restrict that $\mathbf{F}_M \in \mathcal{U}(M_t, M)$ for any chosen value of M. Intuitively, this means that the precoding attempts to diagonalize the channel without any form of power pouring among the spatial parallel channels.

The *key difference* between multi-mode precoding and previously proposed linear precoders is that M *is adapted using current channel conditions* by allowing M to vary between 1 and M_t. We refer to the value of M as the *mode* of the precoder. Usually, only a subset of the M_t possible modes can be chosen. For example, a necessary condition that motivates using only a subset of $\{1, \dots, M_t\}$ is that R/M is an integer for only a few of the modes between 1 and M_t. As well, it might be practical from an implementation complexity point-of-view to support only a small subset of modes if M_t is large. We will denote the set of supported mode values as $\mathcal{M}$. For example, if $R = 8$ bits and $M_t = 4$ then only modes in the set $\mathcal{M} = \{1, 2, 4\}$ can be supported. Another example is *dual-mode precoding* where $\mathcal{M} = \{1,\ M_t\}$.

We assume that a selection function $g : \mathbb{C}^{M_r \times M_t} \to \mathcal{M}$ picks the "best" mode according to some criterion, and we set $M = g(\mathbf{H})$. After M is chosen, the precoder $\mathbf{F}_M$ is selected from a set $\mathcal{F}_M \subseteq \mathcal{U}(M_t, M)$ using a selection function $f_M : \mathbb{C}^{M_r \times M_t} \to \mathcal{F}_M$. Therefore, $\mathbf{F}_M = f_M(\mathbf{H})$. This means that there are $|\mathcal{M}|$ different precoder selection functions.

We assume that the transmitter has no prior knowledge of the matrix $\mathbf{H}$ and that $\mathbf{F}_M$ is designed using data sent from the receiver over a limited capacity feedback channel. This assumption of zero prior channel knowledge approximates a frequency division duplexing (FDD) system where the forward and reverse frequency bands are separated by a frequency bandwidth much larger than the channel coherence bandwidth.

III. Multi-Mode Precoder Selection

The selection of the mode and precoder matrix will determine the performance of the entire system. Because we are interested in constructing a high-rate signaling scheme with low error rates, we will present bounds on the probability of vector symbol error (i.e. the probability that the receiver returns at least one symbol in error). We will also review the capacity results for MIMO systems both with and without transmitter CSI.

A. Performance Discussion

The selection criterion used to choose M and $\mathbf{F}_M$ must tie directly to the resulting performance of the precoded spatial multiplexing system. We will address selection details based on two different performance measures: probability of error and capacity.

1) Probability of Error: The probability of error analysis will be broken down into the three different receiver categories: ML, linear, and V-BLAST. Note that the ML bounds can also be employed for the popular sphere decoder [42], [43] which provides ML performance using a low-complexity search. In order to provide a fair tradeoff, we will use the probability of vector symbol error as the probability of error metric. This choice was made to provide a fair comparison between modes that transmit symbol vectors of different dimensionality.

Maximum Likelihood Receiver

ML performance is commonly characterized as a function of the receive minimum distance defined as

$$d_{min,rec}^2 = \min_{\mathbf{s}', \mathbf{s}'' \in \mathcal{S}^M : \mathbf{s}' \neq \mathbf{s}''} \|\mathbf{HF}_M(\mathbf{s}' - \mathbf{s}'')\|_2^2 . \tag{4}$$

Using the receive minimum distance, the probability of vector symbol error can be bounded using the nearest neighbor union bound (NNUB) as

$$P_e(\mathbf{H}, \mathcal{E}_s/N_0) \le M N_e(M, R) Q\left(\sqrt{\frac{\mathcal{E}_s}{M N_0} \frac{d_{min,rec}^2}{2}}\right) \tag{5}$$

where $N_e(M, R)$ is a nearest neighbor scale factor for the constellation $\mathcal{S}$. Thus to minimize this bound, the receive minimum distance must be maximized. The minimum receive distance, however, can be bounded as

$$\begin{aligned} d_{min,rec}^2 &\ge \left(\min_{\mathbf{u} \in \mathcal{U}(M,1)} \|\mathbf{HF}_M \mathbf{u}\|_2^2\right) d_{min}^2(M, R) \\ &= \lambda_M^2\{\mathbf{HF}_M\} d_{min}^2(M, R) \end{aligned} \tag{6}$$

where $d_{min}(M, R) = \min_{s' \neq s''} |s' - s''|^2$ denotes the minimum distance for the constellation $\mathcal{S}$ used when R/M bits are modulated per substream.

Linear Receiver

It was shown in [22] that the effective SNR of the k^{th} substream after linear processing is given by

$$SNR_k^{(ZF)}(\mathbf{F}_M) = \frac{\mathcal{E}_s}{M N_0 [\mathbf{F}_M^* \mathbf{H}^* \mathbf{HF}_M]_{k,k}^{-1}} \tag{7}$$

for ZF decoding and

$$SNR_k^{(MMSE)}(\mathbf{F}_M) = \frac{\mathcal{E}_s}{MN_0[\mathbf{F}_M^*\mathbf{H}^*\mathbf{H}\mathbf{F}_M + \frac{MN_0}{\mathcal{E}_s}\mathbf{I}_M]_{k,k}^{-1}} - 1 \quad (8)$$

for MMSE decoding where $\mathbf{A}_{k,k}^{-1}$ is entry (k,k) of $\mathbf{A}^{-1}$. The minimum substream SNR, given by

$$SNR_{min}(\mathbf{F}_M) = \min_{1\leq k\leq M} SNR_k(\mathbf{F}_M), \quad (9)$$

is an important parameter that will be used to characterize performance. For ZF and MMSE decoding, $SNR_{min}(\mathbf{F}_M)$ can be bounded by [22]

$$SNR_{min}(\mathbf{F}_M) \geq \lambda_M^2\{\mathbf{H}\mathbf{F}_M\}\frac{\mathcal{E}_s}{MN_0}. \quad (10)$$

Therefore, *the minimum singular value of the effective channel is often an important parameter in linear precoded MIMO systems.*

We are more interested, however, in tight bounds on the probability of vector symbol error. Given a matrix channel $\mathbf{H}$, the conditional probability of vector symbol error can be bounded by the NNUB as

$$\begin{aligned} P_e(\mathbf{H}, \mathcal{E}_s/N_0) &\leq MN_e(M,R)Q\left(\sqrt{SNR_{min}\frac{d_{min}^2(M,R)}{2}}\right) \\ &\leq MN_e(M,R) \cdot \\ &\quad Q\left(\sqrt{\lambda_M^2\{\mathbf{H}\mathbf{F}_M\}\frac{\mathcal{E}_s}{MN_0}\frac{d_{min}^2(M,R)}{2}}\right). \end{aligned} \quad (11)$$

where SNR_{min} is computed for the given linear receiver.

V-BLAST Receiver

Similarly to linear receivers, the V-BLAST receiver probability of error can be bounded using the SNR of the weakest substream after cancellation and before detection. Combining the NNUB with the results in [32], this gives

$$\begin{aligned} P_e(\mathbf{H}, \mathcal{E}_s/N_0) &\leq MN_e(M,R) \cdot \\ &\quad Q\left(\sqrt{\lambda_M^2\{\mathbf{H}\mathbf{F}_M\}\frac{\mathcal{E}_s}{MN_0}\frac{d_{min}^2(M,R)}{2}}\right). \end{aligned} \quad (12)$$

Once again, the minimum singular value of the effective channel $\mathbf{H}\mathbf{F}_M$ plays an important role in system performance.

Relation to Unitary Precoding

It must be noted that each receiver uses the same performance bound that relates back to the minimum singular value of the precoded channel matrix $\lambda_M^2\{\mathbf{H}\mathbf{F}_M\}$. Under the assumption of maximum singular value constrained precoders (i.e. $\lambda_1\{\mathbf{F}_M\} \leq 1$), unitary precoding is optimal with respect to minimizing the probability of error bound.

2) Capacity: The mutual information of the channel $\mathbf{H}\mathbf{F}_M$ assuming uncorrelated Gaussian signaling on each substream, denoted C_{UT}, is known to be

$$C_{UT}(\mathbf{F}_M) = \log_2 det\left(\mathbf{I}_M + \frac{\mathcal{E}_s}{MN_0}\mathbf{F}_M^*\mathbf{H}^*\mathbf{H}\mathbf{F}_M\right). \quad (13)$$

The notation C_{UT} is used because this is commonly called the uninformed transmitter (UT) capacity (i.e. no transmitter CSI) [2]. Note that this is not really a "capacity" expression in the sense of distribution maximization because we assume a fixed distribution [45]. We will, however, refer to (13) as the capacity of the effective channel $\mathbf{H}\mathbf{F}_M$ when there is no form of CSI at the transmitter in order to follow existing terminology in the MIMO literature. When transmitter and receiver both have perfect knowledge of $\mathbf{H}$ and $\mathbf{F}_M$, the capacity is found by waterfilling [1]–[3].

B. Selection Criteria

We will present probability of error and capacity selection. The probability of error selection is based on the previous work in [22], [29], [30] while the capacity selection is similar to work presented in [17], [22].

Probability of Error Selection

Assuming a probability of error selection criterion, the optimal selection criterion would obviously be to choose the mode and precoder that provide the lowest probability of vector symbol error. Selection using this criterion, however, is unrealistic because closed-form expressions for the probability of vector symbol error conditioned on a channel realization are not available to the authors' knowledge. The NNUB can be successfully employed in place of this bound for asymptotically tight selection. Using the NNUB results in Section III-A, the following selection criterion is obtained.

Probability of Error Selection Criterion: Choose M and $\mathbf{F}_M$ such that

$$g^{(SV)}(\mathbf{H}) = \underset{m\in\mathcal{M}}{\operatorname{argmax}} \frac{\lambda_m^2\{\mathbf{H}f_m^{(SV)}(\mathbf{H})\}}{m} d_{min}^2(m,R) \quad (14)$$

$$f_M^{(SV)}(\mathbf{H}) = \underset{\mathbf{F}'\in\mathcal{F}_M}{\operatorname{argmax}} \lambda_M^2\{\mathbf{H}\mathbf{F}'\}. \quad (15)$$

The function (15) corresponds to finding the optimal precoding matrix from a limited feedback codebook $\mathcal{F}_M$ conditioned on a specific mode number M. This optimization, thus, corresponds to a fixed mode limited feedback codebook matrix selection. The mode number is determined in (14) using the optimal codebook precoder matrix for each mode. Thus the receiver would send both the chosen mode number (i.e. $g^{(SV)}(\mathbf{H})$) and the optimal precoder given that mode (i.e. $f_M^{(SV)}(\mathbf{H})$) back to the transmitter.

This criterion is computed by first searching for the precoder (denoted by $f_m^{(SV)}(\mathbf{H})$) in each mode's codebook (i.e. $\mathcal{F}_m$) that maximizes the effective channel minimum squared singular value. The optimal mode is then determined by computing the receive minimum distance bound $\lambda^2\{\mathbf{H}f_m^{(SV)}(\mathbf{H})\}d_{min}^2(m,R)/m$ for each mode in $\mathcal{M}$ and returning the mode with the largest bound. These optimizations correspond to multiple brute force searches.

Capacity Selection

While capacity selection is not optimal from a probability of error point-of-view, it can provide insight into the attainable spectral efficiencies given the multi-mode precoding system model when an ML or V-BLAST receiver is used. Because the uninformed transmitter capacity is evaluated in closed-form given a matrix channel, the following criterion can be succinctly stated.

Capacity Selection Criterion: Choose M and $\mathbf{F}_M$ such that

$$g^{(Cap)}(\mathbf{H}) = \underset{m\in\mathcal{M}}{\operatorname{argmax}}\, C_{UT}(f_m^{(Cap)}(\mathbf{H}))$$
$$f_M^{(Cap)}(\mathbf{H}) = \underset{\mathbf{F}'\in\mathcal{F}_M}{\operatorname{argmax}}\, C_{UT}(\mathbf{F}'). \tag{16}$$

IV. Limited Feedback Multi-Mode Precoding

We now consider the implementation of multi-mode precoding when the transmitter has *no form of channel knowledge* besides feedback. This design makes multi-mode precoding practical even in systems that do not meet the assumption of full transmitter CSI.

A. Codebook Model

The design of an adaptive modulator in a system without transmitter CSI is daunting because we must find a simple method that allows the transmitter to adapt to *current* channel conditions. We will overcome the lack of transmitter CSI by using a *low-rate feedback channel* that can carry a limited number of information bits, denoted by B, from the receiver to the transmitter.

In this limited feedback scenario, the precoder $\mathbf{F}_M$ is chosen from a finite set, or *codebook*, of N_M different $M_t \times M$ precoder matrices $\mathcal{F}_M = \{\mathbf{F}_{M,1}, \mathbf{F}_{M,2}, \ldots, \mathbf{F}_{M,N_M}\}$. *Thus, we assume that there is a codebook for each supported mode value.* Because there are a total of

$$N_{total} = \sum_{m\in\mathcal{M}} N_m$$

codeword matrices, a total of

$$B = \lceil \log_2(N_{total}) \rceil = \left\lceil \log_2\left(\sum_{m\in\mathcal{M}} N_m\right)\right\rceil$$

bits is required for feedback. Feedback can thus be kept to a reasonable amount by varying the size of N_M for each mode.

There are two main problems associated with this codebook-based limited feedback system. First, we must determine how to distribute the N_{total} codewords among the modes in $\mathcal{M}$. The second problem is how to design $\mathbf{F}_M$ given M and N_M. We present solutions for both of these problems in Sections IV-B and IV-C.

B. Diversity-Multiplexing Codeword Distribution

The feedback amount B is often specified offline by general system design constraints. For example, only B bits of control overhead might be available in the reverselink frames. For this reason, we will assume that B *is a fixed system parameter.* Thus, we wish to understand how to distribute $N_{tot} = 2^B$ codeword matrices among the $|\mathcal{M}|$ modes.

In most standardized wireless scenarios, the system can support a wide range of different SNR values. Because we do not want to redesign the codebooks each time the SNR changes, we will take a different approach than [38], [41] where the codebook is explicitly a function of the SNR. Our approach will be based on maximizing the asymptotic performance measures of diversity gain and multiplexing gain.

Diversity gain is a fundamental performance parameter in MIMO communications that has been given a variety of definitions. For probability of error selection, we will define diversity as the negative of the probability of error curve's asymptotic slope. Following [26], [46], [47], we define diversity gain for capacity selection as the negative of the probability of outage curve's asymptotic slope. Mathematically, a MIMO wireless system is said to have diversity gain d [35] if

$$d = \begin{cases} -\lim_{\mathcal{E}_s/N_0\to\infty} \frac{\log(P_e(\mathcal{E}_s/N_0))}{\log(\mathcal{E}_s/N_0)}, & \text{if probability of error selection;} \\ -\lim_{\mathcal{E}_s/N_0\to\infty} \frac{\log(P_{out}(R(\mathcal{E}_s/N_0)))}{\log(\mathcal{E}_s/N_0)}, & \text{if capacity selection.} \end{cases}$$

The diversity gain is always bounded above by the product of the number of transmit and receive antennas, $M_t M_r$. Diversity gain, or diversity order, is one of the fundamental parameters for MIMO systems. Therefore, it will be essential that we maximize the diversity gain using as few bits of feedback as possible.

As in [35], let $\doteq$ denote exponential equality. This means that $f(\mathcal{E}_s/N_0) \doteq (\mathcal{E}_s/N_0)^d$ if

$$\lim_{\mathcal{E}_s/N_0\to\infty} \frac{\log f(\mathcal{E}_s/N_0)}{\log(\mathcal{E}_s/N_0)} = d. \tag{17}$$

The following lemma addresses the conditions sufficient to obtain full diversity order.

Lemma 1.A *If $N_1 \geq M_t$ and the rate is fixed, multi-mode precoding provides full diversity order.*

Proof: Each selection criterion will be treated separately.

Probability of Error Selection

Combining the probability of error results in Section III-A reveals that the probability of vector symbol error with an ML, linear, or V-BLAST receiver can be bounded as

$$\begin{aligned} P_e(\mathcal{E}_s/N_0) &= E_{\mathbf{H}}\left[P_e(\mathbf{H}, \mathcal{E}_s/N_0)\right] \\ &\leq E_{\mathbf{H}}\Bigg[M N_e(M,R) \cdot \\ &\qquad Q\left(\sqrt{\lambda_M^2\{\mathbf{H}\mathbf{F}_M\}\frac{\mathcal{E}_s}{MN_0}\frac{d_{min}^2(M,R)}{2}}\right)\Bigg]. \end{aligned} \tag{18}$$

Bounding this by the transmit diversity case gives

$$\begin{aligned} &P_e(\mathcal{E}_s/N_0) \\ &\leq E_{\mathbf{H}}\left[N_e(1,R) Q\left(\sqrt{\max_{\mathbf{f}\in\mathcal{F}_1}\lambda_1^2\{\mathbf{H}\mathbf{f}\}\frac{\mathcal{E}_s}{N_0}\frac{d_{min}^2(1,R)}{2}}\right)\right]. \\ &\leq E_{\mathbf{H}}\Bigg[N_e(1,R) \cdot \\ &\qquad Q\left(\sqrt{\left(\max_{k,l}|h_{k,l}|_2^2\right)\frac{\mathcal{E}_s}{N_0}\frac{\lambda_{M_t}^2\{\mathbf{W}\}d_{min}^2(1,R)}{2M_rM_t}}\right)\Bigg] \end{aligned} \tag{19}$$

where $\mathbf{W} = [\mathbf{f}_1\ \mathbf{f}_2\ \cdots\ \mathbf{f}_{M_t}]$ is constructed by taking M_t vectors from $\mathcal{F}_1$. The upper-bound in (19) rolls off with full diversity if the vectors in $\mathcal{F}_1$ span $\mathbb{C}^{M_t}$. This condition was shown in [48] to be satisfied when $N_1 \geq M_t$ and $\mathcal{F}_1$ is a non-trivial codebook design.

Capacity Selection

We need to show that

$$P_{out}(R) = Pr\left(\max_{\mathcal{M}} \max_{\mathbf{F}\in\mathcal{F}_M} C_{UT}(\mathbf{F}) < R\right) \tag{20}$$

rolls off with full diversity. Note that

$$P_{out}(R) \leq Pr\left(\max_{\mathbf{f}\in\mathcal{F}_1} \log_2\left(1 + \frac{\mathcal{E}_s}{N_0}\|\mathbf{H}\mathbf{f}\|_2^2\right) \leq R\right). \tag{21}$$

Rearranging gives

$$P_{out}(R) \leq Pr\left(\max_{\mathbf{f}\in\mathcal{F}_1} \frac{\mathcal{E}_s}{N_0}\|\mathbf{H}\mathbf{f}\|_2^2 \leq \left(2^R - 1\right)\right). \tag{22}$$

This is bounded further as

$$\begin{aligned} P_{out}(R) &\leq Pr\left(\frac{\lambda_{M_t}^2\{\mathbf{W}\}}{M_t^2 M_r^2}\frac{\mathcal{E}_s}{N_0}\|\mathbf{H}\|_F^2 \leq \left(2^R - 1\right)\right) \\ &\doteq (\mathcal{E}_s/N_0)^{-M_t M_r} \end{aligned} \tag{23}$$

with $\mathbf{W}$ defined as before. This is simply the outage probability of orthogonal space-time block codes with an SNR shift of $\frac{\lambda_{M_t}^2\{\mathbf{W}\}}{M_t^2 M_r^2}$ and was shown to possess full diversity order in [35]. ■

The achievability of full diversity gain is a substantial benefit. Spatial multiplexing has limited diversity order performance (ex. achieves diversity order M_r for overly complex maximum likelihood decoding), so a large diversity increase such as this adds considerable error rate improvements.

Just as diversity order is often used to characterize probability of error performance, *multiplexing gain* can be used to characterize spectral efficiency. Let $R(\mathcal{E}_s/N_0) = r\log(\mathcal{E}_s/N_0)$ denote the supported data rate as a function of SNR. A MIMO wireless system is said to have a multiplexing gain of c [35] if

$$c = \lim_{\mathcal{E}_s/N_0\to\infty} \frac{R(\mathcal{E}_s/N_0)}{\log(\mathcal{E}_s/N_0)}, \tag{24}$$

and this rate is the supremum of all rates that can be transmitted with a non-zero diversity order. Multiplexing gain is a fundamental property and, in our system, will be bounded from above by $\min(M_t, M_r) = M_t$.

The next lemma addresses multiplexing gain.

Lemma 1.B *If $N_{M_t} > 0$ and the rate is allows to grow with SNR, multi-mode precoding provides full multiplexing gain.*

Proof: We know that the multiplexing gain is upper-bounded by $\min(M_t, M_r) = M_t$. Thus we need to find a suitable lower-bound on multiplexing gain for each selection scenario.

Probability of Error Selection

We can bound the probability of error conditioned on the channel realization by

$$\begin{aligned} &P_e(\mathcal{E}_s/N_0) \\ &\quad\leq M_t N_e(M_t,R) Q\left(\sqrt{\lambda_{min}^2\{\mathbf{H}\}\frac{\mathcal{E}_s}{N_0}\frac{d_{min}^2(M_t,R)}{2M_t}}\right) \\ &\leq M_t N_e(M_t,R)\ \cdot \end{aligned} \tag{25}$$

$$Q\left(\sqrt{\min_{1\leq k\leq M_t}\frac{1}{M_t^3\left[\left(\mathbf{H}^*\mathbf{H}\right)^{-1}\right]_{k,k}}\frac{\mathcal{E}_s}{N_0}\frac{d_{min}^2(M_t,R)}{2}}\right). \tag{26}$$

Lets assume that $R = r\log(\mathcal{E}_s/N_0)$ with $r < M_t$. Following the technique used in [35], we will obtain an upper-bound using a QAM signaling assumption. This means that we can choose the constellation such that the minimum distance satisfies $d_{min}(M_t,R) \geq \alpha(\mathcal{E}_s/N_0)^{-r/(2M_t)}$ where α is a real constant. Using this assumption, the probability of error can be bounded as

$$\begin{aligned} &P_e(\mathcal{E}_s/N_0) \\ &\quad\leq M_t N_e(M_t,R)\ \cdot \end{aligned} \tag{27}$$

$$\begin{aligned} &E_{\mathbf{H}}\left[Q\left(\sqrt{\min_{1\leq k\leq M_t}\frac{\alpha^2}{2M_t^3\left[\left(\mathbf{H}^*\mathbf{H}\right)^{-1}\right]_{k,k}}\left(\frac{\mathcal{E}_s}{N_0}\right)^{1-r/M_t}}\right)\right] \\ &\doteq(\mathcal{E}_s/N_0)^{-(M_r-M_t+1)(1-r/M_t)} \end{aligned} \tag{28}$$

where (28) follows from the results in [49]. Thus multi-mode precoding with probability of error selection can achieve a multiplexing gain of M_t.

Capacity Selection

For the capacity case, we will use the outage probability assuming that $R = r\log(\mathcal{E}_s/N_0)$ with $r < M_t$.

$$\begin{aligned} &P_{out}(r\log(\mathcal{E}_s/N_0)) \\ &\quad\leq Pr\left(\log\left(det\left(\mathbf{I}_{M_t} + \frac{\mathcal{E}_s}{M_t N_0}\mathbf{H}^*\mathbf{H}\right)\right) \leq r\log(\mathcal{E}_s/N_0)\right) \\ &\quad\doteq (\mathcal{E}_s/N_0)^{-M_r(M_t-r)}. \end{aligned} \tag{29}$$

Thus, we have shown that capacity selection yields a multiplexing gain of M_t. ■

In order to satisfy the conditions in Lemma 1, we will require that $N_1 \geq M_t$ and $N_{M_t} = 1$ when $N_{tot} \geq M_t + 1$. Following the uninformed transmitter results in [1], [50], when $B < \log_2(M_t+1)$ we will first set $N_{M_t} = 1$ and allocate the remaining $2^B - 1$ matrices to N_1. Thus we can state the following allocation criterion.

Probability of Error and Capacity Allocation Criterion for $B \leq \log_2(M_t+1)$: Set $N_{M_t} = 1$ and $N_1 = 2^B - 1$.

When $B > \log_2(M_t+1)$, the first step in assigning codewords is the determination of a distortion function. The distortion function must be specific to the selection function used in order to maximize performance. We will design the distortion function by attempting to force the quantized multi-mode precoder to perform identically to an unquantized (or perfect CSI) multi-mode precoder.

To determine the allocation of matrix codewords among the modal codebooks, we will use rate-distortion theory. Each selection criteria will motivate a different definition of distortion. Using the defined distortion, the distortion-rate function is the minimum obtainable distortion for a given feedback rate. We will upper-bound the distortion-rate function for each mode in order to determine an allocation scheme.

Let the channel's singular value decomposition be given by

$$\mathbf{H} = \mathbf{V}_L \mathbf{\Sigma} \mathbf{V}_R^* \tag{30}$$

where $\mathbf{V}_L$ is an $M_r \times M_r$ unitary matrix, $\mathbf{V}_R$ is an $M_t \times M_t$ unitary matrix, and $\mathbf{\Sigma}$ is a diagonal matrix with $\lambda_i\{\mathbf{H}\}$ at position (i,i). We will define the probability of error design distortion to be the average loss of the mode selection function. This can be expressed as

$$\begin{aligned} & D^{(Pe)}(M, \log_2 N_M) \\ & = \min_{\mathcal{F}_M : |\mathcal{F}_M| = N_M} E_{\mathbf{H}} \left[\frac{d^2_{min}(M,R)}{M} \cdot \right. \\ & \qquad \left. \left| \lambda_M^2\{\mathbf{H}\overline{\mathbf{V}}_M\} - \max_{\mathbf{F}\in\mathcal{F}_M} \lambda_M^2\{\mathbf{HF}\} \right|^2 \right] \\ & \leq \frac{d^2_{min}(M,R)}{2M} \min_{\mathcal{F}_M : |\mathcal{F}_M| = N_M} E_{\mathbf{H}} \left[\|\mathbf{H}\|_F^2 \right] \cdot \\ & \qquad E_{\mathbf{H}} \left[\min_{\mathbf{F}\in\mathcal{F}_M} \left\| \overline{\mathbf{V}}_M \overline{\mathbf{V}}_M^* - \mathbf{F}\mathbf{F}^* \right\|_F^2 \right] \\ & \leq \frac{d^2_{min}(M,R)}{M} M_r M_t^3 N_M^{-2/M_t(M_t+1)} \end{aligned} \tag{31}$$

where $\overline{\mathbf{V}}_M$ is the matrix taken from the first M columns of $\mathbf{V}_R$ and (31) follows from the fact that a Gaussian random variable gives the worst possible distortion-rate function (i.e. the largest minimum achievable distortion for a given rate and given variance) with a mean squared error [51]. In particular the bound results from dividing up the N_M matrices (or equivalently $\log_2 N_M$ bits) among the $M_t(M_t+1)$ real parameters in the $M_t \times M_t$ complex matrix $\overline{\mathbf{V}}_M \overline{\mathbf{V}}_M^*$ for scalar quantization. The capacity selection will use the conditional distortion-rate function given by

$$\begin{aligned} & D^{(Cap)}(M, \log_2 N_M) \\ & = E_{\mathbf{H}} \left[\left| \log_2 \left(det \left(\mathbf{I}_M + \frac{\mathcal{E}_s}{MN_0} \overline{\mathbf{V}}_M^* \mathbf{H}^* \mathbf{H} \overline{\mathbf{V}}_M \right) \right) \right. \right. \\ & \qquad \left. \left. - \max_{\mathbf{F}\in\mathcal{F}_M} \log_2 \left(det \left(\mathbf{I}_M + \frac{\mathcal{E}_s}{MN_0} \mathbf{F}^* \mathbf{H}^* \mathbf{H} \mathbf{F} \right) \right) \right|^2 \right] \\ & \leq \frac{1}{2(\ln(2))^2} E_{\mathbf{H}} \left[det \left(\mathbf{I}_M + \frac{\mathcal{E}_s}{MN_0} \overline{\mathbf{\Sigma}}_M^T \overline{\mathbf{\Sigma}}_M \right) \right] \cdot \\ & \qquad E_{\mathbf{H}} \left[\min_{\mathbf{F}\in\mathcal{F}_M} \left\| \overline{\mathbf{V}}_M \overline{\mathbf{V}}_M^* - \mathbf{F}\mathbf{F}^* \right\|_F^2 \right] \end{aligned} \tag{32}$$

$$\leq \frac{M_t^2}{(\ln(2))^2} E_{\mathbf{H}} \left[\left(1 + \frac{\mathcal{E}_s}{N_0} \lambda_1^2\{\mathbf{H}\} \right)^{M_t} \right] N_M^{-2/M_t(M_t+1)} \tag{33}$$

where $\overline{\mathbf{\Sigma}}_M$ is the matrix constructed from the first M columns of $\mathbf{\Sigma}$. Note that (32) follows by using $|\ln(x) - \ln(y)| \leq |x-y|$, $\overline{\mathbf{V}}_M^* \mathbf{H}^* \mathbf{H} \overline{\mathbf{V}}_M = \overline{\mathbf{\Sigma}}_M^T \overline{\mathbf{\Sigma}}_M$, and

$$1 - \max_{\mathbf{F}\in\mathcal{F}_M} \left| det \left(\overline{\mathbf{V}}_M^* \mathbf{F} \right) \right|^2 \leq \frac{1}{2} \min_{\mathbf{F}\in\mathcal{F}_M} \left\| \overline{\mathbf{V}}_M \overline{\mathbf{V}}_M^* - \mathbf{F}\mathbf{F}^* \right\|_F^2 .$$

Let $\mathcal{M} = \{m_1, m_2, ..., m_{|\mathcal{M}|}\}$ denote the set of possible modes. We will allocate the codewords to minimize the total distortion-rate function assuming a uniform mode distribution which is given by

$$\overline{D} \left(\log_2 N_{m_1}, \ldots, \log_2 N_{m_{|\mathcal{M}|}} \right) = \sum_{i=1}^{|\mathcal{M}|} \frac{1}{|\mathcal{M}|} D(m_i, \log_2 N_{m_i}) \tag{34}$$

using the appropriate distortion. By plugging (31) into (34) and removing the common scale factors among all modes, the probability of error selection is equivalent to minimizing the allocation cost function

$$A^{(Pe)} \left(N_{m_1}, \ldots, N_{m_{|\mathcal{M}|}} \right) = \sum_{i=1}^{|\mathcal{M}|} \frac{d^2_{min}(m_i, R)}{m_i} N_{m_i}^{-2/M_t(M_t+1)} . \tag{35}$$

Similarly, the capacity selection allocation should minimize the cost function

$$A^{(Cap)} \left(N_{m_1}, \ldots, N_{m_{|\mathcal{M}|}} \right) = \sum_{i=1}^{|\mathcal{M}|} N_{m_i}^{-2/M_t(M_t+1)} . \tag{36}$$

Both (35) and (36) must be minimized subject to

$$\sum_{i=1}^{|\mathcal{M}|} N_{m_i} = N_{tot}, \tag{37}$$

$N_{m_1} = N_1 \geq M_t$, and $N_{m_{|\mathcal{M}|}} = N_{M_t} = 1$. Thus, this distortion-rate function based on a uniform mode distribution will allow the codeword allocation to be done *independently* of the SNR.

It is easily seen that the following allocation will minimize (36).

Capacity Allocation Criterion for $B > \log_2(M_t+1)$**:** If $B \leq \log_2(M_t(|\mathcal{M}|-1)+1)$, set $N_{M_t} = 1$, $N_1 = M_t$, and $N_{m_i} = \frac{2^B - M_t - 1}{|\mathcal{M}|-2}$ for $1 < i < |\mathcal{M}|$. If $B > \log_2(M_t(|\mathcal{M}|-1)+1)$, set $N_{M_t} = 1$ and $N_{m_i} = \frac{2^B-1}{|\mathcal{M}|-1}$ for $1 \leq i < |\mathcal{M}|$. In the cases where this yields non-integer allocations, the allocation can be adjusted by giving any extra matrices to the lower order modes.

Eq. (35) can be minimized by a numerical search or convex optimization techniques. Let $\tilde{N}_{m_1} = N_1 - M_t$ and $\tilde{N}_{m_i} = N_{m_i}$ for $i > 1$. Using this notation, we can reformulate (35) as

$$\tilde{A}^{(Pe)}\left(\tilde{N}_{m_1},\ldots,\tilde{N}_{m_{|\mathcal{M}|-2}}\right)$$
$$= d^2_{min}(1,R)\left(\tilde{N}_{m_1}+M_t\right)^{-2/M_t(M_t+1)}$$
$$+\sum_{i=2}^{|\mathcal{M}|-2}\frac{d^2_{min}(m_i,R)}{m_i}\tilde{N}_{m_i}^{-2/M_t(M_t+1)}$$
$$+\frac{d^2_{min}(m_{|\mathcal{M}|-1},R)}{m_{|\mathcal{M}|-1}}\cdot$$
$$\left(N_{tot}-M_t-1-\sum_{l=2}^{|\mathcal{M}|-2}\tilde{N}_{m_l}\right)^{-2/M_t(M_t+1)} \quad (38)$$

where we have omitted the $N_{|\mathcal{M}|}$ term. The values of $\tilde{N}_{m_1},\ldots,\tilde{N}_{m_{|\mathcal{M}|-1}}$ that minimize $\tilde{A}$ can be easily determined with $\tilde{N}_{m_{|\mathcal{M}|-1}} = N_{tot}-1-M_t+\sum_{l=2}^{|\mathcal{M}|-2}\tilde{N}_{m_l}$. These values can be used to determine $N_{m_1},\ldots,N_{m_{|\mathcal{M}|-1}}$ with $N_{m_{|\mathcal{M}|}}=1$. Therefore, the following criterion can be used for codeword allocation.

Probability of Error Allocation Criterion for $B > \log_2(M_t+1)$: Minimize (35) such that $N_1 \geq M_t$, $N_{M_t}=1$, and $\sum_{i=1}^{|\mathcal{M}|} N_{m_i} = N_{tot}$. This minimization can be done using a numerical search or by using convex optimization techniques on (38).

It should be noted that the assumption of a uniform mode distribution is a rough approximation. In general, environmental effects such as spatial correlation will play a large role in the notion of an "optimal" mode. As well, the probability of mode selection is inherently dependent on the rate at which the system transmits. Based on simulations, the uniform allocation tends to be overly conservative in that it biases codeword distribution to the lower mode numbers.

C. Codebook Criterion Given the Number of Substreams

Now that we have determined an algorithm that gives an allocation of the possible codebook matrices among the modes, it is now imperative to present the design of $\mathcal{F}_M$ for each mode.

For a given mode, the codebook $\mathcal{F}_M$ is a set of matrices in $\mathcal{U}(M_t,M)$. Each matrix in $\mathcal{F}_M$ defines an M-dimensional subspace in $\mathbb{C}^{M_t}$. The set of all M-dimensional subspaces in $\mathbb{C}^{M_t}$ is the Grassmann manifold $\mathcal{G}(M_t,M)$. A finite set of subspaces in the Grassmann manifold can be thought of as a subspace code [52]. A large variety of different subspaces distances can be used for subspace coding [53]. These include the *projection two-norm distance* given by

$$d_{proj}(\mathbf{F}_1,\mathbf{F}_2)=\|\mathbf{F}_1\mathbf{F}_1^*-\mathbf{F}_2\mathbf{F}_2^*\|_2 \quad (39)$$

and the *Fubini-Study distance* given by

$$d_{FS}(\mathbf{F}_1,\mathbf{F}_2)=\arccos\left|det\left(\mathbf{F}_1^*\mathbf{F}_2\right)\right|. \quad (40)$$

The codebooks will be designed by choosing and then minimizing a distortion function. We will break the presentation into the two different selection cases following the development in [54].

Probability of Error

Following Section IV-B, we will define the distortion as

$$E_{\mathbf{H}}\left[\frac{d^2_{min}(M,R)}{M}\left(\lambda^2_{\min}\{\mathbf{HF}_{opt}\}-\max_{\mathbf{F}_i\in\mathcal{F}}\lambda^2_{\min}\{\mathbf{HF}_i\}\right)\right] \quad (41)$$

where $\mathbf{F}_{opt}$ is the precoder in $\mathcal{U}(M_t,M)$ that maximizes $\lambda^2_{\min}\{\mathbf{HF}_{opt}\}$. This can be bounded as

$$E_{\mathbf{H}}\left[\frac{d^2_{min}(M,R)}{M}\left(\lambda^2_{\min}\{\mathbf{HF}_{opt}\}-\max_{\mathbf{F}_i\in\mathcal{F}}\lambda^2_{\min}\{\mathbf{HF}_i\}\right)\right]$$
$$\leq \frac{d^2_{min}(M,R)}{M}E_{\mathbf{H}}\left[\lambda^2_M\{\mathbf{H}\}\right]\cdot$$
$$E_{\mathbf{H}}\left[\left(1-\max_{\mathbf{F}_i\in\mathcal{F}}\lambda^2_{\min}\{\overline{\mathbf{V}}_R^*\mathbf{F}_i\}\right)\right]$$
$$\leq \frac{d^2_{min}(M,R)}{M}E_{\mathbf{H}}\left[\lambda^2_M\{\mathbf{H}\}\right]\cdot$$
$$\left(\frac{\delta^2_{proj}}{4}\Delta_{proj}(\delta_{proj})+(1-\Delta_{proj}(\delta_{proj}))\right) \quad (42)$$

where

$$\delta_{proj}=\min_{\mathbf{F}_1,\mathbf{F}_2\in\mathcal{F}_M:\mathbf{F}_1\neq\mathbf{F}_2} d_{proj}(\mathbf{F}_1,\mathbf{F}_2) \quad (43)$$

and $\Delta_{proj}(\delta_{proj})$ is N times the normalized volume of a metric ball in $\mathcal{G}(M_t,M)$ of radius $\delta_{proj}/2$.

Using metric ball arguments similar to [33] and the asymptotic metric ball volumes derived in [52], the bound in (42) can be approximately minimized by thinking of the set $\mathcal{F}_M$ as a set of subspaces rather than as a set of matrices. The bound can thus be minimized by maximizing the minimum projection two-norm subspace distance δ_{proj}.

Probability of Error Design Criterion: Design $\mathcal{F}_M$ such that

$$\delta_{proj}=\min_{\mathbf{F}_1,\mathbf{F}_2\in\mathcal{F}_M:\mathbf{F}_1\neq\mathbf{F}_2} d_{proj}(\mathbf{F}_1,\mathbf{F}_2) \quad (44)$$

is maximized.

Capacity

The capacity distortion is defined as

$$E_{\mathbf{H}}\left[\left|C_{UT}(\mathbf{F}_{opt})-\max_{\mathbf{F}_i\in\mathcal{F}}C_{UT}(\mathbf{F}_i)\right|^2\right] \quad (45)$$

where $\mathbf{F}_{opt}$ is the precoder in $\mathcal{U}(M_t,M)$ that maximizes C_{UT}. The distortion cost function can be bounded as

$$E_{\mathbf{H}}\left[\left|C_{UT}\left(\mathbf{F}_{opt}\right)\ -\max_{\mathbf{F}_i\in\mathcal{F}}C_{UT}(\mathbf{F}_i)\right|^2\right]$$
$$\leq (1/\ln(2))^2E_{\mathbf{H}}\left[det\left(\mathbf{I}_M+\frac{\mathcal{E}_s}{MN_0}\overline{\mathbf{\Sigma}}^T\overline{\mathbf{\Sigma}}\right)\right]\cdot$$
$$\left(1-\max_{\mathbf{F}_i\in\mathcal{F}}E_{\mathbf{H}}\left[\left|det\left(\overline{\mathbf{V}}_R^*\mathbf{F}_i\right)\right|^2\right]\right) \quad (46)$$
$$\leq (1/\ln(2))^2E_{\mathbf{H}}\left[det\left(\mathbf{I}_M+\frac{\mathcal{E}_s}{MN_0}\overline{\mathbf{\Sigma}}^T\overline{\mathbf{\Sigma}}\right)\right]\cdot$$
$$\left(1-\cos^2\left(\delta_{FS}/2\right)\Delta_{FS}(\delta_{FS})\right) \quad (47)$$

where δ_{FS} and $\Delta(\delta_{FS})$ are defined using the Fubini-Study distance. Using the metric ball volume approximations from [52] and differentiating the resulting bound tells us that we want to maximize δ_{FS} in order to approximately minimize the distortion.

Capacity Design Criterion: Design $\mathcal{F}_M$ such that

$$\delta_{FS} = \min_{\mathbf{F}_1, \mathbf{F}_2 \in \mathcal{F}_M : \mathbf{F}_1 \neq \mathbf{F}_2} d_{FS}(\mathbf{F}_1, \mathbf{F}_2) \tag{48}$$

is maximized.

Discussion

Note that the column vectors in $\mathcal{F}_1$ correspond to beamforming vectors [33], [34]. The design of limited feedback beamforming was explored in [33], [34], [55]–[58]. In particular, it was shown in [33], [34] that the set of vectors should be designed by thinking of the vectors as representing lines in $\mathbb{C}^{M_t}$. The lines can then be optimally spaced by maximizing the minimum angular separation between any two lines. This is seen by noting that when $M = 1$ both δ_{proj} and δ_{FS} are maximized by minimizing $|\mathbf{f}_1^* \mathbf{f}_2|$ for any two distinct vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ in $\mathcal{F}_1$. The set $\mathcal{F}_{M_t}$ is trivially designed because we will require that $\mathcal{F}_{M_t} = \{\mathbf{I}_{M_t}\}$. This precoder matrix corresponds to sending a standard spatial multiplexing vector.

For $M < M_t$, codebooks can be designed using the matrix codebook design algorithms for non-coherent constellations in [59]. The only modification needed is to use the correct subspace distance when performing the optimization. In addition, algebraic design techniques can be used for certain values of M and N_M [60], [61]. Numerical design techniques have also been studied in [62], [63].

V. Relation to Covariance Quantization

The capacity analysis for MIMO systems with transmitter CSI relies on *optimizing the transmit covariance matrix.* A MIMO system has a general input-output relationship

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

with $\mathbf{H}$ and $\mathbf{v}$ defined as in (1). The mutual information is maximized by optimizing the covariance matrix

$$\mathbf{Q} = E_{\mathbf{x}}\left[\mathbf{x}\mathbf{x}^*\right]. \tag{49}$$

Covariance quantization, proposed in [38], [41], chooses $\mathbf{Q}$ from a codebook $\mathcal{Q} = \{\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_N\}$. Assuming that $\mathbf{s}$ in (1) consists of independent entries distributed according to $\mathcal{CN}(0, \mathcal{E}_s/M)$, the covariance matrix will be $(\mathcal{E}_s/M)\mathbf{F}_M\mathbf{F}_M^*$. *Thus multi-mode precoding quantizes the set of covariance matrices assuming a rank constraint.* Let

$$\mathcal{Q}_M = \{(\mathcal{E}_s/M)\mathbf{F}\mathbf{F}^* \mid \forall\ \mathbf{F} \in \mathcal{F}_M\}. \tag{50}$$

This allows multi-mode precoding to be reformulated as covariance quantization with a codebook

$$\mathcal{Q} = \bigcup_{m \in \mathcal{M}} \mathcal{Q}_m.$$

Multi-mode precoding is a *rank constrained* covariance quantization. While the codebook matrices in [38], [41] attempt to quantize a waterfilling solution, chooses a covariance rank and then allocates equal power among each mode. This avoids the power allocation problems associated with waterfilling.

In addition, multi-mode precoding uses a set of the form in (50) that only requires a multiplicative scale factor when the SNR changes. In contrast, [41] uses the Lloyd algorithm. The Lloyd algorithm, described in [64], uses a set of test channels (usually numbering in the thousands to hundred of thousands) and then repeats the two steps of generating a codebook for each Voronoi region (i.e. the subset of the test channels that map to the same covariance matrix) and redefining the Voronoi regions for the generated codebook. The algorithm converges rather quickly [64] to a locally optimal solution but still suffers because the random matrix being quantized is highly dependent on the SNR.

VI. Simulations

Limited feedback multi-mode precoding was simulated to exhibit the available decrease in probability of error and the increase in capacity. The capacity results are compared with the results in [38], [41]. We also consider both full channel knowledge [2], [3], [65] and limited feedback [33], [34], [58] beamforming. Probability of error simulations used the probability of error selection criterion, while capacity simulations used the capacity selection criterion.

Experiment 1: This experiment addresses the probability of vector symbol error of 4×4 dual-mode precoding with a ZF receiver. The results are shown in Fig. 2. The rate is fixed at $R = 8$ bits per channel use with QAM constellations. Because the system is dual-mode, the set of supported modes is $\mathcal{M} = \{1, 4\}$. Four bits of feedback is assumed to be available. This means that $\mathcal{F}_1$ contains 15 vectors and $\mathcal{F}_4 = \{\mathbf{I}_4\}$. Limited feedback beamforming using four bits (see [33], [34]) and spatial multiplexing are simulated for comparison. Multi-mode precoding provides approximately a 0.6 dB performance improvement over limited feedback beamforming. These gains are modest because of the restriction to dual-mode precoding.

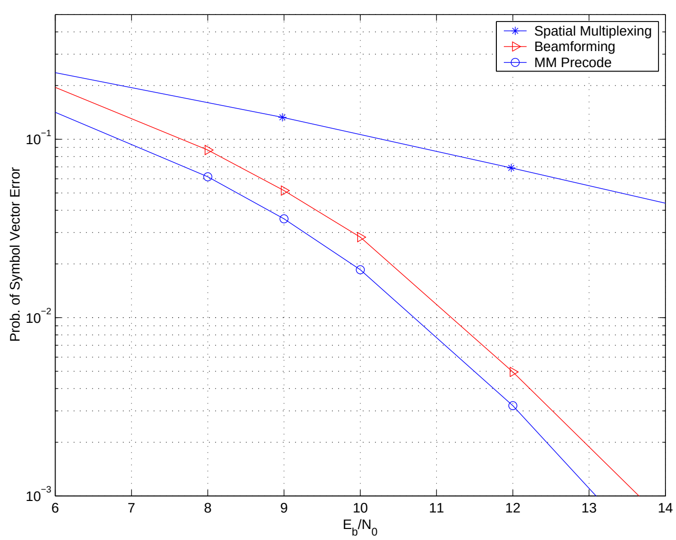

Fig. 2. Probability of vector symbol error performance for limited feedback dual-mode precoding, limited feedback beamforming, and spatial multiplexing.

Experiment 2: In contrast to the first experiment, this experiment considers a 4×4 MIMO system transmitting

$R = 8$ bits per channel use with $\mathcal{M} = \{1, 2, 4\}$. Codebooks were designed using $B = 5$ bits of feedback. This resulted in $N_1 = 7$, $N_2 = 24$, and $N_4 = 1$. Constellations were restricted to be QAM and the receiver was a ZF decoder. Fig. 3 presents the simulation results. Spatial multiplexing, unquantized beamforming (i.e. perfect CSI at the transmitter using maximum ratio transmission/maximum ratio combining [2], [65]), and unquantized MMSE precoding are shown for comparison. MMSE precoding is implemented by transmitting two 16-QAM symbol streams using linear transmit and receive processing (i.e. precoding and a linear receiver). The MMSE precoding was implemented as in [5] with the sum power constraint and the trace cost function. This means that the unquantized MMSE simulation is using power allocation among the modes. Note that all of the selection criteria provide approximately the same probability of vector symbol error performance. Five bit multi-mode precoding provides approximately a 5dB gain over full CSI beamforming. There is more than an 8.5dB gain over spatial multiplexing at an error rate of 10^{-1}. Interestingly, MMSE precoding with full transmit channel knowledge, a less restrictive power constraint, and a superior receiver gives only a 1.2dB gain over limited feedback multi-mode precoding.

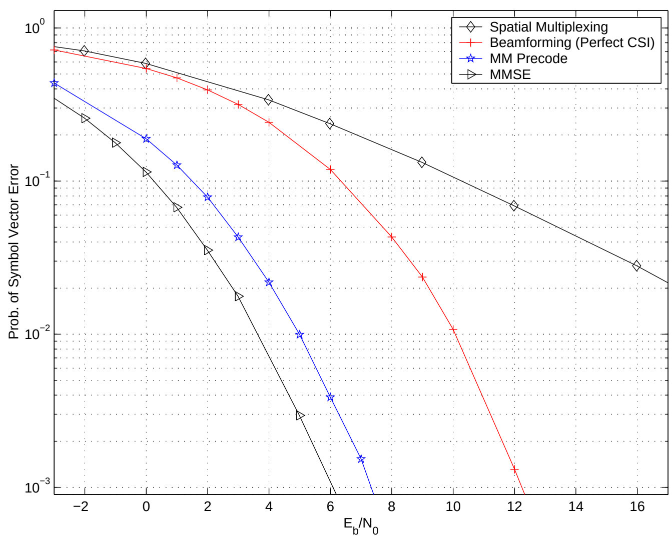

Fig. 3. Probability of vector symbol error performance for limited feedback multi-mode precoding, beamforming, and spatial multiplexing.

Experiment 3: The third experiment, shown in Fig. 4, compares the performance of limited feedback multi-mode precoding with various amounts of feedback. Again, we considered a 4×4 MIMO system transmitting $R = 8$ bits per channel use with $\mathcal{M} = \{1, 2, 4\}$. We used a ZF receiver and QAM constellations. Feedback amounts of $B = 4$, $B = 8$, and ∞ bits were simulated. The $B = 4$ bit codeword allocation used $N_1 = 4$, $N_2 = 11$, and $N_4 = 1$. The $B = 8$ bit allocation was $N_1 = 54$, $N_2 = 201$, and $N_4 = 1$. Note that four bits of feedback performs within 1dB of the infinite feedback scenario. Adding four more bits of feedback adds approximately a 0.6dB gain. For comparison, multi-mode antenna selection [29] using the probability of error selection criterion is presented. Multi-mode antenna selection requires four bits of feedback. Interestingly, multi-mode precoding outperforms multi-mode antenna selection by 0.4dB using the same amount of feedback. Unlike multi-mode antenna selection, multi-mode precoding can use more feedback to add additional array gain as the eight bit curve demonstrates. Perfect CSI beamforming is also shown to demonstrate the multi-mode precoding gain.

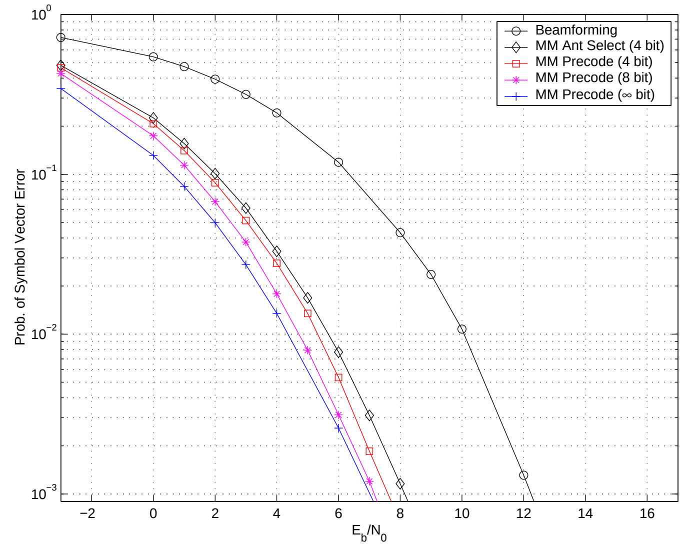

Fig. 4. Probability of vector symbol error performance comparison for limited feedback and perfect CSI multi-mode precoding, multi-mode antenna selection, and beamforming.

Experiment 4: As mentioned in Section III, multi-mode precoding can be employed with ML, V-BLAST, and linear receivers. Fig. 5 compares the performance of limited feedback multi-mode precoding with ML, V-BLAST, and ZF decoding. We simulated a 4×4 MIMO system transmitting $R = 8$ bits per channel use with QAM signaling and $\mathcal{M} = \{1, 2, 4\}$. The feedback bit total was set to $B = 5$ with a codeword allocation of $N_1 = 7$, $N_2 = 24$, and $N_4 = 1$. All receivers perform very closely with ZF within 0.5dB of ML decoding. V-BLAST decoding performs approximately 0.1dB away from ML decoding.

Experiment 5: The capacity gains available with the capacity selection criterion are illustrated in Fig. 6 for a 2×2 MIMO system with $B = 2, 3,$ and 4 averaged over a spatially uncorrelated Rayleigh fading channel. The codewords were allocated as $N_1 = 2^B - 1$ and $N_2 = 1$. The plot shows the ratio of the computed mutual information with the capacity of a transmitter with perfect CSI using waterfilling [1]. The capacity of an uninformed transmitter (UT) and the limited feedback covariance optimization mutual information results published in [38] are shown for comparison. Note that limited feedback multi-mode precoding outperforms limited feedback covariance optimization for both two and three bits of feedback. This result is striking because, unlike covariance optimization, multi-mode precoding *does not require any form of waterfilling.* Thus our scheme, on average, always transmits with the same power on each symbol substream. The high-rate feedback performance difference between limited feedback covariance optimization and multi-mode precoding can be

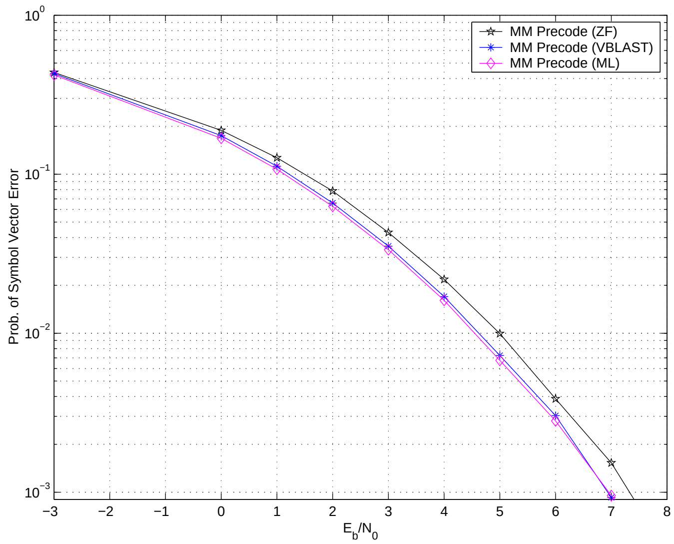

Fig. 5. Probability of vector symbol error performance comparison of different receivers with multi-mode precoding.

most likely attributed to this power-pouring.

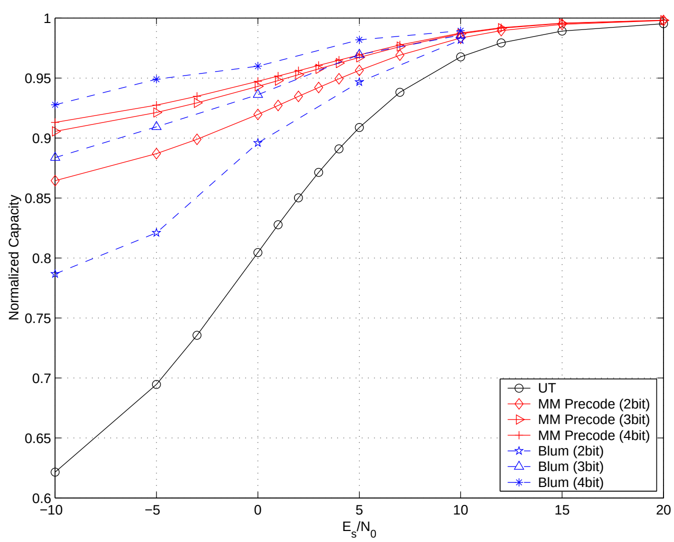

Fig. 6. Capacity comparison of multi-mode precoding, limited feedback covariance optimization [38], and the uninformed transmitter.

Experiment 6: The sixth experiment compares infinite resolution multi-mode precoding, limited feedback multi-mode precoding, and the UT capacity for a 3×3 MIMO system. The capacities have been normalized by the optimal waterfilling capacity at each SNR. The results are shown in Fig. 7. We considered supported modes of $\mathcal{M} = \{1, 2, 3\}$ with four and five feedback bits. We designed the limited feedback codebook using techniques from Section IV. The codeword matrices were allocated with $|\mathcal{F}_1| = \lceil (2^B - 1)/2 \rceil$, $|\mathcal{F}_2| = \lfloor (2^B - 1)/2 \rfloor$, and $|\mathcal{F}_3| = 1$. The infinite resolution multi-mode precoder obtains within 98.5% of the system capacity when the channel is known to both the transmitter and receiver. The limited feedback case obtains more than 84% of the perfect transmitter CSI system capacity with four feedback bits and 87.8% with five feedback bits. This comes with the benefit of only requiring a few bits of feedback.

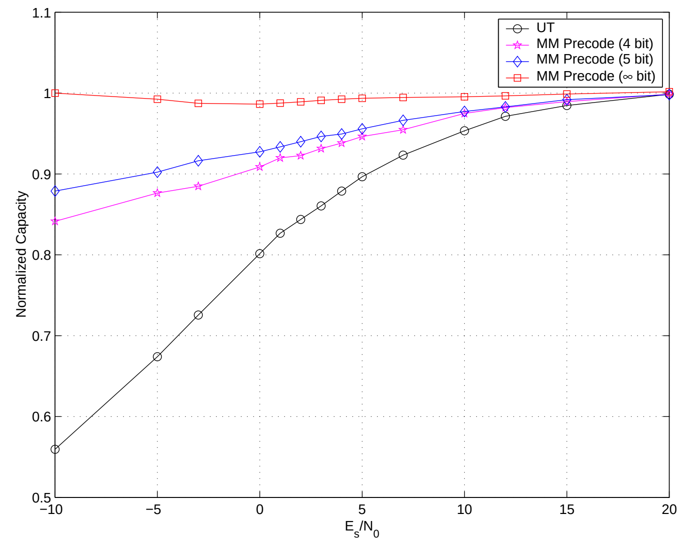

Fig. 7. Capacity comparison of multi-mode precoding with an infinite amount of feedback, multi-mode precoding with four and five feedback bits, and an uninformed transmitter.

Experiment 7: This experiment, shown in Fig. 8, compares limited feedback multi-mode precoding with the feedback technique proposed in [41] on a 4×4 MIMO system. Again, the mutual informations are normalized by the waterfilling capacity. Multi-mode precoding was restricted the modes $\mathcal{M} = \{1, 2, 4\}$, and both schemes used three bits of feedback. Multi-mode precoding performs within approximately 1.5dB of the algorithm in [41] for all SNRs. Note that the algorithm in [41] used a feedback codebook that was redesigned for each SNR while the multi-mode codebooks are fixed for all SNRs.

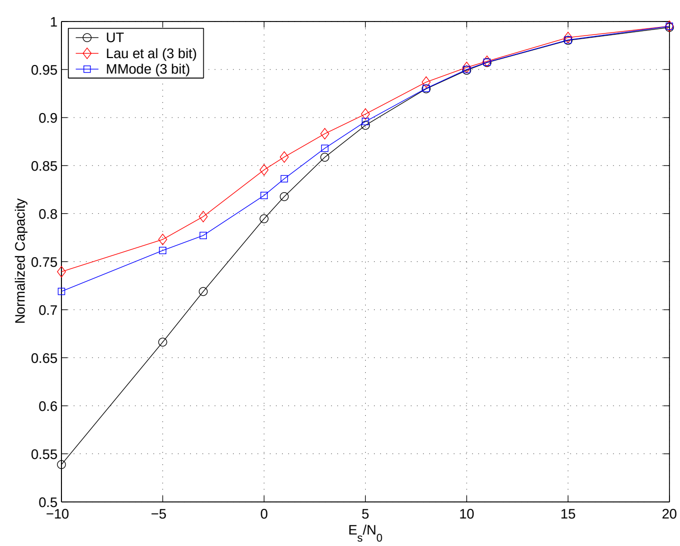

Fig. 8. Capacity comparison of multi-mode precoding and the covariance feedback technique designed in [41].

Experiment 8: The final experiment examines the diversity vs. multiplexing performance using the outage probability. A 4×4 multi-mode precoding system with $N_1 = 4$ and $N_4 = 1$ was simulated. Various rate growths were considered including a fixed rate ($R = 30$), $R = 3\log_2(\mathcal{E}_s/N_0)$, $R = 3.5\log_2(\mathcal{E}_s/N_0)$, $R = 3.75\log_2(\mathcal{E}_s/N_0)$, and $R = 3.9\log_2(\mathcal{E}_s/N_0)$. The results are shown in Fig. 9. The $R = 30$ outage probability curve shows the full diversity rate performance of multi-mode precoding. The other curves demonstrate that the growth can get arbitrarily close to $R = 4\log_2(\mathcal{E}_s/N_0)$ with a non-zero asymptotic slope of the outage curve.

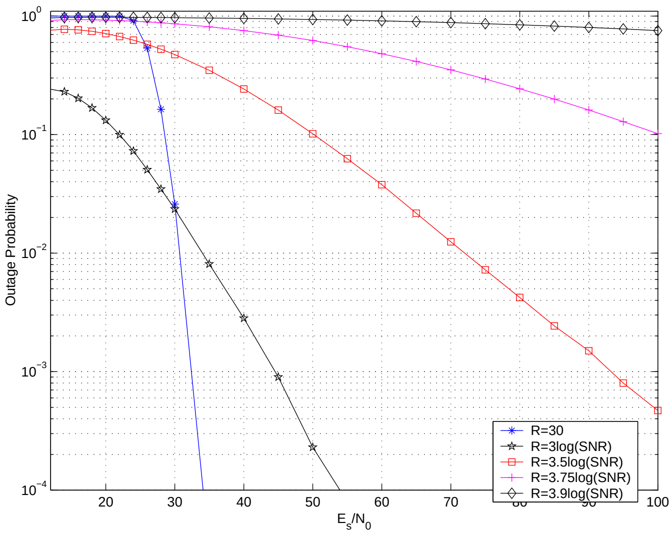

Fig. 9. Outage probability analysis for multi-mode precoding with various transmission rates.

VII. Summary and Conclusions

We presented limited feedback multi-mode precoding for spatial multiplexing MIMO wireless systems. This allows the MIMO system to adaptively vary both the number of substreams and the precoder using current channel conditions. Because only a limited number of feedback bits are used, the algorithm can be successfully implemented in MIMO systems without transmitter CSI. We showed how to design the codebooks needed for the multi-mode precoding. We found that multi-mode precoding provides full diversity gain for a fixed rate and full multiplexing gain when the rate varies with SNR.

A point of future work is to investigate a more specific multi-mode framework that relates to each kind of receiver. The probability of error derivations in this paper used ML, linear, and V-BLAST bounds that all relate to the minimum singular value of the channel. While this may provide high performance for a linear receiver, it is a rather loose performance metric for both V-BLAST and ML. Future research is needed into i.) tight, but computationally simple, performance bounds for MIMO receivers and ii.) improved precoder design for non-linear receivers. We speculate that the performance of a multi-mode system could be greatly improved by more judiciously designing the mode selection framework using tight probability of error expressions.

Another point of future work is in the computation of the optimal precoder matrix from the codebook given a mode number. This paper assumes they are computed via a brute force search. While this is reasonable given a small amount of feedback and a moderate number of transmit antennas, the computational complexity will quickly scale to unacceptable levels. A possible sub-optimal solution might be to analytically determine the mode and/or precoder matrix for an unquantized system and then restrict the brute force optimization to candidate multi-mode configurations around the unquantized case.

An interesting area for future research is the distribution of the codebook matrices among the different modes. In our derivation, we assumed that the modes were equiprobable. This assumption will be unrealistic in most environments. It may be possible to compute an approximate closed-form solution of the number of matrices to be allocated to each mode given i.) the number of feedback bits, ii.) channel correlation, and iii.) rate.

The ideas behind the relationship between multi-mode precoding and covariance quantization [38], [41] are also of interest. Future research is needed to obtain a more definitive capacity analysis when the transmitter is only constrained to vary the rank of the covariance matrix. We conjecture that the losses relative to waterfilling with a trace constraint (as discussed in [1]) will always be minimal.

For practical implementation, the effect of delay and errors in the feedback channel on system performance is something that should be studied. These effects will lead to degradation in the obtainable bit error rate and spectral efficiency. This analysis would provide interesting knowledge into the benefits of multi-mode precoding in practical wireless systems.

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References

[1] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecomm.*, vol. 10, no. 6, pp. 585–595, 1999.

[2] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. New York: Cambridge University Press, 2003.

[3] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. New York: Cambridge University Press, 2003.

[4] D. A. Gore, R. U. Nabar, and A. Paulraj, "Selecting an optimal set of transmit antennas for a low rank matrix channel," in *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, vol. 5, pp. 2785–2788, June 2000.

[5] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 1051–1064, May 2002.

[6] H. Sampath, P. Stoica, and A. Paulraj, "Generalized precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, pp. 2198–2206, Dec. 2001.

[7] L. Collin, O. Berder, P. Rostaing, and G. Burel, "Optimal minimum distance-based precoder for MIMO spatial multiplexing systems," *IEEE Trans. Sig. Proc.*, vol. 52, pp. 617–627, March 2004.

[8] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Trans. Info. Th.*, vol. 47, pp. 2632–2639, Sept. 2001.

[9] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Jour. Select. Areas in Commun.*, vol. 21, pp. 684–702, June 2003.

[10] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Trans. Sig. Proc.*, vol. 50, pp. 2599–2613, Oct. 2002.

[11] H. Sampath and A. Paulraj, "Linear precoding for space-time coded systems with known fading correlations," *IEEE Commun. Lett.*, vol. 6, pp. 239–241, June 2002.

[12] Z. Hong, K. Liu, R. W. Heath, Jr., and A. M. Sayeed, "Spatial multiplexing in correlated fading via the virtual channel representation," *IEEE Jour. Select. Areas in Commun.*, vol. 21, pp. 856–866, June 2003.

[13] S. A. Jafar, S. Vishwanath, and A. Goldsmith, "Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback," in *Proc. IEEE Int. Conf. on Commun.*, vol. 7, pp. 11–14, June 2001.

[14] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel correlations," *IEEE Trans. Info. Th.*, vol. 49, pp. 1673–1690, July 2003.

[15] J. Akhtar and D. Gesbert, "A closed-form precoder for spatial multiplexing over correlated MIMO channels," in *Proc. IEEE Glob. Telecom. Conf.*, vol. 4, pp. 1847–1851, Dec. 2003.

[16] J. C. Roh and B. D. Rao, "Multiple antenna channels with partial channel state information at the transmitter," *IEEE Trans. Wireless Comm.*, vol. 3, pp. 677–688, March 2004.

[17] R. S. Blum and J. H. Winters, "On optimum MIMO with antenna selection," *IEEE Commun. Lett.*, vol. 6, pp. 322–324, Aug. 2002.

[18] M. Gharavi-Alkhansari and A. B. Gershman, "Fast antenna subset selection in wireless MIMO systems," in *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, vol. 5, pp. 57–60, April 2003.

[19] D. A. Gore, R. W. Heath, Jr., and A. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Commun. Lett.*, vol. 6, pp. 491–493, Nov. 2002.

[20] A. Gorokhov, "Antenna selection algorithms for MEA transmission systems," in *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, vol. 3, pp. 2857–2860, May 2002.

[21] A. F. Molisch, M. Z. Win, and J. H. Winters, "Capacity of MIMO systems with antenna selection," in *Proc. IEEE Int. Conf. on Commun.*, vol. 2, pp. 570–574, June 2001.

[22] R. W. Heath Jr., S. Sandhu, and A. Paulraj, "Antenna selection for spatial multiplexing systems with linear receivers," *IEEE Commun. Lett.*, vol. 5, pp. 142–144, April 2001.

[23] D. J. Love and R. W. Heath, Jr., "Limited feedback precoding for spatial multiplexing systems using linear receivers," in *Proc. IEEE Mil. Comm. Conf.*, vol. 1, pp. 627–632, Oct. 2003.

[24] D. J. Love and R. W. Heath, Jr., "Limited feedback precoding for spatial multiplexing systems," in *Proc. IEEE Glob. Telecom. Conf.*, vol. 4, pp. 1857–1861, Dec. 2003.

[25] D. J. Love and R. W. Heath Jr., "Grassmannian precoding for spatial multiplexing systems," in *Proc. of Allerton Conf. on Comm. Cont. and Comp.*, Oct. 2003.

[26] A. Gorokhov, D. Gore, and A. Paulraj, "Diversity versus multiplexing in MIMO systems with antenna selection," in *Proc. of Allerton Conf. on Comm. Cont. and Comp.*, Oct. 2003.

[27] X. Ma and G. B. Giannakis, "Error probability performance of L-branch diversity reception of MQAM in Rayleigh fading," *IEEE Trans. Sig. Proc.*, vol. 51, pp. 2917–2930, Nov. 2003.

[28] H. El Gamal and M. O. Damen, "Universal space-time coding," *IEEE Trans. Info. Th.*, vol. 49, pp. 1097–1119, May 2003.

[29] R. W. Heath Jr. and D. J. Love, "Multi-mode antenna selection for spatial multiplexing," in *Proc. of Allerton Conf. on Comm. Cont. and Comp.*, Oct. 2003.

[30] R. W. Heath Jr. and D. J. Love, "Dual-mode antenna selection for spatial multiplexing systems with linear receivers," in *Proc. 37th Asilomar Conf. on Sig., Sys., and Comp.*, Nov. 2003.

[31] N. Khaled, C. Desset, S. Theon, and H. De Man, "On spatial-mode selection for the joint transmit and receive MMSE design," in *Proc. IEEE Int. Conf. on Commun.*, vol. 5, pp. 2812–2816, June 2004.

[32] R. Narasimhan, "Spatial multiplexing with transmit antenna and constellation selection for correlated MIMO fading channels," *IEEE Trans. Sig. Proc.*, vol. 51, pp. 2829–2838, Nov. 2003.

[33] D. J. Love, R. W. Heath Jr., and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Info. Th.*, vol. 49, pp. 2735–2747, Oct. 2003.

[34] K. K. Mukkavilli, A. Sabharwal, E. Erkip, and B. Aazhang, "On beamforming with finite rate feedback in multiple antenna systems," *IEEE Trans. Info. Th.*, vol. 49, pp. 2562–2579, Oct. 2003.

[35] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Info. Th.*, vol. 49, pp. 1073–1096, May 2003.

[36] R. W. Heath Jr. and A. Paulraj, "Switching between spatial multiplexing and transmit diversity based on constellation distance," in *Proc. of Allerton Conf. on Comm. Cont. and Comp.*, Oct. 2000.

[37] B. Varadarajan and J. R. Barry, "The rate-diversity trade-off for linear space-time codes," in *Proc. IEEE Veh. Technol. Conf.*, vol. 1, pp. 67–71, Sept. 2002.

[38] R. S. Blum, "MIMO with limited feedback of channel state information," in *Proc. IEEE Int. Conf. Acoust., Speech and Sig. Proc.*, vol. 4, pp. 89–92, April 2003.

[39] W. Santipach, Y. Sun, and M. L. Honig, "Benefits of limited feedback for wireless channels," in *Proc. of Allerton Conf. on Comm. Cont. and Comp.*, Oct. 2003.

[40] W. Santipach and M. L. Honig, "Asymptotic performance of MIMO wireless channels with limited feedback," in *Proc. IEEE Mil. Comm. Conf.*, vol. 1, pp. 141–146, Oct. 2003.

[41] V. Lau, Y. Liu, and T.-A. Chen, "On the design of MIMO block-fading channels with feedback-link capacity constraint," *IEEE Trans. Commun.*, vol. 52, pp. 62–70, Jan. 2004.

[42] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Info. Th.*, vol. 45, pp. 1639–1642, July 1999.

[43] O. Damen, A. Chkeif, and J.-C. Belfiore, "Lattice code decoder for space-time codes," *IEEE Commun. Lett.*, vol. 4, pp. 161–163, May 2000.

[44] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, "Detection algorithm and initial laboratory results using the V-BLAST space-time communication architecture," *Elec. Lett.*, vol. 35, pp. 14–15, Jan. 1999.

[45] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Wiley & Sons, 1992.

[46] A. Gorokhov, D. A. Gore, and A. J. Paulraj, "Receive antenna selection for MIMO spatial multiplexing: theory and algorithms," *IEEE Trans. Sig. Proc.*, vol. 51, pp. 2796–2807, Nov. 2003.

[47] A. Gorokhov, D. Gore, and A. Paulraj, "Receive antenna selection for MIMO flat-fading channels: theory and algorithms," *IEEE Trans. Info. Th.*, vol. 49, pp. 2687–2696, Oct. 2003.

[48] D. J. Love and R. W. Heath, Jr., "Necessary and sufficient conditions for full diversity order in correlated Rayleigh fading beamforming and combining systems," *IEEE Trans. Wireless Comm.*, Jan. 2005.

[49] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Commun.*, vol. 42, pp. 1740–1751, Feb/Mar/Apr 1994.

[50] R. Zamir and U. Erez, "A Gaussian input is not too bad," *IEEE Trans. Info. Th.*, vol. 50, pp. 1362–1367, June 2004.

[51] T. Berger, *Rate Distortion Theory: A Mathematical Basis for Data Compression*. Englewood Cliffs, N.J.: Prentice-Hall, 1971.

[52] A. Barg and D. Y. Nogin, "Bounds on packings of spheres in the Grassmann manifold," *IEEE Trans. Info. Th.*, vol. 48, pp. 2450–2454, Sept. 2002.

[53] A. Edelman, T. A. Arias, and S. T. Smith, "The geometry of algorithms with orthonormality constraints," *SIAM J. Matrix Anal. Appl.*, vol. 20, no. 2, pp. 303–353, 1998.

[54] D. J. Love and R. W. Heath, Jr., "Limited feedback unitary precoding for spatial multiplexing systems." submitted to *IEEE Trans. Info. Th.*, July 2003.

[55] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient use of side information in mulitiple-antenna data transmission over fading channels," *IEEE Jour. Select. Areas in Commun.*, vol. 16, pp. 1423–1436, Oct. 1998.

[56] R. W. Heath Jr. and A. Paulraj, "A simple scheme for transmit diversity using partial channel feedback," in *Proc. of the 32nd Annual Asil. Conf. on Sig. Sys. and Comp.*, vol. 2, pp. 1073–1078, Nov. 1998.

[57] D. J. Love and R. W. Heath, Jr., "Equal gain transmission in multiple-input multiple-output wireless systems," *IEEE Trans. Commun.*, vol. 51, pp. 1102–1110, July 2003.

[58] S. Zhou, Z. Wang, and G. B. Giannakis, "Performance analysis for transmit-beamforming with finite-rate feedback," in *Proc. Conf. on Inform. Sciences and Systems*, March 2004.

[59] B. M. Hochwald, T. L. Marzetta, T. J. Richardson, W. Sweldens, and R. Urbanke, "Systematic design of unitary space-time constellations," *IEEE Trans. Info. Th.*, vol. 46, pp. 1962–1973, Sept. 2000.

[60] T. Strohmer and R. W. Heath, Jr., "Grassmannian frames with applications to coding and communications," *Appl. Comput. Harmonic. Analysis*, vol. 14, pp. 257–275, May 2003.

[61] A. R. Calderbank, R. H. Hardin, E. M. Rains, P. W. Shor, and N. J. A. Sloane, "A group-theoretic framework for the construction of packings in Grassmannian spaces," *J. Algebraic Combin.*, vol. 9, no. 2, pp. 129–140, 1999.

[62] D. Agrawal, T. J. Richardson, and R. L. Urbanke, "Multiple-antenna signal constellations for fading channels," *IEEE Trans. Info. Th.*, vol. 47, pp. 2618–2626, Sept. 2001.

[63] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: Packings in Grassmannian spaces," *Experimental Mathematics*, vol. 5, pp. 139–159, 1996.

[64] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Boston: Kluwer, 1992.

[65] C.-H. Tse, K.-W. Yip, and T.-S. Ng, “Performance tradeoffs between maximum ratio transmission and switched-transmit diversity,” in *Proc. 11th IEEE Int. Symp. on Personal, Indoor and Mobile Radio Comm.*, vol. 2, pp. 1485–1489, Sept. 2000.

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