MULTIPLEXING ANALYSIS FOR DYNAMIC SPECTRUM ACCESS

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ABSTRACT
The problem of wireless spectrum sharing is investigated using a multi-server queueing system that models a group of channels and a multiplexed arrival process formed by a primary and secondary group of users. Motivated by under-utilized capacity in RF spectrum owned by the primary group, this work examines the impact of secondary user access patterns on blocking probabilities and achievable improvement in spectrum utilization with statistical multiplexing. The analytical framework presented allows estimation of the blocking for any renewal process characterization of secondary access times. In this work, Gamma distributed inter-access times are chosen to examine the impact of their index of dispersion (IDI) on the excess blocking probabilities of the Poisson distributed primary group. It is shown that by reducing the IDI value to be below unity, one can increase the secondary system utilization in the shared spectrum with minimal impact on the primary system. Combined with an increase in the number of excess channels available for secondary use, a feasible spectrum sharing paradigm can be proposed by shaping the access patterns of secondary systems.

1 Introduction

Low utilization of licensed radio frequency (RF) spectrum and an increased demand in the unlicensed bands has resulted in proposals for allowing secondary users access to frequency bands under the ownership of a group referred here as the primary system. The frequency chart produced by US Department of Commerce shows that all of the electromagnetic spectrum is allocated, but recent measurements [1] indicate that most of these allocations are utilized in low duty cycles. Spectrum usage is expected to vary significantly in time, geographic location and frequency. Even in the areas of intense activity, studies have shown [2], that just 35% of the spectrum below 2.4 GHz is used. In recent years, with the development of a large number of wireless data applications, the 2.4 GHz and 5 GHz unlicensed bands have grown to be congested and devoid of any interference protection. To overcome these problems, the Federal Communications Commission (FCC) is considering allocation of some of the television bands [3] to secondary users, constrained on a minimum interference basis for primary users operating on the shared bands. Berger [4] identifies two fundamental approaches to this problem. One based on leveraging capacity through physical layer access methods such as wide and ultra wideband spread spectrum and the other based on cognitive radios and mesh networks. In the latter approach, software defined radios are expected to identify usable spectrum in real-time and distribute the demand across multiple frequencies while leveraging transmission diversity in waveform shapes, space and time [5, 2]. In reality, both physical layer and medium access layer will have to be coordinated to ensure that spectrum required to meet the demands of both primary and secondary user groups is available in both time and frequency dimensions.

In recent work, rule based approaches have been proposed for secondary users accessing the spectrum. Conditioned on the knowledge of the spectrum usage of primary users, Zheng and Cao [6] propose a set of rules for collaborative, conflict-free and contention based sharing that can lead to an equilibrium state of spectrum usage for all users in the system. Here the number of available channels, number of spectrum users and their relative location influence the performance of the algorithm. The rules are similar to the channel borrowing schemes that were proposed earlier for dynamic channel assignment in cellular networks [7, 8]. The design of secondary access protocols generally rely on complete knowledge of the primary group channel usage and channel availability in time. These features being typically random, the statistical characterization of the usage variables is an important ingredient in the design of spectrum sharing protocols.

In this work, we present approaches for determining feasibility of admitting secondary users into the spectrum space conditioned on the number of excess channels available and as a function of the access patterns of the primary and secondary group. In this context, we examine the impact of controlling the arrival patterns of secondary users on the excess blocking experienced by the primary group with channel sharing. The design of cognitive radios is predicated on real-time estimation of channel availability conditions and adaptation of the access parameters in multiple dimensions such as time, frequency bandwidth, power and modulation schemes. Here we focus on the admission control problem by considering the impact of the first and second moments of the inter-access times of secondary users. The blocking performance of primary systems is examined as a function of the index of dispersion of the inter-access times (IDI) of secondary users. The IDI is defined as the ratio of variance to the square of the expected value of the inter-access times. This traffic metric is also practical in that it can be readily estimated from measurements of the access times. Having identified the range of IDI values optimal for a given channel availability and primary system load, cognitive radios may also adapt their access patterns to the spectrum so that IDI values required to minimize performance degradation of primary users are achieved.

This paper extends our previous work [9] which addressed the problem of multiplexing Poisson distributed primary and secondary sources. It was shown that Poisson distributed secondary sources render limited flexibility in controlling the increase in blocking of primary sources even at low values of system utilization. To ensure that primary sources experience better performance than secondary users, the primary sources have to be restricted to a finite population group. Under this constraint, the parametric range of secondary system utilization that will bound the blocking probabilities can be obtained. In this work we impose additional constraints on the dispersion of the secondary access times and examine the spectrum sharing performance. The multi-server queueing model of a channel bank that admits primary and secondary access is described in Section 2. The characterization of the multiplexed Poisson distributed primary sources and a general secondary arrival stream modeled as a renewal process is given in Section 3. The results of primary system blocking as a function of system loads, number of excess channels and index of dispersion of Gamma distributed secondary users are given in Section 4. Section 5 concludes the paper.

2 Multi-server Queueing Model for Spectrum Sharing

Consider a wireless system characterized by K_c channels that admits two groups of users referred here as primary and secondary groups. The primary group is assumed to access the system as a Poisson process with an arrival rate λ_p. These sources occupy $M_p \leq K_c$ channels on arrival and release all M_p channels simultaneously. The channels are held for an exponentially distributed holding time with mean value of μ_p^{-1} seconds. The primary system utilization $\rho_p = \frac{\lambda_p}{\mu_p}$. Assuming each channel has a fixed transmission bandwidth, the incorporation of the parameter M_p in the model allows use or reservation of a bank of channels by the primary group. The secondary users admitted to make opportunistic use of the available channels are restricted to the use of a single channel per access time. The system has no waiting room and blocks primary users when less than M_p channels are available. The secondary users are blocked when all K_c channels are occupied.

In the aforementioned framework, excess available bandwidth N_e is realized for cases where $N_e = K_c - N_p M_p > 0$ with $N_p = \lfloor K_c/M_p \rfloor$. It is these N_e excess channels that can become randomly available for use by secondary systems. Channels are not marked or ordered in any way. This allows the primary group to access in a non-contiguous manner, any of the M_p available channels. The secondary user inter-access times are assumed to be characterized by a general renewal process specified by a probability density function $a_s(t)$ and a mean arrival rate λ_s. The channel holding times for secondary users are assumed to be exponentially distributed, as with the primary system, but with a different service rate denoted μ_s. The offered load from secondary sources is $\rho_s = \frac{\lambda_s}{\mu_s}$. The queueing system considered is a variant of the classical $G/M/K_c/0$ [10] blocked calls cleared model with G, the arrival process corresponding to a superposition of a Poisson and a renewal process. The system state of interest is the number of channels occupied and the distribution of busy channels between primary and secondary sources.

The state dynamics are determined using a discrete-time Markov chain embedded at the arrival instants of the sources. Due to the renewal property of the arrival process, the system state as observed by the n^{th} arrival completely determines the transition probabilities to the state observed by the $(n+1)^{th}$ arrival. The state of the system observed by the n^{th} arrival to the system is represented using the variable $\eta = (i_p\ M_p + i_s), \quad i_p = 0, 1, ..N_p, i_s = 0, 1, ...K_c$ where i_p and i_s represent the number of primary and secondary sources occupying the channels respectively. The system state observed by the $(n+1)^{th}$ arrival is represented by $\nu = (j_p\ M_p + j_s) \quad j_p = 0, 1, ..N_p, j_s = 0, 1, ...K_c$. Assuming time-homogeneous arrival processes, the one step Markov transition probabilities $p(i_p, i_s; j_p, j_s)$ based on primary and secondary source occupancies are derived first. From these probabilities, the transition probabilities for aggregate channel occupancy denoted as $p_{\eta\nu},\ \eta = 0, ..K_c, \nu = 0, ...K_c$ will also be determined.

Let $A_p = \frac{\lambda_p}{\lambda_p+\lambda_s}$ and $A_s = \frac{\lambda_s}{\lambda_p+\lambda_s}$ represent the probability that an arrival is of primary or secondary type respectively. Define the function $P_c(x_p, c_p; x_s, c_s)$ to represent the probabilities of x_p primary service completions conditional on c_p primary servers occupied and x_s secondary service completions conditional on c_s secondary servers occupied. The state $\eta = i_p M_p + i_s$ is generated by combinations of $(i_p, i_s),\ i_p = 0, \lfloor \eta/M_p \rfloor,\ i_s = \eta - i_p M_p$. Let $path[i_p, i_s|\eta]$ represent the probability of reaching the state (i_p, i_s), conditional on $\eta = i_p M_p + i_s$. Due to the Markovian property being satisfied at the arrival instants, one can choose any computationally tractable path to state η. Here, the shortest path, that is, one where no departures have occurred and starting from state of zero occupancy will be considered. There are $N_s = \frac{(i_p+i_s)!}{i_p!\ i_s!}$ sequential access patterns that lead to a state with i_p primary and i_s secondary sources, starting from the zero state. The probability of occurrence of each sequence is determined by the $n = (i_p + i_s)$-fold product of the one-step transition probabilities given by $p(0, 0; i_{p_1}, i_{s_1}) \times p(i_{p_1}, i_{s_1}; i_{p_2}, i_{s_2}) \times .... \times p(i_{p_{n-1}}, i_{s_{n-1}}; i_p, i_s)$, where the arguments $i_{p_j} = i_{p_{j-1}} + 1, i_{s_j} = i_{s_{j-1}}$ if the j^{th} arrival in the sequence is a primary source or $i_{p_j} = i_{p_{j-1}}, i_{s_j} = i_{s_{j-1}} + 1$ if the j^{th} arrival in the sequence is a secondary source. Denote this product relation as $path_{seq}[l_s|(i_p, i_s; \eta)],\quad l_s = 1, 2...N_s$. Then the total probability of reaching state η under the chosen path constraint is given by,

$$total_{path}[\eta] = \sum_{i_p=0}^{\lfloor \eta/M_p \rfloor} path[i_p, \eta - M_p i_p|\eta] \quad (1)$$

with the conditional path probabilities,

$$path[i_p, i_s|\eta] = \frac{\sum_{l_s=1}^{N_s} path_{seq}[l_s|(i_p, i_s; \eta)]}{total_{path}[\eta]} \quad (2)$$

Note that the calculation of the path probabilities for (i_p, i_s) involves transition probabilities with arguments $i_{p_j} < i_p$ and $i_{s_j} < i_s$ allowing a recursive algorithm to be constructed starting from $(i_p, i_s) : (0, 0)$. Given the conditional path probabilities, the transition probabilities from state $(i_p, i_s)|\eta$ to state $(j_p, j_s)|\nu$ can be determined. Denote by $p^p(i_p, i_s; j_p, j_s)$ and $p^s(i_p, i_s; j_p, j_s)$ the transition probabilities conditional on a primary and secondary arrival respectively. To accomodate the effects of blocking of primary and secondary sources which can take place for different values of state η, consider the binary variables $i_{p_{blk}},\ i_{s_{blk}}$. For the non-blocking case, where $\eta \leq K_c - M_p$, $i_{p_{blk}} = 0, i_{s_{blk}} = 0$. For the states when the primary source is blocked, i,e. $K_c - M_p < \eta < K_c$, $i_{p_{blk}} = 1, i_{s_{blk}} = 0$, and finally for the state $\eta = K_c$ when both primary and secondary sources are blocked, $i_{p_{blk}} = 1, i_{s_{blk}} = 1$. This leads to the following representation for the transition probabilities for $i_p = 0, ..\lfloor \eta/M_p \rfloor,\ i_s = \eta - (i_p M_p)$.

On arrival of a primary source and $j_p = 0, ..i_p + 1 - i_{p_{blk}}, j_s = \nu - (j_p M_p)$, the conditional probability $p^p(i_p, i_s; j_p, j_s) = P_c(j_p - (i_p + 1 - i_{p_{blk}}), i_p + 1 - i_{p_{blk}}; j_s - i_s, i_s)$. On arrival of a secondary source and $j_p = 0, ..i_p, j_s = \nu + 1 - i_{s_{blk}} - (j_p M_p)$, the conditional probability $p^s(i_p, i_s; j_p, j_s) = P_c(j_p - i_p, i_p; j_s - (i_s + 1 - i_{s_{blk}}), i_s + 1 - i_{s_{blk}})$ From these probabilities, the transition probabilities,

$$\begin{aligned} p(i_p, i_s; j_p, j_s) &= path[i_p, i_s|\eta] * [A_p p^p(i_p, i_s; j_p, j_s) \\ &+ A_s p^s(i_p, i_s; j_p, j_s)] \end{aligned} \quad (3)$$

From Eq. (3) the transition probabilities for aggregate number of busy channels are obtained for $\nu = 0, ..K_c$,

$$p_{\eta\nu} = \sum_{i_p=0}^{\lfloor \eta/M_p \rfloor} \sum_{j_p=0}^{j_{p_{max}}} p(i_p, \eta - i_p M_p; j_p, \nu - j_p M_p) \quad (4)$$

for $j_{p_{max}} = [min(\lfloor \nu/M_p \rfloor), i_p + 1 - i_{p_{blk}}]$ The transition probabilities will allow the computation of the steady-state probability distribution of the channel occupancy, from which the blocking performance can be derived.

The computational approach presented above will be briefly illustrated with an example considering, $K_c = 3, M_p = 2$. For the case when $\eta = 0$, there is one combination of $(i_p, i_s) : (0,0)$ with $path[0,0|0] = 1.0$ Application of Eq. 3 leads to,

$$\begin{aligned} p(0,0,0,0) &= A_p P_c(1,1;0,0) + A_s P_c(0,0;1,1) \\ p(0,0,1,0) &= A_p P_c(0,1;0,0) \\ p(0,0,0,1) &= A_s P_c(0,0;0,1) \end{aligned}$$

and applying Eq. (4), $p_{00} = p(0,0,0,0)$, $p_{01} = p(0,0,0,1)$, $p_{02} = p(0,0,1,0)$,$p_{03} = 0$. Next, for $\eta = 1$, there is only one sequence $(i_p, i_s) : (0,1)$ consisting of one secondary source. Therefore $path[0,1|1] = 1.0$ and the transition probabilities,

$$\begin{aligned} p(0,1,0,0) &= A_p P_c(1,1;1,1) + A_s P_c(0,0;2,2) \\ p(0,1,0,1) &= A_p P_c(1,1;0,1) + A_s P_c(0,0;1,2) \\ p(0,1,1,0) &= A_p P_c(0,1;1,1) \\ p(0,1,0,2) &= A_s P_c(0,0;0,2) \\ p(0,1,1,1) &= A_p P_c(0,1;0,1) \end{aligned}$$

from which $p_{10} = p(0,1;0,0)$, $p_{11} = p(0,1;0,1)$, $p_{12} = p(0,1,1,0) + p(0,1;0,2)$, $p_{13} = p(0,1;1,1)$. For state $\eta = 2$, there are two combinations $(i_p, i_s) : (0,2),(1,0)$ each consisting of one possible sequence such that, $path[0,2|2] = \frac{p(0,0,0,1)*p(0,1,0,2)}{total_{path}[2]}$ and $path[1,0|2] = \frac{p(0,0,1,0)}{total_{path}[2]}$ where $total_{path}[2] = p(0,0,0,1) * p(0,1,0,2) + p(0,0,1,0)$. The calculation of the transition probabilities from state $\eta = 2$ then proceeds as in the previous states. For state $\eta = 3$, there are two combinations, $(i_p, i_s) : (1,1),(0,3)$ each having path probabilities, $path[1,1|3] = \frac{p(0,0,1,0)*p(1,0,1,1)+p(0,0,0,1)*p(0,1,1,1)}{total_{path}[3]}$, $path[0,3|3] = \frac{p(0,0,0,1)*p(0,1,0,2)*p(0,2,0,3)}{total_{path}[3]}$ with $total_{path}[3] = p(0,0,1,0) * p(1,0,1,1) + p(0,0,0,1)*p(0,1,1,1)+p(0,0,0,1)*p(0,1,0,2)*p(0,2,0,3)$.

The service completion probabilities are governed by the service rate attributed to each type of user and the distribution of inter-access times generated by the superposition of the Poisson process and the renewal process $a_s(t)$ of the secondary users. A general technique for computing the multiplexed arrival process is given in the next section. Denoting the probability density function (pdf) of the inter-access times of the multiplexed process as $a_m(t)$, the service completion probabilities can be determined considering that the number of completions in a given interval will be Poisson distributed with the rates μ_p and μ_s for the primary and secondary groups respectively. Since the primary and secondary source servers are independent,

$$\begin{aligned} P_c(x_p, c_p; x_s, c_s) &= \frac{c_p!}{(c_p - x_p)!x_p!} P_{cp}(x_p, c_p) \times \\ &\quad \frac{c_s!}{(c_s - x_s)!x_s!} P_{cs}(x_s, c_s) \quad (5) \end{aligned}$$

where

$$\begin{aligned} P_{cp}(x_p, c_p) &= \int_0^\infty a_m(t) \frac{(c_p \mu_p t)^{x_p}}{x_p!} e^{-c_p \mu_p t} dt \\ P_{cs}(x_s, c_s) &= \int_0^\infty a_m(t) \frac{(c_s \mu_s t)^{x_s}}{x_s!} e^{-c_s \mu_s t} dt \end{aligned}$$

Eq. (5) can be exactly determined for both Poisson and Gamma distributed secondary source inter-access times.

3 Model for Multiplexed Primary and Secondary Sources

This section provides a general derivation for computing the distribution of inter-arrival times generated by multiplexing Poisson processes with arrival rate λ_p and a renewal process with pdf $a_s(t)$. Denote the complementary cumulative distribution function as $A_s^c(t)$. The approach follows that presented by Akimaru et al.[11]. The exponentially distributed inter-access times of the primary user is denoted T_p, whereas that of the secondary user is denoted as T_s. The forward recurrence times of the renewal process given by $a_s(t)$ are represented by T_{s^r}. Let τ_p and τ_s represent the inter-access times in the multiplexed process as observed by a primary and secondary arrival respectively. Then $\tau_p = min(T_p, T_{s^r})$ and $\tau_s = min(T_p, T_s)$. From these transformations, the cumulative distribution functions (cdfs) for T_p and T_s are given as,

$$F_p(t) = 1 - e^{-\lambda_p t} A_{s^r}^c(t) \quad (6)$$

$$F_s(t) = 1 - e^{-\lambda_p t} A_s^c(t) \quad (7)$$

The cdf and pdf of the multiplexed arrival process are then given by weighting each distribution by the arrival probabilities, A_p, A_s.

$$A_m(t) = A_pF_p(t) \ + \ A_sF_s(t)$$
$$a_m(t) = A_pF_p'(t) \ + \ A_sF_s'(t) \tag{8}$$

The forward recurrence times are distributed with pdf $a_{s^r}(t) = \frac{A_s^c(t)}{E[T_s]}$ where $E[T_s]$ is the average inter-access time of the secondary arrival process, given by $1/\lambda_s$. Making the substitutions in Eqs. (6) and (7) so that the result for $a_m(t)$ can be derived using the specified $a_s(t)$ and $A_s^c(t)$ and substituting

$$F_p'(t) = e^{-\lambda_p t}\left[\lambda_p\lambda_s\int_t^\infty A_s^c(\tau)d\tau + \lambda_sA_s^c(t)\right] \tag{9}$$

$$F_s'(t) = e^{-\lambda_p t}\left[\lambda_pA_s^c(t) \ + \ a_s(t)\right] \tag{10}$$

for $a_m(t)$ in Eq. 8 yields the required pdf that allows computation of the service completion probabilities given in. Eq. 5.

4 Primary System Blocking Performance

The blocking performance of primary users when secondary sources are allowed to share excess channels is examined in this section. The analysis here considers Gamma distributed inter-arrival times for the secondary sources. The Gamma distribution is specified as $a_s(t) = \frac{(\lambda_s t)^{n-1}}{(n-1)!}\lambda_s e^{-\lambda_s t}$ with average inter-arrival time $\lambda_s{}^{-1} = \frac{n}{\lambda}$ and a variance of $\frac{n}{\lambda_s^2}$. The index of dispersion (IDI) of this renewal process is therefore $1/n$. Note that by considering $n = 1$ the analysis also includes the effect of a Poisson distributed secondary source. By considering values of $n > 1$ one can allow the secondary process to access the channel pool with indices of dispersion less than one. This effect is seen to significantly reduce the performance impact on the primary group blocking.

In the absence of secondary users sharing the channels, the blocking probability of primary system approaches the Erlang-B blocking given by the $M/M/N_p$ loss system [10]. This will serve as the reference blocking level and is denoted as P_{blk}^o. Note that since all K_c channels are not utilized in this system the system utilization will generally be less than the offered load ρ_p. The objective of allowing secondary access is to improve this underutilized spectrum. When the IDI of the secondary users is controlled to be below unity, the superposition process results in a structure for which the probability of small inter-arrival times is reduced and probability of larger inter-arrival times is increased. An example is shown in Fig. 1.

This feature will reduce collisions with the primary arrivals and increase the overall utilization of the spectrum. The excess blocking of primary source $P_e = P_{blk}^p - P_{blk}^o$ due to the secondary users accessing the system will be used to understand the influence of secondary group access parameters. Fig. 2 compares the excess blocking probability of primary users for Poisson distributed secondary users as a function of ρ_p and ρ_s. The different surfaces represent the results for increasing $N_e = 1, 2, 4, 8$. The blocking probability increases exponentially from the reference value for a given N_e as ρ_s is increased. With increase in N_e both the reference and the excess blocking level can be decreased.

Fig. 3 and Figure. 4 compare the excess blocking probability P_e of primary users for Gamma distributed secondary users as a function of ρ_p, ρ_s for cases $n = 2$ and $n = 10$ resulting in IDI values of 0.5 and 0.1 respectively. An observation of the rate at which excess blocking probability increases with increase in ρ_s shows important differences between the Poisson and Gamma distributed cases. When the IDI reduces below unity value, a sub-exponential trend is evident in the rate of increase in Figs. 3 and 4. For $N_e = 1$, the excess blocking probability of primary group tends to be zero until $\rho_s > 0.2$ in Fig. 3 and until $\rho_s > 0.3$ in Fig. 4. With increase in $N_e = 2$, the range of admittable ρ_s is extended to 0.6 and 0.8 in the cases of IDI equal to 0.5 and 0.1 respectively. This is in contrast to the Poisson distributed case of Fig. 2 where the excess blocking increases almost immediately as ρ_s increases from zero. The influence of IDI in improving the blocking performance of primary with increasing ρ_s is demonstrated in Fig. 5. With $N_e = 2$ excess channels, the blocking probability of primary with secondary source admission is very close to P_{blk}^o until ρ_s = 0.6 for IDI = 0.5 and ρ_s = 0.8 for IDI = 0.1. Fig. 6 compares the increase in spectrum utilization $\rho = 1 - \Pi[0]$ of the system as the primary and secondary load ρ_p and ρ_s is increased. The reference surface depicts the system utilization in the absence of secondary sources.

5 Conclusions

The problem of sharing spectrum between a primary and secondary group of sources has been analyzed using a multi-server queueing model for the available channel pool. The effect of changing the access patterns of secondary sources on blocking performance of primary group is examined. An analytical framework is presented that will allow the blocking probabilities to be computationally estimated given the renewal process characterization of the secondary access times. It is shown that a sub-exponential rate of increase in the excess blocking experienced by the primary group can be achieved as a function of increased offered load by considering indices of dispersion of the secondary source arrival process that are less than unity. This feature combined with an increase in number of excess channels available for secondary use can render feasible operating regimes for the co-existence of primary and secondary sources in the shared spectrum. By controlling the dispersion, the overall spectrum utilization is also found to be significantly improved.

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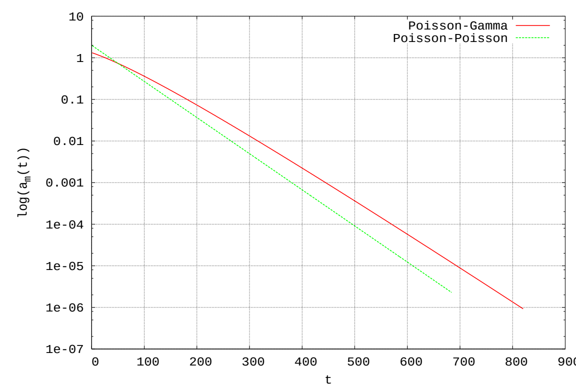

Figure 1. Inter-arrival time pdf for multiplexed Poisson-Poisson and Poisson-Gamma cases.

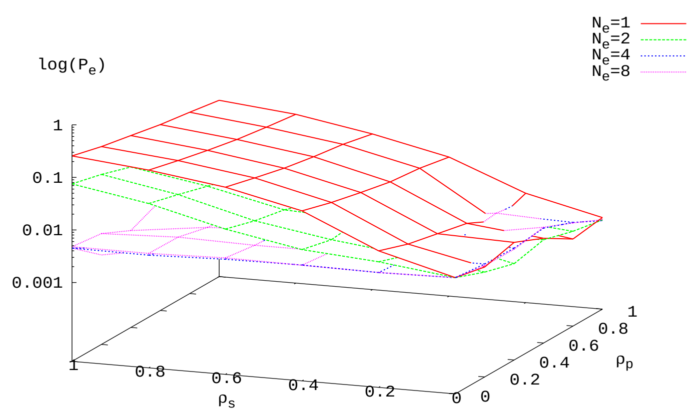

Figure 2. P_e of primary with Poisson distributed secondary for K_c = 18 and N_e = 1,2,4,8

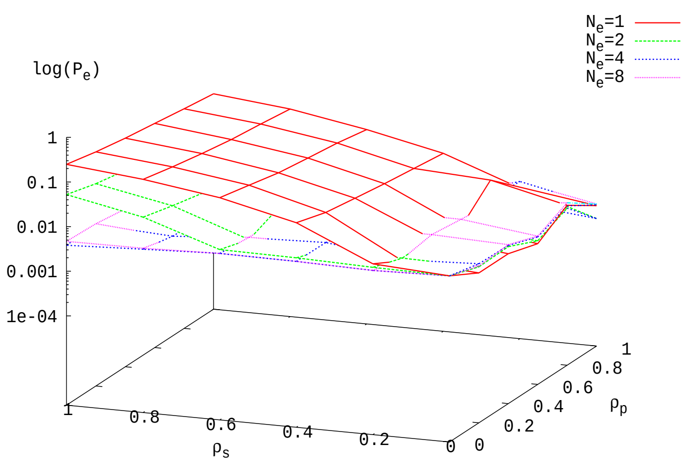

Figure 3. P_e of primary with Gamma distributed secondary for N_e = 1,2,4,8 and IDI = 0.5

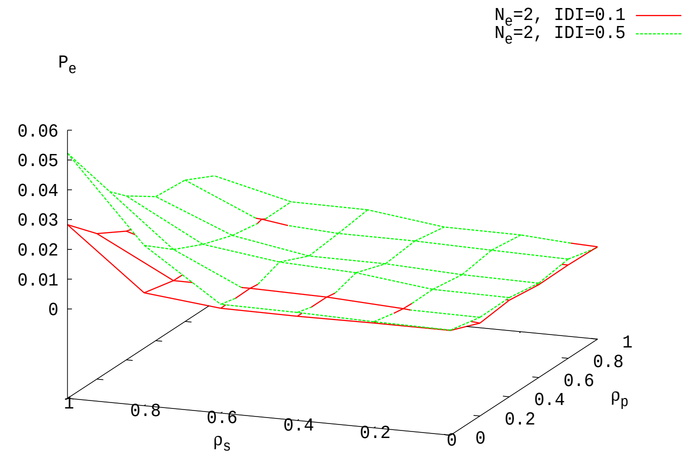

Figure 5. Influence of IDI on blocking of primary with Gamma distributed secondary for N_e = 2 and IDI = 0.1,0.5

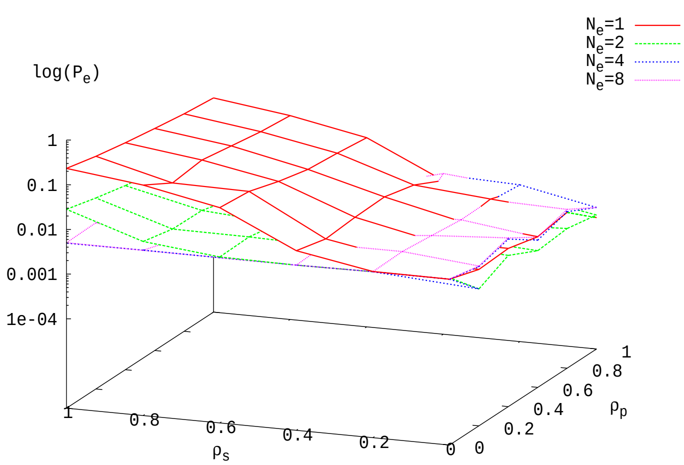

Figure 4. P_e of primary with Gamma distributed secondary for N_e = 1,2,4,8 and IDI = 0.1

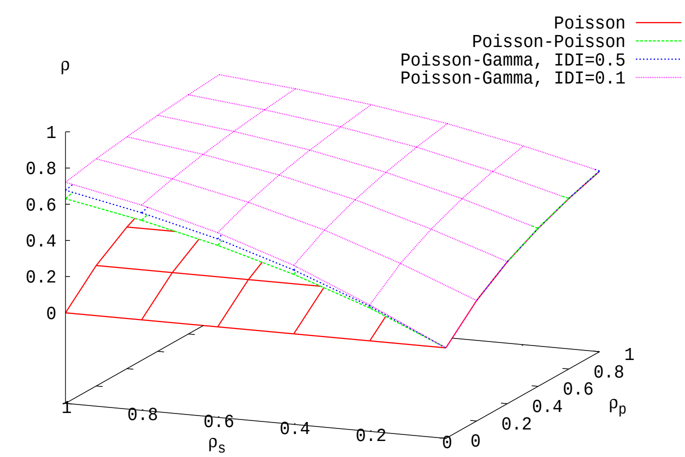

Figure 6. Total utilization of the system with only primary and Poisson, Gamma distributed secondary for N_e = 1 and IDI = 1,0.5,0.1