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# Discussion on: "Application of Logarithmic-based Parameter and Upper Bounding Estimation Rules to Adaptive-Robust Control of Robot Manipulators

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## Discussion

A combination of robust and adaptive controllers is proposed in this paper to perform tracking for robot manipulators with uncertainty in parameter values. It is shown by simulation studies involving a two-link planar robot that the proposed controller performs better tracking than either the adaptive or the robust controller alone.

### 1. Main Ideas and Contributions

The general dynamic model of an *n*-link manipulator is considered. It is assumed that the joint positions, velocities and accelerations are measurable without error but there is uncertainty regarding the system parameters (including payload, etc.). The problem is to track desired values of joint positions, velocities and accelerations under this uncertainty. Motivated by the partial and perhaps complementary success of previous adaptive [1], [2], robust [3] and robust-adaptive [4,5] control designs, a new robust-adaptive control design is introduced.

To facilitate the development of a robust-adaptive controller, a new term is added to the usual Lyapunov function which is quadratic in the position and the velocity errors. Since it is known that the usual Lyapunov function has a negative semidefinite derivative along the trajectory when a PD type controller is used, the remaining terms in the derivative of the new Lyapunov function need to be negative-semidefinite to maintain at least stability under additional control action. So the additional terms are divided into two groups according to whether they contain the estimate of the parameters or the estimate of the upper bounds to parameter errors and are set to zero separately. Then estimation algorithms are given for the parameters (adaptive part) and the parameter error upper bounds (adaptive-robust part). Simulation results are provided to justify the use of combined adaptive and adaptive-robust controllers in improving tracking capability.

The notation used becomes confusing at times, which detracts from the readability of the paper. For example, in Eq. (23), two different symbols are introduced for the definition of the same quantity i.e. $\tilde{\pi} = \hat{\pi} - \pi = \rho(t)$. Also $\hat{\rho}(t)$ is defined two lines below equation (23) as the upper bound on the parameter estimation error and also later as the estimate of the same quantity. Another confusion regarding notation arises when $\pi$, which was originally defined above Eq. (5) as unknown, is used in the applied torque expression (53) as if it were a known or a measurable quantity. Our guess, based on the previous section on robust control and the simulation section later, is that this new $\pi$ is some known nominal value $\pi_0$ of the uncertain parameter vector.

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**2. Directions for Future Research**

Splitting Eq. (36), which is in the form A + B = 0 into two equations (37) and (38) which are each in the form A = 0 and B = 0, that are sufficient when considered together but not necessary for (36), obviously adds conservativeness to the result. So the possibility of dealing with the two sets of terms in (36) together can be explored.

Another issue in splitting Eq. (36) is, in which of the Eqs (37) or (38) (or both), should the term $Y^{T}\sigma$ be included. In the present case, it is only included in (37). The question, therefore, is, for example, if we were to include fractions of this term, in both (37) and (38), and derive the controllers accordingly, would this result in anything different (better or worse) than the present performance?

Another investigation can be conducted regarding the limits of additional on-line computation. Since the apparent improvement in performance shown by simulation studies is obtained at the expense of combining multiple (adaptive and robust) control actions using multiple parameter estimates, the trade-off between the additional on-line computational requirement and the improvement in tracking performance should be seriously looked into from a practical implementation viewpoint.

## References

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# Discussion on: "Application of Logarithmic-based Parameter and Upper Bounding Estimation Rules to Adaptive-Robust Control of Robot Manipulators"

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## Discussion

The paper by R. Burkan and I. Uzmay deals with the classical problem of the control of robot manipulators under parameter uncertainty which has been extensively studied in the literature as the list of references in the paper indicates. The main proposed contribution of the paper consists of combining the well known adaptive approach of Slotine and Li [6] and a robust approach inspired from Spong [7]. The result is what is referred to as the "adaptive-robust" approach. The authors performed extensive simulation studies to compare the adaptive [6] and robust [7] approaches against the proposed control law.

This discussion focuses on two areas. The first is related to well-known technical results that would make the proof of the main result in the paper more rigorous. The second addresses the conclusions presented in the simulation section of the paper.

The proof of convergence of tracking error to zero in the adaptive control law of Slotine and Li [6] has been rigorously established in the literature. Using the notation of the paper, it is well known that for the dynamic model (1) with the adaptive control law (5), and the Lyapunov Function candidate $V(\sigma, \tilde{q}, \tilde{\pi})$ given by (8) and satisfying $\dot{V} \leq 0$ given by (11), the result is a "stable" controlled system (1)–(5) in the sense of Lyapunov. Since tracking is considered, and hence (1)–(5) is a non-autonomous ordinary differential equation system, LaSalle's Theorem [3] cannot be used to conclude asymptotic convergence of tracking error to zero. Instead, additional arguments need to be

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used [4] as follows. We first introduce the following standard notation and terminology [2]: $\Re_+$ will denote the set of non-negative real numbers, and $\Re^n$ will denote the usual $n$-dimensional vector space over $\Re$ endowed with the Euclidean norm $\|\mathbf{x}\| = \left\{\sum_{i=1}^{n} \mathbf{x}_i^2\right\}^{1/2}$. We define the standard Lebesgue spaces $L_\infty^n$ and $L_2^n$ as

$$L_\infty^n(\Re_+) = \{f : \Re_+ \to \Re^n \text{ such that } f \text{ is Lebesgue measurable and } \|f\|_\infty < \infty\},$$

where $\|f\|_\infty = ess \sup_{t\in[0,\infty)} \|f(t)\|$, and

$$L_2^n(\Re_+) = \{f : \Re_+ \to \Re^n \text{ such that } f \text{ is Lebesgue measurable and } \|f\|_2 < \infty\},$$

where $\|f\|_2 = \left\{\int_0^\infty \|f(t)\|^2 \, dt\right\}^{1/2}$

From (11), we conclude that $\tilde{q}, \dot{\tilde{q}} \in L_\infty^n$ and $\tilde{\pi} \in L_\infty^p$. Furthermore, $\tilde{q}$, $\dot{\tilde{q}} \in L_2^n$. Since $\tilde{q} \in L_\infty^n \bigcap L_2^n$ and $\dot{\tilde{q}} \in L_\infty^n$, and as a corollary to Barbalat Lemma [5], it follows that $\tilde{q} \to 0$. Using the boundedness of $\tilde{q}$, $\dot{\tilde{q}}$, and $\tilde{\pi}$ along with (6), it is easy to verify that $\ddot{\tilde{q}} \in L_\infty^n$. Since in addition $\dot{\tilde{q}} \in L_2^n$, it follows that $\dot{\tilde{q}} \to 0$. This type of argument could also be used in the proof of main result in Section 4 to conclude the convergence of the tracking error to zero.

The authors proposed to cancel the constant $C$ in (47). It was assumed that the initial condition of the estimate of the parameter vector is the true (unknown) value of the parameter vector, namely, $\hat{\pi}(0) = \pi$. This is of course not feasible; if we knew the true parameters, there would have been no need to consider adaptive and robust control. In fact, it is not necessary to force $C$ to be zero and the results of the paper could be maintained without this assumption.

The simulation section of the paper is based on tracking error comparison of the proposed adaptive-robust control law with Spong's robust control law [7] and Slotine and Li's adaptive control law [6]. The authors succeed in demonstrating that their control law is superior in the sense of tracking error in certain simulations. However, they are weakening their case by failing to consider other important measures. One such measure is torque input $\tau(t)$ which gives an indication of the required actuator effort to achieve tracking convergence. It seems that the authors' input torque $\tau$ is at best competitive. Another measure is the real-time computational burden which gives an indication of the complexity and practicality of the control law. Clearly, the authors' computational burden is less attractive. Another important point to be noted is that Spong's robust control law [7] guarantees only "uniform ultimate boundedness" [1], making the comparison of tracking error unfair.

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# Final comments by the authors

R. Burkan, I. Uzman

## 1. Response to Ghorbel's Comment

The method presented in our paper provides both a stable controlled system and also global convergence of tracking error to zero. It can be seen from different studies, such as Refs [1] (pp. 233–236) and [4] (in Section V). Using the main result in the adaptive control law [1,3] and robust control law [4] (Section V), the tracking error converges to zero for the proposed adaptive-robust control law.

As well known and used in adaptive [1,3] and robust control law [4], it is assumed that the initial value of link parameter estimation is known at the begining [1] (on page 243), [4] (in Section V), and during the control process, these links parameters are tried to adapt to their true values for tracking error to converge to zero. Consequently, it does not mean that link parameters are known perfectly.

Considering the previous results and assumptions [1,4] which provide the basis of this study, initial estimation of the parameter vector is taken as $\hat{\pi}(0) = \pi$. As a result, $C$ is forced to be zero.

For explanation, the function of $\hat{\pi}$ is given as;

$$\hat{\pi} = \Gamma^{-1}\left(\int \Gamma^{-1} Y^{\mathrm{T}} \sigma \, \mathrm{d}t\right) + \pi + C\Gamma^{-1}$$

As seen from the equation, it is also possible to solve this equation using other assumption.

The robust control algorithm is also simple but suffers from uncertainity, if uncertainity is large and chattering happens. Adaptive approaches are feasible for wider variation range of parameter, but it is not sensible for unmodelled dynamics and disturbances. Another disadvantage of adaptive controller is to exhibit poor transient behaviour. Adaptive-robust control law eliminates the disadvantage of the pure adaptive and robust controllers and improves the transient performance and decreases the steady-state error [2]. From the expressions, performance of the adaptive-robust control law is better than a pure adaptive and a pure robust control law.

It is the expected result that the performance of the proposed adaptive-robust controller should be better than the pure adaptive and pure robust control law. Since Spong model (2 DOF planar robot) provides the basis of this study, computer simulation is carried out on the model (2 DOF planar robot) under the same conditions and the simulation results verify the expected result.

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## 2. Response to Yaz's Comment

$\hat{\rho}(\mathrm{t})$ is defined as upper bounding function in order to derive adaptive robust control input. The $\rho(\mathrm{t})$ and $\hat{\rho}(\mathrm{t})$ are different. In order to explain the new adaptive-robust controller well, the pure adaptive control law [1,2], the pure robust control law [3], the Spong's model [3] and explanations about the parameters are given in the paper, and computer simulation is carried out on the Spong's model (2 DOF planar robot) under the same conditions.

As well-known and used in adaptive [1,2] and robust control law [3], it is assumed that the initial value of link parameter estimation is known at the beginning, [1] (on page 243), [3] (in Section V), and during the control process, these links parameters are tried to adapt to their true values for tracking error to converge to zero. Consequently, it does not mean that link parameters are known perfectly.

Considering the previous results and assumptions [1–3] which provide the basis of this study, initial estimation of the parameter vector is taken as $\hat{\pi}(0) = \pi$.

In Eq. (36), there are three unknowns such as $\Gamma$, $\hat{\pi}$ and $\hat{\rho}(\mathrm{t})$. The $\hat{\pi}$ and $\hat{\rho}(\mathrm{t})$ are derived after selection of appropriate function of $\Gamma$ then there are two unknown function such as $\hat{\pi}$ and $\hat{\rho}(\mathrm{t})$. Since there are two unknown functions, we splited Eq. (36) into two parts.

For explanation, $\hat{\pi}$ is given as

$$\hat{\pi} = \Gamma^{-1}\left[\int \Gamma^{-1} Y^{\mathrm{T}} \sigma \, \mathrm{d}t\right] + \pi$$

$\hat{\rho}(\mathrm{t})$ is derived as

$$\hat{\rho}(\mathrm{t}) = -\Gamma^{-1} \rho$$

The control input is

$$\begin{aligned} \tau &= Y^{\mathrm{T}} \sigma(\hat{\pi} + u(t)) \\ &= Y^{\mathrm{T}} \sigma\left(\Gamma^{1}\left(\int \Gamma^{-1} Y^{\mathrm{T}} \sigma \, \mathrm{d}t\right) + \pi + \Gamma^{-1} \rho\right) \end{aligned}$$

If we include fractions of terms $Y^{\mathrm{T}}\sigma$ in both (37) and (38), the same result is obtained.

For explanation, Eq. (36) is given.

$$-Y^{\mathrm{T}} \sigma + \Gamma \dot{\Gamma}(\hat{\pi} - \pi) + \Gamma^2 \dot{\hat{\pi}} - [\Gamma \dot{\Gamma} \hat{\rho}(\mathrm{t}) + \Gamma^2 \dot{\hat{\rho}}] = 0 \quad (36)$$

If fractions of the terms $Y^{\mathrm{T}}\sigma$ are include in both Eqs (37) and (38), the two different equations can be obtained from Eq. (36) as follows.

$$-\frac{1}{2} Y^{\mathrm{T}} \sigma + \Gamma \dot{\Gamma}(\hat{\pi} - \pi) + \Gamma^2 \dot{\hat{\pi}} = 0 \quad (37)$$

$$-(\Gamma \dot{\Gamma} \hat{\rho}(\mathrm{t}) + \Gamma^2 \dot{\hat{\rho}}(\mathrm{t})) = \frac{1}{2} Y^{\mathrm{T}} \sigma \quad (38)$$

Then $\hat{\pi}$ is derived as

$$\hat{\pi} = \frac{1}{2}\Gamma^{1}[\int \Gamma^{-1} Y^{T}\sigma \, \mathrm{d}t] + \pi$$

$\hat{\rho}(t)$ is derived from Eq. (38) as

$$\hat{\rho}(\mathrm{t}) = -\frac{1}{2}\Gamma^{-1}[\int \Gamma^{-1} Y^{T}\sigma \, \mathrm{d}t] - \Gamma^{-1}\rho$$

The control input $\tau = Y^{T}\sigma(\hat{\pi} + u(t))$ is written as [since $u(t) = -\hat{\rho}(t)$)

$$\tau = Y^{T}\sigma(\hat{\pi} + u(t)) = Y^{T}\sigma\left(\frac{1}{2}\Gamma^{1}[\int \Gamma^{-1} Y^{T}\sigma \, \mathrm{d}t] + \pi + \frac{1}{2}\Gamma^{-1}[\int \Gamma^{-1} Y^{T}\sigma \, \mathrm{d}t] + \Gamma^{-1}\rho\right)$$

Then,

$$\tau = Y^{T}\sigma(\hat{\pi} + u(t)) = Y^{T}\sigma(\Gamma^{1}[\int \Gamma^{-1} Y^{T}\sigma \, \mathrm{d}t] + \pi + \Gamma^{-1}\rho)$$

The same result is obtained.

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