Resource Allocation in Wireless Networks with Multiple Relays

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***Abstract*— A cooperative network where transmission between two nodes that have no direct link, but assisted by many relays is considered. We assume a broadband system, such as OFDM, modeled by multiple parallel Gaussian subchannels between the source and each relay, and also between each relay and the destination. We formulate the optimization problem for joint power and subchannel allocation under a short term per-node power constraint to maximize the instantaneous total transmission rate between the source and the destination. To solve this optimization problem, first we find the optimal power allocation for a given subchannel allocation. Then we focus on a greedy algorithm that jointly allocates subchannels and power. Simulation results reveal that our proposed algorithm results in rates close to the optimum allocation.**

I. INTODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a promising technique for broadband wireless networks. OFDM can mitigate the adverse effects of frequency selective multipath fading by transmitting signals over a number of narrowband channels which typically experience different fading levels. When the subchannel gains are known at the transmitter, significant performance improvement can be achieved by dynamic power allocation [1].

In a wireless environment, the source and the destination nodes can be assisted by intermediate nodes when the direct link is not available. The information from the source to the destination is then transmitted by multi-hopping. Multihop transmission is being considered to improve coverage and increase throughput for the next generation of wireless networks, for instance 802.16j.

In this paper, we study a broadband system where the source-destination pair with no direct link is assisted by multiple relays which utilize decode-and-forward. The transmission is done in two hops where the source forwards the information to the relays and the relays forward the decoded information to the destination. The channels between nodes are assumed to be frequency selective and OFDM is used in both hops. We study resource allocation, namely subchannel and power allocation to maximize the end-to-end rate.

An overview of resource allocation techniques applied to relay networks is provided by [2]. In [3], [4] a cooperative single relay network is considered where nodes have partial channel state information (CSIT) about the mean channel attenuation. Optimal power allocation for amplify and forward (AF) cooperation has been investigated in [5]. For a decode and forward (DF) strategy, time and power are optimized in [6], under a constraint on the average total system power, with the goal of either maximizing the delay-limited capacity or minimizing the outage probability. Ergodic capacity for a DF relay system has been also explored under various CSIT assumptions (see [7], [6] and references therein). Some of these papers do not consider OFDM. It would be classify them as so, that makes our contributions more significant. In [8], a broadband relay channel where each link is composed of many parallel, independent Rayleigh fading channels is analyzed. Under a long term total average power constraint per node, power and transmission time are dynamically allocated either to improve the delay limited capacity or to decrease the outage probability. However, [8] assumes the availability of direct link.

Resource allocation is even more critical when multiple relays are available, however only a few papers in literature consider this scenario. In [3], [4], multiple relays are only considered when available in clusters around the source and the destination node. Opportunistic AF relaying has been investigated for multiple-relay networks in [9] and [10]. Resource allocation for a wireless multihop network with multiple sources, multiple relays and one destination is studied in [15]. A multihop network with one source, one destination and multiple relays where only one relay is utilized in each hop is investigated in detail.

The problem we address can be thought of as a combination of downlink and uplink resource allocation problems for multiuser OFDM. On the other hand, rate matching in each relay, that is the constraint that outgoing rate at each relay is bounded by the incoming rate has to be maintained. This constraint makes the resource allocation problem more challenging. In this context, we formulate an optimization problem over resource allocation strategies, namely subchannel and power allocation, over the source and relays to maximize the end-to-end rate. We first establish the optimal power allocation for a given subchannel assignment. Then, we propose a greedy algorithm for end-to-end rate maximization problem which jointly allocates subchannels and power.

Downlink and uplink resource allocation for multiuser OFDM are well investigated topics. In [13] the sum rate is maximized in downlink. Sum rate maximization with fairness and proportionally fairness among users is considered in [11] and [12] respectively. Also in [14] uplink resource allocation problem is studied. These papers studied downlink and uplink problems separately, the challenging part, rate matching, is not considered.

The remainder of this paper is organized as follows: In Section II system model is introduced and an optimization problem to maximize the instantaneous end-to-end rate is formulated. In Section III, the optimum power allocation is established for a given subchannel allocation. In Section IV, we propose a greedy

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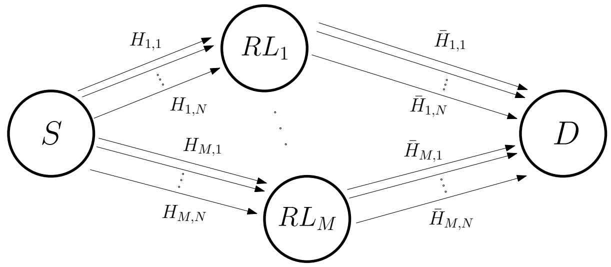

Fig. 1. System model

algorithm which jointly allocates subchannels and power. Then numerical results and conclusion follow.

II. System Model and Problem Formulation

We consider a relay network with a source node $\mathtt{S}$, a destination node $\mathtt{D}$ and M relay nodes $\mathtt{RL}_m$, $m = 1, 2, \ldots, M$, employing decode and forward. We assume that there is no direct link between $\mathtt{S}$ and $\mathtt{D}$ and all transmissions are performed in two hops with the assistance of relays. The links among source/destination and each relay are composed of N parallel subchannels corrupted by independent, unit variance complex additive white Gaussian noise as shown in Fig. 1. For subchannel n, $h_{m,n}$ denotes the complex channel gain of the $S - RL_m$ link and $\bar{h}_{m,n}$ denotes the channel gain of the $RL_m - D$ link. The corresponding power gains are $H_{m,n} = |h_{m,n}|^2$ and $\bar{H}_{m,n} = |\bar{h}_{m,n}|^2$. The source and relay nodes are subject to short term power constraints, P_S and P_R, respectively.

The relays are assumed to be half-duplex, thus receiving and transmitting in orthogonal time slots. To this end, communication takes place in two phases comprising of equal time slots: Phase 1 and phase 2. In phase 1, the source transmits to relays and in phase 2 relays forward the information they decode to the destination. All subchannels can be used in both phases. In order to keep the implementation simple, we do not consider distributed space-time coding and impose a constraint on subchannel allocation where only one relay can be assigned to each subchannel in both phase 1 and 2.

Let the set $\mathtt{E}_m$ and $\bar{\mathtt{E}}_m$ be the sets of subchannels which are assigned to $\mathtt{RL}_m$ in phase 1 and phase 2, respectively. Moreover, source power allocated to subchannel n in phase 1 and $\mathtt{RL}_m$ power allocated to subchannel n in phase 2 are P_n and $\bar{P}_{m,n}$, respectively.

Defining the total rate from $\mathtt{S}$ to $\mathtt{RL}_m$ in phase 1 as R_m and the total rate from $\mathtt{RL}_m$ to $\mathtt{D}$ phase 2 as $\bar{R}_m$, we have

$$R_m = \frac{1}{2} \sum_{n \in \mathtt{E}_m} \log_2(1 + H_{m,n} P_n) \quad (1)$$

$$\bar{R}_m = \frac{1}{2} \sum_{n \in \bar{\mathtt{E}}_m} \log_2(1 + \bar{H}_{m,n} \bar{P}_{m,n}) \quad (2)$$

Since relays utilize decode-and-forward, the contribution of $\mathtt{RL}_m$ to the end-to-end rate is limited by the minimum of phase 1 and phase 2 rates, R_m and $\bar{R}_m$, respectively. Our aim is to find optimum subchannel allocation, $\mathtt{E}_m$ and $\bar{\mathtt{E}}_m$ and power allocations, P_n and $\bar{P}_{m,n}$, which maximize the instantaneous end-to-end achievable rate, R_{total}. The resulting optimization problem can be formulated as

$$\max_{P_n, \bar{P}_{m,n}, \mathtt{E}_m, \bar{\mathtt{E}}_m} R_{total} = \max_{P_n, \bar{P}_{m,n}, \mathtt{E}_m, \bar{\mathtt{E}}_m} \sum_{m=1}^{M} \min\left(R_m, \bar{R}_m\right) \quad (3a)$$

subject to

$$\sum_{n=1}^{N} P_n \le P_S \quad (3b)$$

$$\sum_{n=1}^{N} \bar{P}_{m,n} \le P_m \ \forall m \in \{1, \ldots, M\} \quad (3c)$$

$$P_n, \bar{P}_{m,n} \ge 0 \text{ for all } m,\ n \quad (3d)$$

$$\mathtt{E}_1, \mathtt{E}_2, \cdots \mathtt{E}_M \text{ are disjoint} \quad (3e)$$

$$\bar{\mathtt{E}}_1, \bar{\mathtt{E}}_2, \cdots \bar{\mathtt{E}}_M \text{ are disjoint} \quad (3f)$$

$$\mathtt{E}_1 \cup \mathtt{E}_2 \cup \cdots \cup \mathtt{E}_M \subset \{1, \cdots, N\} \quad (3g)$$

$$\bar{\mathtt{E}}_1 \cup \bar{\mathtt{E}}_2 \cup \cdots \cup \bar{\mathtt{E}}_M \subset \{1, \cdots, N\} \quad (3h)$$

Note that, in this optimization problem, the sum of the relay bottleneck rates is maximized. This results in a combination of downlink and uplink resource allocation for multiuser OFDM. With rate matching in each relay, the coupling of phase 1 and phase 2 leads to a challenging problem since an optimal algorithm has to solve phase 1, phase 2 and rate matching problems jointly. Moreover, the optimization includes set selection, i.e. decision of $\mathtt{E}_m$ and $\bar{\mathtt{E}}_m$ and the objective function involves minimum of two functions, and hence not a convex problem. Instead of solving this problem optimally, we focus on providing a suboptimal solution in two steps.

We first find optimal power allocation for given subchannel assignments in phase 1 and 2. Then, based on this power allocation, a greedy algorithm is proposed which jointly allocates subchannels and power.

III. Optimal Power Allocation for a given subchannel assignment

In this section, we investigate the optimum power allocation that achieves the maximum end-to-end rate for a given subchannel assignment. In other words, we focus on the optimization problem (3a) together with power constraints, (3b), (3c) and (3d) , when the sets $\mathtt{E_m}$ and $\bar{\mathtt{E}}_\mathtt{m}$ are given for all $m \in \{1, \cdots, M\}$. Then, the optimization problem for power allocation can be written as

$$\max_{P_n, \bar{P}_{m,n}} \sum_{m=1}^{M} \min\left(R_m, \bar{R}_m\right) \quad (4a)$$

$$\sum_{n=1}^{N} P_n \le P_S \quad (4b)$$

$$\sum_{n=1}^{N} \bar{P}_{m,n} \le P_m \ \forall m \in \{1, \ldots, M\} \quad (4c)$$

$$P_n, \bar{P}_{m,n} \ge 0 \text{ for all } m,\ n \quad (4d)$$

where R_m and $\bar{R}_m$ are the rates of the $\mathtt{RL}_m$ in phase 1 and phase 2, respectively and are given in (1) and (2).

Theorem 3.1: For a given subchannel assignment, $\mathtt{E_m}$ and $\bar{\mathtt{E}}_\mathtt{m}$ for all $m \in \{1, \cdots, M\}$, the optimal power allocation that solves (4a) - (4d) for $\mathtt{RL}_m$ is found by waterfilling

$$\bar{P}^*_{m,n} = \left(\frac{1}{\bar{\lambda}_m} - \frac{1}{\bar{H}_{m,n}}\right)^+ \text{ for } n \in \bar{\mathtt{E}}_\mathtt{m} \quad (5)$$

where $1/\bar{\lambda}_m$ is chosen to satisfy $\mathtt{RL}_m$ power budget, P_R. The optimal power allocation of the source is given by

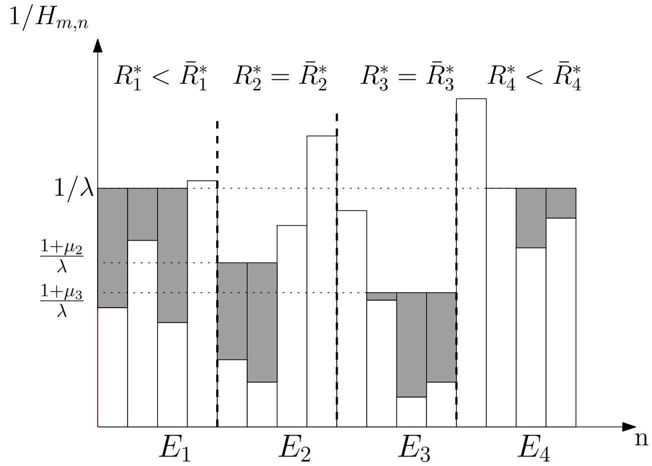

Fig. 2. Optimal source power allocation for a given phase 1 subchannel assignment, $\mathtt{E}_m$, and given optimal phase 2 rates, $\bar{R}_m^*$, $m = 1, \cdots, M$ with $M = 4$ relays and $N = 16$ subchannels.

$$P_n^* = \begin{cases} \left(\frac{1}{\lambda} - \frac{1}{H_{m,n}}\right)^+, & R_m^* < \bar{R}_m^* \text{ for } n \in \mathtt{E}_m; \\ \left(\frac{1+\mu_m}{\lambda} - \frac{1}{H_{m,n}}\right)^+, & R_m^* = \bar{R}_m^* \text{ for } n \in \mathtt{E}_m. \end{cases} \quad (6)$$

where $R_m^* = \frac{1}{2}\sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n^*)$ and $\bar{R}_m^* = \frac{1}{2}\sum_{n\in\bar{\mathtt{E}}_m} \log_2(1 + \bar{H}_{m,n}\bar{P}_{m,n}^*)$ are the rates corresponding to $\mathtt{RL}_m$ in phase 1 and phase 2, respectively, with optimal power allocation and λ, μ_m chosen to satisfy source power budget, P_S.

Proof: First, we consider optimum relay power allocation for given $\bar{\mathtt{E}}_m$. In $(4a)$, each term in the summation is subject to a separate relay power constraint (4c). Hence, relay power allocation $\bar{P}_{m,n}$, $n =\in \{1, \ldots, N\}$ and $m =\in \{1, \ldots, M\}$, can be done only considering (4c). This results in M optimization problems $m = 1, \cdots, M$

$$\max_{\bar{P}_{m,n}} \bar{R}_m = \max_{\bar{P}_{m,n}} \frac{1}{2} \sum_{n\in\bar{\mathtt{E}}_m} \log_2(1 + \bar{H}_{m,n}\bar{P}_{m,n}) \quad (7a)$$

$$\sum_{n=1}^{N} \bar{P}_{m,n} \leq P_R \quad (7b)$$

$$\bar{P}_{m,n} \geq 0 \text{ for all } m,\, n \quad (7c)$$

The problem in (7) is equivalent to the standard problem of communication over parallel channels and the optimum solution is found by allocating $\mathtt{RL}_m$ power among subchannels $n \in \bar{\mathtt{E}}$ according to the waterfilling procedure [16]. The optimum power allocation for relays can be written as

$$\bar{P}_{m,n}^* = \left(\frac{1}{\bar{\lambda}_m} - \frac{1}{\bar{H}_{m,n}}\right)^+ \; n \in \bar{\mathtt{E}}_{\mathtt{m}} \quad (8)$$

where $1/\bar{\lambda}_m$ is chosen to satisfy power constraint, P_R. The resulting optimal phase 2 rate of $\mathtt{RL}_m$ can be written as

$$\bar{R}_m^* = \frac{1}{2} \sum_{n\in\bar{\mathtt{E}}_m} \log_2(1 + \bar{H}_{m,n}\bar{P}_{m,n}^*) \quad (9)$$

The optimal source power allocation in phase 1 can be now written as

$$\max_{P_n} \sum_{m=1}^{M} \min\left(R_m, \bar{R}_m^*\right) \quad (10a)$$

$$\sum_{n=1}^{N} P_n \leq P_S \quad (10b)$$

$$P_n \geq 0 \text{ for all } n \quad (10c)$$

Note that (10a) is the downlink power allocation problem constrained to meet relay phase 2 rates $\bar{R}_m^*$. This phase 1 problem strictly depends on rates achieved in phase 2 since the source has to allocate power according to phase 2 optimum rates of each relay.

The optimization in (10a) can be written as

$$\max_{P_n} \frac{1}{2} \sum_{m=1}^{M} \sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n) \quad (11a)$$

$$\frac{1}{2} \sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n) - \bar{R}_m^* \leq 0 \quad (11b)$$

$$\sum_{n=1}^{N} P_n - P_S \leq 0 \quad (11c)$$

$$P_n \geq 0 \text{ for all } n \quad (11d)$$

To solve (11a)-(11d), the Lagrangian is

$$\begin{aligned} L \;=\; & \frac{1}{2} \sum_{m=1}^{M} \sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n) \quad (12) \\ + \; & \sum_{m=1}^{M} \mu_m \left(\frac{1}{2} \sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n) - \bar{R}_m^*\right) \\ - \; & \nu\Big(\sum_{n=1}^{N} P_n - P_S\Big) \end{aligned}$$

Using Karush-Kuhn-Tucker (KKT) conditions, the optimal source power allocation is found as

$$P_n^* = \left(\frac{1+\mu_m}{\lambda} - \frac{1}{H_{m,n}}\right)^+ \quad (13)$$

where $\lambda = 2\ln 2\nu$ and the complementary slackness condition is $\mu_m(\sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n^*) - \bar{R}_m^*) = 0$ for all $m = 1, \ldots, M$. When $\sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n^*) < \bar{R}_m^*$, in order to satisfy the complementary slackness condition, we choose $\mu_m = 0$. Thus, optimal source power allocation can be written as

$$P_n^* = \begin{cases} \left(\frac{1}{\lambda} - \frac{1}{H_{m,n}}\right)^+, & R_m^* < \bar{R}_m^*; \\ \left(\frac{1+\mu_m}{\lambda} - \frac{1}{H_{m,n}}\right)^+, & R_m^* = \bar{R}_m^*. \end{cases} \quad (14)$$

where $R_m^* = \sum_{n\in\mathtt{E}_m} \log_2(1 + H_{m,n}P_n^*)$

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As seen in the proof, the optimal relay power allocation for $\mathtt{RL}_m$is to do waterfilling on $\bar{\mathtt{E}}_m$, using power P_R resulting in $\bar{R}_m^*$, $m = 1, \cdots, M$.

The optimal source power allocation is illustrated in Fig. 2 for $M = 4$ relays and $N = 16$ subchannels when phase 1 subchannel assignment, $\mathtt{E}_m$ and optimal phase 2 rates, $\bar{R}_m^*$, are given for all $m = 1, \cdots, M$. In the figure, subchannels allocated to the relays are ordered, the heights of the bars denote $1/H_{m,n}$ and gray bars show the amount of power allocated to each subchannel. The source assigns power to all subchannels maintaining an equal water level, until the total phase 1 rate R_m for RL_m meets the phase 2 rate $\bar{R}_m^*$. When R_m is matched with $\bar{R}_m^*$, the optimal source power allocation is to apply waterfilling with water level $(1+\mu_m)/\lambda$ to the subchannels allocated to $\texttt{RL}_m$. When the rate of phase 1 can not be matched with phase 2 rate for $\texttt{RL}_m$ within the source power budget, P_S, the source allocates power using waterfilling with water level $1/\lambda$ to the subchannels allocated to $\texttt{RL}_m$. Note that, the waterfilling level is the same for all subchannels allocated to the relays whose phase 1 rate is not matched with phase 2 rate. If the source has more than the power needed to match all relay rates, the remaining power is not allocated although it can not be used in the next channel realizations.

IV. Greedy algorithm for a joint subchannel and power allocation

In the previous section, the optimal power allocation for a given subchannel assignment is found. In this section, we propose a subchannel assignment algorithm along with the corresponding optimal power allocation in greedy fashion. Since the relays decode and forward each relay can only forward the minimum of phase 1 and phase 2 rates. Thus, a mismatch of phase 1 and phase 2 rates for each relay is not favorable. A good algorithm has to decrease the mismatch by allocating more resources to the bottleneck side of the transmission. Motivated by this observation we propose a polynomial time greedy algorithm which jointly allocates subchannels and powers. The main idea of this algorithm is to increase the end-to-end rate at each step of the algorithm by keeping the relay rate mismatch small. Let $R_{opt}(\texttt{E})$ denote the end-to-end rate obtained by optimal power allocation for a given subchannel allocation $\texttt{E} = [\texttt{E}_1, \ldots, \texttt{E}_M; \bar{\texttt{E}}_1, \ldots, \bar{\texttt{E}}_M]$ as described in Section III.

Greedy Algorithm

1) Initialization
   a) Set $A = \{1, \ldots, N\}$ and $\bar{A} = \{1, \ldots, N\}$ which are the available subchannels in the phase 1 and phase 2, respectively.
   b) Set $\texttt{E}_m = \emptyset$ and $\bar{\texttt{E}}_m = \emptyset$
2) Until $A = \emptyset$ and $\bar{A} = \emptyset$
   a) Set $S_m = \emptyset$ and (or) $\bar{S}_m = \emptyset$
   b) For $m = 1$ to M
      i) Find $n^* = \arg\max H_{m,n}$ for $n \in A$ and $\bar{n}^* = \arg\max \bar{H}_{m,n}$ for $\bar{n} \in \bar{A}$
      ii) Find R^1 using R_{opt} when n^* is tentatively allocated to $\texttt{RL}_m$ in phase 1, that is $\texttt{E}_m = \texttt{E}_m \cup \{n^*\}$
      iii) Find R^2 using R_{opt} when $\bar{n}^*$ is tentatively allocated to $\texttt{RL}_m$ in phase 2, that is $\bar{\texttt{E}}_m = \bar{\texttt{E}}_m \cup \{\bar{n}^*\}$
      iv) Find R^3 using R_{opt} when both n^* and $\bar{n}^*$ are tentatively allocated to $\texttt{RL}_m$ in phase 1 and phase 2, respectively, that is $\texttt{E}_m = \texttt{E}_m \cup \{n^*\}$ and $\bar{\texttt{E}}_m = \bar{\texttt{E}}_m \cup \{\bar{n}^*\}$
      v) Find $R_m = \max(R^1, R^2, R^3)$. The maximum suggests which phase(s) to allocate an additional subchannel to $\texttt{RL}_m$
      vi) Based on the maximum in step 2(b)v above, set $S_m = \{n^*\}$ and (or) $\bar{S}_m = \{\bar{n}^*\}$
   c) Find $m^* = \arg\max(R_m)$
   d) Update $\texttt{E}_{m^*} = \texttt{E}_{m^*} \cup S_{m^*}$ and (or) $\bar{\texttt{E}}_{m^*} = \bar{\texttt{E}}_{m^*} \cup \bar{S}_{m^*}$
   e) Update $A = A \setminus S_{m^*}$ and (or) $\bar{A} = \bar{A} \setminus \bar{S}_{m^*}$

In the greedy algorithm we first initialize the sets A, $\bar{A}$, $\texttt{E}_m$ and $\bar{\texttt{E}}_m$ for all $m = 1, \ldots, M$ where A and $\bar{A}$ are the available subchannel sets in the phase 1 and phase 2 and $\texttt{E}_m$, $\bar{\texttt{E}}_m$ denote the already allocated subchannels. At each step of the algorithm at most one subchannel for phase 1 and phase 2 are allocated. For each relay, the best subchannel among available subchannels are tentatively chosen in phase 1 and phase 2. In steps 2(b)ii-2(b)v, phase(s) to allocate subchannels is decided. The corresponding rate maximizing subchannel(s) for $\texttt{RL}_m$ are stored in sets S_m and (or) $\bar{S}_m$. After following same procedure for all relays, the relay, $\texttt{RL}_m^*$, which has the largest end-to-end rate increase is chosen. The subchannels stored in the sets S_{m^*} and (or) $\bar{S}_{m^*}$ are allocated to $\texttt{RL}_m^*$ and the available subchannel sets are updated. This algorithm continues until all subchannels are allocated. Note that the algorithm is greedy since at each step the R_{opt} improves the rates when additional subchannels are available.

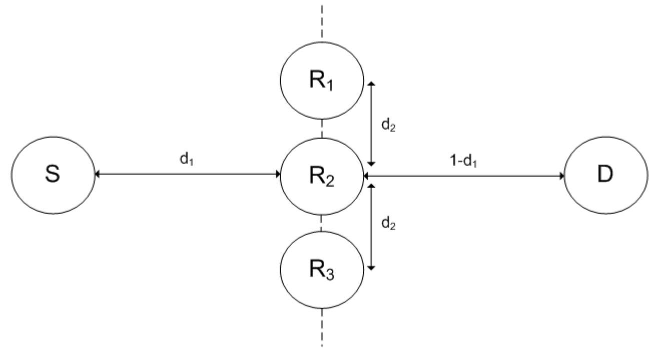

Fig. 3. The model for the source, the relay and the destination locations.

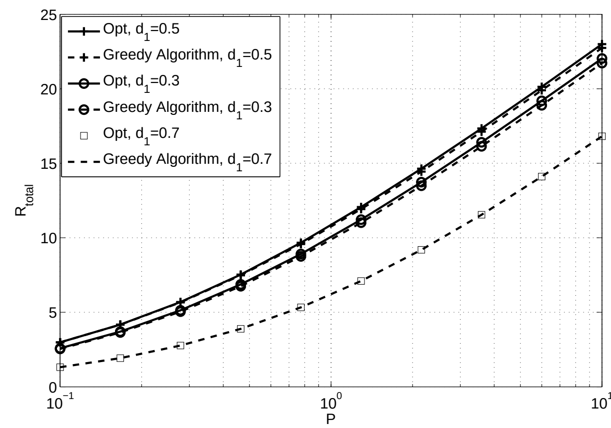

Fig. 4. Total rate vs node power where $P_S = P_R = P$ for $M = 3$, $N = 4$ and $d_2 = 0.1$.

V. Numerical Results

In order to evaluate the performance of proposed greedy algorithm, we consider the topology shown in Fig. 3. The source, the center relay and the destination are located on a straight line and the distance between the source and the destination is normalized to 1. All relays are located on a line perpendicular to $\texttt{S - D}$ axis. The source-center relay and the center relay-destination distances are d_1 and $1 - d_1$, respectively, where $0 < d_1 < 1$. The distance between the relays are d_2. The source-relay and relay-destination channels are modeled as N independent flat Rayleigh fading subchannels. Pathloss among the nodes are taken into account by adjusting the means of the channel coefficients and the pathloss exponent is assumed to be 4. In our numerical results, the end-to-end rate is optimized for each channel realization and the average end-to-end rate over all channel realizations are showed in Figures 4, 5 and 6.

To evaluate how well the greedy algorithm does, we compare the performance with the optimum resource allocation. The optimum resource allocation is found by full search where all possible subchannel allocations are tried with their corresponding optimal power allocation of Sec. III. Hence computing the optimal solution using full search becomes highly intractable when a large number of subchannels are available. Thus, optimal solution is given for only small number of subchannels.

Fig. 4 shows the total rate sum as a function of the node power $P = P_S = P_R$ for different d where we set $d_2 = 0.1$, $M = 3$ and $N = 4$. The figure shows that the greedy algorithm performs very close to the optimal resource allocation. The gap between the optimum solution and the greedy algorithm diminishes when the relays are located close to the destination. On the other hand, for $M = 3$, highest end-to-end rate is achieved when the relays are close to the mid-point of source destination pair, that is $d_1 = 0.5$. In Fig. 5, relay power is fixed to $P_R = 1$ and $d_2 = 0.1$. We plot total rate sum as a function of source power, P_S, for different d_1 and $N = 4$. In the region where source power is less than relay power the end-to-end rate is higher when the relays are close to the source. This is an expected results since the relays utilize decode-and-forward. Furthermore, the proposed greedy algorithm performs very close to the optimal solution when the relays are close to the destination and the source power is small. Fig. 6 shows the effect of number of relays M on total rate sum, R_{total} for three different source power values where relay power $P_R = 1$ is fixed and $N = 4$. When the source power is $P_S = 0.1$, R_{total} slightly increases as M increases. This is because phase 1 is the bottleneck, thus increasing number of relays does not affect R_{total}. However, in the case of $P_S = 10$, R_{total} increases significantly since now phase 2 becomes bottleneck.

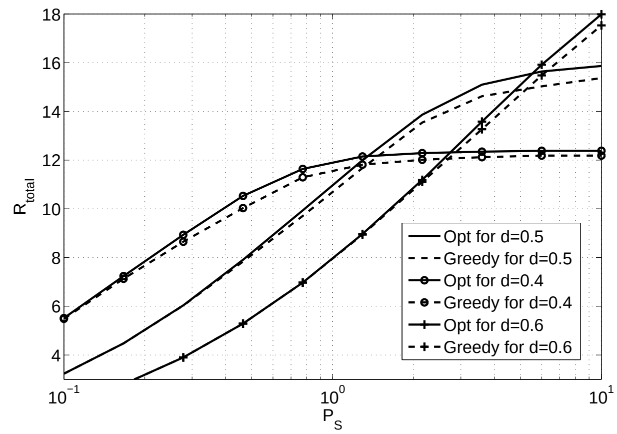

Fig. 5. Total rate vs source power where $P_R = 1$ for $M = 3$, $N = 4$ and $d_2 = 0.1$.

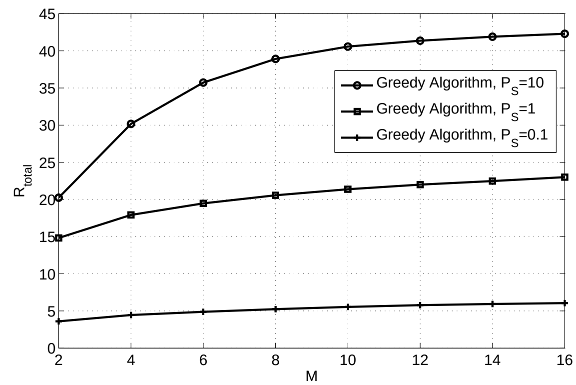

Fig. 6. Total rate vs number of relays M where $P_R = 1$, $N = 8$, $d_1 = 0.5$ and $d_2 = 0$.

VI. Conclusion

In this paper, we analyze resource allocation, namely subchannel and power allocation, in an OFDM system employing multiple relays in a two-hop system to facilitate communication among a source and a destination. We formulate an optimization problem to maximize the instantaneous end-to-end rate. This problem is solved in two steps. First, we establish the optimal power allocation given subchannel assignment. Then a greedy algorithm is proposed which allocates subchannels and power jointly. Numerical results show that proposed greedy algorithm performs very close to the optimum solution. Our future work includes investigation of the resource allocation problem for multiple relays when the direct link between the source and the destination is present in the system.

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