Multirelay Channel with Non-Ergodic Link Failures

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***Abstract*—A multi-relay network is considered in which communication from source to relays takes place over a (discrete or Gaussian) broadcast channel, while the relays are connected to the receiver via orthogonal finite-capacity links. Unbeknownst to the source and relays, link failures may take place between any subset of relays and destination in a non-ergodic fashion. Upper and lower bounds are derived on average achievable rates with respect to the prior distribution of the link failures, assuming the relays to be oblivious to the source codebook. The lower bounds are obtained via strategies that combine the broadcast coding approach, previously investigated for quasi-static fading channels, and various robust distributed compression techniques.**

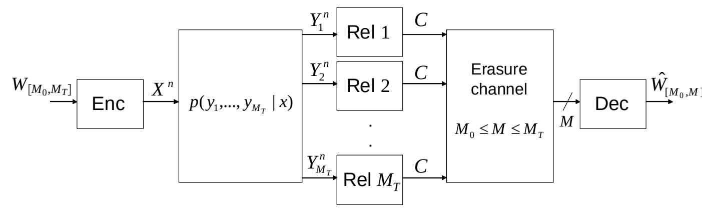

Fig. 1. A single transmitter communicates to a remote receiver via M_T relays connected to the destination through unreliable finite-capacity links (non-ergodic erasures). The number of functioning links M is unknown to source and relays (*uninformed source and relays*), and the relays are oblivious to the codebook used by the source (*oblivious agents*) as in [6].

I. Introduction

In modern packet data networks serving delay-sensitive applications, link failures are often appropriately modelled as being unpredictable and non-ergodic. The conventional transmission design targets worst-case scenarios by transmitting at a judiciously selected constant rate that guarantees an acceptable outage probability. However, it is often possible, and desirable, to deploy transmission strategies that are able to provide *variable-rate* data delivery depending on the current state of the involved links [1]-[3]. Moreover, data communication networks are typically envisaged to include distributed nodes, whose operation is decentralized. In this paper, we consider a baseline model for communication networks that include these two basic elements of non-ergodic link failures and decentralized operation.

Consider a scenario in which a single source communicates with a remote destination via a number of relays (also referred to as "agents" in related literature), with no multi-access interference at the destination (i.e., orthogonal finite-capacity links, see Fig. 1). In [6] the multi-relay network described above was studied under the assumption that the relays are *oblivious* to the codebook used by the source; that is, processing at the relays cannot depend on the specific codebook selected by the source (as in, e.g., compress-and-forward or amplify-and-forward achievable strategies). This assumption is of particular relevance for nomadic applications (in which no signalling is in place to exchange information regarding modulation and coding used at the source) or in networks with inexpensive relays whose processing cannot adapt to the specific source operation. A related model with *unreliable (non-ergodic) connectivity*, in which, unbeknownst to the agents, the links to the destination may not be functioning, was studied in [4] and [5] in the context of *distributed source compression* (the CEO problem). In this work, we extend the analysis in [6] by accounting for *unreliable links* between relays and destinations (*non-ergodic failures*) in the sense of [4] and [5] (see Fig. 1).

The basic idea behind our approach to the analysis of the system in Fig. 1 is to exploit the synergy between the broadcast (BC) coding approach of [3] at the source, which allows for variable-data delivery to the destination depending on the current connectivity conditions, and the robust distributed compression strategies of [4] and [5]. It is noted that a related idea was put forth in [1] and [2] (see also references therein), in which the BC coding approach was combined with successive-description compression techniques for transmission of a Gaussian source over a slowly fading channel without channel state information. For lack of space, in this paper results are provided without formal proofs, which can be found in [7].

Notation: The notation $[a,b]$ with a,b integers represents the interval $[a,a+1,...,b]$, with the convention that if $a>b$ then $[a,b]=\emptyset$. Similarly, the subscript notation $X_{[a,b]}$ denotes the vector $[X_a,...,X_b]$ with the same convention that, if $a>b$, $X_{[a,b]}=\emptyset$. In general, lower-case letters represent instances of the random variables denoted by the corresponding upper-case letters. Moreover, using standard notation, we will sometimes use superscripts to denote index bounds in sequences as in $x^i=[x_1,\cdots,x_i]$. The use of the superscript will be made clear by the context. Probability distributions are identified by their arguments, e.g., $p_X(x)=\Pr[X=x]\triangleq p(x)$.

II. System Model

We consider the decentralized communication scenario of Fig. 1, in which a source communicates to a destination via M_T "agents" or relays, connected to the receiver via orthogonal finite-capacity (backhaul) links of capacity C. No direct connection from the source to the destination is available. The channel from source to relays is memoryless and either discrete or Gaussian. For the former case, the signal $Y_{i,j} \in \mathcal{Y}$ received by the agent $i \in [1, M_T]$ at time instant $j \in [1, n]$ is the output of a *symmetric* memoryless channel defined by the conditional distribution $p(y_1, ..., y_{M_T}|x)$, with input $x \in \mathcal{X}$ and block length n. Symmetric here means that the observations $Y_{i,j}$ for different i are statistically exchangeable (see, e.g., [4]). For the Gaussian case, we similarly have the input-output relation

$$Y_{i,j} = X_j + Z_{i,j}, \tag{1}$$

with X_j being the jth transmitted symbol and the noise $Z_{i,j} \sim \mathcal{N}(0, 1)$ being independent and identically distributed (i.i.d.) over both i and j. We assume an average input power constraint of P: $1/n \sum_{j=1}^{n} x_j^2 \leq P$. In describing the model below, we will use the notation for the discrete model, but it is understood that the extension to the Gaussian model (1) is immediate. To account for a nomadic scenario and/or to simplify the operations at the relays, we assume, as in [6], that the relays are not informed about the codebooks used by the transmitter (i.e., they are **oblivious agents**); see below for details.

The model described above coincides with the one studied in [6]. Here, however, we are interested in investigating the scenario in which the backhaul links from relays to destination present **non-ergodic failures**. Specifically, following [4], we assume that only a number $M \leq M_T$ of links are functioning at a given coding block, while the remaining $M_T - M$ are *erased* (e.g., in outage) for the entire duration of the current transmission (non-ergodic scenario). We define the probability that $M = m$ as p_m and collect the probabilities p_m in vector $\mathbf{p} = (p_{M_0}, ..., p_{M_T})$, where M_0 represents the minimum guaranteed number of active links [4]. We remark that, by the symmetry of $p(y_1, ..., y_M|x)$ (discrete model) and (1) (Gaussian model), the system configuration for a given M depends only on the number M of active links active and not on which links are active. Finally, in keeping with the models of [4] and [5] (for distributed source coding), we are interested in scenarios in which *no instantaneous information regarding the current state of the unreliable links (i.e., the value of M) is available a priori to the source and the agents* (i.e., we assume **uninformed source and agents**). More precisely, the only information that is available at source and relays is the probability mass vector $\mathbf{p}$.

We are interested in *average* achievable rates, where the average is taken with respect to the a priori probability vector $\mathbf{p}$. Specifically, we consider a *degraded message* structure in which the overall source message of rate T_{M_T} [bits/ channel use] is split into submessages $(W_{M_0}, ..., W_{M_T}) \triangleq W_{[M_0,M_T]}$ of rates $R_{M_0}, ..., R_{M_T}$, respectively, i.e., $W_m \in [1, 2^{nR_m}]$. When $M = m$ links are active, with $m \in [M_0, M_T]$, the receiver decodes messages $W_{[M_0,m]} = (W_{M_0}, ..., W_m)$ of total rate $T_m = \sum_{i=M_0}^{m} R_i$. Notice that the more links are active the more bits (and messages) are decoded. The *average rate* R is defined as

$$R = \sum_{m=M_0}^{M_T} p_m T_m. \tag{2}$$

We remark that, as in [3], the average rate (2) does not have the operational significance of an ergodic rate, the channel being non-ergodic. It is instead a measure of the rate that could be accrued with repeated, and independent, transmission blocks, or of the "expected" rate or throughput. The setting is briefly formalized in the following (see [7] for details).

(*i*) The *encoder* performs a (stochastic) mapping $\phi_F^{(E)}$ (the superscript (E) denotes the encoder) from the messages $W_{[M_0,M_T]}$ to a codeword x^n, namely $x^n = \phi_F^{(E)}(W_{[M_0,M_T]})$ with $F \in \mathcal{F} = [1, |\mathcal{X}|^{n2^{nT_{M_T}}}]$ being a random key that runs over all possible codebooks of size $2^{nT_{M_T}}$. The key $F \in \mathcal{F}$ is revealed to the destination, but *not* to the relays (oblivious relays), and formalizes the fact that the relays have no prior knowledge of the codebook. As detailed in [6], by appropriately choosing the probability $\Pr[F = f]$ of selecting a given codebook $\phi_f^{(E)}$, one can model a scenario in which the signal transmitted by the source X^n, in the absence of knowledge of F (i.e., at the relays), is distributed i.i.d. according to a distribution $p_{X^n}(x^n) = \prod_{i=1}^{n} p_X(x_i)$ and similarly the received signals Y_j^n at the relays appear i.i.d.; (*ii*) Each ith *relay* ($i \in [1, M_T]$), unaware of the codebook F (oblivious relays) and of M, maps the received sequence $y_i \in \mathcal{Y}^n$ into an index $s_i \in [1, 2^{nC}]$ via a given mapping $s_i = \phi^{(i)}(y_i^n)$; (*iii*) The *decoder, if $M = m$ links are active*, decodes messages $W_{[M_0,m]} = (W_{M_0}, ..., W_m)$ based on its knowledge of the codebook key F and the received indices s_i over the m active links (these can be assumed by symmetry to be $s_1, ..., s_m$) via a decoding function $\phi_F^{(D)}$; (*iv*) The probability of error when $M = m$ links are active (averaged over F) is defined as $P_{e,m}^n = \Pr[\phi_F^{(D)}(S_{[1,m]}) \neq W_{[M_0,m]}]$. An average rate R (2) is *achievable* if there exists a sequence of codes such that all rates $T_m = \sum_{j=M_0}^{m} R_j$ for $m \in [M_0, M_T]$ are achievable, i.e., $\max_m P_{e,m}^n \to 0$ as $n \to \infty$. The *average capacity* $\mathrm{C}_{\mathrm{avg}}$ is the supremum of all average achievable rates (2).

III. Reference Results

In this section, we start the study of the system presented above by deriving an upper bound on the capacity $\mathrm{C}_{\mathrm{avg}}$. It is noted that, as in [6], for the Gaussian model, *we restrict the input distribution to be Gaussian* with no claim of optimality.

Proposition 1: (Cooperative relays) The following is an upper bound on the capacity $\mathrm{C}_{\mathrm{avg}}$ for the discrete model:

$$\mathrm{C}_{\mathrm{avg}} \leq \max \sum_{m=M_0}^{M_T} p_m \left(\sum_{j=M_0}^{m} R_j \right), \tag{3}$$

where the rates

$$R_{M_0} = I(U_{M_0}; V_{M_0}) \tag{4a}$$
$$R_m = I(U_m; V_m|U_{m-1}) \text{ for } m \in [M_0, M_T - 1] \tag{4b}$$
$$R_{M_T} = I(X; V_{M_T}|U_{M_T-1}), \tag{4c}$$

are calculated with respect to a joint distribution

$$\begin{aligned} &p(u_{[M_0,M_T-1]}, x, y_{[1,M_T]}, v_{[M_0,M_T]}) \\ &= p(u_{[M_0,M_T-1]}, x)p(y_{[1,M_T]}|x)p(v_{[M_0,M_T]}|y_{[1,M_T]}), \end{aligned} \tag{5}$$

and the maximization is taken with respect to the marginals $p(u_{[M_0,M_T-1]}, x)$ and $p(v_{[M_0,M_T]}|y_{[1,M_T]})$ that factorize as

$$p(u_{[M_0,M_T-1]}, x) = \prod_{m=M_0}^{M_T-1} p(u_m|u_{m-1})p(x|u_{M_T-1}), \tag{6a}$$

$$p(v_{[M_0,M_T]}|y_{[1,M_T]}) = \prod_{m=M_0}^{M_T} p(v_m|y_{[1,m]}) \tag{6b}$$

and satisfy the condition

$$mC \geq I(V_m; Y_{[1,m]}). \tag{7}$$

Moreover, for the Gaussian model, the relationship (3) is an upper bound (under the constraint that the input distribution is Gaussian) with

$$R_m = \frac{1}{2}\log_2\left(1 + \frac{m\beta_m P}{1 + m\sigma_m^2 + mP\sum_{k=m+1}^{M_T}\beta_k}\right), \tag{8}$$

for $m \in [M_0, M_T]$, where the maximization is taken with respect to parameters $\beta_{M_0}, ..., \beta_{M_T} \geq 0$ with $\beta_{M_0} + ... + \beta_{M_T} = 1$ and $\sigma_m^2 = (1/m + P)/(2^{2mC} - 1)$.

Remark 1: The upper bounds of Proposition 1 are obtained by assuming that all of the M relays that are connected to the corresponding active links can fully cooperate in processing their received signals (notice that this implies that they are also informed of which links are active). The upper bounds can be interpreted as stating that, under this assumption, the best way to operate at the source is to use a standard BC code characterized by auxiliary random variables U_m ($m \in [M_0, M_T - 1]$) for the discrete case or powers $\beta_m P$ ($m \in [M_0, M_T]$) for the Gaussian case. Such variables (or powers) correspond to the transmission of message W_m to be decoded at the receiver when $M = m$. Notice that variables U_m satisfy the Markov chain condition (6a), or equivalently $U_1 - U_2 - ... - U_{M_T-1} - X$ as for a regular degraded broadcast channel [8]. Moreover, the result in Proposition 1 also proves that the $M = m$ fully cooperative relays can employ without loss of optimality compress-and-forward (CF) techniques to communicate to the receiver, where the auxiliary variables V_m account for the quantization codebook used when $M = m$ and parameter σ_m^2 is the corresponding compression noise power for the Gaussian case. In fact, from standard rate-distortion considerations, condition (7) is easily interpreted in this sense as being necessary and sufficient to guarantee successful compression for all m. Notice that the optimality of CF in this context is a consequence of the obliviousness assumption (see also [6]).

IV. Achievable Rates

In the following, motivated by the upper bound of Proposition 1, we propose achievable schemes based on the BC coding strategy of [3] and CF at the relays. The source transmits a superposition of $M_T - M_0 + 1$ codewords of rates R_m for $m \in [M_0, M_T]$. When $M = m$, the receiver decodes $W_{[M_0,m]}$. The two techniques proposed in the following differ in the way the CF strategy is implemented in terms of compression at the agents and decompression/ decoding at the receiver, and entail increasing levels of complexity.

A. Broadcast Coding and Single-Description Compression (BC-SD)

In this section, we consider a transmission strategy based on BC coding and single-description (SD) compression at the relays. In other words, each relay sends over the backhaul link a single index (description), which is a function of the received signal. The compression/ decompression scheme is inspired by the technique used in [4] for robust distributed source coding in a CEO problem. The technique works by performing random binning at the agents, as is standard in distributed compression. Moreover, the binning scheme is designed so that the receiver can recover with high probability the compressed signals on the M active links irrespective of the realized value of M as long as it is $M \geq M_0$ (as guaranteed by assumption). In other words, design of the compression scheme targets the *worst-case scenario* of $M = M_0$. Notice that, should more than M_0 links be active ($M > M_0$), the corresponding compressed signals would also be recoverable at the receiver, since, by design of the binning scheme, any subset of M_0 descriptions can be decompressed [4]. After decompression is performed, the receiver uses all the M signals obtained from the relays to decode the codewords up to the Mth layer (that is, the layers with rates R_m with $M_0 \leq m \leq M$).

Proposition 2: (BC-SD) The average rate (2) is achievable for the discrete model with

$$R_{M_0} \leq I(U_{M_0}; V_{[1,M_0]}) \tag{9a}$$
$$R_m \leq I(U_m; V_{[1,m]}|U_{m-1}) \text{ for } m \in [M_0 + 1, M_T - 1] \tag{9b}$$
$$R_M \leq I(X; V_{[1,M_T]}|U_{M_T-1}). \tag{9c}$$

where the variables at hand satisfy the joint distribution

$$\begin{aligned} &p(u_{[M_0,M_T-1]}, x, v_{[1,M_T]}, y_{[1,M_T]}) \\ &= \prod_{m=M_0}^{M_T-1} p(u_m|u_{m-1})p(x|u_{M_T-1})p(y_{[1,M_T]}|x)\prod_{i=1}^{M_T} p(v_i|y_i), \end{aligned} \tag{10}$$

with $p(v_i|y_i)$ being the same for every $i \in [1, M_T]$, and the condition

$$C \geq \frac{1}{M_0}\left[H(V_{[1,M_0]}) - M_0 H(V_i|Y_i)\right]. \tag{11}$$

Moreover, for the Gaussian model, the average rate (2) is achievable with

$$R_m \leq \frac{1}{2}\log_2\left(1+\frac{m\beta_m P}{1+\sigma^2+mP\sum_{k=m+1}^{M_T}\beta_k}\right) \quad (12)$$

and σ^2 satisfying

$$C \geq \frac{1}{2}\log_2\left[\left(1+\frac{M_0 P}{1+\sigma^2}\right)^{\frac{1}{M_0}}\left(1+\frac{1}{\sigma^2}\right)\right], \quad (13)$$

for any power allocation $\beta_{M_0},...,\beta_{M_T}\geq 0$ with $\beta_{M_0}+...+\beta_{M_T}=1$.

Remark 2: Similar to the discussion around Proposition 1, the auxiliary random variable U_m for the discrete case and power $\beta_m P$ for the Gaussian case, $m\in[M_0,M_T-1]$, represents the codebook used for the transmission of the mth layer to be decoded at the receiver when $M=m$. Moreover, the variable V_i represents the compression codebook v_i^n used at each agent i. Notice that by symmetry the same distribution $p(v_i|y_i)$ is selected for all $i\in[1,M_T]$. Conditions (11) for the discrete case and (13) for the Gaussian case are shown in [4] to guarantee that the decoder is able to decompress the signals corresponding to any set of M_0 agents. We finally notice that the only difference between the achievable rate of Proposition 2 obtained with BC-SD and the upper bound of Proposition 1 is related to the variables V_m used for compression, and in the Gaussian case to the power of the equivalent compression noise (compare (12) with (8)).

Remark 3: For $M_T=M_0$ (fully reliable links), the achievable rate of Proposition 2 coincides with the one presented in Theorem 1 of [6].

Remark 4: (Joint Decompression/ Decoding) A potentially more efficient (but also more complex) implementation of a system working with BC coding and SD compression can be designed based on joint decompression/ decoding, similarly to the scheme proposed in [6]. We refer to [7] for further analysis and discussion.

B. Broadcast Coding and Multi-Description Robust Compression (BC-MD)

In this section, we propose to couple the BC coding approach considered throughout the paper with multi-description (MD), rather than SD, compression at the agents. The idea follows the work in [5] , which focused on the CEO problem. Accordingly, each relay shares the nC bits it can convey to the destination between multiple descriptions of the received signal to the decoder. The basic idea is that different descriptions are designed to be recoverable only if certain connectivity conditions are met (that is, if the number of functioning links M is sufficiently large). This adds flexibility and robustness to the compression strategy.

To simplify the presentation, here we focus on the two-agent case ($M_T=2$). Dealing with the more general setup requires a somewhat more cumbersome notation, but is conceptually a straightforward extension. Moreover, without loss of generality, we assume $M_0=0$ or $M_0=1$, since with $M_0=M_T=2$ the system coincides with the one with fully reliable links studied in [6]. The two agents send two descriptions: a basic one to be used at the receiver in case the number of active links turns out to be $M=M_0=1$ and a "refined" one that will be used only if $M=M_T=2$. It is also noted that for the scheme at hand the only difference between the cases $M_0=0$ and $M_0=1$ is in the prior $\mathbf{p}=(p_0,p_1,p_2)$, where in the former case, unlike the latter, we have $p_0>0$.

Proposition 3: (*BC-MD*) For $M_T=2$, $M_0=0$ or 1, the average rate (2) is achievable for the discrete model for

$$R_1 \leq I(U;V_{1i}) \quad (14a)$$

$$R_2 \leq I(X;V_{11},V_{12},V_{21},V_{22}|U) \quad (14b)$$

with joint distribution

$$\begin{aligned} &p(u,x,v_{11},v_{12},v_{21},v_{22},y_1,y_2) \quad (15)\\ &= p(u,x)p(y_1,y_2|x)\prod_{i=1}^{2}p(v_{1i},v_{2i}|y_1), \end{aligned}$$

where $p(v_{1i},v_{2i}|y_1)$ is the same for $i=1,2$, satisfying the constraint

$$C \geq I(V_{1i};Y_1)+\frac{1}{2}I(V_{21},V_{22};Y_1,Y_2|V_{11},V_{12}). \quad (16)$$

Moreover, for $M_T=2$, $M_0=0$ or 1 and the Gaussian model, the average rate (2) is achievable for

$$R_1 \leq \frac{1}{2}\log_2\left(1+\frac{\beta P}{1+(1-\beta)P+\sigma_1^2+\sigma_2^2}\right) \quad (17a)$$

$$R_2 \leq \frac{1}{2}\log_2\left(1+\frac{2(1-\beta)P}{1+\sigma_2^2}\right) \quad (17b)$$

with any power allocation $0\leq\beta\leq 1$, and any σ_1^2 and σ_2^2 such that

$$\begin{aligned} C \quad \geq \quad & \frac{1}{2}\log\left(1+\frac{P+1}{\sigma_1^2+\sigma_2^2}\right) \quad (18)\\ &+\frac{1}{4}\log\left(\frac{\left(\sigma_1^2+\sigma_2^2\right)^2\left(2P+\sigma_2^2+1\right)\left(\sigma_2^2+1\right)}{\left(2P+\sigma_1^2+\sigma_2^2+1\right)\left(\sigma_1^2+\sigma_2^2+1\right)\sigma_2^4}\right). \end{aligned}$$

Remark 5: In the MD scheme achieving the rate above, each transmitter divides its capacity C into two parts, say with a fraction $0\leq\lambda\leq 1$ devoted to the first ($m=1$) and $(1-\lambda)$ to the second ($m=2$) description. Auxiliary variables V_{mi} in (14) represent the quantization codebooks corresponding to the mth description ($m=1,2$) of the ith terminal ($i=1,2$). As explained above, the binning scheme for the mth description is designed so that the description is recoverable at the destination whenever $M=m$. To ensure this, it is sufficient to impose the condition $\lambda C\geq I(V_{1i};Y_1)$ for $m=1$ from standard rate-distortion theoretic arguments, and $2(1-\lambda)C\geq I(V_{21},V_{22};Y_1,Y_2|V_{11},V_{12})$ for $m=2$, from distributed lossy distortion theory, see, e.g., [5]. Notice that the latter inequality exploits the fact that the first descriptions V_{11} and V_{12} have been correctly decompressed at the decoder when $M=2$, and thus provide side information. In the Gaussian model, variances σ_1^2 and σ_2^2 in (17)-(18) account for the compression noises for the first and second description, respectively, and condition (18) corresponds to (16). The auxiliary random variable U in the discrete model and powers $(\beta P, (1-\beta)P)$ represent, as in the rest of the paper, the BC code.

Remark 6: On setting V_{21} and V_{22} to be constant for the discrete model or letting $\sigma_2^2 \rightarrow \infty$ for the Gaussian model, Proposition 3 reduces to Proposition 2 for $M_T = 2$, $M_0 = 0$ or 1.

V. Numerical results

Consider a two-agent system $(M_T = 2)$ with $M_0 = 1$ guaranteed functioning links. We compare the performance of the schemes described above, with single description (SD) or multi-description (MD) compression. For reference, we consider the upper bound (8) corresponding to cooperative relays (labelled as "cooperative"). To assess the impact of non-ergodic link outage, we also show the performance of a system in which the link outages occur in an ergodic fashion so that the agents effectively see a link capacity equal to the average $\bar{C} = (1-p_1/2)C$ (labelled "ergodic"). This rate clearly sets another upper bound on the average capacity, and can be found from [6] to be $\mathrm{C}_{\mathrm{avg}} \leq 1/2\log_2(1 + 2P(1-2^{-4\bar{C}}(\sqrt{P^2+2^{4\bar{C}}(1+2P)}-P)))$. Finally, the rate of a baseline single-layer (SL), or non-broadcast, transmission in which the source only sends one information layer to be decoded in the worst case scenario $M_0 = 1$ and the relays perform SD compression is shown for reference. The rate of this SL-SD scheme is easily seen to be $R_{SL-SD} = \frac{1}{2}\log_2\left(1+P/(1+\sigma^2)\right)$, with $\sigma^2 = (1+P)/(2^{2C}-1)$.

Fig. 2 shows the average rates of the proposed schemes for $P = 15\mathrm{dB}$ and $C = 0.5$ versus the probability $p_2 = 1-p_1$ of having $M = 2$ active links (rather than the minimum guaranteed $M_0 = 1$). The rates are optimized numerically over the parameters at hand (i.e., the compression noise variances σ_i^2 and power allocation β). It can be seen that the BC coding strategy provides relevant advantages over SL as long as the probability p_2 is sufficiently large, since it offers the possibility to exploit better connectivity conditions when they arise. Moreover, MD compression clearly outperforms the SD-based approach for all values of p_2 for which BC coding is advantageous, due to the added flexibility in allocating part of the backhaul link rate for the case of full connectivity $(M = M_T)$. In particular, BC-MD performs very close to the upper bound of cooperative relays and for $p_2 = 1$ achieves the capacity for $M_0 = M_T = 2$ of [6] (that is, the ergodic bound above with $p_1 = 0$).

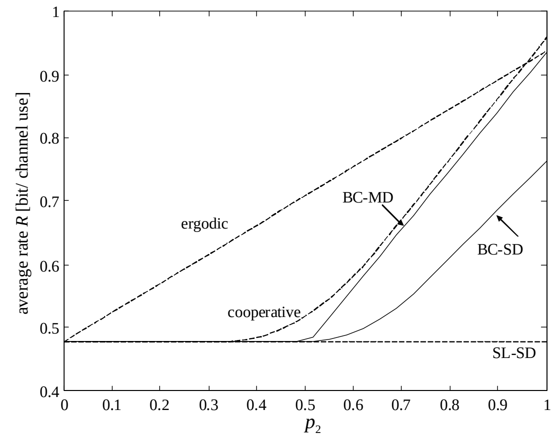

Fig. 2. Average achievable rates (2) for the proposed BC-based schemes with single description (SD) or multi-description (MD) compression, versus the probability $p_2 = 1-p_1$ of having $M = 2$ active links. For reference, the upper bound (8) achievable with cooperative relay, the upper bound corresponding to ergodic link failures and the rate of single-layer (SL), or non-broadcast, transmission with SD compression are also shown ($P = 15\mathrm{dB}$ and $C = 0.5$).

VI. Concluding Remarks

Focusing on a multi-relay network with one transmitter-receiver pair and unreliable orthogonal link between each relay and the destination, we have exploited the synergy between the BC approach of [3] and the distributed source coding techniques of [4] and [5] to propose a number of robust communication strategies. Via comparison with performance upper bounds, we have shown that the proposed techniques are almost optimal for the model in which the relays are oblivious to the source codebooks. This work opens a number of possible avenues for future research, such as the extension to multi-user scenarios with more than one source.

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References

[1] C. Tian, A. Steiner, S. Shamai (Shitz) and S. N. Diggavi, “Successive refinement via broadcast: Optimizing expected distortion of a Gaussian source over a Gaussian fading channel,” *IEEE Trans. Inform. Theory*, vol. 54, no. 7, pp. 2903-2918, Jul. 2008.

[2] C. T. K. Ng, D. Gündüz, A. Goldsmith, and E. Erkip, “Minimum expected distortion in Gaussian layered broadcast coding with successive refinement,” in *Proc. IEEE International Symposium on Information Theory* (ISIT 2007), pp. 2226-2230, Nice, France, June 24-29, 2007.

[3] S. Shamai (Shitz) and A. Steiner, “A broadcast approach for a single-user slowly fading MIMO channel,” *IEEE Trans. Inform. Theory*, vol. 49, no. 10, pp. 2617-2635, Oct. 2003.

[4] P. Ishwar, R. Puri, K. Ramchandran and S. S. Pradhan, “On rate-constrained distributed estimation in unreliable sensor networks,” *IEEE J. Selected Areas in Commun.*, vol. 23, no. 4, pp. 765-775, Apr. 2005.

[5] J. Chen and T. Berger, “Robust distributed source coding,” *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3385-3398, Aug. 2008.

[6] A. Sanderovich, S. Shamai (Shitz), Y. Steinberg and G. Kramer, “Communication via decentralized processing,” *IEEE Trans. Inform. Theory*, vol. 54, no. 7, pp. 3008-3023, Jul. 2008.

[7] O. Simeone, O. Somekh, E. Erkip, H. V. Poor and S. Shamai (Shitz), “Robust communication via decentralized processing with unreliable backhaul links,” submitted.

[8] P. P. Bergmans, “Random coding theorem for broadcast channels with degraded components,” *IEEE Trans. Inform. Theory*, vol. IT-19, pp. 197-207, Mar. 1973.