Is global warming injecting randomness into the climate system?

By

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Recently, data analyses and model simulations have indicated that as the planet is warming, the chance for extreme events increases. Karl et al. [1995] examined precipitation records over the 20th century and showed that the high-frequency (up to interannual) variability has increased. Subsequently, Tsonis [1996] showed that the low-frequency variability has also increased. These variability trends indicate that the frequency of extremes (more drought events and more heavy precipitation events) has increased whereas the mean has remained approximately the same. Such a tendency is observed with other variables and is consistent with model projections of a warmer planet.

Interestingly, a tendency for increased extremes is often translated as increased randomness (simply because the fluctuations increase). Strictly speaking, however, this is incorrect. An increase in the extremes affects the probability distribution of a *random* variable, but the variable is still *random* and thus it is equally unpredictable. This is in agreement with the Chaitin- Kolmogorov-Solomonoff complexity definition of randomness [*Casti, 1990*]. According to this definition the degree of randomness of a given sequence is determined by the length of the computer program written to reproduce it. If the program involves as many steps as the length of the sequence, then the sequence is called maximally random. Random sequences generated from probability distributions are all equally maximally random because their values appear with no particular order or repetition, regardless of the form of the distribution. As such to describe such sequences one must write a program that involves as many steps as the length of the sequence. It follows that changes in the degree of randomness cannot be assessed by changes in the probability distribution. Changes in the degree of randomness can only be probed by changes in the dynamical properties of a system with complex behavior. If the dynamics change the system may become more (less) complex, which will imply that a longer (shorter) program will be needed to describe it.

Changes in predictability

A common element in any definition of randomness is unpredictability. Simply, a process is random if we cannot predict it. If changes in global temperature affect the degree of randomness in the climate system, then predictability should vary according to temperature trends. Toward this end we need to consider a strong signal of our climate system with demonstrated complex structure and investigate its predictability as the global temperature varies. A good candidate is El Nino/ Southern Oscillation (ENSO). Because of the established nonlinear character of ENSO and its connection to global dynamics it represents an excellent candidate to empirically investigate the relation between predictability and global temperature.

Dynamically speaking predictability is equal to the inverse of the Kolmogorov entropy (K), which is equal to the sum of all the positive Lyapunov exponents. Lyapunov exponents relate to the divergence of nearby states at a specific location in the attractor. The inverse of K is a measure of the predictability of the system. Thus, changes in predictability can be assessed by probing the local structure of the attractor (or the local Lyapunov exponents) along the trajectory generated by the Southern Oscillation Index (SOI) [*Abarbanel et al., 1991*]. For SOI we find [*Tsonis and Elsner, 1997*] that there exist two positive exponents. Their sum ranges from a minimum of about 0.3 to a maximum of about 0.5 (months^{-1}) (Figure 1 top). A careful examination of Figure 1 (top) reveals striking similarities with global temperature records. It exhibits an overall positive trend with the following features: a decrease up to about 1905, a steady increase up to about 1940, a subsequent decrease up to about 1970 and a rise afterwards. Such features are identified in almost all global temperature records as, for example, the global marine air temperature record [*Newell, 1989*] (Figure 1 bottom). The two signals in figure 1 correlate highly but this could be due to the presence of the overall slight positive trends. However, coherence analysis [*Tsonis and Elsner, 1997*] has established that the residuals of the detrended time series are coherent with high confidence for all frequencies less than 0.25 cycles/year. Even though the two signals may differ at short scales, their oscillatory components at low frequencies are linearly related, which means that warmer temperatures correspond to higher K values or to lower predictability. We conclude that as the global temperature increases predictability decreases. According to the definition of randomness this result indicates that as the global temperature increases the randomness of the climate system increases as well. The physical mechanism behind this relation can be understood in terms of a subsystem of the climate system (ENSO) and its connectivity to global temperature [*Tsonis et al., 2003*].

Climate networks

A network is a system of interacting agents. In the literature an agent is called a node. The nodes in a network can be anything. In the network of actors, the nodes are actors that are connected to other actors if they have appeared together in a movie. In a network of species the nodes are species that are connected to other species they interact with. In the network of scientists, the nodes are scientists are connected to other scientists if they have collaborated. There are four basic types of networks.

a) Regular (ordered) networks. These networks are networks with a fixed number of nodes, each node having the same number of links connecting it in a specific way to a number of neighboring nodes (Figure 1a). These networks exhibit high degree of local clustering, meaning that connecting two far away nodes requires many steps.

b) Classical random networks. In these networks [*Erdos and Renyi, 1960*] the nodes are connected at random (Figure 1b). In the case the degree distribution is a Poisson distribution (the degree distribution, p_k, gives the probability that a node in the network is connected to k other nodes). In random networks connecting far away nodes requires only a few steps.

c) Small-world networks.

A ‘small-world’ network is a superposition of regular and classical random graphs. Such networks exhibit a high degree of local clustering, but they also have a small number of random long-range links (Figure 1c). These random links help connect far away nodes with only a few steps *[Watts and Strogatz, 1998]*. Both random and ‘small-world’ networks are rather homogeneous networks in which each node has approximately the same number of links $<k>$. Both have nearly Poison degree distributions that peak at $<k>$ and decay exponentially for large k.

d) Networks with a given degree distribution. These networks have a degree distribution other than Poisson. The most interesting and common of such networks are the so-called *scale-free* networks, in which the degree distribution is the power law $p_k \sim k^{-\gamma}$ (Figure 1d). Like a map showing an airline’s routes, this network has a few hubs connecting to many other points (super nodes) and many points connected to only a few other points. Such a map is highly clustered; yet one can move for a point to another far away point with just a few connections. As such, this network has the *property* of ‘small-world’ networks. Note that scale-free networks have properties of ‘small-world’ networks, but ‘small-world’ networks a la Watts and Strogatz are not scale-free [*Barabasi and Bonabeau, 2003*].

The networks can be either fixed, where the number of nodes and links remains the same, or evolving, where nodes and links may be added or eliminated. Whatever the type of the network, its underlying topology provides clues about the collective dynamics of the network. The structural properties of networks are provided by the clustering coefficient C and the characteristic path length (or diameter) L of the network. The clustering coefficient is defined as follows: Assume that a node i is connected to k_i other nodes. Now consider the k_i closest nodes of i. This defines the neighborhood of i. Then count the number of links, Δ_i, between any two nodes of the neighborhood (excluding node i). The clustering coefficient of node i is then given by $C_\mathrm{i}=2\Delta_i/k_i(k_i-1)$. Since there can be at most $k_i(k_i-1)/2$ links between k_i nodes (which will happen if they formed a fully connected subnetwork), the clustering coefficient is normalized on the interval [0,1]. The average C_i over all nodes provides C. As such C provides a measure of local “cliqueness”. The diameter of the network is defined by the average number of connections needed to connect any two nodes in the network. Graph theory predicts that for classical random networks $L_\mathrm{random}\approx \ln n/\ln\langle k\rangle$ and $C_\mathrm{random}\approx\langle k\rangle/n$, where <k> is the average number of connections per node *[Watts and Strogatz, 1999].* The ‘small-world’ property requires that $C \gg C_\mathrm{random}$ and $L\geq L_\mathrm{random}$. *From these conditions it follows that if a* *network is changing in time in a way that C and L decrease, then the network approaches the classical random limit (i.e. its degree of randomness increases).*

Tsonis and Roebber [2004] applied these ideas to a climate network using 500hPa data in the period 1948-1999. The nodes of the networks were points arranged in a $5^o x 5^o$ grid. Any two points were assumed as connected if the correlation between their corresponding time series was above a statistically significant threshold. From all possible 3,547,116 pairs about 350,000 were found to be connected. For this network it was estimated that L=2.7, C=0.69, and <k>=170. For a random network with the same specifications (number of nodes, and average links per node) it is estimated that L_{random}=1.5 and C_{random}=0.08. These values indicate that indeed $L \geq L_{\text{random}}$ and $C >> C_{\text{random}}$ (by a factor of about nine). Thus, this global network appears to have the 'small-world' property.

There are some very interesting implications of the climate system having 'small-world' properties [for more details see *Tsonis and Roebber, 2004*], but here we will stick with the relevant issue, which is that if L and C decrease in time, then the network's degree of randomness increases. The 52-year period used in this preliminary study can be divided into two distinct periods each of length of 26 years. One is the 1948-1973 and the other the 1974-1999 period. During the first period the global temperature shows no significant overall trend. During the second period a very strong positive trend is present. Does this change in the global property of the system affect the dynamics of the network? To answer this question C and L for the two periods were estimated. It was found that C is about 5% smaller and L is about 4% smaller in the second period. This result will indicate that during the warming of the planet the network has acquired more long-range connections and less small range-connections. This is shown in Figure 3, which shows the distribution of the connections according to their distance. The thick line represents the distribution in the first period and the thin line the distribution in the second period. This figure shows that the frequency of long-range connections (>7,500 Km) has increased whereas the frequency of shorter-range connections (2,500-7,500 Km) has decreased. One may argue that visually these differences are not impressive, but that with a network having hundreds of thousand of connections these distributions are statistically different at the 99% confidence level (according to Kolmogorov-Smirnov test) or at the 95% confidence level (from bootstrapping; see inset). A tendency for smaller C and L implies that the network is becoming more *random*. Therefore, this analysis indicates that as the global temperature increases the properties of the climate network tend to the properties of a network with increased degree of randomness. A possible mechanism that explains this result and ties it with the first analysis is that a warmer planet makes the large scales more coherent (as temperature increases at all places). At the same time, fluctuations at small scales increase thereby decreasing short-range correlations. This follows from thermodynamic arguments: the higher the temperature of the system the larger the fluctuations in the system.

An investigation into some of the dynamical properties of the climate system is presented. Two different approaches are considered. One approach finds predictability as a function of time of a very strong signal of the system. It is found that predictability is highly correlated to global temperature. More specifically, as the global temperature increases predictability decreases. This translates to increasing degree of randomness. The other approach studies the collective behavior of the climate system using network dynamics and concludes that this behavior is consistent with a network of increasing randomness. Thus, both approaches agree that global warming has resulted in an increase of randomness in the climate system.

References

Abarbanel, H.D.I., R. Brown, and M.B. Kennel, Lyapunov exponents in chaotic systems. *Int. J. Mod. Phys., 5*, 1347-1375, 1991.

Barabasi, A-L., and E. Bonabeau, Scale-free networks. *Scientific American 288*, 60-69, 2003

Casti, J.L., *Searching for uncertainty*. Morrow, New York, 1990, 496 pp.

Erdos, P., and A. Renyi, On the evolution of random graphs. *Publ. Math. Inst. Hung. Acad. Sci. 5*, 17-61, 1960.

Karl, T.R., R.W. Knight, and N. Plummer, Trends in high-frequency variability in the twentieth century. *Nature 377*, 217-220, 1995

Newell, N.E., R.E. Newell, J. Hsiung, and W. Zhong-Xiang, Global marine temperature variation and the solar magnetic cycle. *Geophys. Res. Lett. 16*, 311-314, 1989.

Strogatz, S.H., Exploring complex networks. *Nature 410*, 268-276, 2001.

Tsonis, A.A, Widespread increases in low-frequency variability of precipitation over the last century. *Nature 382*, 700-702, 1996.

Tsonis, A.A., and J.B. Elsner, Global temperature as a regulator of climate predictability. *Physica D 108*, 191-196, 1997.

Tsonis, A.A., A.G. Hunt, and J.B. Elsner, On the relation between ENSO and global climate change. *Meteorol. Atmos. Phys. 84*, 229-242, 2003.

Tsonis, A.A., and P.J. Roebber, The architecture of the climate system. *Physica A* 333, 497-504, 2004

Watts, D. J., and S. H. Strogatz, Collective dynamics of ‘small-world’ networks. *Nature 393,* 440-442, 1999.

Figure captions

Figure 1: Top: The sum of the positive Lyapunov exponents (months^{-1}) along the trajectory (i.e. as a function of time) generated by the SOI index. The inverse of this sum is a measure of the predictability of the system. Bottom: The global marine temperature record. As explained in the text these two signals are coherent at all frequencies less than 0.25 cycles/year.

Figure 2: Example of an ordered, a random network, a ‘small-world’, and a scale-free network (adopted from Watts and Strogatz 1999 and Strogatz 2001).

Figure 3: The relative frequency distribution of the connections according to their distance for the period 1948-1973 (thick line) and for the period 1974-1999 (thin line). The inset shows results from bootstrapping in the range between 10,000 and 16,500 Km. We randomly selected two 26 years samples from our 52-year period and produced similar distributions. We then repeated this 1000 times and produced the 2.5% and 97.5% confidence intervals of these distributions indicated by the broken lines. The distributions

of the first and second 26-year period are outside these intervals indicating that their differences are statistically significant at the 95% level. This conclusion is also valid for most of the range between 2,500 and 7,500 km, but for clarity we only show the magnification at longer distances.

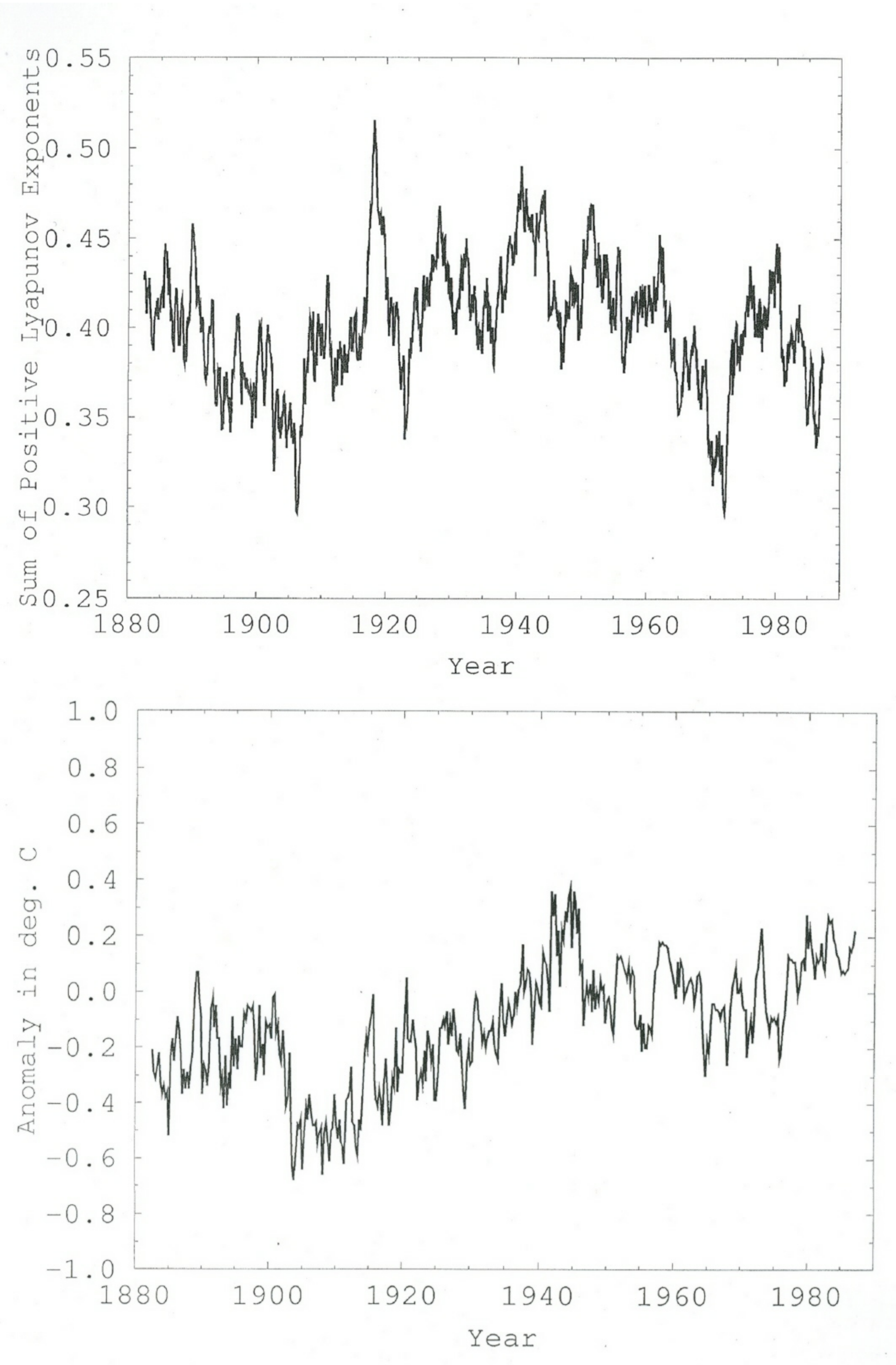
Sum of Positive Lyapunov Exponents
0.55
0.50
0.45
0.40
0.35
0.30
0.25
1880
1900
1920
1940
1960
1980
Year
Anomaly in deg. C
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
1880
1900
1920
1940
1960
1980
Year

Figure 1

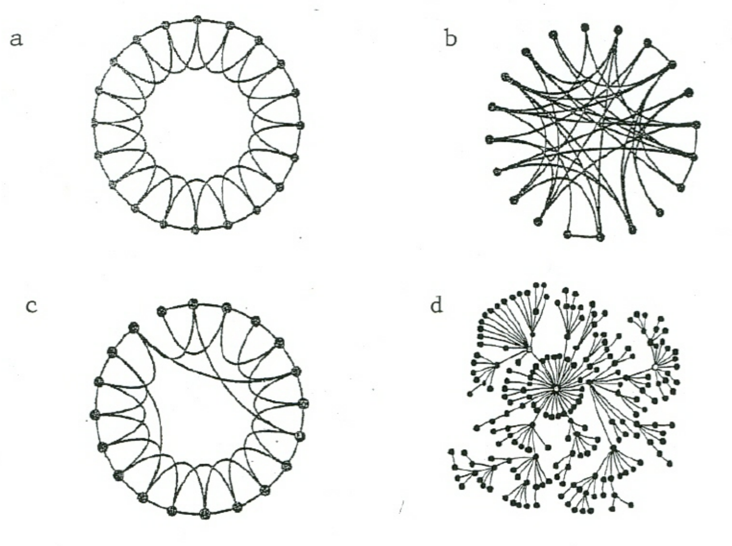

Figure 2

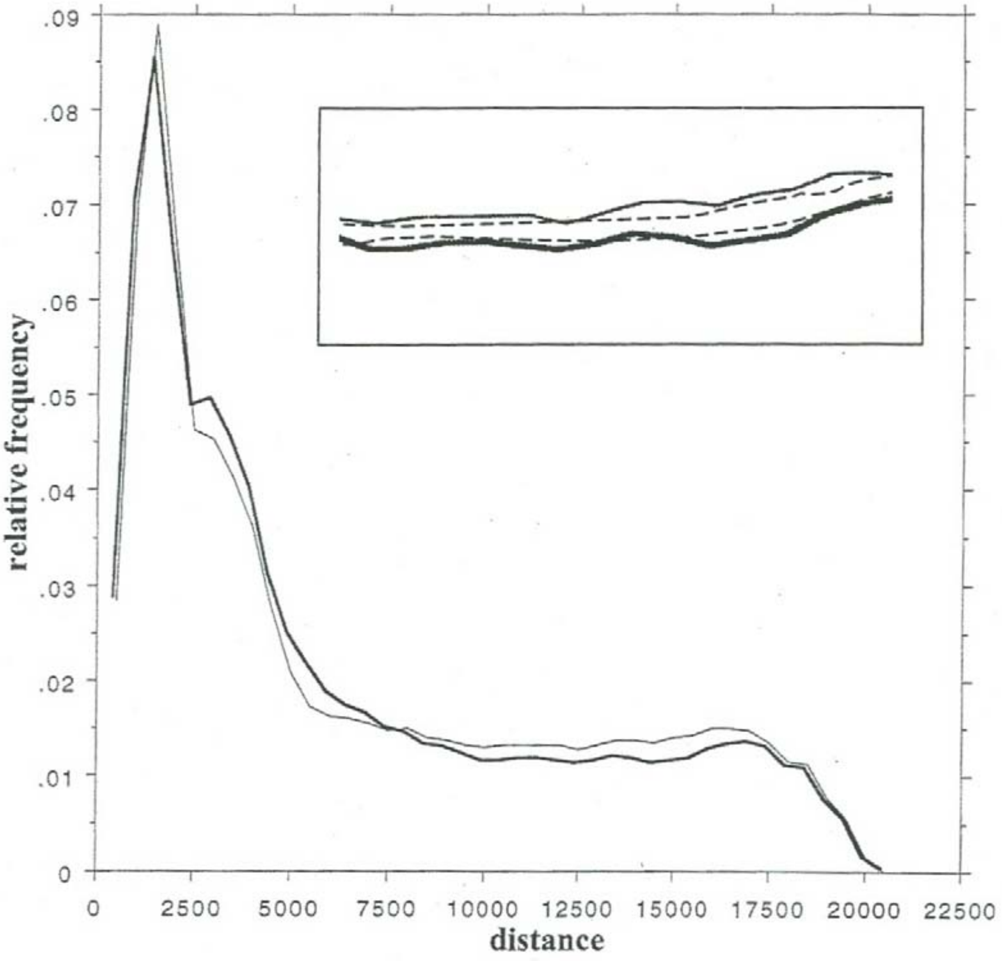
.09
.08
.07
.06
.05
.04
.03
.02
.01
0
relative frequency
0
2500
5000
7500
10000
12500
15000
17500
20000
22500
distance

Figure 3