Wireless Diversity through Network Coding

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***Abstract* – This paper investigates the diversity gain offered by implementing network coding [1] over wireless communication links. The network coding algorithm is applied to both a wireless network containing a distributed antenna system (DAS) as well as one that supports user cooperation between users. The results show that DAS with network coding leads to better diversity performance, at a lower hardware cost and higher spectral efficiency. In the case of user cooperation, network coding yields additional diversity, especially when there are multiple network users.**

***Index Terms* – Network coding, distributed antenna system, user cooperation, wireless communication.**

I. Introduction

Spatial diversity has been widely accepted as one of the most effective ways to combat fading over wireless channels. Such consensus has inspired extensive research on multiple-input-multiple-output (MIMO) over the past decade [3]. Achieving full diversity gain, however, requires multiple antennas be placed sufficiently far apart [10], which may be problematic for size-limited mobile terminals. In recognition of such practical limits, alternate approaches, such as distributed antenna system (DAS) [9] and user cooperative transmission [4], have been proposed to provide spatial diversity.

Although the term "coded cooperation diversity" has been adopted in some literature on cooperation diversity [2], current implementations of coded user cooperation essentially apply channel coding to achieve coding gain, and have not yet fully explored the potential of *network coding* [1] facilitated by the presence of multiple mobile users in the wireless network. The idea of network coding was first proposed by Yeung *et al.* [1] to enhance the capacity of the *noiseless* wired network. In this paper, we investigate the additional diversity gain facilitated by this type of network coding[1] in wireless networks. As an initial study, we do not incooperate other distributed channel coding in this paper, however, it is possible to apply existing channel coding techniques on top of the network coding scheme studied here for further performance improvement.

The *probability of system outage* is adopted as the criterion for our analysis of small networks (containing two or three nodes). System outage occurs when the destination (e.g., the base station) is unable to correctly receive data from any one of its users. We then extend our investigation to larger, multi-user networks and compute outage probability of a typical system user. In both cases, we study the application of network coding assuming distributed antenna systems (DAS) and user cooperation. Through theoretical analysis and numerical evaluation, we show that network coding offers improved diversity and more design flexility in wireless networks.

Section II describes how network coding may be applied to DAS and systems supporing user cooperation. The performance of such applications is analyzed in Section III and Section IV, respectively. We also provide the numerical evaluation of system performance in Section V and conclude our work in Section VI.

II. System Description

The core idea of network coding is to allow simple coding capability at *relaying nodes*, in exchange for network capacity gain. For example, it has been shown [1] that considerable capacity gain can be obtained as such over *wired* communication links. Such problems have been studied in the name of "network information flow" since [1]; interested readers are referred to [5] and the references within.

The pioneering work in [1] has thereafter inspired considerable research efforts in computer networking and communication communities. Though most of these studies have focused on wired networks [5], there is some initial work investigating network coding in wireless scenarios, e.g., exchange of information between independent wireless nodes [7]. Although the noiseless assumption used in [1] is no longer valid in wireless communication, the wireless medium does provide some desirable characteristics that facilitate the application of network coding, e.g., broadcasting without additional cost. In this work, we show that proper application of network coding can lead to improved system performance in wireless networks via additional diversity gain.

Distributed antenna systems (DASs) were originally proposed to provide coverage to blind-spots in shielded buildings [8]. In cases where employing multiple antennas at a single mobile terminal is impractical, DAS has also been used as an alternate way to implement MIMO communication [9]. In a DAS-MIMO system, antenna units are distributed over the network to facilitate spatial diversity among users. We borrow such a model in this paper, i.e., we assume assisting antennas (AAs) are deployed over a geographic region to aid wireless users communicate with a local base station. We assume each AA unit is equipped with a single antenna and is able to perform decoding and *simple* encoding; the AAs are thus comparable in cost and complexity to user terminals.

The distributed AA units function as relays that improve the reliability of the communication link between a user terminal and the base station. In the simplest configuration, which we call *plain-DAS*, each user is coupled with one AA, which decodes the incoming signal from the user and

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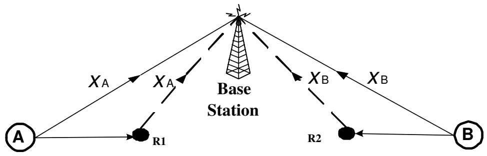
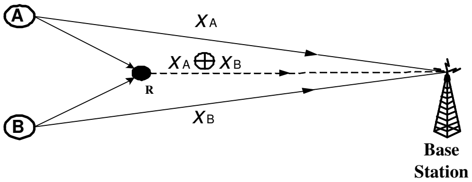
[1]The term "network coding gain" has been used in some recent literature on multi-user cooperative scheme [12] to refer to the gain obtain by multiple-relays. However, we follow the convention from [5] in this paper and use "network coding" to mean the coding scheme proposed in [1].



(a) Plain-DAS without network coding



(b) DAS with network coding

Fig. 1. Different implementations of distributed antenna systems (DASs)

forwards it to the base station. In addition to this relayed copy, the base station also receives the original signal along the direct path from the user terminal, e.g., see Fig. 1 (a). We assume the original and relayed signals are transmitted over orthogonal channels and experience independent fading.[2] Despite its simplicity, the benefits of plain-DAS are somewhat limited: at any given time, each antenna can only assist one user. As a result, users have to compete by queuing to enjoy the possible diversity gain offered by DAS. In addition, this scheme requires twice the bandwidth of the unassisted system.

In contrast, Fig. 1 (b) shows the DAS with network coding. Each of user A and user B has its own data, x_A and x_B, to send to the base station. Due to the broadcast nature of the wireless medium, one assisting antenna R will receive signals x_A and x_B, and may relay these signals simultaneously to the base station. Specially, if linear network coding is used at R, then instead of relaying for only A or B, R can assist both terminals simultaneously by transmitting $x_R = x_A \oplus x_B$. With only one assisting antenna R, the information transmitted from *both* A and B can now be retrieved correctly, even if the direct uplink of user A *or* user B fails. For example, if the base station fails to decode x_A, yet x_B and x_R both arrive correctly, then the base station can recover x_A from $x_A = x_B \oplus x_R = x_B \oplus (x_A \oplus x_B)$. Similarly, if the transmission from B fails, x_B can be retrieved if neither x_A nor x_R fails. Further analysis in Section III shows that both schemes in Fig. 1 achieve a diversity order of 2; however, DAS with network coding does so with lower complexity and spectrum cost.

Another natural application of network coding in wireless networks arises from transmissions based on user cooperation [4]. In such systems, each user is equipped with one antenna, and spatial diversity is achieved across multiple users via user cooperation. The cooperation schemes used by the terminals can be categorized into two main classes: amplify-and-forward (AF) and decode-and-forward (DF) [6]. When a coding scheme other than repetition coding is adopted, the DF cooperation is also know as coded cooperation [2]. In this paper, we study user cooperation assuming DF cooperation as it offers the design flexibility for implementing network coding. In the DF mode, each data block is segmented into two time slots of length M_1 and M_2. During the first time slot, each user transmits its own data while its partner receives this data and tries to decode it. During the second time slot, each user uses its own antenna to transmit the data for its partner if its partner's data has been correctly decoded at the end of slot 1. If the partner's data was not decoded correctly, the terminal will simply transmit its own data in the second time slot.

The non-cooperative communication and the conventional cooperative communication are clearly distinguished by the different strategies adopted during the second time slot. For non-cooperative user, the *entire* power is used for oneself in the second time slot, while the conventional cooperative user commits *all* of its own power to its cooperating partner (provided its partner's data was received correctly). Network coding, on the other hand, provides an alternative between the two extremes, and leads to the scheme illustrated in Fig. 2.

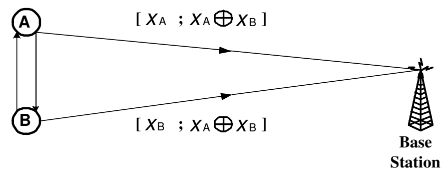


Fig. 2. User cooperation between two users with network coding

For the user cooperation scheme with network coding, each user still transmits its own data in the first time slot. In the second time slot, the *network-coded* data of both users is transmitted using the two separate antennas of the two cooperating users. We will show in section IV that the network-coded scheme yields lower system outage.

III. Performance analysis of network-coded DAS

We begin our study of the DAS with and without network coding by looking at a simple network of two users. We assume distributed antennas are deployed such that each user is assisted by one AA, i.e., each user's data can be reliably decoded by one AA unit. For fair comparison, this assumption applies to both plain-DAS and network-coded DAS.

To compare the performance of the two DAS schemes, we say that a *system outage* occurs when the data from user A and user B cannot *both* be correctly recovered at the base station. We denote the probability of such system outage as $\mathbb{P}_{\mathcal{S}}$. Next, we assume the BERs of the unlink channel of user A and user B are p_A and p_B, respectively, and that the BERs of the uplink channels of the assisting antennas for users A and B are p_1 and p_2, respectively. The system outage probability for the plain-DAS can then be computed as:

$$\begin{aligned}\mathbb{P}_{\mathcal{S}1} &= p_A p_1 (1 - p_B p_2) + p_B p_2 (1 - p_A p_1) + p_A p_1 \cdot p_B p_2 \\ &= p_A p_1 + p_B p_2 - p_A p_B \cdot p_1 p_2. \qquad (1)\end{aligned}$$

For the DAS with network coding, we assume the BER of the uplink of the AA is p_R, and we have:

$$\begin{aligned}\mathbb{P}_{\mathcal{S}2} &= p_A p_R (1 - p_B) + p_B p_R (1 - p_A) \\ &\quad + p_A p_B (1 - p_R) + p_A p_B p_R \qquad (2) \\ &= p_A p_B + p_R (p_A + p_B) - 2 p_A p_B p_R.\end{aligned}$$

[2]From the standpoint of coding, such forwarding is simply distributed repetition coding; more powerful distributed channel coding (such as turbo coding) can be applied and better performance can be expected. Such improvement, however, is just additional coding gain. Therefore, we will simply assume repetition coding in this study.

If we simply assume that each of the two users and the assisting antennas have the same uplink BER p and $p \ll 1$, the probabilities (1) and (2) then imply

$$\mathbb{P}_{\mathcal{S}1} = 2p^2 - p^4 \backsim p^2 \quad \text{and} \quad \mathbb{P}_{\mathcal{S}2} = 3p^2 - 2p^3 \backsim p^2.$$

That is, both schemes achieve a diversity order of 2. However, such diversity is achieved in plain-DAS at higher hardware cost (two assisting antennas as opposed to one for the network-coded scheme), more power ($\frac{4}{3}$ of the network-coded DAS), and more bandwidth expenditure (four orthogonal channels as compared to three for the network-coded DAS).

Next, we assume the total system power is constrained to E_T. For the plain-DAS, each user and antenna consume power E_{U1} and E_{A1}, respectively; for network-coded DAS, they consume powers E_{U2} and E_{A2}, respectively. We assume the fractions of power used by *all* mobile users in the plain-DAS and network-coded DAS are α_1 and α_2, respectively, $(0 < \alpha_1, \alpha_2 < 1)$. Thus,

$$\begin{aligned} E_{U1} &= \frac{\alpha_1}{2} E_T; \qquad E_{A1} = \frac{1-\alpha_1}{2} E_T; \\ E_{U2} &= \frac{\alpha_2}{2} E_T; \qquad E_{A2} = (1-\alpha_2) E_T. \end{aligned} \tag{3}$$

We assume all user-to-base and AA-to-base channel gains are i.i.d. as $h \sim \mathcal{CN}(0,1)$. Assuming BPSK modulation, the BERs of such uplinks can be computed as [10]:

$$p_e = \mathbb{E}\left[\mathrm{Q}\left(\sqrt{2|h|^2 \mathrm{SNR}}\right)\right] = \frac{1}{2}\left(1 - \sqrt{\frac{\mathrm{SNR}}{1+\mathrm{SNR}}}\right). \tag{4}$$

When $\mathrm{SNR} \gg 1$, the *Taylor expansion* gives

$$p_e \approx \frac{1}{4 \cdot \mathrm{SNR}} = \frac{N_0}{4E}, \tag{5}$$

where E is the transmit power and N_0 is the one-side Gaussian noise power. Using (3) and (5), $\mathbb{P}_{\mathcal{S}1}$ and $\mathbb{P}_{\mathcal{S}2}$ are:

$$\mathbb{P}_{\mathcal{S}1} = 2\gamma - \gamma^2 \tag{6}$$

and

$$\mathbb{P}_{\mathcal{S}2} = \left(\frac{\beta}{\alpha_2}\right)^2 + \frac{\beta^2}{\alpha_2(1-\alpha_2)} - \frac{\beta^3}{(1-\alpha_2)\alpha_2^2} = \frac{\beta^2(1-\beta)}{\alpha_2^2(1-\alpha_2)}, \tag{7}$$

where $\beta = \frac{N_0}{2E_T} = \frac{1}{2\mathrm{SNR_T}}$ and $\gamma = p_A p_1 = \frac{\beta^2}{\alpha_1(1-\alpha_1)}$.

Since $0 < \gamma < 1$, $\mathbb{P}_{\mathcal{S}1}(\gamma)$ monotonically increases with γ; minimizing $\mathbb{P}_{\mathcal{S}1}$ is then equivalent to minimizing $\gamma(\alpha_1)$. Further, $\gamma(\alpha_1)$ is minimum when $\alpha_1 = 1/2$. In other words,

$$\begin{aligned} (\mathbb{P}_{\mathcal{S}1})_{\min} &= \mathbb{P}_{\mathcal{S}1}(\alpha_1 = 1/2) = 8(\beta^2 - 2\beta^4) \qquad (8) \\ &= \frac{2}{\mathrm{SNR_T}^2} - \frac{1}{\mathrm{SNR_T}^3} \sim \mathcal{O}\left(\frac{1}{\mathrm{SNR_T}}\right)^2. \end{aligned}$$

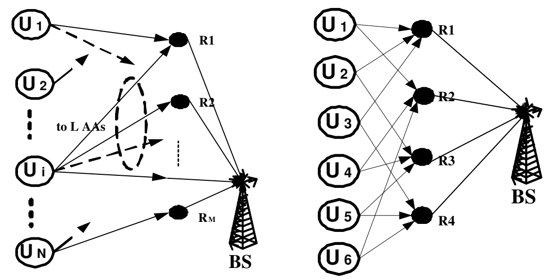
On the other hand, for the network-coded DAS, $\mathbb{P}_{\mathcal{S}2}$ is minimized when $\alpha_2 = 2/3$:

$$\begin{aligned} (\mathbb{P}_{\mathcal{S}2})_{\min} &= \mathbb{P}_{\mathcal{S}2}(\alpha_2 = 2/3) = \frac{27}{4}\beta^2 - \frac{27}{4}\beta^3 \qquad (9) \\ &= \frac{27}{16 \cdot \mathrm{SNR}_\mathrm{T}^2} - \frac{27}{32 \cdot \mathrm{SNR_T}^3} \sim \mathcal{O}\left(\frac{1}{\mathrm{SNR_T}}\right)^2. \end{aligned}$$

Assuming $\mathrm{SNR_T} > 1$,[3] which implies $0 < \beta < \frac{1}{2}$, and comparing the minimum system outage probabilities of the two systems, we have:

$$\Delta(\mathbb{P}_{\mathcal{S}}) = \mathbb{P}_{\mathcal{S}1} - \mathbb{P}_{\mathcal{S}2} = \frac{5}{4}\beta^2 + \frac{27}{4}\beta^3 - 16\beta^4 > 0 \tag{10}$$

We can see that network-coded DAS not only achieves the same diversity order of 2 as plain-DAS; it actually comes with lower system outage. More importantly, this performance improvement is achieved with lower hardware and less bandwidth cost.



(a) General network-coded DAS (b) $N = 6$, $M = 4$, $L = 2$

Fig. 3. Extension of network-coded DAS

We now extend network-coded DAS to the general scenario where there are total of N users and M AA units in the network. We assume that the data from user i can be reliably received by some L_i AAs; specially, we let $L_i = L \leq M, \forall i$, and $(N \times L) | M$. An AA unit R_j can then reliably decode data from a set of K users, $K = \frac{NL}{M}$, and transmit $x_{Rj} = \sum_{k=1}^{K} x_{jk}$ to the base station, where x_{jk} denotes the signal received by antenna j from the mobile terminal k.[4] We write the outage probability for an individual user i as $\mathbb{P}_i$ and denote the uplink channel gain for user i as h_i, and that of j^{th} AA unit as $\widetilde{h}_j$. Without loss of generality, we assume that $\{h_i, \widetilde{h}_j\} \sim \mathcal{CN}(0,1)$, $\forall i, j$, and all units transmit at the same power. Thus, the BER of each uplink satisfies $\{p_i, \widetilde{p}_j\} \sim \frac{1}{\mathrm{SNR}}$, where p_i is the BER along the direct path from user i to the base station and $\widetilde{p}_j$ is the BER along the path from AA j to the base station. Overall, the signal transmitted by each AA combines information from K users (via network coding) and the data of each user is embedded in the signal transmitted by L AA units, as well as in the signal along the direct path to the base station. $\mathbb{P}_i$ can then be computed as

$$\mathbb{P}_i = p_i \prod_{j=1}^{L} \left(\widetilde{p}_j + \sum_{j \neq i}^{K} \mathbb{P}_j - \biguplus_{\{\widetilde{p}_j, \mathbb{P}_j, \forall j \neq i\}} \right), \tag{11}$$

where $\biguplus_{\mathcal{A}} = \sum_{\ell} (\prod_i a_i), a_i \in \mathcal{B}_\ell, \forall \mathcal{B}_\ell \subseteq \mathcal{A}, |\mathcal{B}| \geq 2$. Similar to the approaches taken in the Appendix, we can have $\mathbb{P}_i \sim p_i^{L+1} \sim \mathcal{O}\left(\frac{1}{\mathrm{SNR}}\right)^{L+1}$.

A specific example of a *multi-user* network is shown in Fig. 3(b) for $N = 6$, $M = 4$ and $L = 2$.[5] Here $K = \frac{NL}{M} = 3$, and each individual user can expect a diversity order of $L + 1 = 3$. We see that user 1 experiences outage only when *all* the following three conditions hold:

1) h_1 fails with probability p_1.

[3]Note that approximation (5) is obtained under the assumption that $\mathrm{SNR} \gg 1$ for each individual uplink channel, i.e., $\mathrm{SNR_T} = \sum \mathrm{SNR} \gg 1$. Our assumption $\mathrm{SNR_T} > 1$ is therefore quite loose.

[4]The summation denotes the XOR operation.

[5]We have omitted the direct uplink path for each user in Fig.3(b) for the sake of clarity.

2) $\tilde{h}_1$ fails with probability $\tilde{p}_1$, or either of user 2 and user 3 experience outage with probability $\mathbb{P}_2$, and $\mathbb{P}_3$, respectively. This condition would compromise the recovery of x_1 via $x_1 = \tilde{x}_1 \oplus x_2 \oplus x_3$.
3) $\tilde{h}_2$ fails with probability $\tilde{p}_2$, or either of user 4 and user 5 experience outage with probability $\mathbb{P}_4$, and $\mathbb{P}_5$, respectively. This condition would compromise the recovery of x_1 via $x_1 = \tilde{x}_2 \oplus x_4 \oplus x_5$.

The outage probability of user 1 can be computed as:

$$\mathbb{P}_1 = p_1 \cdot (\tilde{p}_1 + \mathbb{P}_2 + \mathbb{P}_3 - \biguplus_{\{\tilde{p}_1, \mathbb{P}_2, \mathbb{P}_3\}}) \cdot (\tilde{p}_2 + \mathbb{P}_4 + \mathbb{P}_5 - \biguplus_{\{\tilde{p}_2, \mathbb{P}_4, \mathbb{P}_5\}}). \tag{12}$$

Assuming the symmetric case where $\mathbb{P}_i = \mathbb{P}_1 \ll 1, \forall i$ and $\{p_i, \tilde{p}_j\} \sim p \ll 1, \forall i, j$, we have $\biguplus_{\mathcal{A}} \ll a_k, \forall a_k \in \biguplus_{\mathcal{A}}$, (12) can then be rewritten as

$$\mathbb{P}_1 = p \cdot (p + 2\mathbb{P}_1)^2. \tag{13}$$

Solving for $\mathbb{P}_1$ and using Taylor expansion, we have

$$\mathbb{P}_1 \simeq \frac{p^3}{8} \sim \mathcal{O}\left(\frac{1}{\mathrm{SNR}}\right)^3, \tag{14}$$

which agrees with (11). Other users will have similar performance due to the symmetry of the configuration.

The plain-DAS (as described earlier) for a network of N users would require N AA units but offers each user only two independent paths to the base station, i.e., a diversity order of 2. If we simply let a user terminal couple with L different AAs (but without network coding) for higher diversity, the diversity order of $L+1$ can only be achieved at the expense of LN AAs. Furthermore, the plain-DAS system requires $(L+1)N$ orthogonal channels as compared to $N + M < (L+1)N$ channels in network-coded DAS, to achieve the same diversity order of $L+1$. As noted earlier, the diversity gain offered by network coding comes at reduced hardware and spectral costs.

IV. Performance analysis of user cooperation with network coding

Even in the absence of distributed antennas, the additional diversity gain of network coding can still be implemented via user cooperation. In this section, we first study the gains offered when two users cooperate and then extend the results for larger cooperative configurations.

We begin by assuming that users cooperate with each other only in presence of reliable inter-user channel conditions. The slot length M_1 and M_2 are chosen such that the channels experience slow fading within each slot duration and are statistically independent from the first slot to the second.[6] For both user cooperation schemes (with or without network coding), we assume user A and user B transmit with BER p_a and p_b during the first time slot, respectively. During the second time slot, transmission from the two cooperating terminals have BERs p_{Ra} and p_{Rb}, respectively. Assuming the same definition of system outage as before, $\mathbb{P}_{\mathcal{S}1}$, the outage probability for conventional cooperation can be expressed as:

$$\begin{aligned}\mathbb{P}_{\mathcal{S}1} \;=\; & p_a p_{Rb}(1-p_b)(1-p_{Ra}) + p_b p_{Ra}(1-p_a)(1-p_{Rb}) \\ & + p_a p_b \cdot p_{Ra} p_{Rb}.\end{aligned} \tag{15}$$

For the network-coded cooperation scheme, we have

$$\mathbb{P}_{\mathcal{S}2} = (1-p_b)p_a \cdot p_{Ra}p_{Rb} + (1-p_a)p_b \cdot p_{Ra}p_{Rb} + p_a p_b. \tag{16}$$

Assuming $\{p_a, p_b, p_{Ra}, p_{Rb}\} \sim p \sim \frac{1}{\mathrm{SNR}}$, (15) and (16) can then be simplified as:

$$\mathbb{P}_{\mathcal{S}1} = p^2(2 - 2p + 3p^2) \sim \frac{1}{\mathrm{SNR}^2} \tag{17}$$

$$\mathbb{P}_{\mathcal{S}2} = p^2(1 + 2p - 2p^2) \sim \frac{1}{\mathrm{SNR}^2}, \tag{18}$$

Simple algebra yields

$$\Delta(\mathbb{P}_{\mathcal{S}}) = P_{\mathcal{S}1} - P_{\mathcal{S}2} = p^2(1 - 4p + 5p^2) > 0, \forall p \in (0,1).$$

We can see that while both schemes provide a diversity order of 2, network coding has *strictly* lower system outage probability. The power of network coding is in fact more pronounced in presence of multiple users. The user cooperation with network coding in a wireless network with three mobile users is illustrated in Fig. 4. The conventional cooperative scheme, on the other hand, cannot properly address (or be extended to address) cooperation between multiple ($N > 2$) users in two time slots. For this reason, we focus the remainder of our discussion on network-coded user cooperation only.

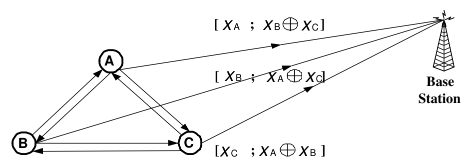


Fig. 4. User cooperation with network coding in presence of 3 users

The network-coded scheme provides "fair" cooperation between all users in Fig. 4. Each user transmits its own message during the first time slot. In the second slot, each user combines the data received from the other two users via network coding and transmits the result. Again, we define system outage probability $\mathbb{P}_{\mathcal{S}}$ as the probability that the data from at least one of the three users cannot be correctly recovered at the BS. We construct $\mathcal{M} = \{a, b, c, a \oplus b, b \oplus c, a \oplus c\}$ as the set of transmitted signals over the two time slots. We assume quasi-slow Rayleigh fading uplinks, i.e., for $s \in \mathcal{M}$, $h_s \sim \mathcal{CN}(0,1)$ where h_s is channel gain for signal s. As before, we assume transmit power is uniformly allocated among the three users.

We let the set $\mathcal{G}$ ($\mathcal{G} \subseteq \mathcal{M}$) be those signals *incorrectly* received by the base station. It is straightforward to verify that when $|\mathcal{G}| \leq 2$, the data from the *all* three users can be correctly retrieved with the help of network coding. When $|\mathcal{G}| \geq 4$, a system outage will occur *with probability* 1. For the special case of $|\mathcal{G}| = 3$, data from the three users can still

[6] When the fading statistics in M_1 and M_2 frames are not independent, the network-coded scheme reverts to the conventional user cooperation scheme.

be fully recovered except for a few special cases:

$$\begin{aligned}\mathcal{G} &= \{a,b,c\}, \text{i.e., all data is un-retrievable;}\\ \mathcal{G} &= \{a, a\oplus b, a\oplus c\}, \text{i.e., } a \text{ is un-retrievable;}\\ \mathcal{G} &= \{b, a\oplus b, b\oplus c\}, \text{i.e., } b \text{ is un-retrievable;}\\ \mathcal{G} &= \{c, b\oplus c, a\oplus c\}, \text{i.e., } c \text{ is un-retrievable;}\end{aligned}$$

If $\mathcal{G}$ is *not* one of these cases, for example, if $\mathcal{G} = \{b, c, a\oplus c\}$, i.e. the set $\mathcal{G}^C = \{a, a\oplus b, b\oplus c\}$ is received correclty, then a is directly recovered; b can be recovered by $b = a\oplus(a\oplus b) = b$; and c can be recovered after b is recovered via $c = b\oplus(b\oplus c)$. Thus, for the system shown in Fig. 4, ($|\mathbb{N}| = 3, |\mathcal{M}| = 6$), the probability of system outage can be expressed as:

$$\begin{aligned}\mathbb{P}_{\mathcal{S}} &= \frac{|\mathbb{N}|+1}{\binom{|\mathcal{M}|}{|\mathbb{N}|}} p_s^3(1-p_s)^3 + p_s^4(1-p_s)^2 \qquad (19)\\ &\quad + p_s^5(1-p_s) + p_s^6 = \frac{1}{5}p_s^3 + \mathcal{O}(p_s^4) \sim \mathcal{O}(\frac{1}{\text{SNR}})^3,\end{aligned}$$

which corresponds to a system diversity order of 3.

A natural extension of Fig.4 with N ($N > 2$) users would require that user i transmits $(x_i \oplus \sum_{j=1}^{N} x_j)$ in the second time slot. Following the analysis above, the system outage probability of such a generalized network can be written as:

$$\begin{aligned}\mathbb{P}_{\mathcal{S}} &= \frac{1}{\binom{2N}{N+1}} p_s^N(1-p_s)^N + \sum_{k=N+1}^{2N} p_s^k(1-p_s)^{2N-k} \qquad (20)\\ &= \frac{1}{\binom{2N}{N+1}} p_s^N(1-p_s)^N + \mathcal{O}(p_s^{N+1}) \sim \mathcal{O}(\frac{1}{\text{SNR}})^N,\end{aligned}$$

which, theoretically, provides a diversity order N. However, when $N \to \infty$, the underlying assumptions that lead to (20) are strongly weakened. For example, to achieve (20), each user needs to make sure that all the other $N-1$ users can reliably decode its data. As $N \to \infty$, this would require unreasonably high transmit powers. Alternatively, we may assume user i transmits the combined information for only a *subset* of users, instead of *all* users, in the second time slot. Let us denote $\mathbb{N}$ as the full set of N users, we assume user i is able to reliably decode the data from a subset of users $\mathbb{M}_i$, where $\mathbb{M}_i \subseteq \mathbb{N}$ and $|\mathbb{M}_i| = m_i$. Each user then uses certain criterion to select a subset $\mathbb{N}_i$ of n_i ($n_i \leq m_i$) cooperating users from $\mathbb{M}_i$; we assume $n_i = n, \forall i$, (e.g., the n users with the highest inter-user SNR). In the second time slot, user i transmits network-coded data for these n users. Fig. 5 shows such scheme when $n = 2$. Assuming user locations are random and uniformly distributed, we see (via the law of large numbers) that when $N \to \infty$, each user's data will be embedded in n copies of network-coded data from other users w.p.1. during the second time slot. A diversity of $n+1$ can then be reasonably achieved (more details in Appendix).

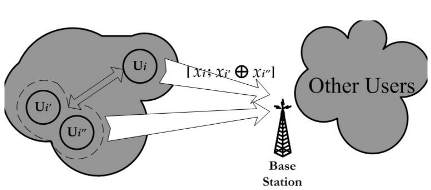


Fig. 5. Subset user cooperation with network coding.

V. Performance Evaluation

We now present the numerical results for the network-coded wireless systems discussed in the previous sections. For simplicity, no coding (such as distributed channel coding) is used other than the network coding. All uplink channels are modeled as normalized Rayleigh fading with $h \sim \mathcal{CN}(0,1)$. Single receive antenna at the base station is assumed and the total transmit power for each scheme is constrained to E_T. The system outage probabilities of the two-user plain-DAS

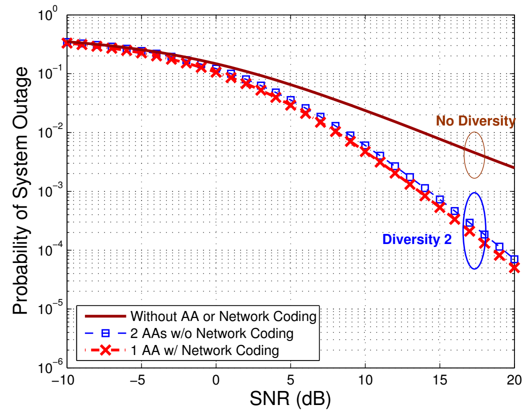


Fig. 6. System outage probabilities for different schemes with AAs, $|\mathbb{N}| = 2$.

and DAS with network coding are shown in Fig. 6, as is the outage probability of a system without AAs or network coding. The total power E_T is allocated among user terminals and AA units according to the optimization results derived in Section III. Compared to the outage performance of the system without assisting antennas, both DAS-based schemes achieve a diversity of 2, with network coding enabling a lower outage probability than the plain-DAS, especially at high SNR. As discussed earlier, the improvement of network-coded DAS is achieved with fewer hardware and bandwidth resources.

Fig. 7 shows the system outage probabilities of the conventional cooperative scheme and of the scheme with network coding, assuming two network users. In addition, we include (for comparison purposes) the outage probability when no cooperation is supported. A diversity order of 2 is achieved for both cooperation schemes, with network-coded user cooperation offering further reduced system outage probability.

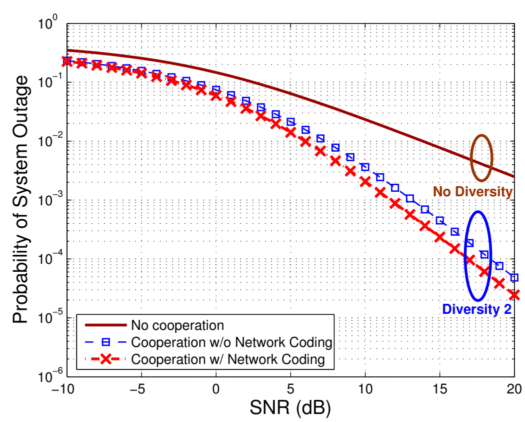


Fig. 7. System outage probabilities for two-user cooperation schemes.

The advantage of applying network coding in a multi-user cooperative network is demonstrated in Fig. 8, which shows the outage probability of the cooperation network in Fig. 4. Although the conventional user cooperation scheme *cannot* provide an appropriate approach to support three co-operating users simultaneously, the theoretical system outage probability for two-user cooperation is also shown in Fig. 8 for comparison purpose. Also shown is the outage probability without cooperation or network coding. We observe that while conventional cooperation scheme achieves a diversity of 2 (although it may not be even applicable in presence of three users), network coding between three nodes leads to improved diversity performance of order 3, as is predicted by (19).

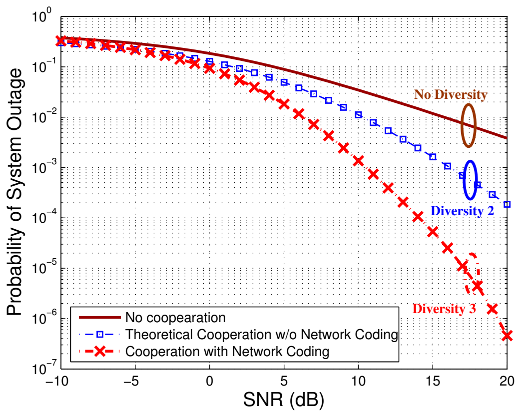


Fig. 8. System outage probabilities for different three-node systems with and without network coding.

VI. Conclusion and Remarks

We discussed the application of network coding [1] in wireless networks that either contain distributed antenna systems or support user cooperation between user terminals. In both cases, improved diversity gains are achievable with the introduction of network coding. In the practical case where only a few nodes are located over a small geographical area and long channel codes are unapplicable, network coding provides a feasible method for performance improvement. For large networks, distributed channel coding can be used on top of network coding, and further improvement would be possible. We have concentrated on the application of *linear network coding* in this work. However, other non-linear network coding schemes have also been developed for wired networks [5], and may be applicable for wireless communications.

Appendix

Achieving diversity $n+1$ in network-coded cooperative network

We begin with the case when $n = 2, \forall i$. We let $\mathbb{P}_i$ denote the outage probability of an individual user i, i.e., the probability that data of user i cannot be retrieved at the base station. During the second time slot, the data x_i is also embedded in the transmission from a user j and a user k, in the form of $x_i \oplus x_{j'}$ and $x_i \oplus x_{k'}$, respectively, where j' is the *other* cooperating partner for user j and k' is the other cooperating partner for user k. Assuming that the transmission of the x_i, $x_i \oplus x_{j'}$, and $x_i \oplus x_{k'}$ is subject to the BER of p_i, $\widetilde{p}_j$ and $\widetilde{p}_k$, respectively, we have:

$$\mathbb{P}_i = p_i(\tilde{p_j} + \mathbb{P}'_j - \tilde{p_j}\mathbb{P}'_j)(\tilde{p_k} + \mathbb{P}'_k - \tilde{p_k}\mathbb{P}'_k). \tag{21}$$

We assume that $\mathbb{P}_i \simeq \mathbb{P}_{j'} \simeq \mathbb{P}_{k'} = \mathbb{P}$, and $0 < \{p_i, p_{k'}, p_{j'}\} \sim p \ll 1$, a closed-form solution for (21) can then be given as:

$$\mathbb{P} = \frac{1 + x - \sqrt{1+2x}}{y} \tag{22}$$

where $x = 2p^2(p-1)$ and $y = 2p(p-1)^2$. Assuming $0 < \{x, y\} \ll 1$, (22) can be rewritten by Taylor expansion as:

$$\mathbb{P} \;=\; \frac{x^2}{2y} + \mathcal{O}\left(\frac{x^3}{2y}\right) = p^3 + \mathcal{O}(p^5) \sim \mathcal{O}\left(\frac{1}{\mathrm{SNR}}\right)^3,$$

i.e., when $n = 2$, a diversity of $n + 1 = 3$ is achieved.

For any arbitrary $n \leq N - 1$, a more general form of (21) can be written as

$$\mathbb{P}_i = p_i \prod_{j \in \mathbb{N}_i} \left(p_{j2} + \sum_{k=1}^{n} \mathbb{P}_{jk} - \biguplus_{\{\tilde{p}_{j2}, \mathbb{P}_{jk}(\forall k=1\cdots n)\}} \right), \tag{23}$$

where p_{j2} is the BER of the transmission in the second time slot of user j, $\mathbb{P}_{jk}$ denotes the outage probability of the k^{th} user in user j's cooperation set $\mathbb{N}_j$. When $n = 3$ or 4, the degree-n polynomial equation (23) can be solved to obtain a closed-form expression for $\mathbb{P}$; it can be shown that a diversity of 4 and 5 can be achievable, respectively. However, there is no algebraic solution for a general polynomial equations higher or equal to degree five, according to the *Abel's Impossibility Theorem* [11]. While numerical analysis can easily verify that diversity $n + 1$ is possible for arbitrary n, we can also note from the solutions to $\mathbb{P}$ when $n = 2, 3$ and 4 that $\mathbb{P} \ll p$. Thus, by assuming $p + n\mathbb{P} \simeq p$ and $p\mathbb{P} \ll p$, equation (23) implies

$$\mathbb{P} \sim p^{n+1} \sim \mathcal{O}\left(\frac{1}{\mathrm{SNR}}\right)^{n+1}, \tag{24}$$

i.e., diversity of $n + 1$ is achievable. □

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