Mitigating Error Propagation in Decision-Feedback Equalization for Multiuser CDMA

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***Abstract*—This letter presents a robust decision-feedback equalization design that mitigates the error-propagation problem for multiuser direct-sequence code-division multiple-access systems under multipath fading. Explicit constraints for signal energy preservation are imposed on the filter weight vector to monitor and maintain the quality of the hard decisions in the nonlinear feedback loop. Such a measure protects the desired signal power against the detrimental effect of erroneous past decisions, thus providing the leverage to curb error propagation.**

***Index Terms*—Decision-feedback equalization (DFE), error propagation, multiuser code-division multiple access (CDMA), recursive least-square (RLS) implementation, robust constraining.**

I. Introduction

DECISION-feedback equalization (DFE) [1]–[3] is a very effective receiver component that cancels intersymbol interference (ISI) under diverse channel conditions without causing noise amplification. However, for DFE and decision-directed adaptation in general [1], noise-induced symbol decision errors may propagate through the feedback loop, leading to unreliable transmissions with error bursts. Various *ad hoc* techniques for mitigating error propagation have been proposed, including taking preventive measures against detectable feedback errors [4]–[6], and adopting equalizer structures with a reduced probability of long error bursts [7]. In practice, a typical operation of adaptive DFE detection involves mode switching between a decision-directed blind transmission phase and a training phase to avoid unreliable adaptation.

In this letter, we propose a constrained optimization approach to tackle the error-propagation problem existing in the decision-directed class of adaptive filtering methods. The root to error propagation lies in the quality of the hard decisions in the feedback loop. As a countermeasure, we impose explicit signal-preserving constraints on DFE to monitor and sustain the accuracy of tentative decisions, such that detection errors are less likely to happen or propagate. No extra error detection or *ad hoc* monitoring devices are resorted to. Note that *implicit* signal-preserving constraining is present in the basic single-user and multiuser DFEs [2], [8]. We establish that it is the *explicit* constraining that makes DFE robust to error propagation. While this constraining strategy applies to any decision-directed adaptive filter, its realization relies on the feasibility of constructing practical signal-preserving constraints, which take on various forms in different signal-processing applications. For a multiuser direct-sequence code-division multiple-access (DS-CDMA) system under multipath fading, it amounts to constructing appropriate constraints on the feedforward filter weights of DFE from the knowledge of the desired user's spreading code. When the constraints are properly constructed to match the channel, they are able to pull an equalizer out of a bad minimum by forcing energy preservation of the desired signal. It naturally leads to robust blind multiuser DFE.

The constrained optimization idea has been developed for blind equalization [9], and in linear minimum output energy (MOE) detection for blind multiuser detection in CDMA [10]–[12]. This letter takes a fresh look at the constrained approach and recognizes its efficacy in mitigating error propagation for DFE. Based on this observation, along with the existing literature on signal-preservation constraint construction for multiuser CDMA, we develop robust blind constrained DFE algorithms. The adaptive implementations of these DFE algorithms will be described using recursive least-squares (RLS) adaptation rules, and the corresponding analytic steady-state signal-to-interference-and-noise ratio (SINR) performance will be presented.

II. DFE for Multiuser CDMA

Consider a multiuser DS-CDMA system with a spreading gain of L. The multipath channel experienced by the kth user can be modeled as a tapped-delay-line filter in the form of $g^{(k)}(t) = \sum_{i=0}^{N_g-1} g_i^{(k)}\delta(t - iT_c)$, where T_c is the chip period, N_g denotes the (maximum) number of chips spanned by the multipath channel, and $g_i^{(k)}$ are the unknown channel tap coefficients. Let us assume the desired user to be user 1, for which we drop the superscript $^{(1)}$ for notational simplicity. The received chip-rate baseband data vector $\mathbf{x}(k)$ can be modeled as $\mathbf{x}(k) = \sum_{i=0}^{N_b} \mathbf{h}_i b(k-i) + \mathbf{i}(k) + \mathbf{n}(k)$, which combines the desired user's current symbol $b(k)$, its surrounding N_b ISI symbols $\{b(k-i)\}_{i=1}^{N_b}$, each weighted by its impulse response signature vector $\mathbf{h}_i$, and the noise component $\mathbf{v}(k) := \mathbf{i}(k) + \mathbf{n}(k)$, consisting of both multiple-access interference (MAI) $\mathbf{i}(k)$ and the ambient noise $\mathbf{n}(k)$. Synchronization is assumed for the desired user only, and no information on the interfering users is required for demodulating $b(k)$. The length of $\mathbf{x}(k)$ is N_c set to be $N_c \geq L + N_g$, which represents the memory length of the ensuing chip-rate digital receiver. A matrix-vector model arises in the form of $\mathbf{x}(k) = \mathbf{h}b(k) + \mathbf{H_d}\mathbf{d}(k) + \mathbf{v}(k)$, where $\mathbf{h} := \mathbf{h}_0$, $\mathbf{H_d} := [\mathbf{h}_1 \ \cdots \ \mathbf{h}_{N_b}]$, and $\mathbf{d}(k) := [b(k-1), \ldots, b(k-N_b)]^T$.

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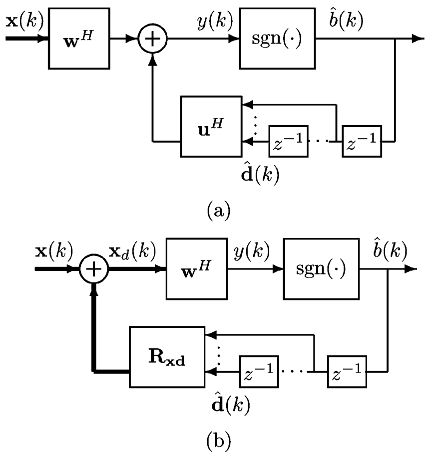

Fig. 1. Standard DFE and its equivalent MMSE filter.

It can be shown that the multipath channel vector $\mathbf{h}$ takes on the form of $\mathbf{h} = \mathbf{C}\mathbf{g}$, where $\mathbf{C}$ is an $N_c \times N_g$ matrix whose jth column contains the spreading code vector delay shifted by $(j-1)$ chips [11], and $\mathbf{g} := \left[g_0^{(1)}, \ldots, g_{N_g-1}^{(1)}\right]^T$ is the fading gain vector of the N_g delayed paths. The matrix $\mathbf{C}$ is known to the receiver, while the multipath channel parameters of the desired user, $\mathbf{g}$, and all the parameters of other users, are assumed unknown.

DFE is achieved via a chip-rate feedforward filter $\mathbf{w} = [w_1, \ldots, w_{N_c}]^T$, and a symbol-rate feedback filter $\mathbf{u} = [u_1, \ldots, u_{N_d}]^T$, of length $N_d \geq N_b$ to cover all ISI symbols, as illustrated in Fig. 1(a). The filters yield soft estimates $y(k) = \mathbf{w}^H\mathbf{x}(k) - \mathbf{u}^H\hat{\mathbf{d}}(k)$, in which $\hat{\mathbf{d}}(k)$ are the decisions of past ISI symbols. The filter output $y(t)$ is quantized by a decision device into a hard decision on $b(k)$, e.g., $\hat{b}(k) = \mathrm{sgn}\left[\Re\{y(k)\}\right]$ for binary transmission. Existing ISI can be exactly cancelled by matching the feedback taps to the combined channel and feedforward filter impulse response, using the minimum mean-square error (MMSE) principle (assuming correct past decisions $\hat{\mathbf{d}}(k) = \mathbf{d}(k)$)

$$\begin{aligned}\min_{\mathbf{w},\mathbf{u}} \quad \epsilon^2 &:= E\left\{|b(k) - y(k)|^2\right\} \\ &= E\Big\{|b(k) - \mathbf{w}^H\mathbf{h}b(k) - \mathbf{w}^H\mathbf{H_d}\mathbf{d}(k) \\ &\qquad - \mathbf{w}^H\mathbf{n}(k) + \mathbf{u}^H\hat{\mathbf{d}}(k)|^2\Big\}. \end{aligned} \tag{1}$$

The optimum DFE solution to (1) imposes the following implicit soft constraints on $\mathbf{w}$ and $\mathbf{u}$:

$$\mathbf{w}^H\mathbf{h} = 1 \tag{2}$$

$$\mathbf{w}^H\mathbf{H_d} = \mathbf{u}^H \tag{3}$$

$$\min_{\mathbf{w}} E\left\{\left|\mathbf{w}^H\mathbf{v}(k)\right|^2\right\}. \tag{4}$$

Equations (2) and (3) recover the current symbol and eliminate ISI, while (4) suppresses the noise and interference output.

III. Robust Constrained DFE

The dissection (2)–(4) on the intrinsic optimization process in DFE reveals that signal energy preservation, implicitly performed in (2), holds the key to alleviate the error-propagation problem. Unfortunately, when noise-induced estimation errors occur, i.e., $\hat{\mathbf{d}}(k) \neq \mathbf{d}(k)$, the design strategy in (3) no longer cancels ISI. Incorrect ISI cancellation at this step may destroy the balanced optimization performed by (2) and (4), leading to decision errors in the current symbols that may propagate into future estimates. For robustness against error propagation, it is vital to monitor and maintain the decision quality inside the feedback loop.

We propose to take out the *implicit* soft signal-preserving constraint in (2) and *explicitly* impose it as a hard constraint on the feedforward filter, while (3) and (4) are still enforced as implicit constraints by minimizing over (1). This constrained DFE (C-DFE) detector is formulated as

$$\min_{\mathbf{w},\mathbf{u}} \quad E\left\{\left|b(k) - \mathbf{w}^H\mathbf{x}(k) + \mathbf{u}^H\hat{\mathbf{d}}(k)\right|^2\right\} \tag{5}$$

$$\text{s.t.} \quad \mathbf{w}^H\mathbf{h} = 1. \tag{6}$$

In the C-DFE structure, the quality of the feedback decisions, namely, $\hat{\mathbf{d}}(k)$, is monitored by the signal-preserving constraint (2). Under adverse channel conditions, this constraint is expected to prevent the output signal power of the desired user from dropping dramatically, therefore, decision errors are less likely to happen or propagate. On the other hand, when the standard DFE is operating under stable transmissions, this constraint is inherently complied to the maximum extent, and will only be minimally activated.

Note that the constraint $\mathbf{w}^H\mathbf{h} = 1$ itself is not a new contribution. In fact, it is implicit in the basic single-user and multiuser DFEs in the absence of error propagation, as indicated by (2). Our approach of imposing it as an explicit constraint is seemingly redundant. However, when feedback errors occur, this implicit constraint is no longer executed by (1) due to incorrect ISI cancellation from another implicit constraint (3). In contrast, the explicit constraint in (6) is still enforced to achieve signal preservation. It is the *explicit constraining* that makes DFE robust to error propagation. Explicit constraining on DFE enjoys other benefits as well: 1) enhanced MAI suppression capacity, since the MAI minimization in (4) is less likely to be compromised by incorrect ISI cancellation; and 2) improved steady-state performance for RLS-based adaptive DFE filtering, as will be shown in Section VI.

IV. Blind Formulations of Robust DFE

To implement the robust C-DFE in (6), both the desired symbol $b(k)$ and the channel information $\mathbf{h}$ are required to be known to the receiver. Typically, $b(k)$ is substituted by its estimate $\hat{b}(k)$ via decision direction [1]. To get around the unknown channel knowledge $\mathbf{h}$, it has been suggested in the linear MOE literature that the signal-preserving constraint (6) be alternatively implemented as $\mathbf{w}^H\mathbf{C} = \mathbf{f}^H$ [10]. Such a contraint preserves the signal components that are projected onto the delayed paths described by the columns of $\mathbf{C}$. The detection performance depends on how the constraint vector $\mathbf{f}$ is specified, while the optimal value for $\mathbf{f}$ depends on not only the data covariance of $\mathbf{x}(k)$, but also the channel gain $\mathbf{g}$, both of which are generally unavailable to the receiver. Common practice is to choose a nominal value for $\mathbf{f}$ [10], or to optimize it via a max/min formulation [11], [12]. We adopt these two constraint construction methods to develop robust blind DFE algorithms as follows.

A. C-DFE With Fixed Linear and Quadratic Constraints

A simple strategy for constructing $\mathbf{f}$ is to set it to a fixed value which is reasonably close to the optimal value over a wide range of scenarios. To compensate for residual modeling mismatch, we add a quadratic inequality constraint in the form of $\mathbf{w}^H\mathbf{w} \leq T_o$ to avoid excessive noise enhancement and to ensure adequate ISI cancellation. This robust decision-directed C-DFE detector is described by

$$\min_{\mathbf{w},\mathbf{u}} \quad E\left\{\left|\hat{b}(k) - \mathbf{w}^H\mathbf{x}(k) + \mathbf{u}^H\hat{\mathbf{d}}(k)\right|^2\right\}$$
$$\text{s.t.} \quad \mathbf{w}^H\mathbf{C} = \mathbf{f}_o, \mathbf{w}^H\mathbf{w} \leq T_o. \tag{7}$$

Some reasonable *ad hoc* choices of the constraint parameters $\mathbf{f}_o$ and T_o are discussed in [12]. Practical values for the fixed vector $\mathbf{f} \in \mathcal{C}^{N_g \times 1}$ could be $[1 \quad 0 \quad \ldots \quad 0]^T$, $[1 \quad 1 \quad \ldots \quad 1]^T$, or $[1 \quad (1/N_g - 1) \quad \ldots \quad (1/N_g - 1)]^T$, while a heuristic value for T_o is given by $2\mathbf{f}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{f}$.

To express the optimum solution to (7), let us define several correlation quantities: $\mathbf{R}_\mathbf{x} := E\{\mathbf{x}\mathbf{x}^H\}$, $\mathbf{R}_{\mathbf{xd}} := E\{\mathbf{x}\hat{\mathbf{d}}^H\}$, $\mathbf{p}_{\mathbf{x}b^*} := E\{\mathbf{x}b^*\}$, $\mathbf{R}_{\mathbf{x},M} := \mathbf{R}_\mathbf{x} - \mathbf{R}_{\mathbf{xd}}\mathbf{R}_{\mathbf{xd}}^H$, and $\mathbf{P}_M = \mathbf{R}_{\mathbf{x},M}^{-1} - \mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C})^{-1}\mathbf{C}^H\mathbf{R}_{\mathbf{x},M}^{-1}$. Without the quadratic constraint, the optimal solution to $\mathbf{w}$ is given by a linear combination of two weight vectors $\mathbf{w} = \bar{\mathbf{w}} + \tilde{\mathbf{w}}$, in which $\bar{\mathbf{w}} = \mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C})^{-1}\mathbf{f}$ is in the form of a standard linearly constrained minimum power (LCMP) weight vector that satisfies the linear signal-preserving constraints, and $\tilde{\mathbf{w}} = \mathbf{P}_M\mathbf{p}_{\mathbf{x}b^*}$ is the unconstrained DFE feedforward weight vector projected onto the space orthogonal to the constraints. Note that the matrix $\mathbf{P}_M$ lies in the subspace orthogonal to $\mathbf{C}$, i.e., $\mathbf{P}_M\mathbf{C} = \mathbf{0}$, but it can be updated from the data vector $\mathbf{x}(k)$ in the direct form [12].

With an additional quadratic constraint, the optimal solution to $\mathbf{w}$ has the same form as above, but with $\mathbf{R}_{\mathbf{x},M}$ replaced by its diagonally loaded version $(\mathbf{R}_{\mathbf{x},M} + \lambda\mathbf{I})$. The loading term λ is determined by quadratic constraint parameter T_o. Having obtained $\mathbf{w}$, the optimal solution to $\mathbf{u}$ can be deduced as $\mathbf{u} = \mathbf{R}_{\mathbf{xd}}^H\mathbf{w}$.

B. C-DFE With Variable Linear Constraints

In an effort to obtain a close-to-optimal constraint vector $\mathbf{f}$, a max/min approach with variable constraint parameters [11] can be applied to C-DFE as follows:

$$\max_{||\mathbf{f}||=1}\min_{\mathbf{w},\mathbf{u}} \quad E\left\{\left|b(k) - \mathbf{w}^H\mathbf{x}(k) + \mathbf{u}^H\hat{\mathbf{d}}(k)\right|^2\right\}$$
$$\text{s.t.} \quad \mathbf{w}^H\mathbf{C} = \mathbf{f}^H. \tag{8}$$

The minimization process finds the weight vectors $\mathbf{w}$ and $\mathbf{u}$ to suppress ISI and MAI while preserving the desired symbol, and the maximization process finds a constraint vector $\mathbf{f}$ to maximize the signal energy after interference suppression. After some manipulations, it is found that the optimized weight vectors are $\mathbf{w} = \mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C})^{-1}\mathbf{e} + \mathbf{P}_M\mathbf{p}_{\mathbf{x}b^*}$ and $\mathbf{u} = \mathbf{R}_{\mathbf{xd}}^H\mathbf{w}$, respectively. The vector $\mathbf{e}$ is the principal eigenvector of $(\mathbf{C}^H\mathbf{R}_{\mathbf{x},M}^{-1}\mathbf{C})^{-1}$, while the optimized constraint vector is given by $\mathbf{f} = \mathbf{e}$.

TABLE I
DIRECT FORM RLS IMPLEMENTATION OF LINEARLY AND QUADRATICALLY CONSTRAINED DD-DFE DETECTOR

Init.	$\bar{\mathbf{w}}(0) = \mathbf{w}_q,\ \tilde{\mathbf{w}}(0) = \mathbf{0},\ \mathbf{w}(0) = \mathbf{w}_q,\ \mathbf{u}(0) = \mathbf{0}$ $\mathbf{P}_M(0) = \frac{1}{\sigma_o^2}\mathbf{P}_c^\perp,\ \mathbf{R}_{\mathbf{xd}}(0) = \mathbf{0},\ \hat{\mathbf{d}}(0) = \mathbf{0}$
Data Updating	$\mathbf{R}_{\mathbf{xd}}(k) = \mu\mathbf{R}_{\mathbf{xd}}(k-1) + \mathbf{x}(k)\hat{\mathbf{d}}^H(k)$ $\mathbf{x}_d(k) = \mathbf{x}(k) - \mathbf{R}_{\mathbf{xd}}(k)\hat{\mathbf{d}}(k)$
Decision	$y(k) = \mathbf{w}^H(k-1)\mathbf{x}(k) - \mathbf{u}^H(k-1)\hat{\mathbf{d}}(k)$ $\hat{b}(k) = \operatorname{sgn}[Re\{y(k)\}]$
RLS Update	$\mathbf{g}_M(k) = \dfrac{\mathbf{P}_c^\perp\mathbf{P}_M(k-1)\mathbf{x}_d(k)}{\mu + \mathbf{x}_d^H(k)\mathbf{P}_M(k-1)\mathbf{x}_d(k)}$ $\mathbf{P}_M(k) = \mu^{-1}[\mathbf{P}_M(k-1) - \mathbf{g}_M(k)\mathbf{x}_d^H(k)\mathbf{P}_M(k-1)]$
Tentative Weight Update	$y_p(k) = \bar{\mathbf{w}}(k-1)^H\mathbf{x}_d(k)$ $\bar{\mathbf{w}}(k) = \bar{\mathbf{w}}(k-1) - \mathbf{g}_M(k)y_p^*(k)$ $e_b(k) = \hat{b}(k) - \tilde{\mathbf{w}}^H(k-1)\mathbf{x}_d(k)$ $\tilde{\mathbf{w}}(k) = \tilde{\mathbf{w}}(k-1) + \mathbf{g}_M(k)e_b^*(k)$ $\mathbf{w}(k) = \bar{\mathbf{w}}(k) + \tilde{\mathbf{w}}(k)$
Variable Loading	if $\|\|\mathbf{w}(k)\|\|^2 > T_o$ $\mathbf{v}(k) = \mathbf{P}_M(k)\mathbf{w}(k)$ $a = \|\|\mathbf{v}(k)\|\|^2$, $b = -2Re\{\mathbf{v}^H(k)\mathbf{w}(k)\}$, $c = \|\|\mathbf{w}(k)\|\|^2 - T_o$ $\lambda(k) = \frac{1}{2a}\left(-b - \Re e\left\{\sqrt{b^2-4ac}\right\}\right)$ $\mathbf{w}(k) = \mathbf{w}(k) - \lambda(k)\mathbf{v}(k)$ $\mathbf{u}(k) = \mathbf{R}_{\mathbf{xd}}^H(k)\mathbf{w}(k)$

V. RLS ADAPTATION ALGORITHMS

Literature abounds on adaptive *linear* filtering in the RLS form [13]. To make these techniques available for *nonlinear* DFE adaptive filters, we develop a key transform that bridges a nonlinear DFE filter to an equivalent linear MMSE filter, as discussed below.

Observe that the optimum feedback filters in all DFE formulations (1), (7), and (8) are given by $\mathbf{u} = \mathbf{R}_{\mathbf{xd}}^H\mathbf{w}$. We introduce a new transformed data vector

$$\mathbf{x}_d(k) := \mathbf{x}(k) - \mathbf{R}_{\mathbf{xd}}\hat{\mathbf{d}}(k). \tag{9}$$

When the feedback filter $\mathbf{u}$ is optimized, the mean square error (MSE) value of DFE in (1) becomes [17]

$$\epsilon^2 = E\left\{\left|b(k) - \left(\mathbf{w}^H\mathbf{x}(k) - \mathbf{w}^H\mathbf{R}_{\mathbf{xd}}\hat{\mathbf{d}}(k)\right)\right|^2\right\}$$
$$= E\left\{\left|b(k) - \mathbf{w}^H\mathbf{x}_d(k)\right|^2\right\}. \tag{10}$$

This implies that a nonlinear DFE filter based on the input data vector $\mathbf{x}(k)$ is equivalent to a linear MMSE filter based on the input $\mathbf{x}_d(k)$.

This transformation process, illustrated in Fig. 1, enables the construction of RLS adaptation rules for various DFE detectors, based on well-studied RLS procedures for their equivalent MMSE formulations. Accordingly, the robust C-DFE receivers in Sections IV-A and B can be implemented as constrained MMSE, whose adaptive implementations have been developed in [12].

The direct-form adaptive RLS implementations of robust DFE filters are summarized in Tables I and II. The algorithms use a variable loading technique [12] to implement the quadratic inequality constraint in (7), and the projection approximation subspace tracking with deflation (PASTd) method [16] to update the principle eigenvector $\mathbf{e}$ in (8). The constrained implementation in the generalized sidelobe canceller (GSC) form

TABLE II
DIRECT FORM RLS IMPLEMENTATION OF OPTIMIZED LINEARLY CONSTRAINED DD-DFE DETECTOR

Init.	$\mathbf{e}(0)=\mathbf{f}/\|\|\mathbf{f}\|\|,\ \eta(0)=\frac{1}{L},\ \tilde{\mathbf{w}}(0)=\mathbf{0},\ \mathbf{u}(0)=\mathbf{0}$ $\mathbf{T}(0)=\mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1},\ \mathbf{w}(0)=T(0)\mathbf{e}(0),$ $\mathbf{P}_M(0)=\frac{1}{\sigma_o^2}\mathbf{P}_c^{\perp},\ \mathbf{R}_{\mathbf{xd}}(0)=\mathbf{0},\ \hat{\mathbf{d}}(0)=\mathbf{0}$
Data Updating	$\mathbf{R}_{\mathbf{xd}}(k)=\mu\mathbf{R}_{\mathbf{xd}}(k-1)+\mathbf{x}(k)\hat{\mathbf{d}}^H(k)$ $\mathbf{x}_d(k)=\mathbf{x}(k)-\mathbf{R}_{\mathbf{xd}}(k)\hat{\mathbf{d}}(k)$
Decision	$y(k)=\mathbf{w}^H(k-1)\mathbf{x}(k)-\mathbf{u}^H(k-1)\hat{\mathbf{d}}(k)$ $\hat{b}(k)=\mathrm{sgn}\left[Re\left\{y(k)\right\}\right]$
RLS Update	$\mathbf{g}_M(k)=\dfrac{\mathbf{P}_c^{\perp}\mathbf{P}_M(k-1)\mathbf{x}_d(k)}{\mu+\mathbf{x}_d^H(k)\mathbf{P}_M(k-1)\mathbf{x}_d(k)}$ $\mathbf{P}_M(k)=\mu^{-1}\left[\mathbf{P}_M(k-1)-\mathbf{g}_M(k)\mathbf{x}_d^H(k)\mathbf{P}_M(k-1)\right]$
Subspace Tracking	$\bar{\mathbf{z}}_p(k)=\mathbf{T}(k-1)^H\mathbf{x}_d(k)$ $\rho(k)=\dfrac{\mathbf{e}(k-1)^H\bar{\mathbf{z}}_p(k)}{\sqrt{(1+\mu^{-1}\mathbf{x}_d(k)^H\mathbf{P}_M(k-1)\mathbf{x}_d(k))}}$ $\eta(k)=\mu\eta(k-1)+\|\rho(k)\|^2$ $\mathbf{e}(k)=\mathbf{e}(k-1)+\left[\bar{\mathbf{z}}_p(k)-\mathbf{e}(k-1)\rho(k)\right]\dfrac{\rho^*(k)}{\eta(k)}$
Weight Update	$\mathbf{T}(k)=\mathbf{T}(k-1)-\mathbf{g}_M(k)\bar{\mathbf{z}}_p^H(k)$ $e_b(k)=\hat{b}(k)-\tilde{\mathbf{w}}^H(k-1)\mathbf{x}_d(k)$ $\tilde{\mathbf{w}}(k)=\tilde{\mathbf{w}}(k-1)+\mathbf{g}_M(k)e_b^*(k)$ $\mathbf{w}(k)=\mathbf{T}(k)\mathbf{e}(k)+\tilde{\mathbf{w}}(k)$ $\mathbf{u}(k)=\mathbf{R}_{\mathbf{xd}}^H(k)\mathbf{w}(k)$

TABLE III
PERFORMANCE ANALYSIS OF ADAPTIVE DFE FILTERS ($\gamma=(1-\mu/1+\mu)L$)

	RLS-DFE	RLS-CDFE
SINR^*	$\mathbf{h}^H(\mathbf{R}_\mathbf{x}-\mathbf{h}\mathbf{h}^H-\mathbf{H}_\mathbf{d}\mathbf{H}_\mathbf{d}^H)^{-1}\mathbf{h}$	
SINR^∞	$\dfrac{\mathrm{SINR}^*}{(1+\gamma)+\gamma/\mathrm{SINR}^*}$	$\dfrac{\mathrm{SINR}^*}{\frac{1+2\gamma+\mu^2}{1+\mu}}\approx\dfrac{\mathrm{SINR}^*}{\mu+\gamma}$

can also be derived straightforwardly based on the adaptive algorithms in [12].

VI. STEADY-STATE RLS PERFORMANCE ANALYSIS

This section presents the theoretical SINRs for DFE adaptive filters under stable transmissions. Accurate past decisions are assumed for the steady-state analysis. The two DFE receivers for comparison are RLS-DFE and RLS-CDFE, which are the RLS versions of the unconstrained DFE in (1) and the constrained C-DFE in (6), respectively. The analytic results will also answer the following question: Is explicit constraining redundant under stable transmissions?

Let us define $\mathrm{SINR}^* := (E^2\{y(k)\}/\mathrm{var}\{y(k)\})$ and $\mathrm{SINR}^\infty := \lim_{k\to\infty}(E^2\left\{\mathbf{w}^H(k-1)\mathbf{x}(k)-\mathbf{u}^H(k-1)\mathbf{d}(k)\right\}$ $/\mathrm{var}\left\{\mathbf{w}^H(k-1)\mathbf{x}(k)-\mathbf{u}^H(k-1)\mathbf{d}(k)\right\})$ as the optimum SINR and steady-state SINR, respectively. Our analysis on the steady-state performance of $\mathbf{w}(k)$ and $\mathbf{u}(k)$ builds upon the SINR measures for $\tilde{\mathbf{w}}(k)$ and $\bar{\mathbf{w}}(k)$, which have been studied under the context of unconstrained MMSE filtering and LCMP filtering, respectively [13]–[15]. The detailed derivations are referred to in [18], and the major analytic results are summarized in Table III.

The analytic results are verified using a CDMA setup with seven equal-powered users. The spreading gain is $L=31$, and the feedback filter length is $Nd=5$. For both RLS-DFE and RLS-CDFE, a forgetting factor $\mu=0.9$ is used, and the experimental SINR^∞ curves are obtained using 200 Monte Carlo simulations. Fig. 2 shows that the simulation results match well with the analytic SINR values. The SINR performance gap between the optimal DFE/CDFE and the steady state RLS-CDFE is smaller than that of the unconstrained RLS-DFE. This indicates that appropriate constraining on adaptive DFE improves its steady-state performance.

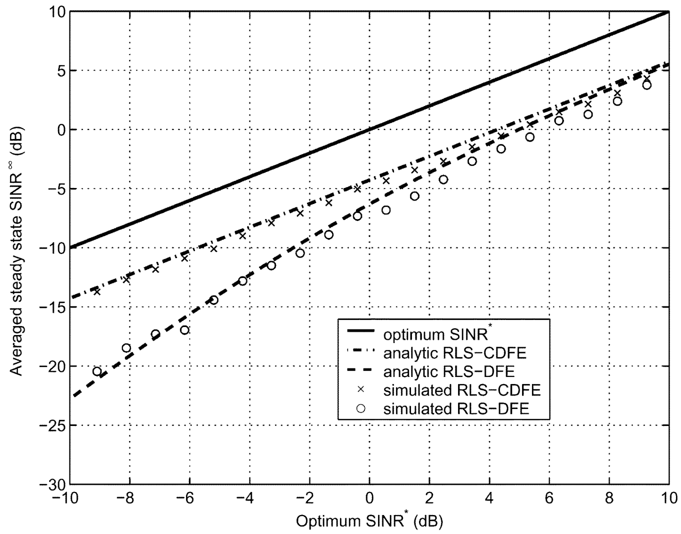

Fig. 2. Steady-state SINR^∞ performance.

Although our performance analysis does not take into account decision-feedback errors, it offers justification for incorporating our constraints even under stable transmissions. The advantage of explicit constraining in error-prone environments will be tested by simulations next.

VII. SIMULATIONS

We compare the standard decision-directed DFE (DD-DFE) receiver against our decision-directed C-DFE with fixed linear and quadratic constraints (DD-CDFE-LQC), and with optimized variable constraints (DD-CDFE-VC). RLS adaptation rules discussed in Section V are applied to all these DFE detectors.

The first example examines the dynamic behavior of these DFE detectors in a multiuser DS-CDMA communication system with a spreading gain of $L=31$. The spreading codes for all the users are generated randomly. The desired user experiences frequency-selective multipath fading. The channel response of each path is generated using the Jakes' method with a maximum Doppler spread corresponding to a terminal speed $v=3$ km/h. The path delays and path variances are taken from the land-mobile model of GSM Rec 05.05, resulting in a channel delay spread of $N_g=8$ chips. The channels for interferers are assumed to be Rayleigh faded. The simulation starts with one desired signal and six MAI signals, each of SNR $=5$ dB. At time $k=500$, a strong MAI signal of 20 dB is added to the system, and the SNR of the desired user drops to 2.5 dB. The channel of the desired user remains time invariant during the first block of 500-chip duration, but changes to a different realization for the rest time. Such a change in the environment may lead to detrimental error propagation. Fig. 3 shows the average bit-error rate (BER) versus time for each

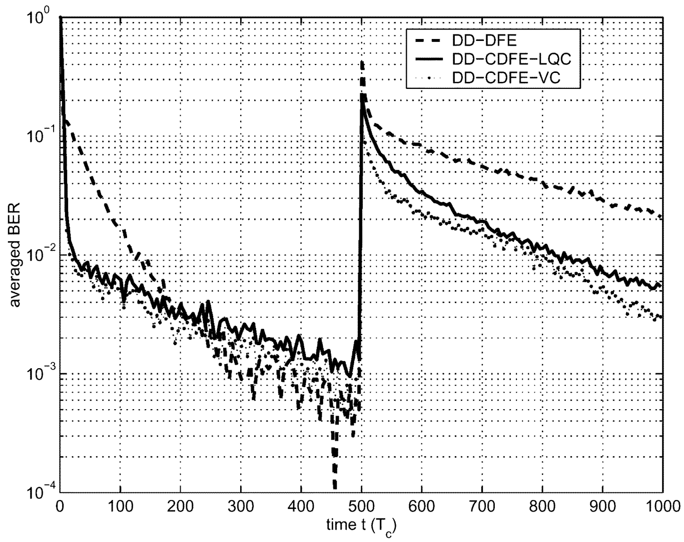

Fig. 3. Average BER of RLS-DFE filters in a dynamic environment.

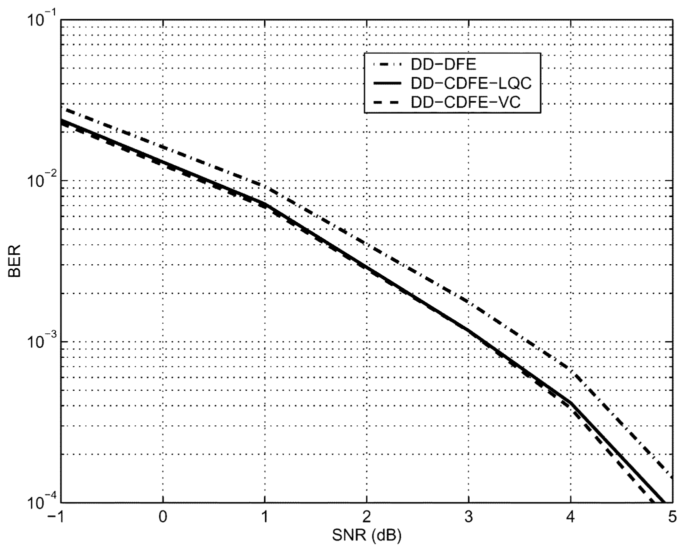

Fig. 4. Steady-state BER performance of RLS-DFE filters.

of the RLS adaptive DFE algorithms using a forgetting factor $\mu = 0.995$. The curves are generated using 10 000 simulations. It is shown that the unsupervised DD-DFE has much higher BER in the changing environment, as a result of its vulnerability to error propagation. Both of the constrained DFE receivers, on the other hand, sustain the impact of error propagation and MAI. This is primarily because the constrained portion of each weight vector works against past decision errors to preserve the desired signal output energy, thus providing the leverage to prevent error propagation and maintain a stable transmission.

Fig. 4 depicts the BER performance of these DFE receivers under stable transmission, averaged over 500 simulations. A similar channel model is used as in the first example, but without a sudden change of environment. The BER performance of robust C-DFEs is slightly better than that of the unconstrained DFE, when error propagation is not a concern. The error performance demonstrated here is consistent with the SINR behavior analyzed in Section VI.

VIII. Summary

We have observed that explicit signal energy-preserving constraints on DFE filters are capable of mitigating the perplexed error-propagation problem. This robust constraining idea leads to various blind constrained DFE formulations, in which erroneous decision errors in the feedback loop are inherently monitored and curbed by our constraints, eliminating the need for training or mode switching under adverse channel conditions. We have also developed adaptive RLS algorithms for these constrained DFE filters via transforming them into equivalent linear MMSE filters. Our theoretical study confirms that constrained DFE filters enjoy enhanced steady-state performance under stable transmissions, in addition to their robustness to error propagation.

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