On the Beating of ASE and XPM Noise in Optical Receivers

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***Abstract*—This letter presents a theoretical study on the effects of beating between amplified spontaneous emission (ASE) noise and cross-phase modulation (XPM)-induced noise in optical receivers. Our theory of ASE-XPM beat noise predicts that such beating effects are actually beneficial to receiver performance, resulting in lower error probabilities. The ASE-XPM beat noise produces skewed non-Gaussian probability distributions and shifts optimum receiver threshold closer to the one level. These conclusions are insensitive to the exact noise statistics assumed for XPM or ASE.**

***Index Terms*—Amplified spontaneous emission (ASE), cross-phase modulation (XPM), noise, optical receivers.**

I. INTRODUCTION

OPTICAL RECEIVER design is growing in importance as wavelength-division-multiplexing (WDM) transmission systems scale to ultradense channels spacing [1]. An optimally designed receiver can partially compensate for the greater transmission penalties inherent in ultradense WDM (UDWDM). The performance of an optical receiver in UDWDM transmission systems depends critically on the optical and electrical filter shapes and bandwidths [2]–[5]. These critical receiver parameters must be optimized taking into account the presence of amplified spontaneous emission noise (ASE), linear crosstalk, and fiber nonlinearity. The ultimate goal is to approach the quantum limit of receiver performance, although this is extremely challenging due to fiber nonlinear effects, which distort the pulse shape and produce noise in the receiver.

Cross-phase modulation (XPM) fiber nonlinearity is a particularly important effect in UDWDM systems operating at 10-Gb/s channel rates [6]. XPM produces a signal-dependent amplitude noise in the receiver, with a magnitude that is inversely proportional to channel spacing [7]. Moreover, unlike four-wave mixing, XPM cannot be eliminated by launching adjacent channels with orthogonal polarization. Thus, XPM is a fundamental impairment, along with the ASE noise always present in optically amplified systems. In a well-designed UDWDM transmission system, the optical launch power is optimized to achieve a balance between ASE- and XPM-induced noise at the receiver. Thus, an accurate description of receiver performance must include a model for both ASE and XPM, where the noise contributions from the two effects can be equally strong.

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Systems limited by either ASE or XPM in isolation have been studied extensively [7]–[10], and simple Gaussian noise models have been used to include both effects in a Q model [11]. In this letter, we present a novel theory of receiver performance, which includes the beating effects between ASE- and XPM-induced noise in the receiver. Our theory predicts inherently non-Gaussian probability distributions, with important consequences, such as a shift in optimum receiver threshold and lower error probabilities compared to the Gaussian Q model.

II. THEORY

Consider an optical nonreturn-to-zero or return-to-zero pulse, corresponding to a logical one bit, arriving at a receiver. After square law detection in a PIN or avalanche photodiodes receiver, the optical pulse is converted to an electrical pulse, amplified, filtered, and sampled by a clock and data recovery (CDR) circuit. We assume that the receiver circuitry is well designed so the CDR samples the signal at the optimum sampling instant, corresponding to the peak of the pulse. We model the resulting sample by a normalized random variable (RV)

$$z = 1 + n_{\mathrm{ase}} + n_{\mathrm{xpm}} + \varepsilon \cdot n_{\mathrm{ase}} \cdot n_{\mathrm{xpm}} \tag{1}$$

where n_{ase} is an RV corresponding to ASE signal-spontaneous beat noise, n_{xpm} is an RV corresponding to XPM noise, and $\varepsilon n_{\mathrm{ase}} n_{\mathrm{xpm}}$ is the beating between the two optical noise terms with a strength parameter ε. The addition of the noise beating term $\varepsilon n_{\mathrm{ase}} n_{\mathrm{xpm}}$ is a central hypothesis of our theory, based on the physics of square law detection. The RV z is normalized, without loss of generality, such that $< z >= 1$, which is equivalent to normalizing ASE and XPM noise to peak optical power. We assume that n_{ase} and n_{xpm} are independent RVs, with zero mean, and variance σ_{ase}^2 and σ_{xpm}^2, respectively.

RV z can be written in a particularly simple, and physically revealing, form by first considering the case $\varepsilon = 1$

$$z = (1 + n_{\mathrm{ase}})(1 + n_{\mathrm{xpm}}) = xy. \tag{2}$$

In this form, RV z clearly exhibits the multiplicative noise nature of the ASE-XPM noise beating effect [12]. The new RVs x and y are also independent, with mean one, and variance σ_{ase}^2 and σ_{xpm}^2, respectively. Using a transformation technique [13], the probability density function (pdf) of z can be put into an integral form in terms of x and y

$$\begin{aligned} f_z(z) &= \int_{-\infty}^{+\infty} \frac{dy}{|y|} f_x\left(\frac{z}{y}\right) f_y(y) \\ &= \int_{-\infty}^{+\infty} dy \frac{1}{\sqrt{2\pi y^2 \sigma_{\mathrm{ase}}^2}} e^{-(z-y)^2/2y^2\sigma_{\mathrm{ase}}^2} f_y(y) \end{aligned} \tag{3}$$

where the last equality assumes a Gaussian model for the pdf of x. A Gaussian model for ASE noise is deemed reasonable, while we make no assumptions at this point about XPM noise statistics. The error probability for receiving a "0" given that "1" was transmitted is obtained by integrating over z

$$P(0/1) = \int_{-\infty}^{V_d} dz f_z(z) = \frac{1}{2}\int_{-\infty}^{+\infty} dy f_y(y) \cdot \mathrm{erfc}\left(\frac{y - V_d}{y\sigma_{\mathrm{ase}}\sqrt{2}}\right) \tag{4}$$

where V_d is the threshold voltage. In comparison, ignoring the noise beating effects, and employing Gaussian statistics to describe XPM noise, the error probability $P(0/1)$ would be

$$P_G(0/1) = \int_{-\infty}^{V_d} dz \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-(1-z)^2/2\sigma_1^2} = \frac{1}{2}\mathrm{erfc}\left(\frac{1 - V_d}{\sigma_1\sqrt{2}}\right) \tag{5}$$

where $\sigma_1^2 = \sigma_{\mathrm{ase}}^2 + \sigma_{\mathrm{xpm}}^2$.

The total error probability includes the probability for receiving a "1" given that "0" was transmitted. We assume the transmitted signal has a high extinction ratio, and intersymbol interference on the zero level is negligible. In this ideal case, we can ignore the influence of XPM noise on the zero level, and employ a Gaussian pdf model of zero mean, and variance σ_o^2[11]. Thus, the probability for receiving "1" erroneously is

$$P(1/0) = P_G(1/0) = \frac{1}{2}\mathrm{erfc}\left(\frac{V_d}{\sigma_0\sqrt{2}}\right). \tag{6}$$

The total error probability is obtained from the above expressions as $P_e = 1/2(P(0/1) + P(1/0))$, where the factor of $\frac{1}{2}$ accounts for equal *a priori* probability for transmitting "1" or "0." In the Gaussian approximation of ASE and XPM noise, optimizing the threshold voltage V_d in $P_e = 1/2(P_G(1/0) + P_G(0/1))$ yields the Q model.

III. Results and Discussion

Fig. 1 shows the calculated pdfs, where $\sigma_{\mathrm{ase}} = \sigma_{\mathrm{xpm}} = 0.1$ and $\sigma_0 = 0.05$ (corresponding to $Q \sim 5$). The solid curves represent the standard Gaussian model, which ignores any noise beating effects. The dashed curve shows the calculated pdf when ASE-XPM noise beating is included, and XPM is modeled with a Gaussian pdf. As shown in Fig. 1, the ASE-XPM noise beating tends to skew the pdf toward the one level, with a skew factor [13] given by

$$\xi = \frac{\langle (z - \langle z\rangle)^3\rangle}{\sigma_z^3} = \frac{6\sigma_{\mathrm{ase}}^2\sigma_{\mathrm{xpm}}^2}{(\sigma_{\mathrm{ase}}^2 + \sigma_{\mathrm{xpm}}^2 + \sigma_{\mathrm{ase}}^2\sigma_{\mathrm{xpm}}^2)^{3/2}}. \tag{7}$$

The skew factor is positive definite, indicating that the pdf is always skewed toward the one level as long as both ASE and XPM noises are present in the receiver. In the example shown in Fig. 1, the skew factor $\xi = 0.21$. Note that ASE-XPM beat noise has relatively little impact on the total width of the pdf. This can be clearly seen by considering the variance in RV z, given by $\sigma_z^2 = \sigma_{\mathrm{xpm}}^2 + \sigma_{\mathrm{ase}}^2 + \sigma_{\mathrm{ase}}^2\sigma_{\mathrm{xpm}}^2$, where the ASE-XPM beat noise term $\sigma_{\mathrm{ase}}^2\sigma_{\mathrm{xpm}}^2$ gives a negligible contribution for σ_{xpm}^2, $\sigma_{\mathrm{ase}}^2 \ll 1$.

Fig. 2 compares the calculated error probabilities corresponding to the pdfs in Fig. 1 as a function of receiver threshold. The ASE-XPM beat noise has the effect of shifting optimum receiver threshold toward the one level. ASE-XPM beat noise also results in a lower error probability compared to the Gaussian model. A qualitative explanation for these effects, both of which follow from the skewed pdf, can be given by considering the multiplicative nature of ASE-XPM noise beating. Physically, the impact of ASE noise, being proportional to signal strength, is reduced when XPM results in the signal fluctuating down, and vice versa. Mathematically, a skewed or asymmetric pdf results because the width of the pdf (for $z < 1$) is reduced when signal fluctuates down due to XPM, while the width (for $z > 1$) increases when signal fluctuates up due to XPM. The case of no XPM is also shown in the dashed–dotted curves of Figs. 1 and 2 for comparison.

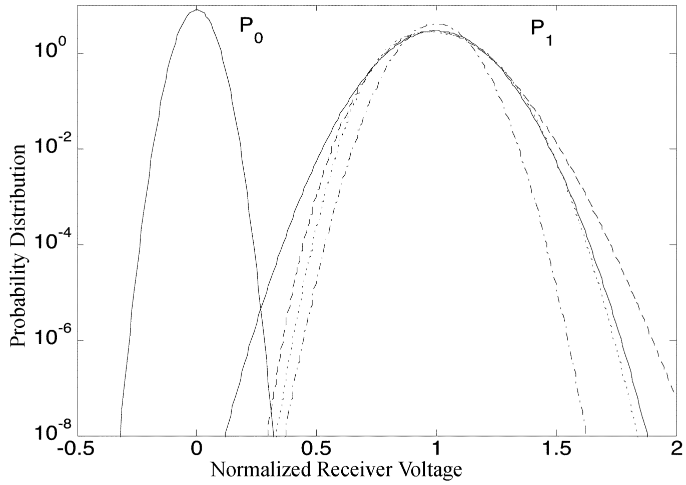

Fig. 1. Calculated probability distributions. Solid curves show Gaussian pdf models for both one and zero level. Dashed curve shows how pdf for one level changes when ASE-XPM beating effects are included with XPM treated as Gaussian RV. Dotted curve is similar to dashed curve but with XPM treated as a uniform RV. Dashed–dotted curve shows the case where XPM is absent.

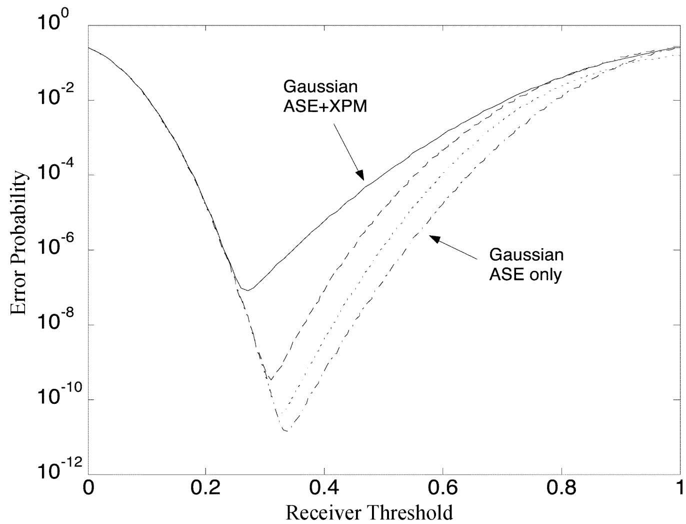

Fig. 2. Calculated error probabilities. Solid curve shows the standard Gaussian model. Dashed curve shows our model including ASE-XPM beating effects with XPM treated as Gaussian RV. Dotted curve is similar to dashed curve but with XPM treated as a uniform RV. Dashed–dotted curve shows the case where XPM is absent.

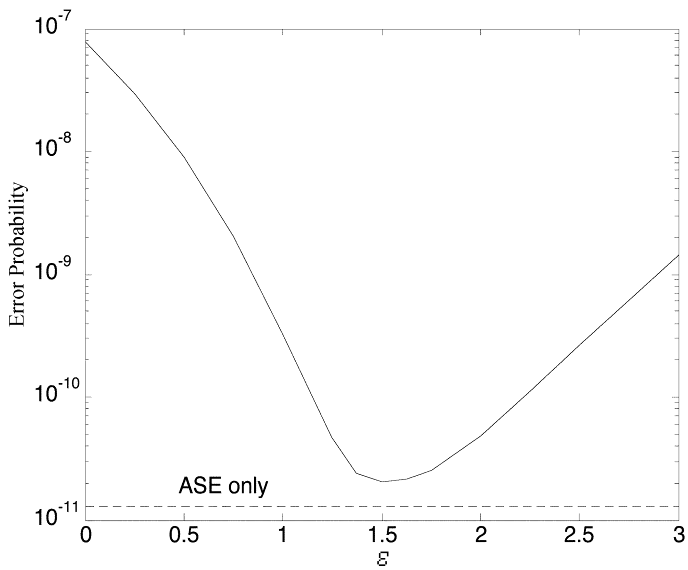

Fig. 3. Calculated error probability at optimum threshold as a function of ASE-XPM beat noise strength parameter ε. Both ASE and XPM are treated as Gaussian RV. Dashed line shows the error probability when XPM is absent.

The dotted curves in Figs. 1 and 2 show the pdf of the normalized sample voltage z and corresponding error probability when XPM noise is modeled by a uniform pdf. The uniform pdf is defined over an interval $[1-d, 1+d]$, where the parameter $d = \sqrt{3}\,\sigma_{\rm xpm}$ to ensure the same variance as in the Gaussian model of XPM noise. The uniform pdf model of XPM noise tends to compress the tails in the pdf of z more than a Gaussian model of XPM. This results in an additional small shift in optimum threshold toward the one level, and a modest reduction in error probability. Note that, due to mathematical symmetry in RV z as a function of x and y, the same error probabilities are obtained by assuming a uniform pdf model of ASE noise, while modeling XPM with a Gaussian pdf. Indeed, ASE-XPM beat noise exhibits similar features for any reasonable statistics assumed for ASE or XPM noise. However, in practical UDWDM systems, where many channels are involved in producing XPM, XPM-induced noise should be well modeled by Gaussian statistics.

It is straightforward to include an arbitrary strength parameter ε for the ASE-XPM beat noise. The analogous result for (4) is

$$P(0/1) = \frac{1}{2}\int_{-\infty}^{+\infty} dy f_y(y)\cdot \mathrm{erfc}\left(\frac{y - V_d}{(1-\varepsilon+\varepsilon y)\sigma_{\rm ase}\sqrt{2}}\right). \quad (8)$$

The corresponding formulas for variance and skew factor remain the same with $\sigma_{\rm ase}^2\sigma_{\rm xpm}^2$ replaced by $\varepsilon\sigma_{\rm ase}^2\sigma_{\rm xpm}^2$. Note that in the limit $\varepsilon \to 0$, (8) reduces as expected to the simple model of ASE and XPM noise developed by Killey *et al.* [14, eq. (3)]. Fig. 3 shows the calculated error probability at optimum threshold voltage as a function of ASE-XPM beat noise strength parameter ε. As long as ε is of order unity, our theory predicts a lower error probability compared to previous models, which ignore ASE-XPM beat noise effects [11], [14, eq. (3)]. The exact numerical value of ASE-XPM beat noise strength parameter ε should be determined experimentally, for example, by finding the best parameter fit of ε to measured data of error probability versus receiver threshold voltage.

IV. Conclusion

We have proposed a novel theory of receiver performance that includes a model for ASE-XPM beat noise. Our theory predicts that ASE-XPM beat noise tends to skew the pdf toward the one level, also shifting the optimum threshold voltage. Such noise beating effects are beneficial in reducing the total error probability at the optimum threshold. These conclusions are relatively insensitive to the exact noise statistics assumed for XPM or ASE. While this work focused on the beating between XPM and ASE noise, our theory may have applications to other multiplicative noise processes, as long as they dominate over the signal independent receiver thermal noise.

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