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Bi-alphabetic pulse compression radar signal design

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Abstract. Ternary sequences have superior merit factors but they cannot be transmitted with existing technology. It is proposed that a ternary sequence be coded into a binary sequence for the purpose of transmission. On reception it can be processed as a binary sequence as received and also decoded into a ternary sequence. These two interpretations provide a coincidence detection scheme for efficient target detection provided that the corresponding signal design problem is solved. Such an algorithm is developed by taking the merit factor as desideratum and the Hamming scan as optimization technique. Merit factor values obtained in some cases are further improved by implementing a back-tracking algorithm for bi-alphabetic sequence.

Keywords. Bi-alphabetic radar; Hamming scan; back-tracking; merit factor; coincidence detection.

1. Introduction

The problem of obtaining long sequences with peaky autocorrelation (Barker 1953) has long been an important problem in the field of radar, sonar and system identification. It is viewed as the problem of optimization (Bernasconi 1987; De Groot *et al* 1992). The signal design problem for radar application is suggested by sequences like binary, polyphase, ternary and quequenary sequences. There has been extensive work on binary sequences for obtaining good merit factor or discrimination (Barker 1953; Turyn 1963, 1968; Baumert 1971; Golay 1972, 1977, 1982, 1983; Moharir 1975; Beenker *et al* 1985; Kerdock *et al* 1986; Bernasconi 1987, 1988; Hoholdt *et al* 1985, 1988; Golay & Harris 1990; Newmann & Byrnes 1990; Jensen *et al* 1991; DeGroot *et al* 1992). The eight best binary sequences have lengths 13, 11, 27, 55, 49, 57, 45 and 51 with merit factors of 14.08, 12.10, 9.85, 8.85, 8.83, 8.86, 8.58 and 8.50 respectively. Beyond $n = 59$ but below $n = 117$, the highest merit factor available is 9.56. Known high merit factors fall considerably beyond this length, though statistical optimization techniques have been used. For example, beyond a length of a few hundreds, a sequence exceeding a merit factor of 5 is rarely known. Ternary sequences were searched and gave higher values of merit factor and discrimination (Moharir *et al* 1996, 1997; Moharir & SubbaRao 1997). This work was based on global optimization techniques such as genetic algorithm (De Groot *et al* 1992), eugenic algorithm (Singh *et al* 1996; Moharir 1998), and SKH (Simon–Kronecker–Hamming) algorithm (Moharir *et al* 1996). Though the ternary sequences resulted in superior merit factors when compared to binary sequences, they had two problems. The ternary alphabet has zero as an element, which implies no transmission during some time slots. Secondly, it is considered difficult to have on–off switching at high power in comparison to phase shifting.

The proposed idea here is that a ternary sequence can be coded into a binary sequence for the purpose of transmission. Further advantage results if after reception it can be subjected to dual ternary–binary interpretation to facilitate a coincidence detection scheme for efficient target detection. The binary sequence transmitted could be so chosen that each of these bi-alphabetic interpretations leads to a high merit factor.

The signal design problem for optimization of bi-alphabetic sequence is carried out in different stages as follows:

- Recursive Hamming scan to obtain a ternary sequence with good merit factor.
- Proposed coding of ternary sequence into a binary sequence is to replace every ternary element with a binary bigram, e.g.: $+ \rightarrow ++$, $- \rightarrow --$ and $0 \rightarrow -+$ or $+-$. Thus, if ternary sequence has nz zeros, there can be 2^{nz} binary sequences. The best one among them can be chosen. In the above proposal the ternary sequence is optimized first and the freedom afforded by 1: many coding into a binary sequence is exploited separately to optimize the performance of a binary sequence.
- It is possible to use the dual interpretation directly so that the two interpretations together give better performance without any of the interpretations being best individually. Bi-alphabetic Hamming scan algorithm is derived for this purpose.
- By employing back-tracking Hamming scan for bi-alphabetic sequence.

The present proposal is made possible due to some recent developments in neural network radar processing. Conventionally, crosscorrelation at the receiver is done in the analog mode without decoding the received waveform into a binary sequence. But now, such decoding will be needed. There is already a precedent as it is also needed for neural network processing (Deergha Rao & Sridhar 1995; Kwan & Lee 1989, 1993) of radar signals. The latter is accepted as a possibility because of enormous improvements in side-lobe suppression.

2. The algorithm and associated concepts

To set up notation and to establish the ideas for design algorithm for bi-alphabetic interpretation of the ternary sequence, let

$$S = [S_0 S_1 S_2 \cdots S_{m-2} S_{m-1}],$$

be the ternary sequence of length m, where the elements S_i are chosen from alphabets $[+\ 0\ -]$

$$\rho(k) = \sum_{i=0}^{m-1-k} S_i \times S_{i+k};$$

is called the aperiodic autocorrelation of the sequence and

$$M =:: \rho^2(0)/2 \times \sum_{k=1}^{m-1} \rho^2(k),$$

where M is the merit factor.

The binary sequence to be transmitted could be derived from a ternary sequence with a high merit factor. When such a sequence is transmitted, it can be subjected to bi-alphabetic interpretation on reception with the element $+$ in the ternary sequence and can be coded as $++$ in the binary sequence, the element $-$ can be coded as $--$ and the element 0 can be coded as $+-$ or $-+$ in the binary sequence. When such a bi-alphabetic sequence is subjected to Hamming scan for recursive search, the sum of the merit factors $m = m_t + m_b$ can be considered as an objective function to be maximized. Here m_t is the merit factor of ternary sequence and m_b is the merit factor of binary sequence.

2.1 *Hamming scan*

The Hamming scan is an algorithm which is more efficient though it is sub-optimal. The Hamming scan looks at all the Hamming1 neighbours and picks up the one with largest merit factor. If it is better that the original sequence, the algorithm is recursively continued from there as long as improvement is possible. The Hamming scan was expedited and made applicable at large lengths by not calculating the aperiodic autocorrelation of the Hamming neighbours *ab initio*, recognizing the fact that as only one element is mutated, only its difference contribution needed to be taken into account. Each of the elements in the ternary sequence $[-1\ 0\ 1]$ can be mutated in two possible ways: $-1 \rightarrow 0$ or 1, $0 \rightarrow -1$ or 1 and $1 \rightarrow -1$ or 0 resulting in two strands of Hamming neighbours. The better neighbour of these two strands could be selected by recursive local search among the Hamming neighbours of resulting ternary sequence. This idea is based on earlier work (Moharir *et al* 1997) that employed Hamming scan and back-tracking algorithms for obtaining the ternary sequences with high merit factor. These results are used in the present proposal.

2.2 *Additional degree of freedom of binary interpretation*

Once the recursive local search among Hamming neighbours of a ternary sequence is completed, it is possible to construct a binary sequence s_b of length $2m$ from the ternary sequence. During this correspondence the ternary element 0 can be coded in two different ways viz. $+-$ or $-+$. Thus the binary sequence can be chosen from 2^{nz} alternatives, where nz is the number of zero elements in the ternary sequence. This extra degree of freedom can be exploited to choose a binary sequence among 2^{nz} alternatives which has the largest merit factor. This problem can itself be organized as a selective Hamming scan which permits bigram mutations $-+ \rightarrow +-$ in the binary sequence only where these bigrams have an origin due to 0 element in the ternary sequence.

Some of the results of binary interpretation are listed in table 1. The merit factors of ternary sequence remain unaltered during this process.

Table 1. Results of binary sequence interpretations with the merit factor values of ternary sequences unaltered.

Length of ternary sequence (n_t)	Merit factor values		Sum of merit factor values ($m = m_t + m_b$)
	Ternary sequence (m_t)	Binary interpretation (m_b)	
298	10.9845	3.5242	14.5087
312	10.4719	3.0932	13.5651
572	15.1136	3.7390	18.8526
580	15.1720	3.9021	19.0741
586	16.0245	3.6737	19.6982
588	15.2772	3.7966	19.0738
610	11.2895	3.3967	14.6862
613	11.5900	3.3428	14.9335
617	11.3790	3.4937	14.8727
619	11.8714	3.5764	15.4478
622	12.8247	3.6057	16.4303
628	13.2609	3.7425	17.0034
631	13.1459	3.4113	16.5572
640	11.7064	3.6565	15.3629
646	10.2573	2.5339	12.7912
658	10.3142	3.3589	13.6731
726	11.7805	3.5097	15.2902
867	10.1742	3.2039	13.3781
892	10.0958	3.4022	13.4980

The sum of the merit factors (m) due to individual binary and ternary sequences are listed for comparison of the results obtained in the next section.

2.3 *Bi-alphabetic Hamming scan*

The performance of the binary sequence can be further improved if it is subjected to a bi-alphabetic interpretation in terms of dual ternary–binary representation. This, in turn, facilitates an effective coincidence scheme for target detection in which the sum of the merit factors of individual contributions due to bi-alphabetic sequence can considered as a joint objective function to be maximized. Each mutation in the ternary element induces mutations in the elements of the binary interpretation as shown in table 2. In view of the fact that the first and the last elements of the ternary sequence do not possess a zero as an element, the only possible mutations on these terminal elements are $+ \rightarrow -$ or $- \rightarrow +$ and the corresponding mutations on binary elements are $++ \rightarrow --$ or $-- \rightarrow ++$.

As indicated in table 2, the first two mutations in ternary elements resulted in two elemental changes in binary elements on mutation. As a result, the Hamming scan needed to take into account the difference in autocorrelation contribution due to these two elemental changes. Other mutations of the ternary elements result in one elemental change in terms of binary sequence. Hence, only the difference contribution of this changed element has to be taken into account while calculating the autocorrelation. Once the recursive local search among the first order Hamming neighbours of the ternary sequence is completed, the resulting binary sequence can be further optimized from 2^{nz} alternatives as explained in § 2.2. Therefore, if one starts with a good ternary sequence and subjects it to a bi-alphabetic Hamming scan, the ternary interpretation has the advantage of initial good

Table 2. Details of ternary and corresponding binary elemental mutations.

Ternary element mutation	Binary element mutation	No. of binary elements changed on mutation
$+ \to -$	$++ \to --$	2
$- \to +$	$-- \to ++$	2
$+ \to 0$	$++ \to +-$ or $-+$	1
$- \to 0$	$-- \to +-$ or $-+$	1
$0 \to +$	$+-$ or $-+ \to ++$	1
$0 \to -$	$+-$ or $-+ \to --$	1

choice and complete search among Hamming neighbours. Some of the results for bi-alphabetic sequences are listed in table 3.

2.4 *Back-tracking Hamming scan for bi-alphabetic sequence*

When a bi-alphabetic Hamming scan yields no sequence with a merit factor superior to the starting sequence, the Hamming back-track still looks at the prescribed number n (called

Table 3. Some of the bi-alphabetic sequences with improved merit factors.

Length of ternary sequence (n_t)	Merit factor values: Ternary interpretation (m_t)	Merit factor values: Binary interpretation (m_b)	Sum of merit factor values with Hamming scan ($m = m_t + m_b$)
298	12.0563	3.3409	15.3971
302	12.6012	3.3629	15.9645
308	12.1275	3.4925	15.6200
310	12.0237	3.2206	15.2443
312	11.7399	3.4094	15.1493
572	15.2064	3.9793	19.1857
580	16.0231	3.7113	19.7344
586	15.7473	3.8981	19.6454
588	15.4099	3.8177	19.2276
610	15.9542	3.7894	19.7436
613	15.1553	3.7789	18.9342
617	14.8327	3.8943	18.7269
619	15.1969	3.8475	19.0444
622	15.8401	3.7273	19.5674
628	14.9825	3.6576	18.6401
631	14.8609	3.6263	18.4872
640	14.8290	3.5593	18.3883
646	15.538	3.5514	18.6052
649	14.5415	3.4630	18.0045
658	13.0003	3.7703	16.7707
726	12.2162	3.9209	16.1371
849	11.4143	3.7494	15.1637
867	11.6895	3.4175	15.1070
892	10.8174	3.3295	14.1469
910	11.0041	3.5565	14.5606

Table 4. Some of the results of back-tracking algorithm for bi-alphabetic sequences.

	Sum of merit factor values of bi-alphabetic sequence	
Length(n_t)	With Hamming scan	With back-tracking Hamming scan
586	19.6454	19.8421
649	18.0045	18.3631
726	16.1371	16.2149
892	14.1469	14.5376
910	14.5606	14.7775

span) of the best Hamming neighbours (Moharir *et al* 1997). The algorithm then obtains the best sequence obtainable from them by a prescribed number (called the height) of recursive Hamming scan and selects it, if it is superior to the starting sequence. For the ternary sequence there are two strands of Hamming neighbours, corresponding to the two alphabet values obtained by mutation and n of the best neighbours are picked up on each strand. A span of $n = 6$ is used in this work. If the Hamming back-tracking succeeds in improving the merit factor, the search can resume by further application of bi-alphabetic Hamming scan. Each time the binary sequence corresponding to the improved ternary sequence is chosen among the 2^{nz} alternatives as explained in § 2.2.

Application of back-tracking algorithm for bi-alphabetic sequence has further improved the merit factor values for some sequences as shown in table 4.

3. Conclusion

A long-standing problem in the field of pulse-compression radar has been solved. It had been established that ternary sequences can yield peakier autocorrelation than binary sequences. This was made possible by zero elements in the ternary sequence, which contributed zero to lagged products in the autocorrelation so that only fewer contributions had to balance each other out. But the existence of zero elements posed an engineering problem of on/off transmission at high powers. Thus the theoretical advantage was unavailable in practice. Some recent work proposing use of neural networks in radar opened the possibility of post-detection computations of crosscorrelation. This has a bearing on the problem discussed above. If crosscorrelation is computed after decoding the received signal into a discrete sequence, then the ternary sequence can be transmitted after coding it into a binary sequence, so that the extant radar practice is available. The received signal can then be decoded into a ternary sequence before performing crosscorrelation. But then this opens up additional possibilities. The received signal can be decoded into two interpretations, (a) a binary sequence, and (b) a ternary sequence. The criteria of peaky autocorrelation should then apply to both of them, or, as a compromise, they should be jointly good without necessarily being individually best. This is a new signal design problem and has been solved in two ways in this paper.

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