KURTOSIS BASED MAXIMIZATION/MINIMIZATION APPROACH TO BLIND EQUALIZATION FOR DS/CDMA SYSTEMS IN UNKNOWN MULTIPATH

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ABSTRACT

In this paper, we study blind equalization for a DS/CDMA system in multipath. The kurtosis of the equalizer's output is maximized with respect to the equalizer's parameters subject to multiple linear constraints. The constraints are dependent on the spreading codes of the user of interest. In order to optimally combine signals from multiple propagation paths, we parameterize the kurtosis by the constraint vector as well and further minimize the maxima of kurtosis to obtain the optimal constraints. It is shown that the constraint vector converges to the channel vector of the desired user irrespective of noise. The equalizer guarantees cancellation of both intersymbol interference and multiuser interference.

1. INTRODUCTION

A wireless communication system suffers from multipath channel distortion. Blind equalization has been proposed to estimate the inputs from a single source by designing an equalizer to suppress intersymbol interference (ISI) [7]. Motivated by its appealing property, various high order statistics (HOS) based equalization techniques have appeared in the equalization literature [1, 2, 8].

Recently, there exists significant interest in studying cumulant/kurtosis based multiuser detection techniques due to the prevalence of code division multiple access (CDMA) technique. Different methods can be divided into two categories. One is to design a bank of detectors with each one detecting one user [6]. Thus all users can be detected at the same time. This multiuser detection scheme can be implemented in the base station which is capable of processing large amount of data in parallel. The other one is to consider only the desired user in a flat fading channel [5, 11] or multipath environment [4, 9, 10, 12], such as in the mobile station. With given spreading codes of the desired user, the detector is forced to satisfy a linear constraint such that signals from the user of interest are detected.

However, those algorithms have shown certain disadvantages. It is clear that those approaches suitable for flat channels suffer from signature mismatch. Among approaches for multipath mitigation, [4] exhibits local minima and inability to optimally combine signal components from different paths. The approaches [9, 10] are batch iterative algorithms based on a batch processing of a block of data. Their global convergence has not been established. Our previously developed constant modulus algorithm (CMA) based method [12] requires proper initialization for the constraint vector. One possible solution to solve this problem is to initialize the constraint vector by the minimum output energy (MOE) algorithm [14] due to its excellent convergence property. In such a way, [12] can effectively suppress ISI and multiuser interference (MUI).

In this paper, we consider a DS/CDMA system in multipath, and propose a novel equalization criterion which does not suffer from convergence problem but still holds recursive nature. We maximize the kurtosis of the equalizer's output under multiple unknown constraints. Such linear constraints guarantee no cancellation of the desired signal. The constraint vector can be pre-selected. However, they can be treated as unknowns. Therefore the maxima of the kurtosis are parameterized by the constraint vector and are further minimized. It will be shown that the proposed equalizer can successfully cancel both ISI and MUI. Meanwhile, the constraint vector can be jointly updated with the equalizer's weights. It converges to the channel vector irrespective of noise. This fact well motivates design of another less expensive linear equalizer - minimum mean square error (MMSE) equalizer by treating the constraint vector as an estimate of the channel. Simulation examples are illustrated to validate our analysis.

2. DS/CDMA SYSTEM MODEL

In a DS-CDMA system, each user transmits digital information after being modulated by a distinct spreading sequence. Let user j, $j = 1, \ldots, J$ use spreading codes $c_j(k)$ ($k = 0, \ldots, P-1$) of length P to transmit P chips per information symbol. Let the chip sequence propagate through a linear multipath channel with a baseband impulse response $g_j(n)$. Then the received discrete-time signal $y_j(n)$ due to user j is [14]

$$y_j(n) = \sum_{l=-\infty}^{\infty} w_j(l) h_j(n - d_j - lP) \tag{1}$$

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$$h_j(n) = \sum_{m=-\infty}^{\infty} g_j(m) c_j(n-m) \quad (2)$$

where $w_j(n)$ is the information bearing sequence of user j, $h_j(n)$ is its signature (the convolution of the code with the channel), d_j is its delay in chip periods. If our observation window spans ν symbol intervals with corresponding $L = \nu P$ chip periods from nP to $nP+L-1$, then the data vector due to user j can be easily found to be [14]

$$\boldsymbol{y}_j(n) = \sum_{i=-1-i_0}^{\nu} \widetilde{\boldsymbol{h}}_j^{(i)} w_j(n+i) \quad (3)$$

where $\widetilde{\boldsymbol{h}}_j^{(i)}$ is the signature vector of symbol $w_j(n+i)$, i_0 depends on the delay

$$i_0 = \begin{cases} 0 & \text{if } \ 0 \le d_j \le P-q \\ 1 & \text{otherwise.} \end{cases}$$

According to (2), the signature vector of symbol $w_j(n)$ is given by $\widetilde{\boldsymbol{h}}_j^{(0)} = \boldsymbol{J}^{d_j} \boldsymbol{C}_j \boldsymbol{g}_j$ where $\boldsymbol{g}_j = [g_j(0), \cdots, g_j(q)]^T$,

$$\boldsymbol{C}_j = \begin{bmatrix} c_j(0) & & 0 \\ \vdots & \ddots & c_j(0) \\ c_j(P-1) & & \vdots \\ 0 & \ddots & c_j(P-1) \\ \boldsymbol{0} & \dots & \boldsymbol{0} \end{bmatrix}, \quad (4)$$

$\boldsymbol{J}$ is a matrix with all elements in the sub-diagonal below the main diagonal $1'$s. All other signatures $\widetilde{\boldsymbol{h}}_j^{(i)}$ are related to $\widetilde{\boldsymbol{h}}_j^{(0)}$ by

$$\widetilde{\boldsymbol{h}}_j^{(i)} = \boldsymbol{J}^{iP} \widetilde{\boldsymbol{h}}_j^{(0)}$$

where $\boldsymbol{J}^{-1} \triangleq \boldsymbol{J}^T$, $\boldsymbol{J}^0 \triangleq \boldsymbol{I}$. Pre-multiplying $\widetilde{\boldsymbol{h}}_j^{(0)}$ by $\boldsymbol{J}^{iP}$ will shift elements in $\widetilde{\boldsymbol{h}}_j^{(0)}$ up (if $i < 0$) or down (if $i > 0$) by $|i|P$ positions. If we assume the receiver is synchronized to the user of interest (user 1), then the received data vector becomes

$$\boldsymbol{y}_n = \boldsymbol{C}\boldsymbol{g}_1 w_1(n) + \boldsymbol{H}_{int} \boldsymbol{w}_{int}(n) + \boldsymbol{v}_n \quad (5)$$

where $\boldsymbol{w}_{int}(n) = [w_1(n-1) \ w_1(n+1) \ \dots \ w_1(n+\nu) \ w_2(n-1) \ \dots \ w_J(n+\nu)]^T$ is the interference vector including ISI and MUI, $\boldsymbol{H}_{int}$ is the corresponding signature matrix, $\boldsymbol{v}_n$ is the additive white Gaussian noise (AWGN) vector, $\boldsymbol{g}_1$ is the channel vector and assumed unitary (e.g., under power control), $\boldsymbol{C} \triangleq \boldsymbol{C}_1$ for notational convenience. The particular structure of $\widetilde{\boldsymbol{h}}_1^{(0)}$ will be exploited to derive a blind adaptive multiuser detector which is capable of combating multipath distortions and suppressing MUI, as detailed in the next section.

3. KURTOSIS MINIMIZATION/MAXIMIZATION BASED BLIND EQUALIZATION

Our goal is to design a kurtosis-based multiuser detector $\boldsymbol{f}$ whose output z provides an estimate of $w_1(n)$

$$z = \boldsymbol{f}^H \boldsymbol{C} \boldsymbol{g}_1 w_1(n) + \boldsymbol{f}^H \boldsymbol{H}_{int} \boldsymbol{w}_{int}(n) + \boldsymbol{f}^H \boldsymbol{v}_n. \quad (6)$$

Instead of maximizing kurtosis (the fourth order cumulant $CUM_4(z) = E\{|z|^4\} - 2E^2\{|z|^2\} - |E\{z^2\}|^2$) with output power constraint (or normalized by the power), our approach is to maximize it (or minimizing its absolute value) with multiple constraints $\boldsymbol{C}^H \boldsymbol{f} = \boldsymbol{g}$ which implicitly constrain the power of the desired symbol to be constant

$$\min_{\boldsymbol{f}} \mathcal{J} = |CUM_4(z)|, \ \text{ subject to } \ \boldsymbol{C}^H \boldsymbol{f} = \boldsymbol{g}. \quad (7)$$

After deriving minimum stationary points of $\mathcal{J}$, it can be easily verified that detection of the current symbol from our desired user is ensured. At the minimum point, $\mathcal{J}$ becomes $\mathcal{J}_{min} = |\kappa_1| |\boldsymbol{g}^H \boldsymbol{g}_1|^4$ where κ_1 is the kurtosis of the desired signal. The constraint vector $\boldsymbol{g}$ can be pre-selected. In order to maximize the power of the desired symbol, we treat $\boldsymbol{g}$ as a parameterized vector and opt for maximizing $\mathcal{J}_{min}$ (or minimizing the resulting kurtosis). Therefore, our criterion is described as follows

$$\max_{\boldsymbol{g}} \min_{\boldsymbol{f}} \mathcal{J} = |CUM_4(z)|, \ \text{ subject to } \ \boldsymbol{C}^H \boldsymbol{f} = \boldsymbol{g}. \quad (8)$$

To seek its optimum, we construct a Lagrange cost function

$$\mathcal{J}_1 = |CUM_4(z)| + \boldsymbol{\lambda}^H (\boldsymbol{C}^H \boldsymbol{f} - \boldsymbol{g}) + (\boldsymbol{f}^H \boldsymbol{C} - \boldsymbol{g}^H) \boldsymbol{\lambda} \quad (9)$$

where $\boldsymbol{\lambda}$ is a vectorized Lagrange multiplier. In order to minimize $\mathcal{J}_1$ with respect to (w.r.t.) $\boldsymbol{f}$ and maximize it w.r.t. $\boldsymbol{g}$, we form our gradient based recursions

$$\boldsymbol{f}_{n+1} = \boldsymbol{f}_n - \mu_f \nabla_{\boldsymbol{f}} \mathcal{J}_1 \quad (10)$$

$$\boldsymbol{g}_{n+1} = \boldsymbol{g}_n + \mu_g \left(\boldsymbol{I} - \frac{\boldsymbol{g}_n \boldsymbol{g}_n^H}{\boldsymbol{g}_n^H \boldsymbol{g}_n}\right) \nabla_{\boldsymbol{g}} \mathcal{J}_1 \quad (11)$$

where $\boldsymbol{I} - \frac{\boldsymbol{g}_n \boldsymbol{g}_n^H}{\boldsymbol{g}_n^H \boldsymbol{g}_n}$ suggests update only in the direction orthogonal to $\boldsymbol{g}_n$. Following the similar procedures as in the MOE approach [14], we solve the multiplier at each iteration and finally obtain

$$\boldsymbol{f}_{n+1} = \boldsymbol{\Pi}_{\boldsymbol{C}}^{\perp} (\boldsymbol{f}_n - \mu_f \nabla_{\boldsymbol{f}} |CUM_4(z)|) + \boldsymbol{C}\boldsymbol{A}\boldsymbol{g}_n \quad (12)$$

$$\begin{aligned} \boldsymbol{g}_{n+1} &= \boldsymbol{g}_n + \frac{\mu_g}{\mu_f} \left(\boldsymbol{I} - \frac{\boldsymbol{g}\boldsymbol{g}^H}{\boldsymbol{g}^H \boldsymbol{g}}\right) \boldsymbol{A} (\boldsymbol{g}_n - \boldsymbol{C}^H \boldsymbol{f}_n \\ &+ \mu_f \boldsymbol{C}^H \nabla_{\boldsymbol{f}} |CUM_4(z)|) \end{aligned} \quad (13)$$

where $\nabla_f |CUM_4(z)|$ is the gradient of kurtosis w.r.t. $\boldsymbol{f}$

$$\begin{aligned} \nabla_{\boldsymbol{f}} |CUM_4(z)| &= sign(CUM_4(z)) (2E\{|z|^2 z^* \boldsymbol{y}\} \\ &- 4E\{|z|^2\} E\{z^* \boldsymbol{y}\} \\ &- 4E\{z^2\} E\{z \boldsymbol{y}\}) \end{aligned} \quad (14)$$

$$\boldsymbol{A} = (\boldsymbol{C}^H\boldsymbol{C})^{-1},\ \ \boldsymbol{\Pi}_{\boldsymbol{C}}^{\perp} = \boldsymbol{I} - \boldsymbol{C}\boldsymbol{A}\boldsymbol{C}^H.$$

To make $\max \mathcal{J}_{min}$ meaningful, $\boldsymbol{g}$ is constrained to have unit norm and normalized at each step. During implementation, the received data are used to estimate the stochastic gradient. Unlike in CMA, the gradient of the kurtosis involves the products of statistical averages. Therefore, we can not directly apply instantaneous estimation. As suggested in [3], the ensembled averages are replaced by empirical averages which are then adaptively updated through the use of a forgetting factor α $(0 < \alpha < 1)$. For example, if e_k is a random sequence, the empirical average at k-th iteration is defined as:

$$< e >_k = \alpha < e >_{k-1} + (1-\alpha)e_k.$$

This rule can be applied to $< |z|^2 >$, $< z^2 >$ and $< |z|^4 >$. Then the time-average estimate of the kurtosis can be calculated

$$< CUM_4(z) > = < |z|^4 > -2 < |z|^2 >^2 - < z^2 >^2$$

and the gradient in (14) can be estimated [3].

4. CONVERGENCE ANALYSIS

We now study the convergence property of our algorithm when interference and AWGN are present in the system. Assume the inputs from different users have the same kurtosis κ. Denote the i-th column of signature matrix $[\boldsymbol{C}\boldsymbol{g}_1\ \boldsymbol{H}_{int}]$ by $\boldsymbol{h}_i$. Then considering norm constraint on $\boldsymbol{g}$ and the zero kurtosis for the noise, (9) is transformed to

$$\begin{aligned} \mathcal{J}_1 &= |\kappa| \sum_i |\boldsymbol{f}^H\boldsymbol{h}_i|^4 + \boldsymbol{\lambda}^H(\boldsymbol{C}^H\boldsymbol{f} - \boldsymbol{g}) \\ &+ (\boldsymbol{f}^H\boldsymbol{C} - \boldsymbol{g}^H)\boldsymbol{\lambda} + \rho(\boldsymbol{g}^H\boldsymbol{g} - 1). \end{aligned} \quad (15)$$

where ρ is a multiplier. After taking derivatives, all stationary points can be obtained Under the assumption that $[\boldsymbol{C}\ \boldsymbol{h}_2\ \boldsymbol{h}_3\ \ldots]$ is of full column rank and after some manipulations, these points are found to satisfy the following conditions

$$|\boldsymbol{f}^H\boldsymbol{h}_i| = 0 \ \text{ for } i \neq 1, \quad (16)$$

$$\boldsymbol{g} = \frac{e^{j\theta}}{||\boldsymbol{g}_1||}\boldsymbol{g}_1, \quad (17)$$

$$\boldsymbol{\lambda} = -2|\kappa| e^{j\theta}||\boldsymbol{g}_1||^3\boldsymbol{g}_1,\ \ \rho = -2|\kappa|||\boldsymbol{g}_1||^4, \quad (18)$$

where θ is an arbitrary phase. According to eqs. (16)-(18), we can observe that at stationary points: (a) the optimal receiver $\boldsymbol{f}$ is able to remove all ISI and MUI; (b) our optimal constraint vector converges to the normalized channel vector within a phase ambiguity; and (c) the multiplier $\boldsymbol{\lambda}$ is proportional to the channel vector.

We now check the property of those stationary points. Since the Hessian matrix of $\mathcal{J}$ w.r.t. $\boldsymbol{f}$ is zero at the stationary points, we directly investigate the absolute value of the kurtosis in the neighborhood of stationary points

$$|CUM_4(z)| = |\kappa||\boldsymbol{g}^H\boldsymbol{g}_1|^4 + |\kappa|\sum_{i\neq 1}|\boldsymbol{f}^H\boldsymbol{h}_i|^4. \quad (19)$$

For any constraint $\boldsymbol{g}$, $|CUM_4(z)| \geq |\kappa||\boldsymbol{g}^H\boldsymbol{g}_1|^4$. If $\boldsymbol{f}^H\boldsymbol{h}_i \neq 0$ for some $i \neq 1$, then $|CUM_4(z)| > \mathcal{J}_{min}$. Therefore, we conclude that $\boldsymbol{f}^H\boldsymbol{h}_i = 0$ for $i \neq 1$ is the global minimum point and correspondingly $\mathcal{J}_{min} = |\kappa||\boldsymbol{g}^H\boldsymbol{g}_1|^4$. However, the optimal $\boldsymbol{f}$ might not be unique. Considering the code constraint $\boldsymbol{C}^H\boldsymbol{f} = \boldsymbol{g}$ and (17), the optimal $\boldsymbol{f}$ is then the solution of the following linear equation:

$$\boldsymbol{f}^H[\boldsymbol{C}\ \boldsymbol{h}_2\ \boldsymbol{h}_3\ \ldots] = \Big[\frac{e^{-j\theta}}{||\boldsymbol{g}_1||}\boldsymbol{g}_1^H\ 0\ \ldots\ 0\Big].$$

This equation is under-determined. Thus, the solution for $\boldsymbol{f}$ is not unique. As is known, $\mathcal{J}$ is a function of both $\boldsymbol{f}$ and $\boldsymbol{g}$. It is easily verified that under the norm constraint $||\boldsymbol{g}|| = 1$, $\mathcal{J}_{min}$ reaches its maximal value $|\kappa|||\boldsymbol{g}_1||^4$ when $\boldsymbol{g}$ is proportional to $\boldsymbol{g}_1$. However, $\mathcal{J}_{min}$ is phase insensitive. Hence, $\boldsymbol{g} = \frac{e^{j\theta}}{||\boldsymbol{g}_1||}\boldsymbol{g}_1$ are all maximum points. Motivated by this result, one might treat $\boldsymbol{g}$ as a channel estimate and construct a MMSE equalizer in addition to the proposed kurtosis based equalizer. We can summarize our previous analysis in the following.

- At the optimal points, the detectors can cancel ISI and MUI perfectly irrespective of AWGN noise;
- At the optimal points, the constraint vector $\boldsymbol{g}$ converges to $\boldsymbol{g}_1$ with a phase ambiguity irrespective of AWGN noise;
- Detection of the desired user is ensured.

These results will be verified by our simulations.

5. SIMULATION

The performance of the proposed adaptive kurtosis based max/min detector and corresponding blind MMSE detector constructed from updated constraint vector is studied. Simulation parameters are as follows: processing gain 8, 3 equally-powered users, randomly generated channel of length 4 at each realization, $SNR = 10dB$, equalizer length 16, forgetting factor $\alpha = 0.95$ to estimate the cumulant [3], step sizes $\mu_f = 0.001$ and $\mu_g = 0.0002$. It will be compared with other relevant methods such as [10] once their adaptive versions are available. Fig. 1 shows the average convergence of the proposed detectors versus iteration number over 100 runs. It is seen that the proposed blind MMSE detector converges to the ideal MMSE detector ("Limit"). However, the proposed kurtosis based detector exhibits lower convergence level. This deviation from the optimal solution caused by adaptive implementation needs further investigation. It has been established that $\boldsymbol{g}$ converges to $\boldsymbol{g}_1$ irrespective of noise, we also present in Fig. 2 the mean square channel estimation error for different SNRs from $0dB$ to $20dB$ with a $5dB$ interval. It is observed that the channel estimation error highly depends on SNR, inconsistent with what is expected. It is our conjecture that the larger error for low SNR may be caused by the large variance from kurtosis estimation. The output SINR of the proposed receivers versus the input SNR in the same situation are also plotted in Fig. 3. The proposed blind MMSE receiver demonstrates an asymptotic convergence to the ideal MMSE receiver, while the kurtosis based detector shows worse performance. The reason will be investigated in our future work.

6. REFERENCES

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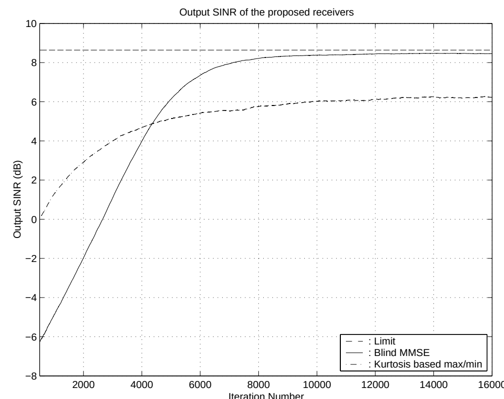

Fig. 1. Performance of the proposed receivers

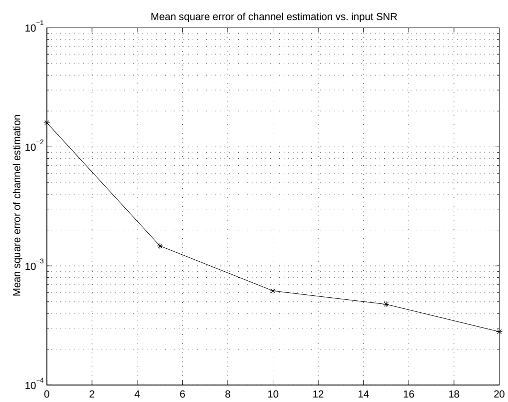

Fig. 2. Mean square channel estimation error versus SNR.

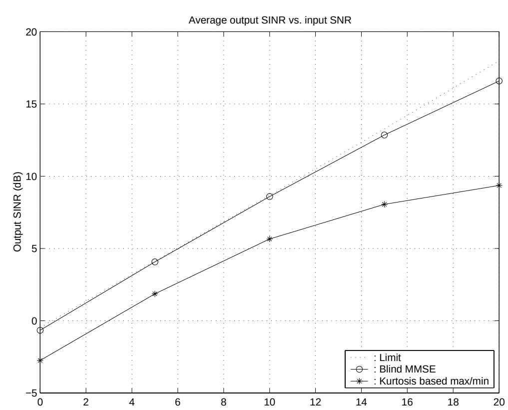

Fig. 3. Output SINR of the proposed receivers versus SNR.