Analytical Level Crossing Rates and Average Fade Durations for Diversity Techniques in Nakagami Fading Channels

Cyril-Daniel Iskander, *Student Member, IEEE*, and P. Takis Mathiopoulos, *Senior Member, IEEE*

***Abstract*—The level-crossing rates (LCRs) and average fade durations (AFDs) of a fading channel find diverse applications in the evaluation and design of wireless communication systems. Analytical expressions for these quantities are available in the literature for certain diversity reception techniques, but are generally limited to the Rayleigh fading channel, with few exceptions. Moreover, the methods employed are usually specific to a certain channel/diversity pair, and thus cannot be applied to all cases of interest. Using a unified methodology, we derive analytical expressions for the LCRs and AFDs for three diversity reception techniques and a general Nakagami fading channel. We provide novel analytical expressions for selection combining (SC) and equal-gain combining (EGC), and rederive in a more general manner the case of maximal-ratio combining (MRC). It is shown that our general results reduce to some specific cases previously published. These results are used to examine the effects of the diversity technique, the number of receiving branches and severity of the fading on the concerned quantities. It is observed that as the Nakagami m-parameter and the diversity order increase, the behavior of the combined received envelope for EGC follows closely the one for MRC, and distances itself from SC.**

***Index Terms*—Average fade durations, diversity, level crossing rates, Nakagami fading channels.**

I. Introduction

The level crossing rates (LCRs) and the average fade durations (AFDs) are two quantities which statistically characterize a fading communication channel. The LCR is defined as the number of times per unit duration that the envelope of a fading channel crosses a given value in the negative direction [1]. The AFD corresponds to the average length of time the envelope remains under this value once it crosses it in the negative direction. These quantities reflect the correlation properties, and thus the second-order statistics, of a fading channel. They provide a dynamic representation of the channel. They complement the probability density function (PDF) and cumulative distribution function (CDF), which are first-order statistics, and can only be used to obtain static metrics associated with the channel, such as the bit error rate (BER). The LCR and the AFD have found a variety of applications in the modeling and design of wireless communication systems, such as the finite-state Markov modeling of fading channels [2], the analysis of handoff algorithms [3], and the estimation of packet error rates [4]. Pioneering work on the subject was done by Rice [5], which examined non-line-of-sight (Rician) channels. Much later, some expressions for the LCR and AFD of the combined envelope of diversity Rayleigh channels were published. For example, Lee [6] derived these quantities for equal-gain combining (EGC). Adachi *et al.* [7] provided general expressions in the case of dual correlated channels with selection combining (SC), maximal-ratio combining (MRC), and EGC diversity; these expressions could be put in closed form for independent channels. Yacoub *et al.* [8] also presented expressions in the case of EGC and MRC with an arbitrary number of independent channels.

While the Rayleigh and Rice distributions can indeed be used to model the envelope of fading channels in many cases of interest, it has been found experimentally [9] that the Nakagami distribution offers a better fit for a wider range of fading conditions. The Nakagami distribution was proposed in the early forties for characterizing urban and suburban fading channels, and was originally deduced from a series of experiments. It was later shown that it constitutes an approximation to the PDF of the amplitude of a sum of phasors with random moduli and phases [9], [10]. Contrary to the Rice PDF, it does not assume a line-of-sight (LOS) condition. Hence, while the Rice distribution can only describe better-than-Rayleigh fading conditions, the Nakagami PDF with parameter $m < 1$ models worse-than-Rayleigh conditions. Moreover, for $m = 1$, the Nakagami PDF reduces to the Rayleigh PDF, and can thus be seen as a generalization of the latter. It was verified in several other independent experimental researches that the Nakagami-m PDF could indeed accurately represent the wide range of commonly encountered fading conditions [11]–[13]. As a result, it has been adopted in some software and hardware fading channel simulators for third generation (3G) cellular networks [14], [15]. It is also increasingly used in the analysis and modeling of wide-band channels, in particular for CDMA systems in frequency-selective fading [16]; this is also encouraged by the fact that its analytical form is more amenable to manipulations, compared to the Rice PDF (which contains a modified Bessel function of the first kind).

While the Nakagami PDF has many attractive features, there is still no widely accepted general and efficient way of simulating a correlated Nakagami fading channel. This is partly due to the fact that no temporal autocorrelation function was specified for the Nakagami PDF when it was proposed, and that, to our

Paper approved by R. A. Valenzuela, the Editor for Transmission Systems of the IEEE Communications Society. Manuscript received May 25, 2001; revised November 14, 2001. This work was supported by the Natural Sciences and Engineering Research Council of Canada and the Science Council of British Columbia. This paper was presented in part at the 7th Canadian Workshop on Information Theory, Vancouver, BC, Canada, June 3–6, 2001.

The authors are with the Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, BC V6T 1Z4, Canada (e-mail: cyrili@ece.ubc.ca., mathio@ece.ubc.ca).

Publisher Item Identifier 10.1109/TCOMM.2002.801465.

knowledge, no analytical expression for it has been proposed yet on a physical basis. Hence, while methods are available to obtain the exact PDF of the envelope [17], most simulators need to make particular assumptions in order to model the temporal correlation of the channel [18], [19]. Despite the lack of a physically justified model for the correlation of a Nakagami channel, some analytical work has been carried out with respect to the latter's second-order statistics. In [20], using results from [21], the authors derived the LCR and AFD for Nakagami channels without diversity, and for a special case of MRC diversity. They also presented an approximate result for EGC, relying on [9, eqs. (81)-(83)]. The LCR and AFD of non-diversity Nakagami channels with isotropic scattering were also later obtained in [22] using a different approach, which relied on the decomposition of the distribution. Field trials were carried out in [24] and [25] for the non-diversity case, and good agreements were reported between the experimental LCR and the analytical expressions. Novel analytical expressions and plots of the LCR for SC and dual-branch EGC in the case of independent identically distributed (i.i.d.) Nakagami fading channels were given in [23], in which the expressions were used to construct a Nakagami channel simulator which had the desired LCR and envelope distribution.

In this paper, we present a very general approach which can be used to analytically evaluate the LCR and AFD for Nakagami channels with diversity reception. Our methodology is the following: we straightforwardly rearrange the expression for the LCR, such that it can be expressed as the product of the PDF of the received signal and an integral involving the conditional PDF of the derivative of this signal. Depending on the cases, the first term either can be found in the literature or has to be derived. The conditional PDF in the second term is found by examining the expression for the derivative of the received signal. It should be noted that the proposed methodology does not have limitations nor makes any simplifying assumptions. However, we shall only present in this paper the cases where a simple analytical closed-form solution for the LCR and AFD is possible, which generally requires the diversity channels to fade independently. It will also be shown that our general analytical expressions for Nakagami fading reduce to previously known results. In contrast to this work, earlier derivations of the LCR and AFD for channels with diversity reception were usually specific to a particular channel/diversity pair, and cannot always be used in obtaining the same quantities for different situations. For example, in [20] the LCR were obtained by finding a closed-form expression for the joint PDF of the received signal and its derivative for the case of Nakagami fading, which is not always possible; whereas in [7] and [8], the analytical derivations were conducted for specific situations only (dual diversity for the former) or under certain assumptions (identical channels for both).

The organization of the paper is as follows. After this introduction, our analytical approach and the steps needed to apply it to Nakagami channels (using the physical insights of [22]) are presented in Section II-A. These results are used to obtain the LCR and AFD for Nakagami channels with SC, MRC, and EGC diversity reception in the following sections. The analytical expressions obtained are evaluated numerically and discussed in Section III. The last section summarizes the contributions of this paper and cites some applications.

II. LCRs AND AFDs FOR DIVERSITY NAKAGAMI CHANNELS

A. General

Let r be the sampled value of the *diversity combined* envelope $R(t)$ of a fading channel. The LCR $N_R(r)$ and AFD $\tau_R(r)$ are defined as a function of r by

$$N_R(r) = \int_0^\infty \dot{r} p_{R,\dot{R}}(r,\dot{r})\, d\dot{r} \tag{1}$$

$$\tau_R(r) = F_R(r)/N_R(r) \tag{2}$$

where $(\dot{\ })$ denotes the derivation operator with respect to time, $F_R(r) = \int_0^r p_R(\alpha) d\alpha$ is the CDF of the fading channel, and $p_R(r)$ is the corresponding PDF. The LCR can be rewritten in terms of $p_R(r)$ and the conditional distribution $p_{\dot{R}}(\dot{r}\,|\,r)$ as

$$\begin{aligned} N_R(r) &= \int_0^\infty \dot{r} p_{\dot{R}}(\dot{r}\,|\,r) p_R(r)\, d\dot{r} \\ &= p_R(r) \int_0^\infty \dot{r} p_{\dot{R}}(\dot{r}\,|\,r)\, d\dot{r}. \end{aligned} \tag{3}$$

This generic expression for $N_R(r)$ will be the basis for all later derivations. It is indeed applicable to all forms of diversity, and can be used in conjunction with any fading distribution. It reduces directly to, for example, [6, eq. (6)], [20, eq. (15)], and [22, eq. (16)] for the special cases treated in these papers.

The output sampled envelope of an L-branch diversity combiner can be expressed in the generic form of

$$r = f(r_1, r_2, \ldots, r_L) \tag{4}$$

where r_l, with $l = 1, 2, \ldots, L$ is the envelope of the lth diversity channel seen by the receiver, and $f(\cdot)$ is a function which depends on the diversity technique used. In the case of Nakagami fading, the PDF of r_l is mathematically expressed as

$$p_{R_l}(r_l) = 2\left(\frac{m_l}{\Omega_l}\right)^{m_l} \frac{r_l^{2m_l-1}}{\Gamma(m_l)} e^{-\frac{m_l}{\Omega_l} r_l^2}, \quad r_l > 0 \tag{5}$$

where $\Omega_l = E[r_l^2]$ and m_l are the average power and the fading figure of the lth channel, respectively, with $E[\cdot]$ denoting statistical expectation. $\Gamma(x)$ is the gamma function [26, eq. (8.310.1)]. The CDF of r_l is given by

$$F_{R_l}(r_l) = \frac{\gamma\left(m_l, \frac{m_l}{\Omega_l} r_l^2\right)}{\Gamma(m_l)} \tag{6}$$

where $\gamma(x, \alpha) = \int_0^\alpha e^{-t} t^{x-1} dt$ is the incomplete gamma function of the first kind [26, eq. (8.350.1)].

By analogy with [22], when $2m_l$ is an integer, the envelope of the lth diversity channel can be written as

$$r_l^2 = \begin{cases} r_{l0}^2 + \sum_{i=1}^{m_l-\frac{1}{2}} r_{li}^2 & \text{with } 2m_l \text{ odd} \\ \sum_{i=1}^{m_l} r_{li}^2 & \text{with } 2m_l \text{ even} \end{cases} \tag{7}$$

where $r_{l0}^2 = x_{l0}^2, r_{li}^2 = x_{li}^2 + y_{li}^2$, and the x_{li}'s and y_{li}'s are Gaussian random variables with zero mean and variance $\sigma_l^2 = \Omega_l/(2m_l)$. The derivatives of the r_l's can then be calculated using

$$\dot{r}_l = \begin{cases} \left(r_{l0}\dot{r}_{l0} + \sum_{i=1}^{m_l-\frac{1}{2}} r_{li}\dot{r}_{li}\right) \Big/ r_l & \text{with } 2m_l \text{ odd} \\ \left(\sum_{i=1}^{m_l} r_{li}\dot{r}_{li}\right)/ r_l & \text{with } 2m_l \text{ even.} \end{cases} \tag{8}$$

From [1], for isotropic scattering, the $\dot{r}_{li}$'s are Gaussian-distributed with zero mean and variance $\hat{\sigma}_{r_{li}}^2 = \mathrm{Var}[\dot{r}_{li}^2] = \sigma_l^2 2\pi^2 f_m^2$, where $f_m = v/\lambda$ is the maximum Doppler shift for a vehicle speed v and carrier wavelength λ. We let $\hat{\sigma}_l^2 = \sigma_l^2 2\pi^2 f_m^2$ to alleviate the notation. Since $\dot{r}_l$ is a sum of zero-mean Gaussian variables, it is also zero-mean Gaussian, conditioned on r_l. Using (8) and (7), its variance is found to be $\hat{\sigma}_{r_l}^2 = \mathrm{Var}[\dot{r}_l] = \hat{\sigma}_l^2$, which is independent of r_l. As asserted in [22], $\dot{r}_l$ and r_l are thus independent, so that $p(r_l, \dot{r}_l) = p(r_l)p(\dot{r}_l)$.

Using the above, the analytical LCR and AFD for the diversity methods of concern will be derived in the next sections.

B. Selection Combining

In [7], the authors present an expression for the LCR for dual SC in Rayleigh fading. It is generalized in [27] for an arbitrary number of i.i.d channels. Below, using (3), we derive an expression for the LCR of SC for L independent but not necessarily identical channels. We then apply it to the case of Nakagami fading. The channel envelope at the output of a SC diversity system is well known to be given by

$$r = \max\{r_l, l = 1, \ldots, L\}. \tag{9}$$

Its derivative is

$$\dot{r} = \dot{r}_j, \quad r_j = \max\{r_l, l = 1, \ldots, L\}. \tag{10}$$

$\dot{r}$ is thus a Gaussian random variable when conditioned on the r_l's, with zero mean and variance

$$\hat{\sigma}_r^2 = \hat{\sigma}_j^2 \text{ if } \left(r_j = \max_{l=1,\ldots,L} r_l \,|\, r_j = r \right). \tag{11}$$

$\hat{\sigma}_r$ is thus a discrete random variable with PDF

$$\begin{aligned} p_{\hat{\Sigma}_r}(\hat{\sigma}_r) &= \sum_{j=1}^{L} P(\hat{\sigma}_r = \hat{\sigma}_j)\delta(\hat{\sigma}_r - \hat{\sigma}_j) \\ &= \sum_{j=1}^{L} P\left(r_j = \max_{l=1,\ldots,L} r_l \,|\, r_j = r \right) \delta(\hat{\sigma}_r - \hat{\sigma}_j). \end{aligned} \tag{12}$$

From (3), the LCR, conditional on $\hat{\sigma}_r$, are given by

$$\begin{aligned} N_R(r \,|\, \hat{\sigma}_r) &= p_R(r) \int_0^\infty \dot{r} \frac{1}{\sqrt{2\pi}\hat{\sigma}_r} e^{-\frac{\dot{r}^2}{2\hat{\sigma}_r^2}} \, d\dot{r} \\ &= p_R(r) \frac{\hat{\sigma}_r}{\sqrt{2\pi}}. \end{aligned} \tag{13}$$

(13) is averaged over the PDF for $\hat{\sigma}_r$, i.e., (12), to obtain

$$\begin{aligned} N_R(r) &= \int_0^\infty N_R(r \,|\, \hat{\sigma}_r) p_{\hat{\Sigma}_r}(\hat{\sigma}_r) d\hat{\sigma}_r \\ &= \sum_{j=1}^{L} p_{R_j}(r) \frac{\hat{\sigma}_j}{\sqrt{2\pi}} P\left(r_j = \max_{l=1,\ldots,L} r_l \,|\, r_j = r \right). \end{aligned} \tag{14}$$

By taking into account the independence assumption, the term $P(r_j = \max_{l=1,\ldots,L} r_l \,|\, r_j = r)$ can be evaluated as

$$\begin{aligned} P&\left(r_j = \max_{l=1,\ldots,L} r_l \,|\, r_j = r \right) \\ &= P(r_l < r_j, l = 1, \ldots, L, l \neq j \,|\, r_j = r) \\ &= P(r_l < r, l = 1, \ldots, L, l \neq j) \\ &= \prod_{\substack{l=1 \\ l \neq j}}^{L} P(r_l < r) = \prod_{\substack{l=1 \\ l \neq j}}^{L} F_{R_l}(r). \end{aligned} \tag{15}$$

From (14) and (15), the LCR can be expressed as

$$N_R(r) = \sum_{j=1}^{L} p_{R_j}(r) \frac{\hat{\sigma}_j}{\sqrt{2\pi}} \prod_{\substack{l=1 \\ l \neq j}}^{L} F_{R_l}(r). \tag{16}$$

Substituting (5) and (6) into (16), and using $\hat{\sigma}_j = \sigma_j \sqrt{2}\pi f_m$ leads to the LCR for a Nakagami fading channel with arbitrary parameters and SC

$$\begin{aligned} N_R(r) = \sqrt{2\pi} f_m \sum_{j=1}^{L} \frac{1}{\Gamma(m_j)} \left(\frac{m_j}{\Omega_j} r^2 \right)^{m_j - \frac{1}{2}} \\ \times e^{-\frac{m_j}{\Omega_j} r^2} \prod_{\substack{l=1 \\ l \neq j}}^{L} \frac{\gamma\left(m_l, \frac{m_l}{\Omega_l} r^2 \right)}{\Gamma(m_l)}. \end{aligned} \tag{17}$$

For identical channel parameters, $m_l = m, \Omega_l = \Omega, l = 1, \ldots, L$, (17) reverts to the expression given in [23], and when $m = 1$, to the one given in [27, eq. (45)], for Rayleigh fading

$$N_R(r) = L\sqrt{2\pi} f_m \frac{r}{\sqrt{\Omega}} e^{-\frac{r^2}{\Omega}} \left(1 - e^{-\frac{r^2}{\Omega}} \right)^{L-1}. \tag{18}$$

It can be verified (cf. Appendix A) that the approach taken in [27] for obtaining the LCR, when extended to include arbitrary parameters, also leads to (17) for Nakagami fading.

(17) gives the average number of times per second that the output $R(t)$ of a selection diversity combiner falls below a specified value r, given $L, f_{\max}$, and the channel parameters $m_l, \Omega_l, l = 1, \ldots, L$. Hence, if the channel conditions can be estimated and a maximum speed for the mobile is assumed, based on (17), one can determine the number of diversity branches needed (L) so that the combined signal $R(t)$ does not fall below a threshold r_T more than a specified maximum number of times N_T. This can be done by evaluating $N_R(r_T)$ for increasing values of L, until $N_R(r_T) < N_T$.

The CDF for SC is given by

$$F_R(r) = P(r_1 < r, r_2 < r, \ldots, r_L < r) \tag{19}$$

which reduces for independent Nakagami channels to

$$F_R(r) = \prod_{l=1}^{L} F_{R_l}(r) = \prod_{l=1}^{L} \frac{\gamma\left(m_l, \frac{m_l}{\Omega_l} r^2 \right)}{\Gamma(m_l)}. \tag{20}$$

The AFD for arbitrary Nakagami channels with SC can then be obtained straightforwardly by substituting (17) and (20) in (2)

$$\tau_R(r) = \frac{\prod_{l=1}^{L} \frac{\gamma\left(m_l, \frac{m_l}{\Omega_l} r^2 \right)}{\Gamma(m_l)}}{\sqrt{2\pi} f_m \sum_{j=1}^{L} \frac{\left(\frac{m_j}{\Omega_j} r^2 \right)^{m_j - \frac{1}{2}}}{\Gamma(m_j)} e^{-\frac{m_j}{\Omega_j} r^2} \prod_{\substack{l=1 \\ l \neq j}}^{L} \frac{\gamma\left(m_l, \frac{m_l}{\Omega_l} r^2 \right)}{\Gamma(m_l)}}. \tag{21}$$

Equation (21) gives the average time (in seconds) that the combined signal $R(t)$ stays below a specified level r, once it has crossed it in the downward direction, again given $L, f_{\max}$ and the channel parameters. Hence, one can again evaluate the required L so that, on average, the combined signal $R(t)$ does not stay below a threshold r_T more than a specified maximum period of time $\tau_{\max}$. The quantity $\tau_{\max}$ can correspond, for example, to the average period of time a receiver can demodulate a signal of amplitude r_T, without going into outage or losing synchronization. Similar insights can be obtained for the cases of maximal ratio and equal-gain diversity, thanks to the expressions for the LCR and AFD derived in the following sections.

C. Maximal-Ratio Combining (MRC)

The output of a MRC diversity system is given by

$$r = \left[\sum_{l=1}^{L} r_l^2\right]^{\frac{1}{2}} \tag{22}$$

and its derivative by

$$\dot{r} = \frac{\sum_{l=1}^{L} r_l \dot{r}_l}{r}. \tag{23}$$

As in the SC case, $\dot{r}$ is a Gaussian random variable when conditioned on the r_l's, with zero mean and variance

$$\begin{aligned}\hat{\sigma}_r^2 &= E\left[\left(\sum_{l=1}^{L} r_l \dot{r}_l\right)^2 \Big/ r^2\right] \\ &= \sum_{l=1}^{L} r_l^2 E[\dot{r}_l{}^2] \Big/ \sum_{l=1}^{L} r_l^2 \end{aligned} \tag{24}$$

where the last equation was obtained using the independence assumption between the branches. If the diversity channels are identically distributed, $E[\dot{r}_l{}^2] = \hat{\sigma}_{r_l}^2 = \sigma^2 2\pi^2 f_m^2$, and (24) reduces to

$$\hat{\sigma}_r^2 = \sigma^2 2\pi^2 f_m^2. \tag{25}$$

In that case, (3) can be solved to give

$$N_R(r) = p_R(r)\frac{\hat{\sigma}_r}{\sqrt{2\pi}}. \tag{26}$$

The PDF of r (again for the special case of i.i.d. channels) is known to be given by [9]

$$p_R(r) = 2\left(\frac{m_T}{\Omega_T}\right)^{m_T} \frac{r^{2m_T-1}}{\Gamma(m_T)} e^{-\frac{m_T}{\Omega_T} r^2}, \quad r > 0 \tag{27}$$

where $m_T = mL, \Omega_T = \Omega L$, and the CDF by

$$F_R(r) = \frac{\gamma\left(m_T, \frac{m_T}{\Omega_T} r^2\right)}{\Gamma(m_T)}. \tag{28}$$

Using (27) and (25) in (26) leads to the following result for the LCR, which was also derived in [20] using the approach mentioned in the introduction

$$N_R(r) = \frac{\sqrt{2\pi} f_m}{\Gamma(m_T)} \left(\frac{m_T}{\Omega_T} r^2\right)^{m_T - \frac{1}{2}} e^{-\frac{m_T}{\Omega_T} r^2}. \tag{29}$$

Substituting (28) and (29) in (2) yields the AFD

$$\tau_R(r) = \frac{\gamma\left(m_T, \frac{m_T}{\Omega_T} r^2\right) e^{\frac{m_T}{\Omega_T} r^2}}{\sqrt{2\pi} f_m \left(\frac{m_T}{\Omega_T} r^2\right)^{m_T - \frac{1}{2}}}. \tag{30}$$

For Rayleigh fading $(m = 1)$ (29) reduces to the following expression[1]:

$$N_R(r) = \frac{\sqrt{2\pi} f_m}{\Gamma(L)} \left(\frac{r^2}{\Omega}\right)^{L - \frac{1}{2}} e^{-\frac{r^2}{\Omega}}. \tag{31}$$

[1]It should be noted that it is similar to [8, eq. (13)], but the latter has possibly a misprint in the exponential term.

D. Equal-Gain Combining (EGC)

The output of a EGC diversity system is given by

$$r = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} r_l \tag{32}$$

whereas its derivative by [6]

$$\dot{r} = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} \dot{r}_l. \tag{33}$$

As opposed to the previous cases, $\dot{r}$ is now a Gaussian random variable *independently* of the r_l's, with zero mean and variance

$$\hat{\sigma}_r^2 = \frac{1}{L} \sum_{l=1}^{L} \dot{r}_l{}^2 = 2\pi^2 f_m^2 \sum_{l=1}^{L} \sigma_l^2 \tag{34}$$

where the last equation was obtained using the independence assumption between the branches. Solving (3) leads to

$$N_R(r) = p_R(r)\frac{\hat{\sigma}_r}{\sqrt{2\pi}}. \tag{35}$$

It is similar to (26) in the previous section, however, for the MRC case, this equation required the i.i.d. assumption in order to be valid, while this is not the case for EGC. For i.i.d. channels, the CDF and PDF of r were presented in [28] and [8], respectively, in integral form, for an arbitrary L. For independent channels with arbitrary parameters, the PDF can be written as

$$\begin{aligned} p_R(r) = \sqrt{L} &\int_0^{\sqrt{L}r} \int_0^{\sqrt{L}r - r_L} \cdots \int_0^{\sqrt{L}r - \sum_{l=3}^{L} r_l} \\ &\times \left[p_{R_1}\left(r\sqrt{L} - \sum_{l=2}^{L} r_l\right) \prod_{l=2}^{L} p_{R_l}(r_l)\right] dr_2 \ldots dr_L. \end{aligned} \tag{36}$$

For Rayleigh fading, a simple closed-form solution is available only for $L \leq 2$, and is implicitly presented in [7]. For Nakagami fading with identical parameters and $L = 2$, (36) reduces to

$$p_R(r) = \sqrt{2} \int_0^{\sqrt{2}r} p_R(r\sqrt{2} - r_2) p_R(r_2) dr_2. \tag{37}$$

After substituting (5) in (37), and following the steps detailed in Appendix B, we obtain the following closed-form solution for the PDF of dual-diversity EGC:

$$\begin{aligned} p_R(r) = \left(\frac{m}{\Omega}\right)^{2m} &\frac{2B\left(2m, \frac{1}{2}\right)}{[\Gamma(m)]^2 2^{2m-2}} r^{4m-1} e^{-2\frac{m}{\Omega} r^2} \\ &\times \Phi\left(2m, 2m + \frac{1}{2}, \frac{m}{\Omega} r^2\right) \end{aligned} \tag{38}$$

where $B(x, y)$ is the beta function and $\Phi(a, c, x)$ the confluent hypergeometric function, given by [26, eqs. (8.380.1), (9.210.1)], respectively. Substituting (37) and (34) in (35) yields the following expression for the LCR of dual-branch EGC:

$$N_R(r) = \frac{\sqrt{2\pi} f_m B\left(2m, \frac{1}{2}\right)}{[\Gamma(m)]^2 2^{2m-2}} \left(\frac{m}{\Omega} r^2\right)^{2m-\frac{1}{2}} e^{-2\frac{m}{\Omega}r^2} \times \Phi\left(2m, 2m+\frac{1}{2}, \frac{m}{\Omega}r^2\right). \quad (39)$$

The derivation of the CDF is carried out in Appendix B and leads to the following infinite series representation:

$$F_R(r) = \frac{\sqrt{\pi}}{[\Gamma(m)]^2 2^{4m-2}} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma\left(2m+n+\frac{1}{2}\right)} \frac{1}{2^n n!} \times \gamma\left(2m+n, 2\frac{m}{\Omega}r^2\right). \quad (40)$$

By substituting (40) and (39) in (2), we obtain the AFD

$$\tau_R(r) = \frac{e^{2\frac{m}{\Omega}r^2} \sum_{n=0}^{\infty} \frac{\Gamma(2m+n)}{\Gamma\left(2m+n+\frac{1}{2}\right)} \frac{1}{2^n n!} \gamma\left(2m+n, 2\frac{m}{\Omega}r^2\right)}{2 f_m B\left(2m, \frac{1}{2}\right) \left(2\frac{m}{\Omega}r^2\right)^{2m-\frac{1}{2}} \Phi\left(2m, 2m+\frac{1}{2}, \frac{m}{\Omega}r^2\right)}. \quad (41)$$

For $m = 1$, (39) can be simplified using [26, eqs. (9.212.2), (9.212.4), (9.212.1), (9.236.1)] for $\Phi(a, c, x)$, in that order, and [26, eq. (8.384.1)] for $B(x, y)$. This reduces to the following expression for the LCR of dual-branch EGC and i.i.d. Rayleigh fading channels, which was also presented in [7]:

$$N_R(r) = \sqrt{2\pi} f_m e^{-\frac{r^2}{\Omega}} \left[\frac{r}{\sqrt{\Omega}} e^{-\frac{r^2}{\Omega}} + \left(\frac{r^2}{\Omega} - \frac{1}{2}\right) \sqrt{\pi} \operatorname{erf}\left(\frac{r}{\sqrt{\Omega}}\right)\right] \quad (42)$$

where $\operatorname{erf}(x) = 2/\sqrt{\pi} \int_0^x e^{-z^2} dz$. For independent but non-identical Rayleigh fading channels, we make use of (36), (34), and (35) to obtain a closed-form expression for the LCR of dual-diversity EGC

$$N_R(r) = \sqrt{2\pi} f_m \frac{\sqrt{\Omega_p}}{\Omega_m} e^{-\frac{r^2}{\Omega_m}} \times \left[\frac{r}{\sqrt{\Omega_m}} \left(\sqrt{\Omega_{12}} e^{-\frac{\Omega_{21}}{\Omega_m} r^2} + \sqrt{\Omega_{21}} e^{-\frac{\Omega_{12}}{\Omega_m} r^2}\right) + \left(\frac{r^2}{\Omega_m} - \frac{1}{2}\right) \sqrt{\pi} \left(\operatorname{erf}\left(\frac{\Omega_{21}}{\sqrt{\Omega_m}} r\right) + \operatorname{erf}\left(\frac{\Omega_{12}}{\sqrt{\Omega_m}} r\right)\right)\right] \quad (43)$$

with $\Omega_m = (\Omega_1 + \Omega_2)/2, \Omega_{21} = \Omega_2/\Omega_1, \Omega_{12} = \Omega_1/\Omega_2$ and $\Omega_p = \Omega_1 \Omega_2 / 4$.

III. Numerical Results and Discussion

The LCR and AFD expressions presented above are plotted in logarithmic scale against the normalized value of the combined received envelope, $r_n = r/\sqrt{\Omega}$ in decibels. Fig. 1–3 compare the LCR (normalized by f_m) for the diversity techniques presented above with $L = 2$ and the no-diversity (ND) case ($L = 1$), for three values of the m parameter. $m = 0.6$ corresponds to severe fading (worse than Rayleigh), $m = 1.3$ to fading conditions slightly better than Rayleigh, and $m = 3.0$ to LOS conditions. For all the curves, it is observed that the LCR for MRC are the lowest for low values of r_n's, and the highest for high values of r_n's, while the opposite is true for the ND case. Indeed, in the ND case, fades occur more frequently due to the absence of diversity. As a consequence, the signal crosses lower values of r_n more often than when diversity is used (with the lowest number of crossings occurring for the optimal diversity scheme, i.e., MRC), whereas it crosses high values of r_n less often. Also, from these curves, the output of an EGC receiver fades less frequently than that of an SC receiver. However, the differences in LCR between MRC, EGC and SC depend on the Nakagami-m parameter (and thus on the severity of the channel in terms of fading), and are commented below.

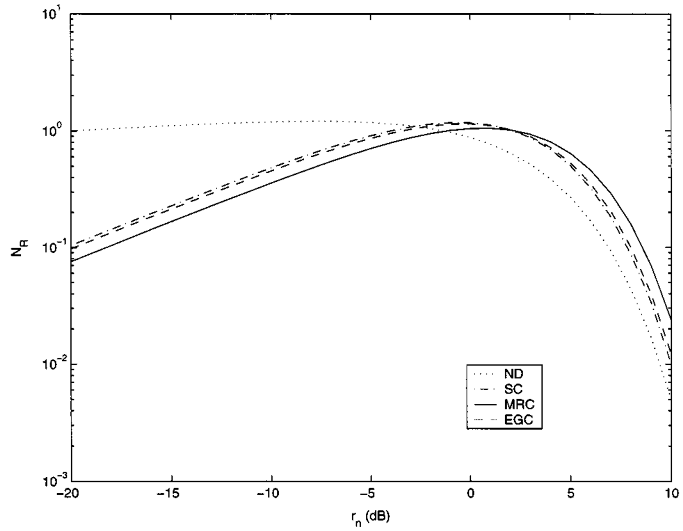

Fig. 1. LCR with SC, MRC, and EGC dual diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 0.6$. The LCR for SC and EGC are similar, but differ from the LCR for MRC.

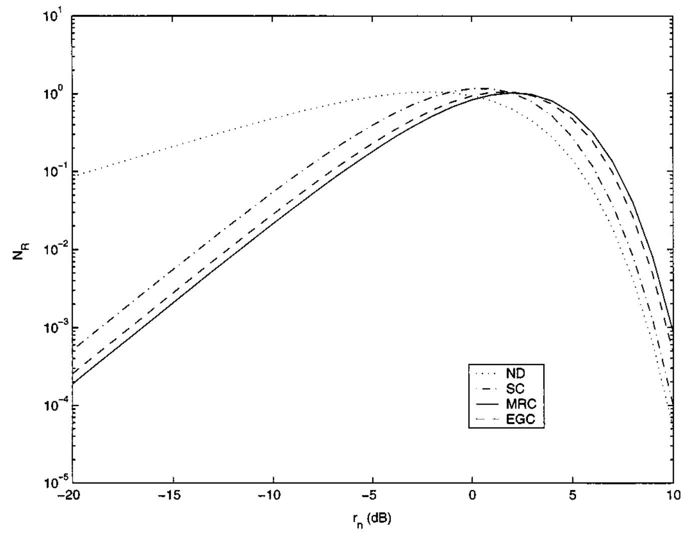

Fig. 2. LCR with SC, MRC, and EGC dual diversity ($L = 2$) and without diversity (ND, $L = 1$); $m = 1.3$. The LCR for EGC are closer to those for MRC, while the LCR for SC are further apart, compared to the case $m = 0.6$.

For $m = 0.6$, the LCR curves for SC and EGC are nearly identical, but differ from those of MRC, for which the combined

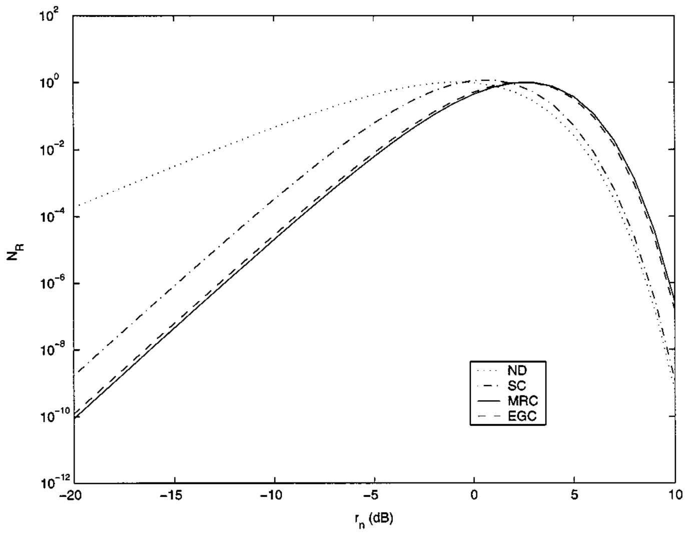

Fig. 3. LCR with SC, MRC, and EGC dual diversity $(L = 2)$ and without diversity (ND, $L = 1$); $m = 3.0$. The LCR for EGC and MRC are similar, but differ significantly from the LCR for SC, compared to the cases $m = 0.6$ and $m = 1.3$.

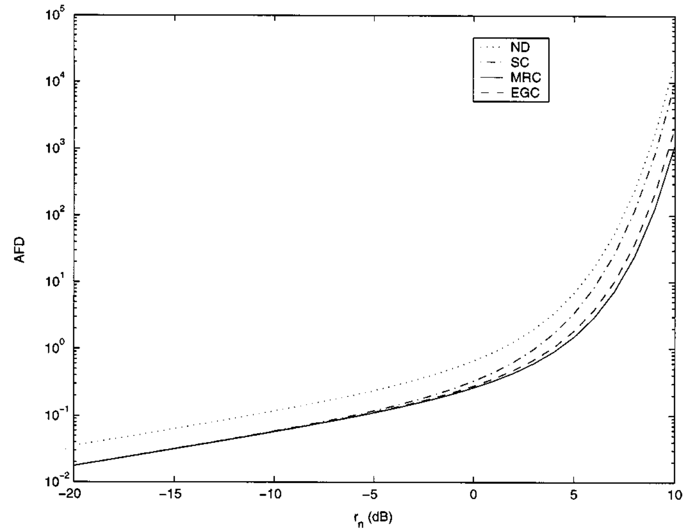

Fig. 5. AFD with SC, MRC, and EGC dual-diversity $(L = 2)$ and without diversity (ND, $L = 1$); $m = 1.3$. The AFD for EGC are closer to those for MRC, while the AFD for SC are further apart (especially for higher r_n's), compared to the case $m = 0.6$.

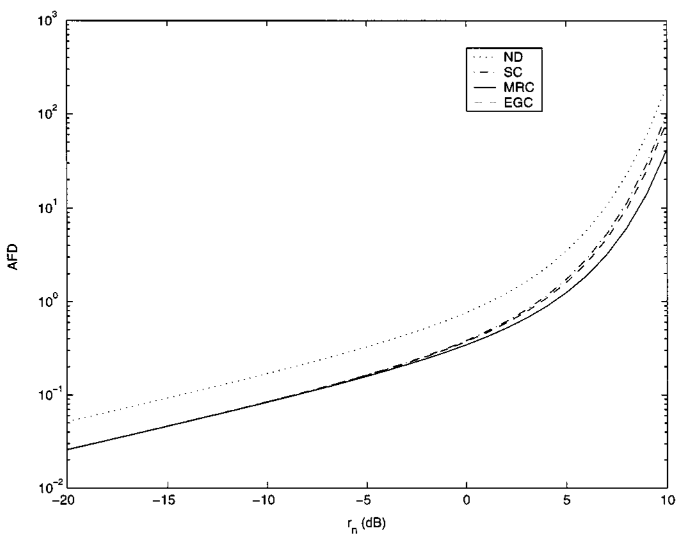

Fig. 4. AFD with SC, MRC, and EGC dual diversity $(L = 2)$ and without diversity (ND, $L = 1$); $m = 0.6$. The AFD for SC and EGC are similar and higher than those for MRC, but the difference gradually reduces for smaller r_n's.

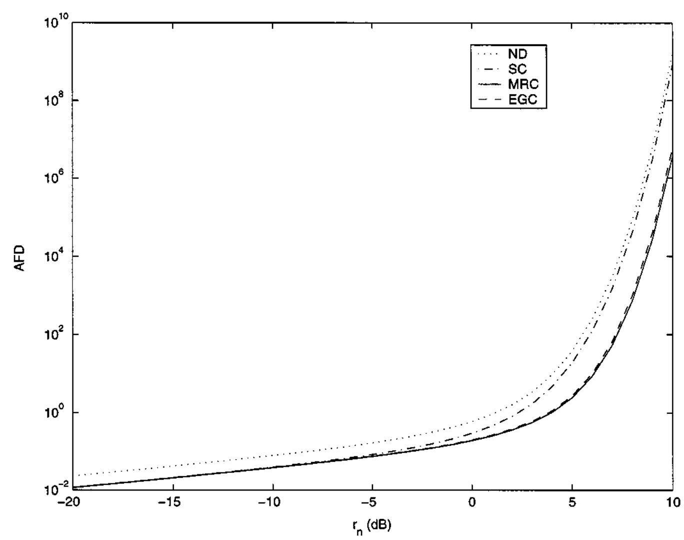

Fig. 6. AFD with SC, MRC, and EGC dual diversity $(L = 2)$ and without diversity (ND, $L = 1$); $m = 3.0$. The AFD for EGC and MRC are very similar, but differ significantly from the AFD for SC (for higher r_n's), compared to the cases $m = 0.6$ and $m = 1.3$.

envelope exhibits less severe fading. As m is increased from 0.6 to 3.0, the LCR curve for EGC gets closer to that for MRC, and further away from that for SC. For $m = 3.0$ the LCR curves for MRC and EGC nearly overlap. This reflects the fact that, as the fading severity decreases (i.e., for higher values of m), the performance of EGC tends to approach that of MRC, while the performance margin between the latter two and SC increases. This could also be observed by comparing plots of the error probabilities for these diversity techniques and different m's.

The results plotted in Fig. 4–6 present the AFD, normalized by $1/f_m$, for the same cases as before. From these curves, it is seen that the AFD for all three diversity techniques remain very close for values of r_n less than about -5 dB. This means that once the combined signal has faded below this value, it remains below for nearly the same amount of time for all of MRC, EGC, or SC. However, from our previous examination of the LCR, since a MRC signal crosses low values of r_n less often than EGC and SC signals, on average it will spend less time into deep fades than the latter two. For higher values of r_n, it is observed that the AFD are lower for MRC than they are for EGC and SC. For each r_n, the combined signals obtained with EGC and SC spend more time below this value than that obtained with MRC, which reflects the fact that on average a stronger signal results from the use of MRC. This agrees with the previous discussion on LCR, in which it was pointed out that a MRC signal is more often in the high end of the signal strength r_n than EGC and SC. As before, we observe that for severe fading $(m = 0.6)$, the behavior of EGC follows closely that of SC, while for mild

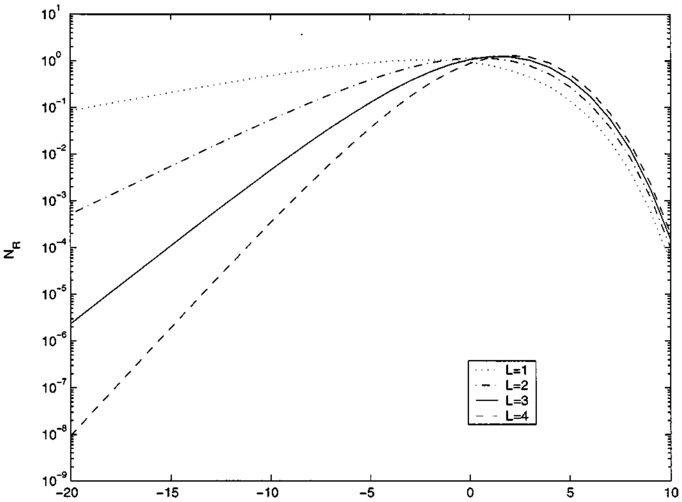

Fig. 7. LCR for SC with different diversity orders (L) and $m = 1.3$. For low r_n's, the LCR decrease dramatically with higher L's.

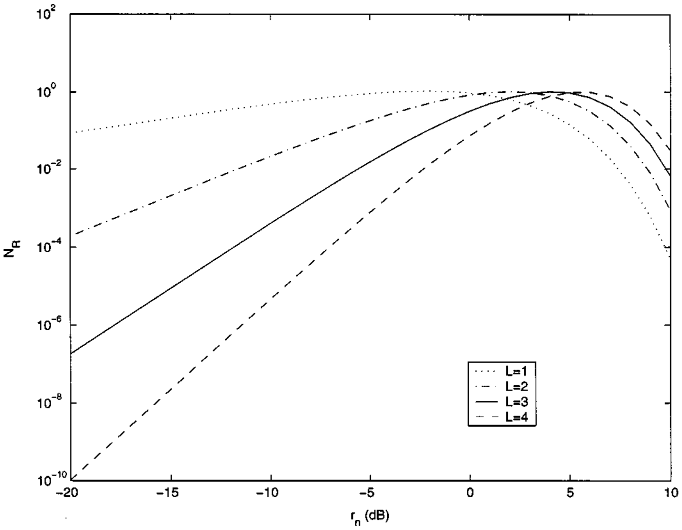

Fig. 8. LCR for MRC with different diversity orders and $m = 1.3$. Compared to the LCR for SC, as L increases, the LCR for MRC decrease faster for low r_n's, and also increase faster for high r_n's.

fading ($m = 3.0$), it compares to that of MRC. Thus, for a fixed set of parameters, there is not a one-to-one correspondence between the behavior of the LCR and that of the AFD (similar observations were reported in [8], for the case of Rayleigh fading). The LCR of MRC differed from that of SC (and EGC for low m's) over the whole range of r_n's, while the AFD are nearly identical for low r_n's. This is due to the term $F_R(r)$, which intervenes in the relation (2) between the latter quantities.

Figs. 7 and 8 illustrate the LCR for SC and MRC, respectively, for a variable number of diversity branches L. In the case of SC, as L increases, the frequency at which the received signal crosses high values (e.g., at approximately $r_n > 0$ dB) stays almost the same. Whereas in the case of MRC, $N_R(r_n)$ increases with L for high values of r_n. Moreover, for low values of r_n, the LCR decrease faster for MRC than for SC as more diversity branches are added, e.g., for $r_n = -20$ dB, the decrease in the dB value of $N_R(r_n)$ is more than sixfold for MRC as L goes from 2 to 4, while it is less than fivefold in the case of SC. This parallels the observations made in [28], according to which the advantage of MRC and EGC over SC (i.e., the strength of the signal) gets more pronounced as the number of diversity branches increases (with EGC following the behavior of MRC).

In summary, the numerical results support the assertion that the gain in performance made possible using MRC and EGC, as compared to using SC, gets more important as the fading gets less severe and the diversity order increases.

IV. Conclusion

In this paper, starting from a common representation for the LCR, we derived generalizations of expressions for the LCR of a diversity received signal in Rayleigh fading, in order to handle the more general Nakagami fading distribution. Closed-form solutions were presented for arbitrary L in the case of SC and MRC, and for $L = 2$ in the case of EGC. The assumption of i.i.d. channels was made throughout the paper (except for SC, where nonidentical parameters were allowed) in order to obtain these results in closed form, however, the methodology used is not limited by this assumption. The correlated case can be dealt with in the same manner, but will require numerical evaluations of the LCR and AFD for most cases of interest. The material we presented can be used in designing finite-state channel simulators [23], analyzing error-correcting schemes for burst error channels [4], determining the minimum duration outages in fading channels [29], or determining the delay spread of frequency-selective channels [30]. [2]

Appendix A

The purpose of this appendix is to show that the analytical approach given in [27], with some adjustments, also leads to (17). For channels with nonidentical parameters, [27, eq. (44)] can be modified as

$$N_R(r) = \sum_{l=1}^{L} \underbrace{\int_0^{2\pi} \cdots \int_0^{2\pi}}_{L-1} \underbrace{\int_0^{r} \cdots \int_0^{r}}_{L-1} \int_0^{\infty} \dot{r}_l \times p_{\dot{R}_l}(\dot{r}_l \,|\, r_l = r, r_{j=1,\ldots,L,j\neq l}, \theta_{l,(j=1,\ldots,L,j\neq l)}) \times p(r_l = r, r_{j=1,\ldots,L,j\neq l}, \theta_{l,(j=1,\ldots,L,j\neq l)}) \times d\dot{r}_1 \prod_{\substack{j=1\\ j\neq l}}^{L} (dr_j d\theta_{lj}). \tag{44}$$

The usual assumption is made that the phases associated with the Nakagami signals are uniformly distributed over $(0, 2\pi)$.

[2]It very recently came to our attention that, since the time our results were submitted (and several months after many of them initially appeared in [23]), other independent contributions dealing with the LCR and AFD of diversity Nakagami channels have been published [31], [32]. The material presented in this paper differs from that of [31] and [32] in the methodology used, and/or the generality (nonidentically versus identically distributed) or the representation (closed form versus integral form) of the original analytical results.

Given that the phases, envelopes, and $\dot{r}_l$ are mutually independent, (44) can be rewritten as

$$N_R(r) = \sum_{l=1}^{L} \left(\prod_{\substack{j=1\\ j\neq l}}^{L} \int_0^{2\pi} p(\theta_{lj}) d\theta_{lj} \right) p_{R_l}(r) \times \left(\prod_{\substack{j=1\\ j\neq l}}^{L} \int_0^{r} p_{R_l}(r_l) dr_l \right) \int_0^{\infty} \dot{r}_l p_{\dot{R}_l}(\dot{r}_l \,|\, r_l = r) d\dot{r}_l = \sum_{l=1}^{L} p_{R_l}(r) \left[\prod_{\substack{j=1\\ j\neq l}}^{L} F_{R_l}(r) \right] n_l \quad (45)$$

where

$$n_l = \int_0^{\infty} \dot{r}_l p_{\dot{R}_l}(\dot{r}_l \,|\, r_l = r) d\dot{r}_l = \frac{\hat{\sigma}_{r_l}}{\sqrt{2\pi}} = \sigma_l \sqrt{\pi} f_m. \quad (46)$$

For a diversity Nakagami channel with arbitrary parameters, (45) reverts to (17).

APPENDIX B

We outline the steps taken to obtain (38) and (40). After substitution for the Nakagami PDF, (37) becomes

$$p_R(r) = \frac{4\sqrt{2}}{[\Gamma(m)]^2} \left(\frac{m}{\Omega}\right)^{2m} e^{-\frac{m}{\Omega} r^2} \times \int_0^{\sqrt{2}r} \left[r_1(\sqrt{2}r - r_1) \right]^{2m-1} \times e^{-\frac{m}{\Omega}(\sqrt{2}r_1 - r)^2} dr_1. \quad (47)$$

The variable transformation $y = (m/\Omega)(\sqrt{2}r_1 - r)$ in (47) gives

$$p_R(r) = \frac{2(\frac{m}{\Omega})^{\frac{1}{2}}}{[\Gamma(m)]^2 2^{2m-2}} \times \int_{-\frac{m}{\Omega}r^2}^{\frac{m}{\Omega}r^2} y^{-\frac{1}{2}} \left[\frac{m}{\Omega} r^2 - y \right]^{2m-1} e^{-y} dy. \quad (48)$$

Using [26, eq. (3.383.1)] then leads to the closed-form solution (38).

The CDF is given by

$$F_R(r) = \left(\frac{m}{\Omega}\right)^{2m} \frac{2B\left(2m, \frac{1}{2}\right)}{[\Gamma(m)]^2 2^{2m-2}} \times \int_0^r \alpha^{4m-1} e^{-2\frac{m}{\Omega}\alpha^2} \Phi\left(2m, 2m + \frac{1}{2}, \frac{m}{\Omega}\alpha^2\right) d\alpha. \quad (49)$$

Making the change of variable $x = 2(m/\Omega)\alpha^2$, using the infinite series expansion

$$\Phi(a, c, x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)}{\Gamma(c+k)} \frac{x^k}{k!} \quad (50)$$

and the relation $B(x, y) = (\Gamma(x)\Gamma(y)/\Gamma(x+y))$ [26, eq. (8.384.1)] in (49) results in

$$F_R(r) = \frac{1}{[\Gamma(m)]^2 2^{4m-2}} \sum_{n=0}^{\infty} \frac{\Gamma(n+2m)}{\Gamma\left(n + 2m + \frac{1}{2}\right) 2^n n!} \times \int_0^{2\frac{m}{\Omega}r^2} e^{-x} x^{2m+n-1} dx. \quad (51)$$

Applying [26, eq. (8.350.1)] to the integral above leads to the desired representation (40).

REFERENCES

[1] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[2] H. S. Wang and N. Moayeri, "Finite-state Markov channel-a useful model for radio communication channels," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 163–171, Feb. 1995.

[3] R. Vijayan and J. M. Holtzman, "Foundations for level crossing analysis of handoff algorithms," in *Proc. ICC*, May 1993, pp. 935–939.

[4] J. Lai and N. B. Mandayam, "Packet error rate for burst-error-correcting codes in Rayleigh fading channels," *Proc. IEEE VTC*, pp. 1568–1572, 1998.

[5] S. O. Rice, "Statistical properties of a sine wave plus random noise," *Bell Sys. Tech. J.*, vol. 27, pp. 109–157, Jan. 1948.

[6] W. C. Y. Lee, "Level crossing rates of an equal-gain predetection diversity combiner," *IEEE Trans. Commun. Technol.*, vol. COM-18, pp. 417–426, Aug. 1970.

[7] F. Adachi, M. T. Feeney, and J. D. Parsons, "Effects of correlated fading on level crossing rates and average fade durations with predetection diversity reception" (in Part F), *Proc. Inst. Elect. Eng.*, vol. 135, pp. 11–17, Feb. 1988.

[8] M. D. Yacoub, C. R. C. M. da Silva, and J. E. Vargas Bautista, "Second-order statistics for equal gain and maximal ratio diversity-combining reception," *Electron. Lett.*, vol. 36, no. 4, pp. 382–384, Feb. 17, 2000.

[9] M. Nakagami, "The m-distribution—A general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, W. C. Hoffman, Ed. New York: Pergamon, 1960, pp. 3–36.

[10] W. R. Braun and U. Dersch, "A physical mobile radio channel model," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 472–482, May 1991.

[11] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. Commun.*, vol. COM-25, pp. 673–680, July 1977.

[12] T. Aulin, "Characteristics of a digital mobile radio channel," *IEEE Trans. Veh. Technol.*, vol. VT-30, pp. 45–53, May 1981.

[13] L. Rubio, N. Cardona, S. Flores, J. Reig, and L. Juan-Llácer, "The use of semi-deterministic propagation models for the prediction of the short-term fading statistics in mobile channels," *Proc. IEEE VTC*, pp. 1460–1464, 1999.

[14] T. Zwick, C. Fischer, D. Didascalou, and W. Wiesbeck, "A stochastic spatial channel model based on wave-propagation modeling," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 6–15, Jan. 2000.

[15] A. Gelonch, F. J. Casadevall, J. J. Olmos, and G. Femenias, "Design and implementation of a wide-band real-time mobile channel emulator," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 746–764, May 1999.

[16] T. Eng and L. B. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," *IEEE Trans. Commun.*, vol. 43, pp. 1134–1143, Feb.-Apr. 1995.

[17] C. Tellambura and A. D. S. Jayalath, "Generation of bivariate Rayleigh and Nakagami-m fading envelopes," *IEEE Commun. Lett.*, vol. 4, pp. 170–172, May 2000.

[18] S. D. Gordon and J. A. Ritcey, "Generating correlated Nakagami fading channels," in *Conf. Rec. 13th Asilomar Conf. on Signals, Systems and Computers*, Nov. 1997, pp. 684–688.

[19] Q. T. Zhang, "A decomposition technique for efficient generation of correlated Nakagami fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 2385–2392, Nov. 2000.

[20] N. Youssef, T. Munakata, and M. Takeda, "Fade statistics in Nakagami fading environments," in *Proc. IEEE 4th Int. Symp. on Spread Spectrum Techniques and Applications*, Mainz, Germany, Sept. 1996, pp. 1244–1247.

[21] R. Barakat, "Level crossing statistics of aperture-integrated isotropic speckle," *J. Opt. Soc. Amer. A.*, vol. 5, no. 8, pp. 1244–1247, 1988.

[22] M. D. Yacoub, J. E. Vargas Bautista, and L. G. de R. Guedes, "On higher order statistics of the Nakagami-m distribution," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 790–794, May 1999.

[23] C. D. Iskander and P. T. Mathiopoulos. Finite-state Markov modeling of diversity Nakagami channels. presented at Seventh Canadian Workshop on Info. Theory, Vancouver, BC, Canada, June 3–6, 2001. [Online]. Available: http://www.cce.com/cwit/

[24] M. D. Yacoub, M. V. Barbin, M. S. de Castro, and J. E. Vargas Bautista, "Level crossing rate of Nakagami-m fading signal: Field trials and validation," *Electron. Lett.*, vol. 36, no. 4, pp. 355–357, Feb. 17, 2000.

[25] A. Abdi, K. Wills, H. Allen Barger, M.-S. Alouini, and M. Kaveh, "Comparison of the level crossing rate and average fade duration of Rayleigh, Rice and Nakagami fading models with mobile channel data," *Proc. IEEE VTC*, pp. 1850–1857, 2000.

[26] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic, 2000.

[27] J. Lu, K. B. Letaief, and M. L. Liou, "Robust video transmission over correlated mobile fading channels," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 9, pp. 737–751, Aug. 1999.

[28] D. G. Brennan, "Linear diversity combining techniques," *Proc. IRE*, vol. 47, pp. 1075–1102, June 1959.
[29] J. Lai and N. B. Mandayam, "Minimum duration outages in Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 49, pp. 1755–1761, Oct. 2001.
[30] K. Witrisal, Y.-H. Kim, and R. Prasad, "A new method to measure parameters of frequency-selective radio channels using power measurements," *IEEE Trans. Commun.*, vol. 49, pp. 1788–1800, Oct. 2001.
[31] X. Dong and N. C. Beaulieu, "Average level crossing rate and average fade duration of selection diversity," *IEEE Commun. Lett.*, vol. 5, pp. 396–398, Oct. 2001.
[32] M. D. Yacoub, C. R. C. M. da Silva, and J. E. Vargas Bautista, "Second-order statistics for diversity-combining techniques in Nakagami-fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, pp. 1464–1470, Nov. 2001.

Cyril-Daniel Iskander (S'99) was born in Toronto, ON, Canada, in 1977. He received the B.S. and M.Sc. degrees in electrical engineering from Université Laval, Québec City, PQ, Canada, in 1998 and 1999, respectively. Since 1999, he has been working toward the Ph.D. degree in electrical engineering at the University of British Columbia, Vancouver, BC, Canada.

His current research topic is video communications over mobile wideband CDMA systems. Other research interests include channel modeling and simulation, adaptive equalization and neural networks.

Mr. Iskander is the recipient of Natural Sciences and Engineering Research Council of Canada (NSERC) PGSA and PGSB, Science Council of British Columbia (SCBC) GREAT, and Industry Canada Fessenden scholarships.

P. Takis Mathiopoulos (S'80–M'89–SM'94) was born in Athens, Greece. He graduated from the German High School of Athens (Dorpfel Gymnasium) in 1974, and received the Diploma from the University of Patras, Patras, Greece, in 1979, and the M.Eng. degree from Carleton University, Ottawa, ON, Canada, in 1989, all in electrical engineering.

From 1981 to 1983, he was with Raytheon Canada Ltd., where he was involved with the analysis, design, implementation, and evaluation of a new generation of air-navigational equipment (DVOR and DME). From 1983 to 1988, he was with the Department of Electrical Engineering, University of Ottawa, Ottawa, ON, Canada, where he served as a Research Engineer and twice as a Sessional Lecturer. In 1989, he joined the Department of Electrical Engineering, University of British Colombia, Vancouver, BC, Canada, where since 1994, he has held the position of Associate Professor. His research interests include RF and microwave digital telecommunications, with recent emphasis on terrestrial and satellite-based wireless personal communications systems and networks. He has co-authored more than 25 IEEE journal papers, and more than 50 papers published in various conference proceedings.

Since 1993, Mr. Mathiopoulos has been the Editor of Wireless Personal Communications for the IEEE TRANSACTIONS ON COMMUNICATIONS. He has been a member of the Technical Program Committee for more than 40 international conferences. From 1992 to 1995, he was an appointed Fellow of the Advanced Systems Institute, and from 1996 to 1997, he was a Killam Research Fellow. He is a Registered Professional Engineer in Greece and the Province of British Colombia, Canada.