

Directional emphasis in ambisonics

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Abstract—We describe an ambisonics enhancement method that increases the signal strength in specified directions at low computational cost. The method can be used in a static setup to emphasize the signal arriving from a particular direction or set of directions. It can also be used in an adaptive arrangement where it sharpens directionality and reduces the distortion in timbre associated with low-degree ambisonics representations. The emphasis operator has very low computational complexity and can be applied to time-domain as well as time-frequency ambisonics representations. The operator upscales a low-degree ambisonics representation to a higher degree representation.

Index Terms—Ambisonics, emphasis, directionality.

I. INTRODUCTION

Ambisonics [1]–[5] is a representation for sound fields that can take the form of a series of countably infinite spatial basis functions, each multiplied by a temporal scalar audio signal. Signal acquisition and rate (storage) constraints lead to truncation of the series, limiting the *degree* of the ambisonics description. A low ambisonics degree bounds the frequency-dependent radius within which the sound field is described accurately and may make the soundfield unnatural outside this region. This can lead to audible artefacts in a signal rendered from the low-degree description, e.g., [3]–[11]. We describe an approach that addresses these issues and additionally facilitates directional emphasis for other purposes.

The artefacts of low-degree ambisonics can be explained as follows [11]. Conventional rendering uses the Moore-Penrose inverse to map the ambisonics representation to loudspeaker signals. Hence it minimizes the energy produced by the virtual or physical loudspeakers subject to the known ambisonics coefficients being correct. The constraint means the sound field is accurate in a *sweet* zone around the origin. The imposed energy efficiency requires the loudspeaker contributions to the sound-field to add coherently in the sweet zone, but not outside. As the radius of the zone is frequency dependent, a low-pass timbre is heard by a listener at the origin. More-over, energy minimization implies the distribution of the acoustic energy over many loudspeakers, reducing the directionality of the sound field outside the sweet zone.

We define our objective more carefully. Consider a sound field in a source-free region around the origin of our coordinate system. The sound field in this region can be generated by a continuous density of monopole sound sources located on a 2-sphere centered at the origin, e.g., [12]. The temporal source signals of the monopole sources form a two-dimensional (2D) scalar *source field* on the 2-sphere that forms an alternative specification of the sound field. The strengthening of the directionality of the sound field can then be defined as the

emphasizing of the source field on the 2-sphere. The emphasis operator can be interpreted as an *acoustic spotlight*.

We aim to implement the emphasis operator directly in the ambisonics representation. Our objective is a low-complexity operator that applies to both time domain and time-frequency domain representations. In addition to the standard goal of *adaptive emphasis* of the sound field (strengthening existing directionality), a secondary goal is *static emphasis* of the sound field (time-invariant emphasis operator).

We are not aware of existing systems that provide static emphasis (time-invariant emphasis) of an ambisonics representation. Adaptive emphasis operators can be classified into two classes. The first class does not change the ambisonics degree. The common $\max r_E$ emphasis operator [3], [4] minimizes sidelobes resulting from the truncation to a low degree representation [6]. The second class upscales the low-degree ambisonics information into a high-degree ambisonics representation [8], [9], [11]. Only the second class facilitates *idempotency*: the analysis of the rendered sound field returns the original ambisonics description. In addition to the fore-mentioned classes, methods exist that are an integral component of rendering, e.g., [7], [13], usually restricted to mapping the ambisonics representation into one or two plane waves [5].

Our contribution is an emphasis operator that has two advantages compared to the state-of-the-art operators for idempotent rendering [8], [9], [11]. First, it can be used in both static and adaptive emphasis applications (existing methods are aimed at adaptive emphasis). Second, our operator, which is based on Clebsch-Gordan coefficients, has low computational complexity. It can be used in the time domain and it raises the ambisonics degree with a matrix multiplication requiring only a handful of multiplies per output sample. To ensure idempotent adaptive rendering, the operator can incorporate a projection [11] without added computational cost.

II. THEORY

This section first describes the source field in section II-A, then defines the emphasis operator in section II-B and methods to compute it in section II-C. The discussion is for complex spherical harmonics but extends to the real case. Similarly to, e.g., [14], we write the spherical harmonics as

$$Y_n^m(\theta, \phi) = (-1)^m \sqrt{\frac{(2n+1)(n-|m|)!}{4\pi(n+|m|)!}} P_n^{|m|}(\cos(\theta)) e^{im\phi}, \quad (1)$$

where the P_n^m are the associated Legendre functions, $\theta \in [0, \pi]$ is elevation and $\phi \in [-\pi, \pi]$ is the azimuth. The $Y_n^m(\cdot, \cdot)$ are orthonormal on the unit 2-sphere and use the Condon-Shortly phase convention. The form of (1) implies that $Y_n^{m*}(\theta, \phi) = Y_n^{-m}(\theta, \phi)$, simplifying derivations.

A. Relating the 3D Sound Field and the 2D Source Field

Our aim in this subsection is to provide the background for deriving an emphasis operator in section II-B. While it is not obvious how to define an emphasis of the sound field directly, it is clear that such an emphasis corresponds to a sharpening of the source field on the 2-sphere defined in section I.

We follow an approach used earlier in [12] and [15]. While the approach is illustrated in the frequency domain, the same reasoning holds in the time domain. We consider an internal sound field expansion of the form

$$p(r, \theta, \phi, k) = \sum_{n=0}^{\infty} \sum_{m=-n}^n B_n^m(k) j_n(kr) Y_n^m(\theta, \phi), \quad (2)$$

where $p(\cdot)$ is pressure, r is radius, $j_n(\cdot)$ is the spherical Bessel function, B_n^m are the ambisonics coefficients and $k = \frac{\omega}{c}$ is the wavenumber (ω is angular frequency and c is soundspeed).

Let us assume the sound field to be generated by the source field $\mu(\theta, \phi, k)$ on a sphere of radius r' :

$$p(r, \theta, \phi, k) = \int_{d\Omega} \mu(\theta', \phi', k) G(x, x', k) \sin(\theta') r'^2 d\theta' d\phi', \quad (3)$$

where $G(x, x', k)$ is a Green's function and $x = (r, \theta, \phi)$.

The Green's function $G(x, x', k)$ can be written as

$$\begin{aligned} G(x, x', k) &= \frac{e^{-jk\|x-x'\|}}{4\pi\|x-x'\|} \\ &= \sum_{n=0}^{\infty} \sum_{m=-n}^n (-j) k h_n^{(2)}(kr') j_n(kr) Y_n^{-m}(\theta', \phi') Y_n^m(\theta, \phi) \\ &\quad \text{for } r' \geq r \end{aligned} \quad (4)$$

where $h_n^{(2)}$ is the spherical Hankel function of the second kind.

Let us define the source field at radius r' by a discrete sequence of spherical harmonics coefficients:

$$\mu(\theta', \phi', k) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \gamma_n^m(k) Y_n^m(\theta', \phi'). \quad (5)$$

Integrating $\mu(\theta', \phi', k) G(x, x', k)$ over the 2-sphere of radius r' , using orthogonality of the spherical harmonics, we obtain an expression for $p(r, \theta, \phi, k)$ in terms of $\gamma_n^m(k)$ that facilitates mode matching. This relates the sound field (2) with the source field on the 2-sphere (5):

$$\gamma_n^m(k) = B_n^m(k) r' j^{-n} e^{jkr'}, \quad kr' \rightarrow \infty \quad (6)$$

where we used the asymptotic behavior of $h_n^{(2)}$ [14], [16]: $\lim_{kr \rightarrow \infty} h_n^{(2)}(kr) = j^{(n+1)} \frac{e^{-jkr}}{kr}$.

(6) is the main result of this section. It shows that emphasizing the source field on the sphere does *not* correspond to a straight emphasizing of the sound field $p(r, \theta, \phi, k)$.

The source field (5) in the frequency domain is

$$\begin{aligned} \mu(\theta, \phi, k) &= r' e^{jkr'} \sum_{n=0}^{\infty} \sum_{m=-n}^n g_n B_n^m(k) Y_n^m(\theta, \phi), \\ &\quad kr' \rightarrow \infty, \end{aligned} \quad (7)$$

where we defined, for later convenience, $g_n = (-j)^n$. Except for a radius-dependent scaling, the vector g_n provides the

mapping from the ambisonics coefficients to the spherical harmonics representation of the source field.

As we also aim to derive time-domain emphasis operators, we note that by applying the inverse Fourier transform $\frac{1}{2\pi} \int \cdot e^{j\omega t} d\omega$ (7) can also be written in the time domain.

B. Emphasizing the Angular Dependency of a Signal

Our objective in this section is to enable us to emphasize a particular direction with low computational complexity. That is, we aim to emphasize (“sharpen”) the angular dependency of the source field μ associated with the sound field. To reduce computational requirements we aim to find expressions for the ambisonics coefficients $B_n^m(\cdot)$ of the sharpened sound field without explicitly calculating the source field μ .

Our emphasis operator uses two time scales. The first time scale resolves the temporal behavior of the monaural source signals and is characterized by their bandwidth. The second time scale captures the rate of change of the parameters of the emphasis operator. For a time-invariant emphasis in a particular direction, these parameters do not change in time. More commonly, the second time scale resolves the changes in the spatial arrangement and loudness of the sound sources. Frequency domain implementations in practice use time-frequency transforms. Hence both frequency and time domain implementations can accommodate time-dependencies on the second time scale.

We first define the emphasis operator for the source field $\mu(\theta, \phi, k)$. We start with a suitable function $v(\theta, \phi, k) : [0, \pi] \times [0, 2\pi] \rightarrow [0, \infty)$ that is real and, ideally, nonnegative and can be used to emphasize the source field $\mu(\cdot, \cdot, k)$ over the 2-sphere. The emphasis operation is then

$$\tilde{\mu}(\theta, \phi, k) = v(\theta, \phi, k) \mu(\theta, \phi, k). \quad (8)$$

For the pressure p , the emphasis operation is not a multiplication. We define ν as a general emphasis operator that also applies to pressure and is the multiplication with v (8) in the source-field domain.

We simplify our notation by introducing a single index for the spherical harmonics and omitting function arguments where that is not ambiguous. Let \tilde{Q} be the degree of the ambisonics expansion for μ . We define $Q = (\tilde{Q} + 1)^2$. Assuming that the source field μ is of finite degree we have

$$\mu = \sum_{q=0}^{Q-1} \gamma_q Y_q. \quad (9)$$

We choose v to be of degree \tilde{L} and define $L = (\tilde{L} + 1)^2$:

$$v = \sum_{l=0}^{L-1} V_l Y_l. \quad (10)$$

Note that the finite degree \tilde{L} prevents strict nonnegativity.

Exploiting that the spherical harmonics form a basis of the 2-sphere, we can write each multiplication of pairs of spherical harmonics as a weighted sum of spherical harmonics. Let $Y^{(Q)}(\theta, \phi)$ be the Q -dimensional column vector $[Y_0(\theta, \phi), Y_1(\theta, \phi), \dots, Y_{Q-1}(\theta, \phi)]^T$. Let us denote the Kronecker product with \otimes . We furthermore use that the

multiplications of two spherical harmonics of degree L and Q can be written as a weighted sum of spherical harmonics with degree less or equal to $Q + L$. Thus, we can write

$$Y^{(Q)} \otimes Y^{(L)} = CY^{(P)}, \quad (11)$$

where $C \in \mathbb{R}^{QL \times P}$ with $\tilde{P} = \tilde{Q} + \tilde{L}$ and $P = (\tilde{P} + 1)^2$ is a real, non-square matrix with Clebsch-Gordan coefficients as elements. The matrix C depends only on the degree of the ambisonics representation and on the degree of the emphasis operator v . Hence it can generally be computed off-line.

The standard formula for the multiplication of spherical harmonics shows that the matrix C is sparse and this can be exploited. However, as will be shown below, for static or slowly varying emphasis (static or slowly varying emphasis operator), optimal computational efficiency can be obtained without consideration of the sparsity of C .

We can use standard formulas for the Clebsch-Gordan coefficients to compute C . However, given that relation (11) exists, we can use it to compute the matrix of Clebsch-Gordan coefficients (C) by creating a set of linear equations corresponding to a set of random (or selected) angles.

The emphasized source field $\tilde{\mu}$ can be written in terms of the spherical harmonics expansions for μ and v :

$$\tilde{\mu} = v\mu = v\mu = \sum_{q=0}^{Q-1} \sum_{l=0}^{L-1} \gamma_q V_l Y_q Y_l \quad (12)$$

$$= \sum_{i=0}^{P-1} Y_i (C^T (\gamma^{(Q)} \otimes V^{(L)}))_i, \quad (13)$$

where we used (11). We now have the ambisonics expansion of $\tilde{\mu}$ in terms of ambisonics expansions for μ and v .

Let \circ be the Hadamard (element-wise) product and $g^{(P)} = [g_{n(0)}, \dots, g_{n(P-1)}]^T$, where with some abuse of notation, $n(i) = \lfloor \sqrt{i} \rfloor$ is the degree n in $Y_n^k = Y_i$. It then follows from (6) that (13) implies that the emphasis operator for the ambisonics coefficients of a pressure field satisfies

$$g^{(P)} \circ \tilde{B}^{(P)} = C^T \left((g^{(Q)} \circ B^{(Q)}) \otimes V^{(L)} \right), \quad (14)$$

which specifies the degree-expanded ambisonics representation of the sound field (2) after emphasis.

Next, we discuss the efficient computation of (14). We will show that if the emphasis operator V is time-invariant, then (14) can be computed with one $P \times Q$ matrix multiply per sample, requiring PQ multiplies to compute all output channels. Thus, for a degree-1 ambisonics representation, $\tilde{Q} = 1$, and a degree-2 emphasis operator, $\tilde{L} = 2$, only four multiplies per output channel are required.

One approach to obtaining high computational efficiency for computing (14) is to exploit that C and V are fixed or slowly varying. Let $\mathbf{1}^{(Q)} = [1, \dots, 1]^T$ be a Q -dimensional vector of ones. Some algebra leads to

$$g^{(P)} \circ \tilde{B}^{(P)} = \bar{C}^T \left((g^{(Q)} \circ B^{(Q)}) \otimes \mathbf{1}^{(L)} \right), \quad (15)$$

where we wrote $\bar{C}^T = C^T \circ (\mathbf{1}^{(P)}(V^{(L)} \otimes \mathbf{1}^{(Q)})^T)$, which is a matrix that retains the dimensionality of C^T . Finally we note that we can define a matrix $A^{(QL)} = I^{(Q)} \otimes \mathbf{1}^{(L)}$, where

$I^{(Q)}$ is the identity matrix with Q rows and columns, such that $B^{(Q)} \otimes \mathbf{1}^{(L)} = A^{(LQ)} B^{(Q)}$. Thus, we can write

$$\tilde{B}^{(P)} = \left(\text{diag}^{-1}(g^{(P)}) \bar{C}^T A^{(LQ)} \text{diag}(g^{(Q)}) \right) B^{(Q)}. \quad (16)$$

As $(\text{diag}^{-1}(g^{(P)}) \bar{C}^T A^{(LQ)} \text{diag}(g^{(Q)})) \in \mathbb{C}^{P \times Q}$ is a time-invariant matrix for a fixed emphasis operator and slowly time-varying for an adaptive emphasis operator, it be computed off-line or at a slow update rate. Thus, we have proven that PQ multiplies for each sample suffice for the emphasis operator.

The usage of (14) without further modification is also relevant, as it facilitates rapid adaptation of the emphasis operator v . In this second approach, the emphasis operator is composed of two components: *i*) a Kronecker product operation, which is an unrolled outer product of a $Q \times 1$ signal vector and an $L \times 1$ emphasis vector, followed by *ii*) a $P \times QL$ matrix multiply. While the size of the matrix C^T is larger than that of $\bar{C}^T A^{(LQ)}$ in (16), it is a sparse matrix. From the explicit formula for the Clebsch-Gordan series for product of two spherical harmonics formulas it follows that the number of multiplies is also for this case PQ . While the formulation (14) is less conveniently structured, the fact that it has no computational overhead may make it more attractive for scenarios that require rapid updates.

For both approaches discussed, the emphasis operation can be performed in the time domain or in the time-frequency domain. The domains result in different outcomes. The methods apply to real and complex spherical harmonics expansions.

C. An Adaptive Emphasis Operator

The emphasis operator can be used to place an acoustic spotlight on a particular direction, in the ambisonics domain. For a time-invariant and source-independent emphasis the tools defined in section II-B suffice. However, a natural application of the emphasis operator is to emphasize an existing source-field power distribution over directions. This section discusses how to find such an *adaptive* emphasis operator v . In most applications, the required adaptation rate is low making both emphasis approaches of section II-B relevant.

Considering the pressure p as a stochastic process, a design for $v(\theta, \phi, k)$ with the desired emphasis result is:

$$v(\theta, \phi, k) = \beta \mathbb{E}[|\mu(\theta, \phi, k)|^\alpha], \quad (17)$$

where \mathbb{E} is ensemble expectation, β is a normalization and $\alpha > 0$ is a real constant. A time-domain representation can also be used. As the time-domain representation averages over frequencies, the results are not the same.

Even integer values for α result in tractable expressions for (17). We illustrate the case $\alpha = 2$. Emphasis strengths can be varied by repeating the procedure and by using a lower degree ambisonics representation as basis. To evaluate (17) for $\alpha = 2$, we first rewrite the complex conjugate of the source field as

$$\begin{aligned} \mu^*(\theta, \phi, k) &= r' e^{-jkr'} \sum_{n=0}^{\infty} \sum_{m=-n}^n j^n \check{B}_n^m(k) Y_n^m(\theta, \phi), \\ kr' &\rightarrow \infty. \end{aligned} \quad (18)$$

where we used $Y_n^{m*} = Y_n^{-m}$ and defined $\check{B}_n^m(k) = B_n^{-m*}(k)$.

Next, we again simplify the notation and write (7) using a single index and without the function arguments:

$$\mu(\theta, \phi, k) = r' e^{jkr'} \sum_{q=0}^{\infty} g_{n(q)} B_q Y_q, \quad kr' \rightarrow \infty, \quad (19)$$

where, with some abuse of notation, we write n as a function of q . Based on (18) and (19) we can rewrite (17) as

$$v(\theta, \phi, k) = \beta r'^2 \sum_{l=0}^{\infty} \sum_{q=0}^{\infty} g_{n(l)} g_{n(q)}^* E[B_l \check{B}_q] Y_l Y_q. \quad (20)$$

The same form is also obtained for the time-domain case.

We rewrite (20) as an expansion in spherical harmonics rather than products of spherical harmonics. We assume that the original source field μ is a degree- \tilde{Q} source field and use $Q = (\tilde{Q} + 1)^2$. Selecting the normalization $\beta = \frac{\beta'}{r'^2}$:

$$\begin{aligned} \frac{v(\theta, \phi, k)}{\beta'} &= \frac{1}{r'^2} E[|\mu(\theta, \phi, k)|^2] \\ &= \sum_{i=0}^{P-1} Y_i (C^T E[(g^{(Q)} \circ B^{(Q)}) \otimes (g^{(Q)*} \circ \check{B}^{(Q)})])_i \end{aligned} \quad (21)$$

where $P = (2\tilde{Q} + 1)^2$, and C is of the form of (11). (21) provides the adaptive emphasis operator in the desired form of an expansion in the spherical harmonics Y_i .

In a practical application the expectation must be approximated. It is natural to assume ergodicity for the signals and approximate the expectation operator with an averaging over time in each bin of a time-frequency representation and simply over time in a time-domain representation.

The expectation contributes most to the computational effort for (21). However, the averaging can be undersampled to satisfy any computational complexity constraint. The remaining computations in (21) are done at the update rate of the emphasis operator, which typically is low. Hence these remaining computations normally do not play a significant role in the computational complexity of finding v . The sparsity of C can be exploited to minimize computational effort.

In general, the emphasis operator changes the sound field also in the sweet zone where the sound field computed from the unemphasized representation is accurate. In the adaptive case, emphasis in this region is usually undesirable. The problem can be removed by using a projection onto the nearest solution for which the sweet zone is unchanged [11]. Because of the orthogonality of the spherical harmonics, the projection can be implemented by overwriting the low-degree ambisonics coefficients with the corresponding original coefficients and requires no additional computational effort.

III. RESULTS

The aim of this letter is to show that static and adaptive directional emphasis can be implemented at negligible computational complexity in the ambisonics domain. For perceptual experiments that show the benefit of directional emphasis we refer to other work: [8], [9] and in particular [11], which implements adaptive emphasis in the source-field domain.

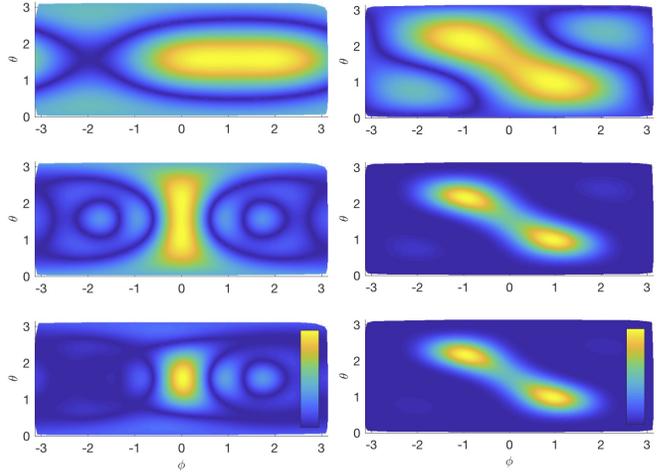


Fig. 1. Mean absolute source field on the 2-sphere for degree-2 signals (top), a degree-4 static emphasis operator (middle left) and an adaptive degree-8 emphasis operator (middle right), and the resulting emphasized signals (bottom). The color bars in the bottom figures show color linearly proportional to distance along the vertical axis, increasing upward.

In this section, we illustrate the operation of the static and adaptive emphasis operator. All computations were performed in the spherical harmonic domain with the methods of section II using complex spherical harmonics. For illustration only, the results were converted to the shown densities on the 2-sphere.

Fig. 1 shows mean source fields and the enhancement operator v (the acoustic emphasis operator) on the 2-sphere for simulated sound fields. On the left is a degree-2 signal enhanced by a static degree-4 ambisonics acoustic emphasis operator. Only the signal highlighted by the emphasis operator is clearly audible. On the right we show the behavior of an adaptive acoustic emphasis operator, for a degree-2 signal enhanced by a degree-8 ambisonics adaptive emphasis operator ($\alpha = 4$). As expected, negative values for v in the source domain were small and away from the high-intensity areas. Their significance reduces further with increasing emphasis operator degree, and increasing emphasis operator smoothness.

IV. CONCLUSION

Practical implementations of ambisonics truncate its series representation of the soundfield because of constraints on estimation and bit rate. For standard rendering, the consequence of the truncation is that the timbre and directionality of the acoustic scenario, as perceived by the listener, are distorted. A strengthening of the directionality of the ambisonics representation can address these problems [8], [9], [11].

We have shown that it is possible to define an emphasis operator that strengthens the directionality of the sound field at negligible computational cost by using Clebsch-Gordan coefficients. In contrast to existing idempotent methods [8], [9], [11], the procedure is attractive for real-time implementation and is particularly suitable for rendering over headsets. Moreover it facilitates a static emphasis.

The new method can be applied to time domain or time-frequency domain ambisonics representations. It can be used for representations based on real and complex spherical harmonics (only the latter was illustrated).

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