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SRTTU

Journal of Electrical and Computer Engineering Innovations

JECEI, Vol. 4, No. 1, 2016

Regular Paper

Passivity-Based Stability Analysis and Robust Practical Stabilization of Nonlinear Affine Systems with Non-vanishing Perturbations

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ARTICLE INFO

ARTICLE HISTORY:
Received 8 August 2016
Revised 10 October 2016
Accepted 10 October 2016

KEYWORDS:
Robust passivity-based Control
Practical stability
Non-vanishing perturbations

ABSTRACT

This paper presents some analyses about the robust practical stability of a class of nonlinear affine systems in the presence of non-vanishing perturbations based on the passivity concept. The given analyses confirm the robust passivity property of the perturbed nonlinear systems in a certain region. Moreover, robust control laws are designed to guarantee the practical stability of the perturbed systems. For this purpose, the control laws are designed in two cases. In the first case, it is assumed that the designer has freedom in choosing the outputs. In the second case, it is assumed that the outputs are predefined. In this case, first it is considered that the nominal system is passive between its inputs and outputs and then the control law is designed as static output feedback law for the perturbed system. Moreover, in the case that the nominal system is not passive, first, a law is designed such that the new nominal system is passive between the virtual inputs and the outputs. Then, the virtual input is designed as a static output feedback law such that the proposed controllers guarantee the practical stability of the perturbed system. Finally, the computer simulations are performed to show the efficacy and applicability of the designed controllers.

1. Introduction

Stabilization problems are one of the important issues in many topics of engineering science. Some stabilization methods are based on the energy of the system like Lyapunov methods, passivity-based control approach, L2 stability and etc.

The concept of passivity provides a good tool for stability analysis of nonlinear systems. A passive system is defined as a system that its internal stored energy is less than the external energy that injected to it. The definition of passive systems is introduced in [1]. In this reference, some concepts such as supply rate and storage function were defined. Also, an important lemma that called Kalman-Yacubovitch-Popov (KYP) was proposed for linear systems [1], [2]. After that, some other researchers, worked on passivity property for nonlinear systems. The authors of [3]-[6] extended the KYP Lemma for nonlinear systems. They proposed the necessary and sufficient conditions for nonlinear passive systems. Furthermore, the passivity and zero-state observability properties were used to design a static output feedback law which guarantees the asymptotic stability of the nonlinear passive systems [7].

Since, all of the nonlinear systems are not passive; therefore, it has been shown that there exists a static state feedback law which changes a non-passive system into a passive one under two circumstances (the system should be minimum phase and its relative degree be one) [8]. Additionally, there are many papers which combined the passivity method with the other nonlinear methods in controller design procedure for special applications. For instance, see

[9]-[13].

In most of practical systems there exist external disturbances or model uncertainties which may cause by model reduction, inaccurate modeling or parametric uncertainties. Therefore, in order to guarantee the robust stability of the perturbed systems, design of robust controllers are necessary. For example, the robust passivity-based control of nonlinear systems (which are weakly minimum phase) was proposed in [14]. The authors of [15], proposed a robust KYP lemma for nonlinear systems with vanishing perturbations. Also, they studied the robust passivity property of minimum phase nonlinear systems having the structural uncertainties. In fact, they proposed new conditions for KYP lemma that if any nonlinear system with vanishing perturbation has these conditions then they are robustly passive. Robust PI (Proportional Integral) passivity based control of nonlinear systems was studied in [16]. Additionally; the authors of [17]-[19] proposed passivity-based robust controllers for uncertain nonlinear feedback systems.

In this paper, the robust passivity-based control strategy is proposed for nonlinear systems with non-vanishing perturbations. These systems do not have equilibrium point(s). Therefore, the stability of nonlinear systems with non-vanishing perturbations turns into the practical stability [20]. In this situation, one cannot expect that the state vector tends to zero as $t \to \infty$. The best thing one can be expected is that the state vector be ultimately bounded by a small bound. Consequently, in this paper certain regions are obtained such that the perturbed systems be robustly passive in these areas and then by designing the output feedback control laws, the stability of the perturbed systems will be guaranteed. In the other word, the state variables of the systems are ultimately bounded near the origin. Since in practice, many of the dynamical systems deal with non-vanishing perturbations, analyzing this kind of stability (practical stability) is so important.

In this paper, passivity based stabilization of nonlinear systems with non-vanishing perturbations is studied for different situations and conditions which may happen in the nonlinear dynamical systems. The main contributions of this paper are as follows:

- Some theorems are given and proved for robust practical stability analyses of the nonlinear affine systems in the presence of non-vanishing perturbations based on the passivity concept.
- The robust passivity-based controllers are designed for the perturbed systems for different situations.

Finally, the computer simulations are performed for different numerical and practical systems to verify the theoretical results and also show the effectiveness of the proposed controllers.

2. Preliminaries

In this section, some basic concepts of passive systems are given.

Consider the following nonlinear affine system.

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector and $y \in R^m$ is the output vector of the system. Moreover, the nonlinear continuous vector function $f : R^n \to R^n$ and the matrix function $g : R^n \to R^{n\times m}$ are locally Lipschitz and $h : R^n \to R^m$ is a continuous vector function ($f(0) = h(0) = 0$).

Definition 1 [3], [8]: System (1) is passive, between input u and output y, if there exists a positive semi-definite function $S : R^n \to R$ ($S(0) = 0$), such that

$$\dot{S} \le y^T u . \tag{2}$$

This function is the storage function.

Lemma 1 [8]: The system (1), is passive, between the input u and the output y , if and only if, there exists a positive semi-definite storage function $S : R^n \to R$ ($S(0) = 0$) such that the following conditions are satisfied.

$$L_f S(x) \le 0 \tag{3}$$

$$L_g S(x) = h^T(x) \tag{4}$$

where $L_f S(x) = \left(\partial S(x)/\partial x\right) f(x)$ and also the definition of $L_g S(x)$ is similar to $L_f S(x)$.

Lemma 2 [8]: Consider the affine nonlinear system (1) and suppose that this system is not passive. Also, it is assumed that the relative degree of this system is one and it is minimum phase. In these situations, there exists a static state feedback as follows:

$$u = \Gamma(x) + \mathrm{B}(x)\omega \tag{5}$$

such that the following new system is passive between the virtual input ω and output y where $\Gamma(.)$ and $\mathrm{B}(.)$ are smooth functions defined near the origin.

$$\begin{aligned} \dot{x} &= f(x) + g(x)\Gamma(x) + g(x)\mathrm{B}(x)\omega \\ y &= h(x) \end{aligned} \tag{6}$$

Thus, there exists a positive semi-definite function $S : R^n \to R$ ($S(0) = 0$) such that:

$$\dot{S}(x) \le y^T \omega . \tag{7}$$

The concept of the minimum phase systems and the relative degree of the nonlinear systems are explained in [21].

Definition 2 [20]: Consider the following perturbed nonlinear system.

$$\dot{x} = f(x) + \Delta f(x,t) \tag{8}$$

where $x \in D$ (assuming $D \subset R^n, \{0\} \in D$) is the state vector. The nonlinear functions f is locally Lipschitz and $f(0)=0$. Also, $\Delta f(x,t)$ is an unknown nonlinear function and denotes the perturbation term which it may cause by model reduction, inaccurate modelling, external disturbances or parameter uncertainties that exist in all practical systems. If $\Delta f(0,t)=0$ for all $t \geq 0$, then the origin is still the equilibrium point for the perturbed system (8) and $\Delta f(x,t)$ is called the vanishing perturbation. However, if $\Delta f(0,t) \neq 0$ for all $t \geq 0$, then the origin is not the equilibrium point for the perturbed system (8), and $\Delta f(x,t)$ is called the non-vanishing perturbation. In this case, it cannot be expected that $x(t) \to 0$ as $t \to \infty$. The best thing that can be expected is that $\|x(t)\|$ be ultimately bounded by a small bound and it is the concept of the practical stability.

Theorem 1: Consider the system (8). It is assumed that there exists a continuous differentiable positive definite function $S(x): D \to R^n$ such that:

$$\alpha_1(\|x\|) \leq S(x) \leq \alpha_2(\|x\|) \tag{9}$$

$$L_f S(x) + L_{\Delta f} S(x) \leq -W(x), \ \forall \|x\| \geq \mu > 0 \tag{10}$$

where $\alpha_1(.)$ and $\alpha_2(.)$ are class K functions and also, $W(x)$ is a continuous positive definite function. It is assumed that there exists $r>0$ in order to $B_r \subset D$ (where $B_r = \{x \in R^n : \|x\| \leq r\}$) and also consider that the upper size of μ is as follows:

$$\mu < \alpha_2^{-1}(\alpha_1(r)). \tag{11}$$

Then, for any initial condition that satisfies $\|x(t_0)\| \leq \alpha_2^{-1}(\alpha_1(r))$, there exists a $T \geq 0$ (where depends on $x(t_0)$ and μ) and a class KL function $\psi(.)$ such that the following inequalities are satisfied:

$$\|x(t)\| \leq \psi(\|x(t_0)\|, t-t_0), \ \forall t_0 \leq t \leq t_0+T \tag{12}$$

$$\|x(t)\| \leq \alpha_1^{-1}(\alpha_2(\mu)), \forall t \geq t_0+T \tag{13}$$

In addition, if $D=R^n$ and $\alpha_1(.), \alpha_2(.)$ are class K_∞ functions, then the restrictions on $\|x(t_0)\|$ and the size of the upper bound of μ will be removed.

Proof [20]: It worth noting that, combining (12) and (13) results in:

$$\|x(t)\| \leq \max\{\psi(\|x(t_0)\|, t-t_0), \alpha_1^{-1}(\alpha_2(\mu))\}, \forall t \geq 0 \tag{14}$$

or in the other words, one has:

$$\|x(t)\| \leq [\psi(\|x(t_0)\|, t-t_0)] + [\alpha_1^{-1}(\alpha_2(\mu))], \forall t \geq 0 \tag{15}$$

3. PASSIVITY-BASED ANALYSES FOR PRACTICAL STABILITY

In this section two theorems and two corollaries are given and proved which include the main contributions of this paper. The variations of the theorems which are presented in this section are related to different situations that may happen in the dynamical systems. The considered problem is the practical stabilization of the following nonlinear system in the presence of non-vanishing perturbations.

$$\dot{x} = f(x) + g(x)u + \Delta f(x,t) \tag{16}$$

where $\Delta f(x,t)$ represents the non-vanishing perturbations of the system (16). This vector function is unknown, however, its upper bound (the positive function $\rho(x)$) is known:

$$\|\Delta f(x,t)\| \leq \rho(x). \tag{17}$$

Theorem 2: Consider the perturbed nonlinear system (16). If there exists a continuous differentiable positive definite function $S(x): R^n \to R$ such that:

$$\alpha_1(\|x\|) \leq S(x) \leq \alpha_2(\|x\|) \tag{18}$$

$$L_f S(x) \leq -\alpha_3(\|x\|) \tag{19}$$

where $\alpha_1(.)$ and $\alpha_2(.)$ are defined in theorem 1 and also, $\alpha_3(.)$ is a class K function. Then by choosing $y_v = (L_g S(x))^T$ as the defined output of the system, the perturbed nonlinear system (16) has the robust passivity property between input u and defined output y_v (i.e., $\dot{S}(x) \leq y_v^T u$) in the following region:

$$\|x\| \geq \alpha_3^{-1}(\|\partial S(x)/\partial x\| \rho(x)/\theta) \tag{20}$$

where $0<\theta<1$. Furthermore, the following control law guarantees the robust practical stability of the closed-loop perturbed system (16), in the region (20).

$$u = -\phi(y_v) = -\phi\left((L_g S(x))^T\right) \tag{21}$$

where $\phi(.)$ is a Lipschitz function with $\phi(0)=0$ and for all $y_v \neq 0$, $y_v^T \phi(y_v) > 0$. (In the whole of this paper, the meaning of the function $\phi(.)$ is a function with these features).

Proof: If condition (19) is satisfied, then according to the lemma 1, by choosing $y_v = (L_g S(x))^T$ the following nominal system is passive between the input u and the defined output y_v.

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y_v = (L_g S(x))^T \end{cases} \tag{22}$$

On the other hand, satisfying the condition (19) is equal to the asymptotic stability of the unforced nominal system (16). Now, consider $S(x)$ as the candidate of Lyapunov function for the perturbed nonlinear system (16). Therefore, one has:

$$\dot{S}(x) = L_f S(x) + L_g S(x)u + L_{\Delta f} S(x)\,. \tag{23}$$

Considering (17), (19) and the definition of the defined output $y_v = \left(L_g S(x)\right)^T$, then:

$$\dot{S}(x) \le -\alpha_3\left(\|x\|\right) + y_v^T u + \left\|\frac{\partial S}{\partial x}\right\|\rho. \tag{24}$$

Thus:

$$\dot{S}(x) \le -(1-\theta)\alpha_3\left(\|x\|\right) - \theta\alpha_3\left(\|x\|\right) + y_v^T u + \left\|\frac{\partial S}{\partial x}\right\|\rho \tag{25}$$

where $0<\theta<1$ is a positive constant. If the term $-\theta\alpha_3\left(\|x\|\right)+\|\partial S/\partial x\|\rho$ be negative definite (or in the other words $\|x\| \ge \alpha_3^{-1}\left(\left\|\left(\partial S/\partial x\right)\right\|\rho/\theta\right)$), then:

$$\dot{S}(x) \le -(1-\theta)\alpha_3\left(\|x\|\right) + y_v^T u\,,\ \forall\ \|x\| \ge \alpha_3^{-1}\left(\left\|\frac{\partial S}{\partial x}\right\|\frac{\rho}{\theta}\right) \tag{26}$$

Therefore, the perturbed system (16) is robustly passive, between input u and defined output y_v (i.e., $\dot{S}(x) \le y_v^T u$), in the region (20). By substituting the control law (21) into (26), obtains:

$$\dot{S}(x) \le -\left[(1-\theta)\alpha_3\left(\|x\|\right) + y_v^T \phi(y_v)\right],\ \forall\ \|x\| \ge \alpha_3^{-1}\left(\|\partial S/\partial x\|\frac{\rho}{\theta}\right)$$

Consequently, condition (10) is satisfied with the following positive definite function

$$W(x) = (1-\theta)\alpha_3\left(\|x\|\right) + \left(L_g S(x)\right)\phi\left(\left(L_g S(x)\right)^T\right) \tag{27}$$

and

$$\mu = \alpha_3^{-1}\left(\left\|\left(\partial S(x)/\partial x\right)^T\right\|\left(\rho(x)/\theta\right)\right)$$

Therefore, according to theorem 1, and expressions (14) and (15); the trajectories of the state variables of the closed-loop perturbed system (16) are bounded for all $t \ge 0$ and the robust practical stability of the closed-loop system is guarantees in the region (20). ■

Theorem 3: Consider the perturbed nonlinear system (16). If there exists a continuous differentiable positive definite function $S(x): R^n \to R$ such that:

$$c_1\|x\|^2 \le S(x) \le c_2\|x\|^2 \tag{28}$$

$$L_f S(x) \le -c_3\|x\|^2 \tag{29}$$

$$\left\|\left(\partial S(x)/\partial x\right)^T\right\| \le c_4\|x\| \tag{30}$$

where c_1, c_2, c_3, and c_4 are positive constants ($c_i > 0$ and $i = 1,\ldots,4$). Then by choosing $y_v = \left(L_g S(x)\right)^T$ as the defined output of the system, the perturbed nonlinear system (16) has the passivity property (i.e., $\dot{S}(x) \le y_v^T u$) in the following region:

$$\|x\| \ge \left(c_4\rho(x)/c_3\theta\right) \tag{31}$$

where $0<\theta<1$ is an arbitrary constant. Also, the following control law guarantees the robust practical stability of the system (16), in the region (31).

$$u = -\phi(y_v) = -\phi\left(\left(L_g S(x)\right)^T\right) \tag{32}$$

Proof: If conditions (28)-(30) are satisfied, then according to the lemma 1, by choosing $y_v = \left(L_g S(x)\right)^T$, the following nominal system is passive between the input u and the output y_v.

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y_v = \left(L_g S(x)\right)^T \end{cases} \tag{33}$$

On the other hand, satisfying the conditions (28)-(30) is equal to the exponential stability of the unforced nominal system (16). Now, consider the positive definite function $S(x)$ as the candidate of Lyapunov function for the nominal system (16). Therefore, the time derivation of $S(x)$ is as follows:

$$\dot{S}(x) = L_f S(x) + L_g S(x) + L_{\Delta f} S(x)\,. \tag{34}$$

According to (17), (29), (30) and the definition of the defined output $y_v = \left(L_g S(x)\right)^T$, one has:

$$\dot{S}(x) \le -c_3\|x\|^2 + y_v^T u + c_4\|x\|\rho(x)\,. \tag{35}$$

Consider $0<\theta<1$, then:

$$\begin{aligned}\dot{S}(x) \le& -(1-\theta)c_3\|x\|^2 - \theta c_3\|x\|^2 \\ &+ c_4\|x\|\rho(x) + y_v^T u\,.\end{aligned} \tag{36}$$

It is obvious that if the expression $-\theta c_3\|x\|^2 + c_4\|x\|\rho(x)$ be negative definite (or in the other word $\|x\| \ge \left(c_4\rho/c_3\theta\right)$), then:

$$\dot{S}(x) \le -(1-\theta)c_3\|x\|^2 + y_v^T u\,,\ \forall\|x\| \ge \left(\frac{c_4\rho}{c_3\theta}\right) \tag{37}$$

Therefore, the perturbed system (16) is robustly passive, between input u and defined output y_v (i.e., $\dot{S}(x) \le y_v^T u$), in the region (31). By substituting the control law (32) into (37), obtains:

$$\dot{S}(x) \le -\left[(1-\theta)c_3\|x\|^2 + y_v^T\phi(y_v)\right],\ \forall\|x\| \ge \left(\frac{c_4\rho}{c_3\theta}\right) \tag{38}$$

Consequently, condition (10) is satisfied for the following positive function

$$W(x) = (1-\theta)c_3\|x\|^2 + \left(L_g S(x)\right)\phi\left(\left(L_g S(x)\right)^T\right)$$

and $\mu = \left[c_4\,\rho(x)/c_3\,\theta\right]$. Consequently, the trajectories of the state variables of the closed-loop perturbed system (16) are bounded for all $t \ge 0$ and the robust practical stability of the closed-loop system is guaranteed in the region (31). ■

Remark 1: There is an important assumption in theorems 2 and 3. It was assumed that the nominal unforced nonlinear system (16) (i.e., $\dot{x} = f(x)$) has the asymptotic (or exponential) stability property. This assumption is restrictive. Now, consider that this assumption is not satisfied. In this case, the state feedback law $u=\gamma(x)+\beta(x)v$ can be designed to satisfy these assumptions for the following system. (where $\gamma(.)$ and $\beta(.)$ are smooth functions)

$$\dot{x}=\underbrace{f(x)+g(x)\gamma(x)}_{f_{new}(x)}+\underbrace{g(x)\beta(x)}_{g_{new}(x)}v \tag{39}$$

or

$$\dot{x}=f_{new}(x)+g_{new}(x)v \tag{40}$$

Therefore, theorem 2 (or 3) can be written in the new form by substituting $f_{new}(x)$, $g_{new}(x)$ and virtual control input v instead of $f(x)$, $g(x)$ and control input u, respectively.

Remark 2: In theorems 2 and 3, the outputs of the system were not considered as the previously known terms (the outputs may be freely chosen). Therefore, the defined outputs of the nominal system were selected such that the nominal system is passive between input vector u and output vector y_v. However, in some cases, the outputs of the system are exactly known. Therefore, the practical stability analyses of such perturbed nonlinear systems are considered in the rest of this paper.

Consider the following nonlinear system in the presence of non-vanishing perturbation:

$$\begin{aligned}\dot{x}&=f(x)+g(x)u+\Delta f(x,t)\\ y&=h(x)\end{aligned} \tag{41}$$

where $y\in R^m$ is the output of the system.

Corollary 1: Consider the nonlinear system (41). If there exists a continuous differentiable positive definite function $S(x):R^n\to R$ such that the nominal system (i.e., system (41) with $\Delta f(x,t)=0$) is passive and also:

$$\alpha_1(\|x\|)\le S(x)\le\alpha_2(\|x\|) \tag{42}$$

$$L_f S(x)\le-\alpha_3(\|x\|) \tag{43}$$

where $\alpha_1(.)$ and $\alpha_2(.)$ are defined in theorem 1, and $\alpha_3(.)$ is class K function. Then, the passivity condition ($\dot{S}(x)\le y^T u$) is satisfied for the perturbed nonlinear system (41) in the following region:

$$\|x\|\ge\alpha_3^{-1}\left(\left\|(\partial S/\partial x)\right\|\rho/\theta\right) \tag{44}$$

where $0<\theta<1$. Also, the control law $u=-\phi(y)$ guarantees the practical stability of the closed-loop perturbed system (41) in the region (44).

Proof: Since the nominal system (41) is passive between the input u and the output y, then the conditions of lemma 1 are satisfied. Therefore, the proof is as the same as the proof of Theorem 2, with replacing y_v with $y=h(x)$. ■

Corollary 2: Consider the nonlinear system (41). If there exists a continuous differentiable positive definite function $S(x):R^n\to R$ such that the nominal system is passive and also:

$$c_1\|x\|^2\le S(x)\le c_2\|x\|^2 \tag{45}$$

$$L_f S(x)\le-c_3\|x\|^2 \tag{46}$$

$$\|\partial S(x)/\partial x\|\le c_4\|x\| \tag{47}$$

where c_1, c_2, c_3, and c_4 are positive constants. Then, the passivity condition (i.e., $\dot{S}(x)\le y^T u$) is satisfied for the perturbed nonlinear system (41) in the following region:

$$\|x\|\ge\left(c_4\rho(x)/c_3\theta\right). \tag{48}$$

Moreover, the control law $u=-\phi(y)$ can guarantees the practical stability of the closed-loop perturbed system (41) in the region (48).

Proof: The proof is as the same as the proof of theorem 3, with replacing y_v with $y=h(x)$ ■

Remark 3: Similar to the remark 1, if the unforced nominal system (41) was not asymptotically (or exponentially) stable, then the static state feedback $u=\gamma(x)+\beta(x)v$ can be used to make the unforced nominal system (41) asymptotically (or exponentially) stable, and then the problem changes to design the virtual input v ($v=-\phi(y)$) for the new system.

Remark 4: In corollaries 1 and 2, it is considered that the nominal system (41) is passive between the input u and the output y. However, in fact, this nominal system may not be passive. In this situation, according to the lemma 2, the non-passive system can be converted to a passive form, using a static state feedback $u=\Gamma(x)+\mathrm{B}(x)\omega$ such that the new system is passive between the new virtual input ω and the output y.

4. Computer Simulations

In this section, computer simulations are performed for different examples to show the applicability of the proposed theorems.

A. First Example

Consider the following perturbed system:

$$\begin{aligned}\dot{x}_1&=-x_1+x_2+\delta_1(t)\sin(x_1)\\ \dot{x}_2&=-x_2\left(1+|x_2|^3\right)-x_1+u+\delta_2(t)\sin(t)\end{aligned} \tag{49}$$

This example is in the form of system (16), where $x=\begin{bmatrix}x_1 & x_2\end{bmatrix}^T$ is the state vector. In this system,

$f(x)=\left[-x_1+x_2 \quad -x_1-x_2\left(1+|x_2|^3\right)\right]^T$, $g(x)=\left[0 \quad 1\right]^T$,

and the vector of non-vanishing perturbations is $\Delta f\,(x,t)=\left[\delta_1(t)\sin(x_1) \quad \delta_2(t)\sin(t)\right]^T$.

Suppose that $|\delta_i(t)|\le 1$ for $i=1,2$. Therefore, the assumption (17) (the upper bound of the perturbations) is satisfied as follows:

$$\|\Delta f\,(x,t)\|=\sqrt{\left(\delta_1(t)\sin(x_1)\right)^2+\left(\delta_1(t)\sin(t)\right)^2}\le \underbrace{\sqrt{2}}_{\rho}$$

The following continuous positive definite function $S(x)$ is considered:

$$S(x)=0.5\left(x_1^2+x_2^2\right) \tag{50}$$

where,

$$L_f S(x)=-x_1^2-x_2^2\left(1+|x_2|^3\right)\le-\|x\|^2. \tag{51}$$

Therefore, condition (19) is satisfied for $\alpha_3(\|x\|)=\|x\|^2$.

Choosing the $y_v=\left(L_g S(x)\right)^T=x_2$, the nominal system (49) is passive between the input u and the output y_v. Taking the time derivation of $S(x)$ in the trajectories of the perturbed system (49), leads to:

$$\begin{aligned}\dot{S}(x)&=L_f S+L_g S u+L_{\Delta f} S\\&\le-\|x\|^2+y_v u+\|x\|\sqrt{2}\\&=-(1-\theta)\|x\|^2+y_v u+\|x\|\sqrt{2}-\theta\|x\|^2.\end{aligned} \tag{52}$$

It is obvious that, if $\|x\|\sqrt{2}-\theta\|x\|^2<0$ then $\dot{S}(x)\le y_v u$. In fact, the perturbed system (49) is robustly passive between its input u and the defined output y_v in the region $\|x\|\ge\sqrt{2}/\theta$. If the θ parameter is selected smaller, the obtained region will be larger and the perturbed system (49) is robustly passive in a larger region. Therefore, according to the theorem 2, the control law $u=-\phi(y_v)$ guarantees the practical stability of the closed-loop perturbed system. The function $\phi(y_v)=k\,y_v$ (that k is a positive constant) can be a candidate for the control law. Computer simulations are done for $u=-10y_v$, $\delta_1=1$, $\delta_2=-1$ and $x(0)=\left[3.25 \quad -4.75\right]^T$.

The trajectories of the state variables of the perturbed system (with control law (closed-loop) and without control law (open-loop)) are presented in Figs. 1 and 2. According to these figures it can be seen that the practical stability of the perturbed system (49) is extremely improved by applying the designed control law and the state variables of the closed-loop systems converge with fast speed. Fig. 3 denotes the time response of the control law $u=-10y_v$.

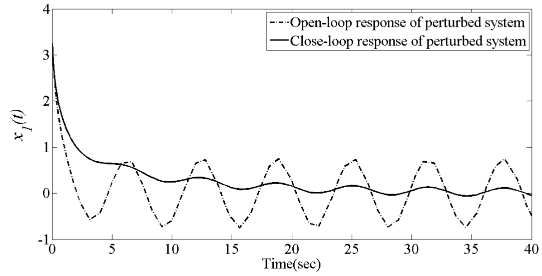

Figure 1: Time history of x_1 for the system (49).

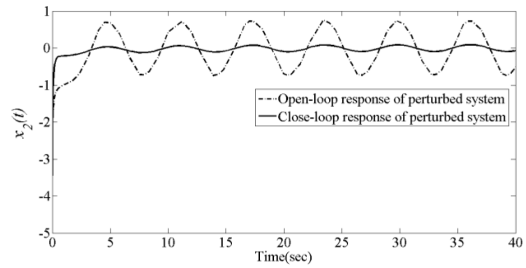

Figure 2: Time history of x_2 for the system (49).

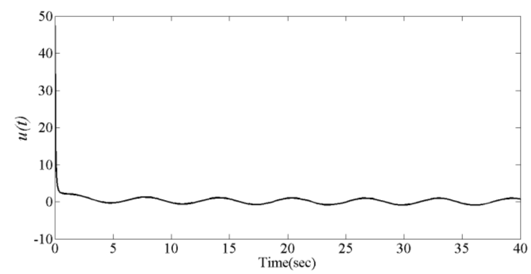

Figure 3: Time history of control input $u=-10y_v$.

B. Second Example

Consider the following system that its output is known (according to the Remark 2) and also the nominal system is not passive (according to the remark 4).

$$\begin{aligned}\dot{x}_1&=x_1\left(x_2-1\right)+d\left(t\right)\\\dot{x}_2&=-x_2+u+\delta_2(t)\sin(x_1)\\y&=x_2\end{aligned} \tag{53}$$

This example is in the form of system (41) with $f(x)=\left[x_1\left(x_2-1\right) \quad -x_2\right]^T$, $g(x)=\left[0 \quad 1\right]^T$, $h=x_2$, $\Delta f(x,t)=\left[d\left(t\right) \quad \delta_2(t)\sin(x_1)\right]^T$ $d(t)=\delta_1(t)\sin(t)\cos(t)$.

Suppose that $|\delta_i(t)|\le 1$ for $i=1,2$. Consequently:

$$\|\Delta f\,(x,t)\|=\sqrt{\left(\delta_1(t)\sin(t)\cos(t)\right)^2+\left(\delta_1(t)\sin(x_1)\right)^2}\le\sqrt{2}\ .$$

Choosing $S(x)=0.5\left(x_1^2+x_2^2\right)$ as the storage function for the nominal system (53), it can be seen that this system is not passive.

$$\dot{S}(x)=x_1^2x_2-\left(x_1^2+x_2^2\right)+yu \tag{54}$$

According to lemma 2, since the nominal system (53) is minimum phase and its relative degree is one, by choosing $\Gamma(x) = -x_1^2$ and $B(x) = 1$, the static state feedback law $u = -x_1^2 + \omega$ can turn the non-passive nominal system (53) into the passive one, between its output y and the new input ω. In other words, by substituting this state feedback law into (54), one obtain that $\dot{S}(x) \le y\omega$. Therefore, the perturbed system (53) can be written in the new form as follows:

$$\dot{x}_1 = x_1\left(x_2 - 1\right) + \delta_1(t)\sin(t)\cos(t)$$
$$\dot{x}_2 = -x_2 - x_1^2 + \omega + \delta_2(t)\sin(x_1) \tag{55}$$
$$y = x_2 .$$

If consider $S(x)$ as the candidate of Lyapunov function for the perturbed system (55), therefore:

$$L_f S(x) = -\left(x_1^2 + x_2^2\right) = -\|x\|^2 . \tag{56}$$

Thus, condition (43) is satisfied for $\alpha_3(\|x\|) = \|x\|^2$. Taking the time derivation of $S(x)$ in the trajectories of the perturbed system (55), leads to:

$$\dot{S}(x) = L_f S + L_g S\,\omega + L_{\Delta f} S$$
$$\le -\|x\|^2 + y\,\omega + \|x\|\sqrt{2} = -(1-\theta)\|x\|^2 + y\,\omega + \|x\|\sqrt{2} - \theta\|x\|^2 .$$

It is obvious that if $\|x\|\sqrt{2} - \theta\|x\|^2 < 0$ then $\dot{S}(x) \le y\,\omega$ for the perturbed system (55). Therefore, according to the corollary 1, the control law $\omega = -\phi(y)$ guarantees the practical stability of the perturbed system (53). Computer simulations are done for, $\phi(y) = 15\,y$, $\delta_1 = 1$ and $\delta_2 = -1$, $x(0) = \begin{bmatrix}-2.3 & 1.8\end{bmatrix}^T$. The trajectories of the state variables of the perturbed system (53) are presented in Fig. 4 and 5. As seen, the practical stability of the perturbed system (53) is improved by applying the designed control law. Fig. 6 denotes the time response of the control law $u = -x_1^2 - 15y$.

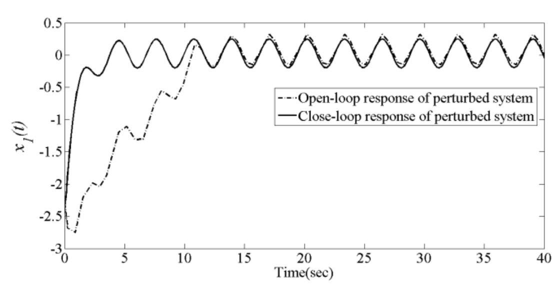

Figure 4: Time history of x_1 for the system (53).

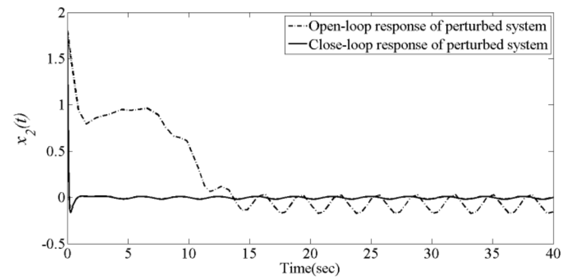

Figure 5: Time history of x_2 for the system (53).

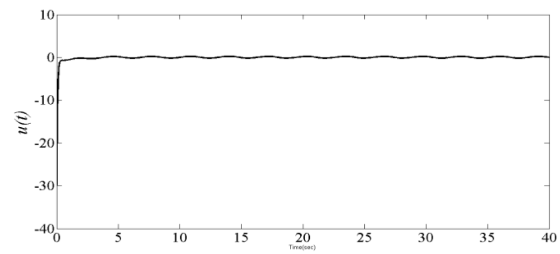

Figure 6: Time history of the control input $u = -x_1^2 - 15y$.

In order to compare the proposed approach with another robust nonlinear controller, a sliding mode method is also applied to the system (53). Here only the control law is presented and the details of calculations are not provided. For more details refer to [20].

Choosing $z = x_2 + 15x_1^2$ as sliding surface where $k > 0$, the sliding mode control law will be obtained as follows:

$$u = x_2 - 2k\,x_1^2\left(x_2 - 1\right) - \left(1 + 2k\,|x_1|\right)sat\left(z\right). \tag{57}$$

Fig. 7 shows the time response of $x_2(t)$. By comparing Fig. 5 with Fig. 7, the effective performance of the proposed controller in practical stabilization of the perturbed systems (in the presence of non-vanishing unmatched perturbations) is clear. As seen the sliding mode controller dose not have the desired performance in face of unmatched non-vanishing uncertainties.

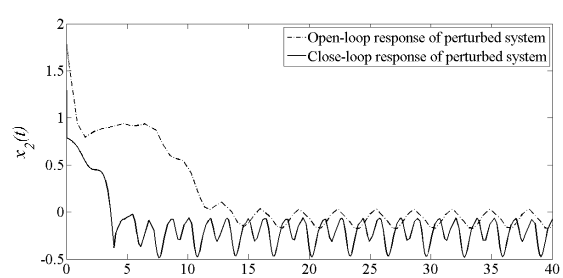

Figure 7: Time history of x_2 for the system (53) with sliding mode controller.

C. Practical Example

This example is concerned on the velocity and body rate stabilization of a spacecraft. The state space equations of these state variables are as follows [22].

$$\underbrace{\begin{bmatrix}\dot v\\ \dot\omega\end{bmatrix}}_{\dot x} = \underbrace{\begin{bmatrix}-\tilde\omega v\\ -J^{-1}\tilde\omega J\omega\end{bmatrix}}_{f} + \underbrace{\begin{bmatrix}(1/m)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & J^{-1}\end{bmatrix}}_{g}(\underbrace{\begin{bmatrix}F\\ T\end{bmatrix}}_{u}+\Delta(v,\omega)) \qquad (58)$$

where $x=[v^T\ w^T]^T$ is the state vector, $v\in R^3$ and $\omega\in R^3$ are velocity and body rates of the spacecraft, respectively. Moreover, $F\in R^3$ includes the elements of control force vector and $T\in R^3$ is a vector of control torque of the spacecraft. The moments of inertia are $J\in R^{3\times3}$, and $\tilde\omega$ is defined as follows:

$$\tilde\omega=\begin{bmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{bmatrix}$$

Also, $\Delta(v,\omega)$ consists of the terms which may cause by inaccurate modeling or model reduction. Consider $S(v,\omega)=0.5v^Tv+0.5\omega^T\omega$ as the storage function for the system (58), the defined outputs of the system are chosen as below:

$$y_v=L_gS=\frac{\partial S}{\partial x}g(x)$$
$$=\begin{bmatrix}\frac{\partial S}{\partial v} & \frac{\partial S}{\partial \omega}\end{bmatrix}.\begin{bmatrix}(1/m)I_{3\times3} & 0_{3\times3}\\ 0_{3\times3} & J^{-1}\end{bmatrix}=\begin{bmatrix}(1/m)v\\ (J^{-1})^T\omega\end{bmatrix}.$$

Simulations are done for $u=\begin{bmatrix}F & T\end{bmatrix}^T=-20y_v$ and

$$J=\begin{bmatrix}3000 & -300 & -500\\ -300 & 3000 & -400\\ -500 & -400 & 3000\end{bmatrix};\quad m=3000kg,\ \|\Delta\|\le 10,$$

$$v(0)=\begin{bmatrix}-0.2 & 0.3 & -0.5\end{bmatrix}^T,\ \omega(0)=\begin{bmatrix}-0.75 & 0.2 & 0.4\end{bmatrix}^T$$

Figs. 8 and 9 show the time-responses of the velocity and body rate vectors, respectively.

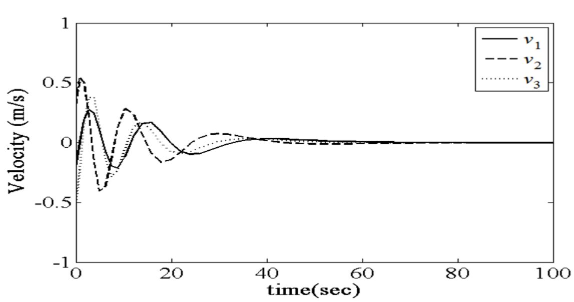

Figure 8: Time history of v for the closed-loop system (58).

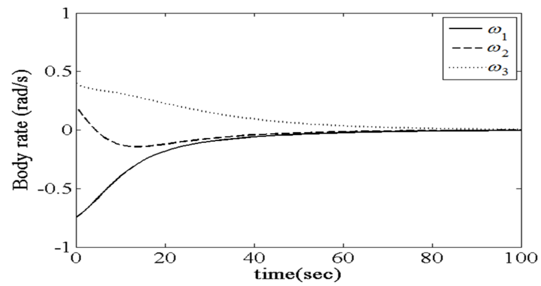

Figure 9: Time history of ω for the closed-loop system (58).

As seen the designed controllers result in the desirable responses in stabilization of v and ω. Furthermore, Figs. 10 and 11 represent the time histories of control forces and control torques, respectively.

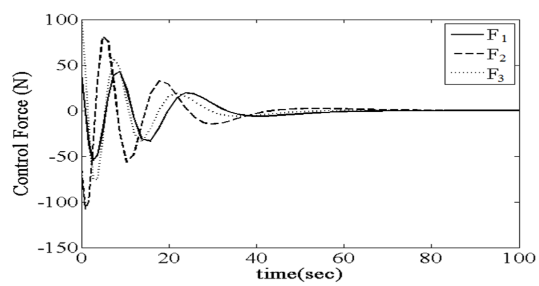

Figure 10: Time history of control forces of the spacecraft.

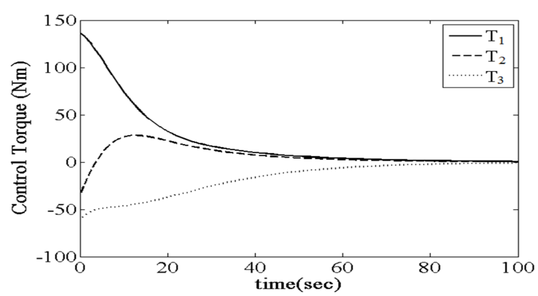

Figure 11: Time history of control torques of the spacecraft.

5. CONCLUSION

In this paper, we have considered the practical stability analyses for the nonlinear perturbed systems with non-vanishing perturbations. Some analyses for robust passivity were performed and some conditions were derived which results in the robust passivity property of the perturbed system in the presence of non-vanishing perturbations in the certain regions. Moreover, robust controllers have been designed to guarantee the practical stability of the closed-loop perturbed systems for different situations. Finally, computer simulations have been carried out and the results illustrate the robust performance of the proposed controllers in practical stabilization and also its applicability for practical systems.

References

[1] J. C. Willems, "Dissipative dynamical systems part I: General theory," *Archive for rational mechanics and analysis*, vol. 45, no. 5, pp. 321–351, 1972.

[2] X. Li, S. Yin, H. Gao, and O. Kaynak, "Robust Static Output-Feedback Control for Uncertain Linear Discrete-Time Systems via the Generalized KYP Lemma," in *World Congress*, 2014, vol. 19, pp. 7430–7435.

[3] D. Hill, P. Moylan, "The stability of nonlinear dissipative systems," *Automatic Control, IEEE Transactions on*, vol. 21, no. 5, pp. 708–711, 1976.

[4] C. King, R. Shorten, "An extension of the KYP-lemma for the design of state-dependent switching systems with uncertainty," *Systems & Control Lett*, vol. 62, no. 8, pp. 626–631, Aug. 2013.

[5] S. You, J. C. Doyle, "A Lagrangian dual approach to the generalized KYP lemma.," in *CDC*, 2013, pp. 2447–2452.

[6] W. Paszke, and E. Rogers, and K. Galkowski, "Experimentally verified generalized KYP lemma based iterative learning control design," *Control Engineering Practice*, Apr. 2016.

[7] C.-C. Tsai, H.-L. Wu, "Passivity, global stabilization and disturbance attenuation of weakly minimum-phase nonlinear uncertain systems with applications to mechatronic systems," in *ICCAS International Conference on Control, Automation and Systems,* 2008, pp. 777–782.

[8] C. I. Byrnes, A. Isidori, and J. C. Willems, "Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems," *IEEE Transactions on Automatic Control,* vol. 36, no. 11, pp. 1228–1240, 1991.

[9] T. Binazadeh, M. J. Yazdanpanah, "Application of passivity based control for partial stabilization," *Nonlinear Dynamics and Systems Theory*, vol. 11, no. 4, pp. 373–382, 2011.

[10] H. Chenarani, T. Binazadeh, "Flexible structure control of unmatched uncertain nonlinear systems via passivity-based sliding mode technique," accepted for publication in *Iranian Journal of Science & Technology, Transactions of Electrical Engineering*.

[11] T. Binazadeh, M. H., Shafiei, "Passivity-based optimal control of discrete-time nonlinear systems," *Control and Cybernetics*, vol. 42, no. 3, pp. 627-637, 2013.

[12] S. Kuntanapreeda, "Adaptive control of fractional-order unified chaotic systems using a passivity-based control approach," *Nonlinear Dynamics*, vol. 84, no. 4, pp. 1–11, 2016.

[13] A. C. Leite, F. Lizarralde, "Passivity-based adaptive 3D visual servoing without depth and image velocity measurements for uncertain robot manipulators," *International Journal of Adaptive Control and Signal Processing*, vol. 30, no. 8-10, pp. 1269-1297, 2016.

[14] C.-C. Tsai, H.-L. Wu, "Robust passivity-based control of weakly minimum phase nonlinear uncertain systems: An application to manipulator," in *Asian Control Conference,* 2009, pp. 919–924.

[15] W. Lin, T. Shen, "Robust passivity and feedback design for minimum-phase nonlinear systems with structural uncertainty," *Automatica*, vol. 35, no. 1, pp. 35–47, 1999.

[16] S. Aranovskiy, R. Ortega, and R. Cisneros, "Robust PI passivity-based control of nonlinear systems: application to port-Hamiltonian systems and temperature regulation," in *American Control Conference (ACC)*, 2015, pp. 434–439.

[17] N. Bu, M. Deng, "Passivity-based robust control for uncertain nonlinear feedback systems," in *International Conference on Advanced Mechatronic Systems (ICAMechS), ,* 2015, pp. 70–74.

[18] A. Donaire, J. Guadalupe Romero, and T. Perez, "Passivity-based trajectory-tracking for marine craft with disturbance rejection," *IFAC-PapersOnLine*, vol. 48, no. 16, pp. 19–24, 2015.

[19] M. S, T. N, "Application of passivity concept for split range control of heat exchanger networks," *Journal of Chemical Engineering & Process Technology*, vol. 07, no. 01, 2015.

[20] H. K. Khalil, J. Grizzle, *Nonlinear Systems*, Prentice Hall, Upper Saddle River, 2002.

[21] N. Kalouptsidis, J. Tsinias, "Stability improvement of nonlinear systems by feedback," *IEEE Transactions on Automatic Control,* vol. 29, no. 4, pp. 364–367, 1984.

[22] D. T. Stansbery, J. R. Cloutier, "Position and attitude control of a spacecraft using the state-dependent Riccati equation technique, "*Proceedings of the American Controls Conference, Chicago, Illinois,* 2000, pp. 1867-1871.

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