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Volt/VAR Optimization: A Survey of Classical and Heuristic Optimization Methods

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ABSTRACT Reactive power optimization and voltage control is one of the most critical components of power system operation, impacting both the economy and security of system operation. It is also one of the most complex optimization problems, being highly nonlinear, and comprising both continuous and discrete decision variables. This paper presents the problem formulation, and a thorough literature review and detailed discussion of the various solution methods that have been applied to the Volt/VAR optimization problem. Each optimization method is described in detail, and its strengths and shortcomings are outlined. The review provides detailed information on classical and heuristic methods that have been applied to the Volt/VAR optimization problem. The classical methods reviewed include (i) first- and second-order gradient-based methods, (ii) Quadratic Programming, (iii) Linear Programming, (iv) Interior-Point Methods, (iv) and mixed-integer programming and decomposition methods. The heuristic methods covered include (i) Genetic Algorithm, (ii) Evolutionary Programming, (iii) Particle Swarm Optimization, (iv) Fuzzy Set Theory, and (v) Expert Systems. A comparative analysis of the key characteristics of the classical and heuristic optimization methods is also presented along with the review.

INDEX TERMS Volt/VAR optimization, reactive power/voltage control, classical/numerical optimization, heuristic methods, artificial intelligence techniques.

I. INTRODUCTION

Optimization of power system operation as a subject of study has quite a long history, enriched over the years by advances in mathematical programming techniques and computational methods, but certainly predating the advent of digital computers which have revolutionized numerical optimization and computation in general. One of the most widely studied power system optimization problems is the Optimal Power Flow (OPF) problem, the first complete formulation of which is generally attributed to Carpentier [1], [2]. The OPF problem seeks to optimize some aspect of power system operation (could be economical, technical, environmental, etc.), while satisfying the physical and operational constraints of the system [3].

Reactive power optimization and voltage control (also known as Volt/VAR Optimization (VVO), or optimal reactive power dispatch (ORPF)) can be considered to be a sub-problem of the OPF problem (or a variant formulation thereof) that is mainly concerned with the determination of the optimal coordinated dispatch of voltage-regulating devices and reactive power sources so as to maintain a secure voltage profile, while also optimizing some aspect of power system operation, subject to physical and operational system constraints [4], [5]. Optimal reactive power dispatch plays a key role in the efficient transfer of real power, especially in the bulk power transmission system, and contributes significantly to the security, reliability, quality and economy of power system operation [6]. The extensive research that has been (and continues to be) conducted in the area of Volt/VAR optimization gives evidence to the continued relevance of research in this aspect of power system operation, particularly in the wake of changes taking place in the electric power system, spurred on by such developments as electric power system deregulation, electric grid modernization under the paradigm of the smart grid, and the rapid growth of renewable and distributed power generation [7]–[13]. Largely progressive as all these developments are, they nonetheless pose a

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significant challenge to the power system operator [14], and hence the growing need for advancements in optimization techniques and computational methods that will support the secure, reliable, and economical operation of the 21st century power system and beyond [15].

Volt/VAR optimization has a number of characteristics that make it a very challenging optimization problem, and much effort has been dedicated over the decades to the study of a variety of problem formulations, as well as the development of solution techniques for the various formulations. Key developments in the treatment of the OPF problem over the years have been presented in a number of review papers, some notable ones being [16]–[23]. Aspects of interest that have been emphasized in these review papers have mainly been the problem formulation, as well solution techniques, considering both the classical/deterministic and the non-deterministic/artificial intelligence-based optimization methods.

A few review papers have focused on solution techniques for Volt/VAR optimization. A review of literature on reactive power planning has been presented in [24]. Reference [25] presents a review of algorithmic and heuristic methods for Volt/VAR control. Reference [26] focuses in their review of Volt/VAR control on reactive power sources and their control devices, as well as discussing a number of solution methods for the Volt/VAR control problem.

Reactive power planning, a problem that is closely related to Volt/VAR optimization (or optimal reactive power dispatch), focuses on optimal investment in new reactive power sources to meet future reactive power compensation needs [24]. The authors of [27]–[31] have carried out extensive work in this area, exploring the application of FACTS devices in reactive power planning [27], [28], and the implementation of heuristic optimization techniques such as Whale optimization, ameliorated Harris Hawk optimization, and hybrid Particle Swarm Optimization (PSO)-Grey Wolf Optimization (GWO) [29]–[31] to reactive power planning. The relevance of effective reactive power planning has become even more pronounced in recent years, due to the need to account for the impact of the growing share of variable renewable generation such as wind and photovoltaic power generation on reactive power compensation. A multi-period, multi-scenario corrective security-constrained OPF has been explored in [32] as a way of dealing with increasing penetration of variable renewable generation. Reference [33] has proposed a probabilistic multi-objective reactive power planning framework that considers large-scale wind generation integration. In [34], the coordination of the reactive power control of large-scale renewable generation with the main grid has been investigated as a way of enhancing the voltage stability of the entire system.

Recognition has continued to increase among utilities and researchers of the role to be played by smart inverters in various forms of grid support. As an example, California Rule 21, which regulates the integration of distributed generation (DG) to the power grid, has implemented an adjustment to the rule that requires the use of advanced (i.e. smart) inverters capable of performing a variety of grid support functions, such as Volt/VAR management [35]. Incorporation of smart inverters in Volt/VAR optimization has been explored as a means of mitigating voltage volatility and voltage fluctuations induced by renewable generation variability [36], [37]. Multi-agent deep reinforcement learning has been applied to the control of DGs via smart inverters in [38] and [39] as a way of adapting to time-varying conditions, as well as the spatial and temporal uncertainties resulting from intermittent generation. The overarching concept underlying many of these works is to exploit the capabilities of modern smart inverters to actively regulate inverter-based DG output so as to support network functions such as voltage regulation, network loss minimization, and electricity market-based day-ahead power dispatch, among others [13], [40], [41].

This article focuses on the operational aspect of reactive power compensation. It presents an up-to-date comprehensive survey of the problem formulation and solution approaches for the Volt/VAR optimization problem. Both classical/conventional and heuristic/intelligent search-based optimization methods are covered. Each optimization method is discussed in detail, its strengths and drawbacks are highlighted, and a thorough comparative analysis of the key characteristics of the classical and heuristic methods is presented.

A pictorial summary of some pertinent aspects of the review is presented in Figs. 1 to 3. Fig. 1 depicts a graph of the number of publications that have been reviewed, plotted against the year of publication, Fig. 2 indicates the number of each of the optimization algorithms that have been covered, and Fig. 3 shows the classification of the algorithms.

The rest of the paper is organized as follows. Section II briefly discusses the main devices for reactive power and voltage control in the power system. The VVO problem formulation is presented in section III, encompassing the objectives, decision variables, and constraints. Then the solution approaches to the VVO problem are presented in section IV, covering both classical and heuristic methods. In section V, a comparative analysis of the solution methods is presented, and the concluding remarks are given in section VI.

II. REACTIVE POWER AND VOLTAGE CONTROL IN THE POWER SYSTEM

As mentioned in the introductory section, reactive power and voltage control plays a pivotal role in the secure and economical operation of the power system. In the course of the operation of a power system, a variety of phenomena occur that need some form of intervention in order to maintain the system voltage, frequency and other vital system parameters within the nominal range. These phenomena may be classified as either steady-state or dynamic, depending on the speed of response required in addressing them. Table 1 lists (not in any order of precedence) some of the main phenomena, the addressing of which typically requires reactive power

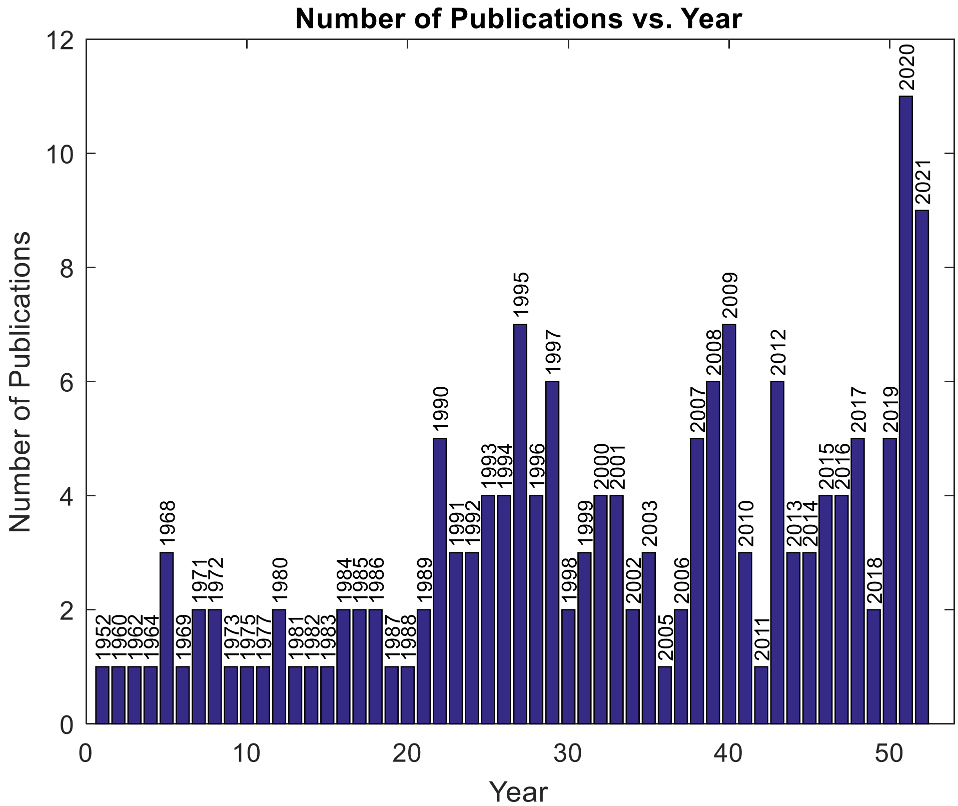

FIGURE 1. Number of publications reviewed plotted against year of publication.

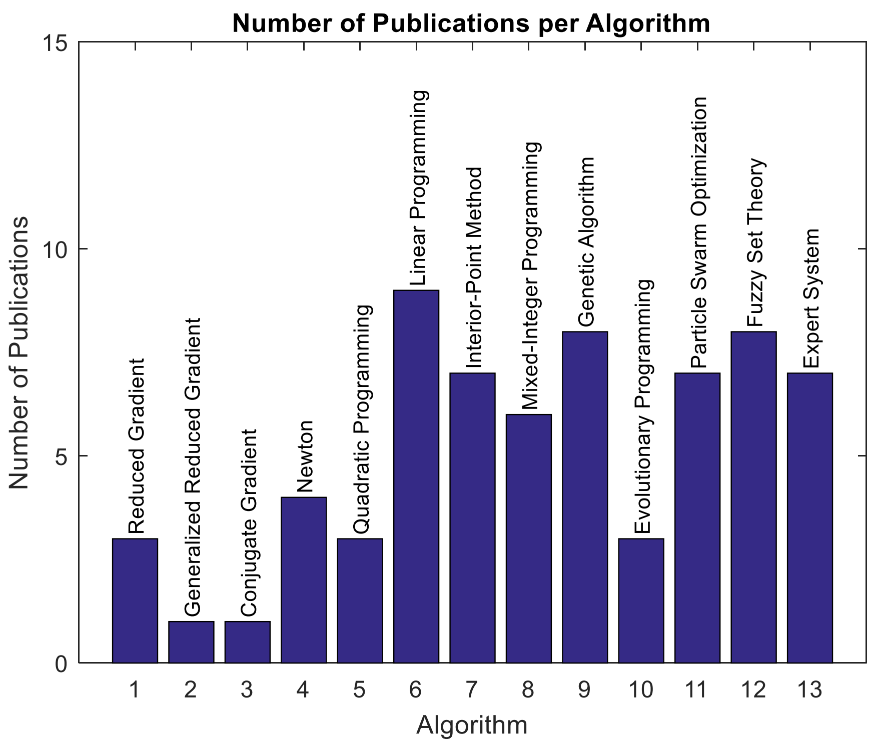

FIGURE 2. Number of publications reviewed in terms of algorithm.

and voltage control of some form [42]. In the following paragraphs, the main power system devices that are typically employed in the provision of reactive power and voltage control are briefly discussed.

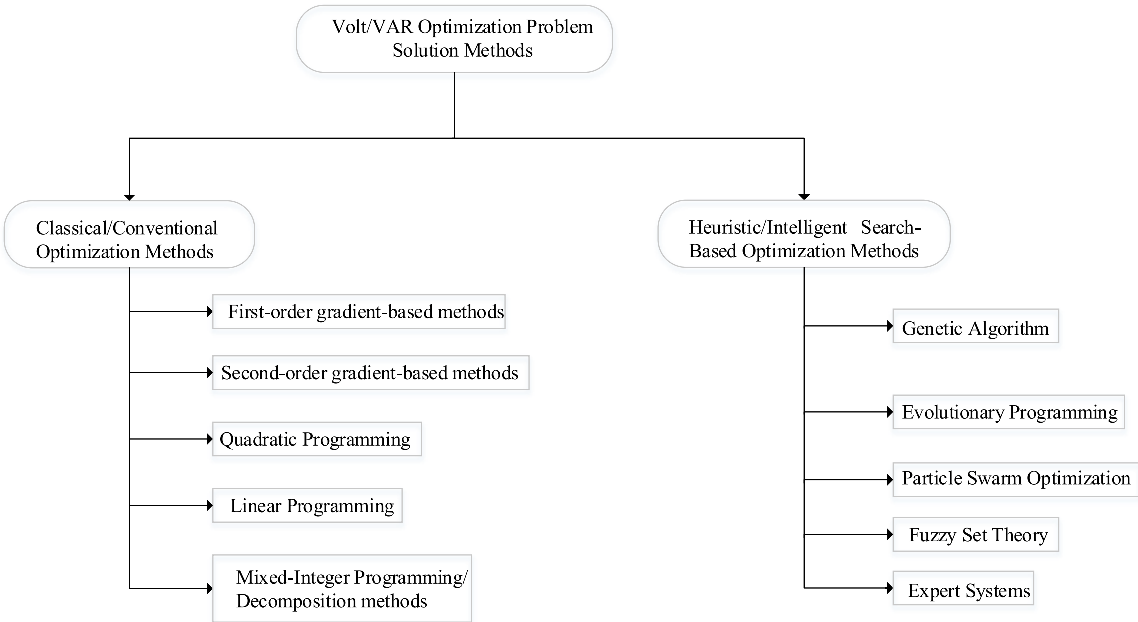

FIGURE 3. Classification of algorithms reviewed.

TABLE 1. Typical power system phenomena requiring Volt/VAR optimization.

Steady-state phenomena (slow response)	Dynamic phenomena (fast response)
Low voltages	Fluctuating loads or impact loads
High voltages	Switching surges or load rejection overvoltages
Large voltage variability	Voltage instability (load voltage collapse)
Excessive reactive power flow (or losses)	Transient or dynamic instability
Normal requirements for HVDC converters	Instability due to subsynchronous resonance (SSR)
Steady-state stability	Variable system phase imbalances
	Dynamic reactive requirements at HVDC terminals
	Small-signal oscillations

A. SYNCHRONOUS GENERATOR

Although the synchronous generator's main role in the power system is to supply active power demand, it is also principally used to regulate system reactive power, with ability to either generate (leading) or absorb (lagging) reactive power, depending on whether it is overexcited or underexcited. An automatic voltage regulator continually adjusts the generator's field excitation in response to system conditions, usually so as to maintain the terminal voltage or voltage at some other system bus at a desired level. The fast response characteristic of the synchronous generator's reactive power generation/absorption implies that it can be used to remedy dynamic system phenomena requiring Volt/VAR control. However, its reactive power supply/absorption capability is limited by the machine thermal and steady-state stability limits, and is a function of the real power output [43].

When a synchronous generator is specially designed and operated so at to generate reactive power only (i.e. real power output set to be zero), it is referred to as a *synchronous condenser*. As a device dedicated to reactive power supply/absorption, it typically has automatic controls that enable fast dynamic response to system anomalies, and has a short-time overload capability that can be utilized in extreme situations. The main disadvantage of the synchronous condenser is its higher capital and maintenance costs compared to other solutions for reactive power supply and absorption [43].

B. SHUNT CAPACITORS

Shunt capacitors constitute a flexible and economical means of providing leading reactive power, which is typically required to boost system voltages during heavy loading periods, or to improve system power factor. Their flexibility stems from their modular nature, large banks can be constructed from several small-size units, which in turn gives them the characteristics of greater control, expansion capability, transportability, and availability. Compared to synchronous generators, shunt capacitors, being static components, have lower maintenance costs, and are generally a cheaper source of reactive power. Their response characteristics, however, make them a lot less effective than synchronous generators in responding to dynamic system phenomena [44]. Also, unlike synchronous generators, they supply discrete (rather than continuous) reactive power, which may affect their treatment in optimization problems, as the corresponding control variable will be discrete rather than continuous [5].

C. SHUNT REACTORS

Shunt reactors are employed in the bulk transmission system to remedy abnormally high transmission voltages, often in lightly loaded conditions, when the capacitive line-charging effects of high-voltage transmission lines tend to lead to conditions exceeding design levels. They are typically required in extra high voltage lines longer than 200 km, where the effects of capacitive line charging can be quite pronounced [43].

D. FACTS DEVICES

Flexible AC Transmission System (FACTS) devices have in recent times emerged as a vital component in the efficient control of active/reactive power and voltage magnitude and frequency. A static VAR compensator (SVC), for example, has ability to continuously vary inductive or capacitive reactive power injection into the system, making use of power electronic technologies. In terms of construction, an SVC can be thought of as being comprised of a controllable reactor and a fixed capacitor, both of which are controlled by means of power electronic switches in accordance with the required reactive power injection, the main purpose being to maintain bus voltage at some specified level. Use of power electronic switches gives FACTS devices ability to provide continuous, instantaneous reactive power, and are thus suitable for addressing many of the dynamic system phenomena associated with Volt/VAR control. Some drawbacks of FACTS devices are their relatively higher cost, and possible negative impact on system power quality due to harmonic generation by power electronic switches [42].

E. UNDER-LOAD TAP-CHANGING TRANSFORMER

A transformer equipped with a load tap-changing mechanism (LTC) can adjust the transformer turns ratio in response to system conditions so as to keep the system voltage within desired ranges. So unlike the devices discussed in the preceding sub-sections, the LTC is not a reactive power source, but rather a voltage-regulating device. Tap positions are discrete points on the windings of the transformer which can be varied so as to realize different transformer turns-ratios, and correspondingly different voltage transformations. The voltage can thus only be varied in discrete steps (rather than continuously). Equipping a transformer with an LTC adds significantly to the cost, and thus requires the utility provided thereby to justify the added cost, which is typically the case in the bulk voltage system where effective voltage regulation is of paramount importance to the secure and efficient operation of the system [43].

F. DISTRIBUTED GENERATION

The proliferation of diverse distributed generation technologies in the power system has been one of the most noteworthy developments in the electric power industry in recent years. Along with their growth, the need for their contribution to the provision of grid ancillary services has been identified as key to their sustained growth and overall improvement in grid operation [45]. Thus, the consideration of distributed generation in Volt/VAR optimization has become an active area of research [38], [40], [41], [46], [47]. The diversity of the technologies (incorporating both conventional synchronous generators and newer technologies in the form of inverter-based generation systems) certainly presents an opportunity for exploiting this form of system resource in the meeting of the various steady-state and dynamic system requirements for the provision of reactive power and voltage control [44], [48].

Table 2 summarizes the principal characteristics of the major devices for reactive power and voltage control that have been discussed in this section. It is the operation of these devices that has to be optimized in order to realize the secure, efficient and economical operation of the power system, as discussed in detail in section IV. The next section addresses the problem formulation for the Volt/VAR optimization problem.

III. VOLT/VAR OPTIMIZATION PROBLEM FORMULATION

Volt/VAR optimization is a constrained optimization problem. The main components of the problem formulation are the objective function, the decision or control variables, and the constraints to be satisfied by the optimal solution to the problem. Mathematically, the objective and constraint functions can be either linear or nonlinear, the decision variables can be either continuous or discrete. Various combinations of these choices will lead to different formulations of the problem. The salient aspects of these components of the VVO problem formulation are briefly discussed in the following sub-sections.

A. OBJECTIVES AND DECISION VARIABLES OF THE VOLT/VAR OPTIMIZATION PROBLEM

There are multiple ways in which optimal reactive power dispatch contributes to the economical, secure and efficient operation of the power system. This can directly be related to the objectives of Volt/VAR optimization. Power loss minimization has featured as the main objective in many research works over the years, both in earlier publications [49]–[52], and in more recent ones [53]–[57]. Maintaining network voltages within the specified range of nominal values constitutes another key objective for Volt/VAR optimization [56], [58]–[60]. Then there is maximization of voltage security [61]–[63], and minimization of the frequency of operation of the Volt/VAR control devices [64]–[66]. Each of these objectives enhances in one way or another the economics, security, power quality, and efficiency of power system operation.

As there are several objectives that can be considered, the VVO problem may be formulated as a single-objective optimization problem (the most prevalent formulation, based on the reviewed literature) or as a multi-objective

TABLE 2. Main characteristics of reactive power and voltage control devices.

Volt/VAR Device	*Relative Cost Per MVA*	*Reactive Supplied*	*Continuous /Discrete*	*Dynamic response*	*Advantages*	*Disadvantages*	*Application (dynamic/steady-state)*
Synchronous Generator/Condenser	High	Lag/lead	Continuous	Fast	Fast response, flexible, strong stabilizing effect	High cost, complex controls	Dynamic
Shunt Capacitor	Moderate	Lead	Discrete	Slow	Flexible, modular, low maintenance requirement	Slow response, non-continuous (i.e. discrete)	Steady-state
Shunt Reactor	Moderate	Lag	Continuous	Slow	Simple, low maintenance requirement	Slow response, non-continuous (i.e. discrete)	Steady-state
FACTS	High	Lag/lead	Near-continuous	Fast	Fast response dynamics, flexible VAR supply/absorption	High cost, complex controls	Dynamic
ULTC Transformer	High	N/A	Discrete	Slow	Effective means of regulating system voltage	High cost, frequent operation may lead to high maintenance costs	Steady-state
Distributed Generation	Technology-dependent	Lag/lead	Continuous	Generally fast	Flexible, modular, can provide VAR support locally	VAR support may impact revenue from active demand supply	Dynamic

optimization problem, examples of which can be found in [50], [51], [57], [65], [67]. A multi-objective formulation permits the simultaneous consideration of economic and security objectives, for example. A major concern in multi-objective optimization is how to formulate the problem in such a way that the obtained solution is optimal for all the considered (and potentially conflicting) objectives. The most common approach is to reduce the multiple objectives to a single objective function by a weighted summation of the individual objectives. This approach has the desirable characteristic of being simple to implement, but also has a number of drawbacks, such as the dependence of the obtained solution on the choice of the weighting vector, with considerable reliance on user expertise and experience. The subject of multi-objective optimization is discussed in detail in [68].

A key consideration regarding the objective function of the VVO problem is its dynamic characteristics, particularly in terms of whether it is linear or nonlinear. Taking the real power transmission losses as an example, the mathematical expression thereof can be stated as [69]:

$$P_{Loss} = \sum_{k=1}^{N_L} G_k \left[V_i^2 + V_j^2 - 2V_i V_j \cos\theta_{ij} \right] \tag{1}$$

where the symbols are defined as follows:

G_k	*series conductance of branch k*
N_L	*number of branches in the network*
P_{Loss}	*total power transmission losses*
V_i, V_j	*voltage magnitude at buses i and j*
θ_{ij}	*phase angle of ij^{th} Y-matrix component*

It can be deduced from (1) that the expression for the real power transmission losses is both nonlinear and nonconvex, being quadratic in terms of the bus voltage magnitudes, in addition to having trigonometric function components. The inherent difficulty of evaluating a nonlinear objective function of this nature has motivated the devising of alternative (i.e. simpler) formulations of the objective function, chiefly by means of linearization. Thus, a number of linear objective functions for the loss minimization-based VVO problem have been proposed in the literature, for example [50], [51], [69]–[71]. Linearization is normally performed about some desired operating point. In [50], [51] and [70], a sensitivity-based method is used to develop a linearized form of the objective function for the VVO problem, where (small) changes in state variables (bus voltage magnitudes, phase angles and slack-bus real power) are expressed as linear functions of corresponding changes in the control variables, and this forms the basis for the solution algorithm development for the optimization problem. In [69] the loss-minimization objective function is linearized by taking partial derivatives of (1) with respect to bus voltage magnitudes, so that incremental losses are the ones to be minimized, expressed as linear functions of the bus voltage magnitudes.

As for the decision or control variables for the VVO problem, these can be classified into those derived from voltage-regulating devices, and those derived from reactive power sources, as has been briefly presented in section II. Voltage regulation is mainly through synchronous generator terminal voltage magnitude adjustments and Under-Load Tap-Changing (ULTC) transformers. Reactive power injection/consumption devices are synchronous generators, synchronous condensers, shunt capacitors and reactors, Flexible AC Transmission System (FACTS) devices, and Distributed Generation to the extent that is dependent on the specific technology [72]. Some of these devices generate continuous variables, others discrete variables. A complete and most accurate formulation of the VVO problem would thus be a Mixed Integer Nonlinear Programming (MINLP) problem formulation [65].

B. CONSTRAINTS OF THE VOLT/VAR OPTIMIZATION PROBLEM

The constraints of the Volt/VAR optimization problem essentially consist of limits on the permissible range of values for the control variables (e.g. transformer tap limits, shunt capacitor range), operating limits on the power system state variables (e.g. generator real and reactive power outputs, voltage magnitudes and phase angles, line and transformer flow limits, etc.) [73]. The standard set of constraints considered in most formulations of the VVO problem can be stated as:

Network Power Balance Equations:

$$P_i(V,\delta,\tau) - P_{Gi} + P_{Li} = 0 \tag{2}$$

$$Q_i(V,\delta,\tau) - Q_{Gi} \pm Q_{Si} + Q_{Li} = 0 \tag{3}$$

where (expressed in polar form):

$$P_i(V,\delta,\tau) = V_i \sum_{j=1}^{N_{Li}} V_j Y_{ij} \cos\left(\delta_{ij} - \theta_{ij}\right)$$

$$Q_i(V,\delta,\tau) = V_i \sum_{j=1}^{N_{Li}} V_j Y_{ij} \sin\left(\delta_{ij} - \theta_{ij}\right)$$

Control Variable Limits:

$$V_{Gi}^{\min} \le V_{Gi} \le V_{Gi}^{\max} \tag{4}$$

$$\tau_k^{\min} \le \tau_k \le \tau_k^{\max} \tag{5}$$

$$Q_{Si}^{\min} \le Q_{Si} \le Q_{Si}^{\max} \tag{6}$$

State Variable Constraints:

$$P_{G1}^{\min} \le P_{G1} \le P_{G1}^{\max} \tag{7}$$

$$Q_{Gi}^{\min} \le Q_{Gi} \le Q_{Gi}^{\max} \tag{8}$$

$$V_{Li}^{\min} \le V_{Li} \le V_{Li}^{\max} \tag{9}$$

$$|S_k|^2 \le \left(S_k^{\max}\right)^2 \tag{10}$$

The symbols in the above expressions have the following definitions:

$P_i(V,\delta,\tau)$	*active power injection at bus i*
P_{Gi}	*generator active power output at bus i*
P_{G1}	*generator active power output of slack bus*
P_{Li}	*active power demand at bus i*
$Q_i(V,\delta,\tau)$	*reactive power injection at bus i*
Q_{Gi}	*generator reactive power output at bus i*
Q_{Li}	*reactive power demand at bus i*
Q_{Si}	*reactive power source/sink magnitude at bus i*
V_{Gi}	*generator terminal voltage magnitude at bus i*
V_{Li}	*voltagemagnitudeatPQbusi*
τ_k	*tap position of ULTC connected in branch k*
S_k	*apparent power flow in branch k*
Y_{ij}	*ij^{th} component of admittance matrix*
δ	*voltage phase angle*
N_{Li}	*number of branches connected to bus i*

The set of constraints (2)-(10) defines the feasible region for the VVO problem, and a solution for the problem (i.e. a set of control variables that minimizes (1)) is admissible only if it is feasible with respect to the constraint set. It can be observed that this constraint set is nonlinear and non-convex, because the constraint equations (2) and (3), for example, have trigonometric terms, and (4), (7)-(9) are non-convex quadratic [2], [3]. Moreover, some control variables (specifically ULTC tap positions and shunt reactive power sources, represented by (5) and (6) respectively) can only take on discrete values. This gives the constraint set (indeed the overall problem formulation) for the VVO problem the characteristic of being highly nonlinear [18], and poses special challenges for any solution algorithm that may be applied to solve the problem.

In the following section, a detailed discussion of the variety of solution approaches that have been applied over the decades to the VVO problem is presented, based on the surveyed literature.

IV. OPTIMIZATION METHODS FOR THE VOLT/VAR OPTIMIZATION PROBLEM

The problem formulation for VVO, and the accompanying discussion that has been presented in section III, clearly shows that it is a complex optimization problem. The complexity is in part due to the nonlinearity and non-convexity of both the objective function and the constraint set, as well as the mixed continuous-discrete nature of the decision variables.

As a result, this section presents an overview of the classical and heuristic optimization methods for supporting voltage and reactive power regulation using the control devices as discussed in section II, and its problem formulation, as presented in section III.

The various approaches that have been proposed over the years for the solution of the VVO problem may be taken to fall into two main categories: classical/conventional methods, and heuristic/intelligent search-based techniques. The merit of any candidate solution approach can be gauged on the basis of its ability to address the performance characteristics relevant to the VVO problem, among them being (in no particular order of importance) [21], [73]:

- Accuracy requirement of problem formulation
- Computation time and memory requirements
- Possibility for real-time implementation

- Scalability of solution approach
- Global convergence characteristics
- Global optimality characteristics
- Reliability of solution
- Robustness of solution method
- Ability to handle both continuous and discrete decision variables
- Ability to (simultaneously) address multiple objectives
- Simplicity of solution method

Model accuracy is a very important consideration in an optimization problem, from the perspective of the accuracy (and usability) of the obtained solution, as well as the complexity of the optimization problem, which has a bearing on the choice of the solution algorithm for application to the problem [74]. Indeed, different solution algorithms require different levels of accuracy (or detail) of the problem formulation. With VVO being an operational optimization problem, speed of computation is also an important consideration, especially in the context of real-time implementation, where control decisions need to be generated quickly in response to dynamic system variations so as to maintain the reliability of system operation. Similar observations can be made about each of the other performance characteristic requirements of solution approaches for the VVO problem outlined above. The interested reader may refer (for example) to [75] for a more detailed discussion of desirable performance characteristics of optimization algorithms.

It is evidently hardly practical to find a single solution algorithm that effectively addresses all of the performance characteristics listed above, in part due to the inherent mutual conflict that they may exhibit. Commonly, the various solution algorithms are differentiated by how well they address some (and not necessarily all) of these requirements. In the following sub-sections, some of the solution algorithms that have been proposed in the literature are discussed under the two main categories as stated earlier (i.e. classical/conventional methods, and heuristic/intelligent search-based techniques).

A. CLASSICAL/CONVENTIONAL METHODS FOR VOLT/VAR OPTIMIZATION

A wide variety of solution methods falling under the category of classical/conventional optimization techniques have been applied to the VVO problem, among them being first-order and second-order gradient-based methods, Quadratic Programming (QP), Linear Programming (LP), Interior-Point Methods (IPM), and Mixed-Integer Programming (MIP), along with decomposition techniques.

Gradient-based methods are iterative optimization techniques that seek to extremize (*i.e.* minimize or maximize) a differentiable nonlinear function by generating a sequence of improving estimates of the decision vector, moving in such a direction as to achieve progressively lower values (in the case of minimization) of the objective function, until the sequence hopefully terminates at the solution (*i.e.* the minimum of the objective function to be optimized) [75]. Some of the earliest efforts to algorithmically solve the VVO problem applied gradient-based methods, examples of which can be found in [49], [50], [76].

1) FIRST-ORDER GRADIENT-BASED METHODS

The principal first-order gradient-based methods that have been applied to the solution of the VVO problem are the Reduced Gradient (RG) [78], [79], Generalized Reduced Gradient (GRG) [80], and Conjugate Gradient (CG) [50] methods.

In the *Reduced Gradient* (RG) method, first applied to the OPF problem in [76], the functional and equality (i.e. power flow equation) constraints are handled by means of penalty terms and Lagrangian multipliers respectively, forming a linear combination with the objective function to construct the Lagrangian function, to which the Karush-Kuhn-Tucker (KKT) conditions are then applied to solve the minimization problem [77]. The RG method provides a way to reduce the problem size, where the problem variables are divided into decision variables and state variables, the objective function expressed as a function of the decision variables, while the state variables are adjusted to maintain solution feasibility [78].

The *Generalized Reduced Gradient* (GRG) method is an extension of the RG method that allows for the direct handling of nonlinear and inequality constraints. Inequality constraints are turned into equality constraints by the introduction of nonnegative slack variables, and the (nonlinear) constraints are then linearized about the operating point. The generalized reduced gradient is then defined as the gradient of the linear combination of the objective function and the linearized constraints [80]. Each such linearization is treated as a subproblem, which can be solved by a gradient-based method such as the RG method, and a series of such subproblem solutions should lead to the solution of the original problem. The GRG method was applied in [79] to the solution of a variety of optimal power flow problems, chiefly power loss minimization and network voltage profile optimization. Some of the attractive features of the GRG method are the avoidance of penalty terms in dealing with the functional constraints, the convenient way it provides for transforming a nonlinear constrained optimization problem into an unconstrained one that can be solved by a gradient-based method, and the reduced dimensionality of the resulting problem [81].

The *Conjugate Gradient* (CG) method was proposed in the 1950s as an iterative method for solving linear systems with symmetric positive definite matrices [82], offering an alternative to existing methods such as Gaussian elimination, and especially well-suited to solving large-scale problems. Extension of the method to the application to nonlinear problems was developed in the 1960s [83], and constituted one of the earliest known methods for solving large-scale nonlinear optimization problems [75]. The nonlinear CG method was applied in [50] to the minimization of the node voltage magnitude deviations from their nominal values. The conjugate gradient vector (which establishes the search direction in the CG method) is computed as a linear combination of successive previous search directions. This method of constructing the search direction ensures non-interference of consecutive search directions, consequently leading to greater advance of the algorithm towards the solution. Key features of the CG method are low storage requirements, and more rapid advance towards the solution relative to the steepest gradient method.

First-order gradient-based methods offer a reliable and fairly unsophisticated way to optimize a differentiable nonlinear function, without being computationally expensive. Their main drawback is the slow rate of convergence, as they rely solely on first-order information of the function to be optimized in advancing toward the solution. The second-order methods, discussed in the next sub-section, constitute an improvement in this aspect.

2) SECOND-ORDER GRADIENT-BASED METHODS

2^{nd}-order methods differ from 1^{st}-order methods chiefly in the construction of the search direction for the optimization algorithm. Whereas 1^{st}-order methods rely solely on the 1^{st}-order (partial) derivatives of the objective and constraint functions, 2^{nd}-order methods additionally incorporate 2^{nd}-order information. The second partial derivatives carry the function's curvature information, and incorporation of this information leads to faster convergence of the algorithm to the solution. Newton's method, the representative 2^{nd}-order gradient-based method, applies a 2^{nd}-order Taylor series expansion to the objective function about the current iterate x_k, which leads to the search direction d_k at x_k being defined by $d_k = -(H(x_k))^{-1} \nabla f(x_k)$, where $H(x_k)$ is the Hessian matrix (i.e. the matrix of 2^{nd}-order partial derivatives of the objective function), and $\nabla f(x_k)$ is the vector of 1^{st}-order derivatives of the objective function at x_k. Examples of the application of Newton's method to the VVO problem can be found in [84]–[86].

The distinguishing feature of 2^{nd}-order methods is their quadratic rate of convergence, much faster than the convergence rate of 1^{st}-order methods, although this comes at the expense of additionally having to compute the inverse of the Hessian matrix, which may be a cumbersome, error-prone, and computationally expensive process, especially in the case of problems with a dense Hessian matrix. An alternative is Quasi-Newton methods, which avoid the exact computation of the Hessian matrix by approximating it using information about the change in the 1^{st}-order derivatives [75]. Two other issues with 2^{nd}-order methods are the need for the Hessian matrix to be positive definite to ensure the search direction is a descent direction, and the difficulty in dealing with the inequality constraints of the VVO problem [16], [85]. The reliability of Newton's method particularly requires that the difference between the objective function and its 2^{nd}-order approximation at the current iterate not be too large. Despite these issues, Newton's method is not only a classical method for nonlinear optimization, but also represents an important optimization approach, both efficient and robust for a large class of problems [86].

3) QUADRATIC PROGRAMMING

Quadratic Programming (QP) is a special case of nonlinear programming in which the objective function is quadratic and the constraint set is linear. When applied to the VVO problem, a technique known as sequential quadratic programming (SQP) is employed, involving iteratively generating a quadratic approximation of the objective function, and linearizing the constraints about the current operating point. The solution of these QP subproblems should converge to the optimal solution of the original nonlinear problem [87]. Quadratic programming is somewhat of a compromise between the general nonlinear programming problem and a linear programming formulation, trying to achieve some balance between the accuracy of the model representation and the computational complexity of the solution of the problem [88].

Depending on whether the QP model formulation is convex or nonconvex, a variety of solution techniques exist, among them being active set methods and interior point methods. Examples of the QP model formulation of the VVO problem can be found in [88]–[90].

In [88], a convex QP formulation has been developed for the real power loss minimization reactive power dispatch problem, and solved by the active-set projection method. In [89], the real and reactive power dispatch problem is solved by quadratic programming, considering a quadratic cost function for the generation and transmission line losses, and a linear approximation of the system constraints. An improvement in accuracy is obtained over the linear programming-based model, and faster solutions compared to the exact nonlinear model of the combined active/reactive power optimization problem. Reference [90] also applies SQP to reactive power optimization, and develops a quadratic multi-objective optimization problem, combining economic and security objectives, which is also solved by the Newton-based active-set method.

The attractiveness of the quadratic programming solution technique lies in its providing a means to achieve a good balance between the requirements of a reasonably accurate model of the VVO problem, and the computational expense associated with the exact nonlinear model formulation. The quadratic approximation of the nonlinear system power loss function is sufficiently accurate, and permits the application of efficient QP solution techniques to the problem [88], [90].

4) LINEAR PROGRAMMING

An optimization problem is classified as a linear programming (LP) problem when both the objective function and the constraint set are linear functions of the decision variables. Because the VVO problem is inherently nonlinear (as discussed in section III), an LP formulation of the problem entails the linearization of both the objective function and the constraint set (section III, equations 1 and 2-10 respectively). As pointed out in section III, linearization is typically performed around some desired operating point, and can be done based on the first-order Taylor series expansion (i.e. taking the first-order partial derivatives of the nonlinear power loss function with respect to the control variables) [69], or on the basis of sensitivity relationships devised to relate changes in the state variables to changes in the control variables [50], [51], [70].

Linear programming has traditionally been a popular approach for the solution of the OPF problem, which includes economic dispatch and reactive power dispatch [91]. The approach has many desirable characteristics, such as reliability, very good convergence properties even for large-scale problems, faster computation speed, and availability of very efficient algorithms for solving the problem [75]. The main solution techniques for the LP problem are several variants of the Simplex method, and Interior Point Methods (IPM).

Examples of LP formulations of the VVO problem are to be found in [50], [69]–[71], [92]–[96]. In [92], an LP formulation was devised for the reactive power dispatch problem incorporating voltage stability, to minimize the risk of voltage collapse in the system, and solved by the dual revised simplex method. The linearization and solution of the problem is done in an iterative manner, leading to what is commonly referred to as sequential linear programming (SLP). The desirable characteristic of this technique that is highlighted is convergence of the solution that is independent of the problem size, whereas in the case of the original nonlinear problem formulation, depending on the solution algorithm, global convergence may not be guaranteed [21]. An LP-based network-constrained reactive power control problem is presented in [93], which is found to be suitable for real-time application in large-scale power systems, with speed of solution and convergence characteristics that are difficult to achieve in the case of the classic nonlinear formulation of the problem.

With all the desirable characteristics of the LP approach to VVO, it must be borne in mind that this comes along with the compromise of the accurate representation of the otherwise highly nonlinear model of the VVO problem. Efforts to devise more efficient solution techniques for the original nonlinear problem formulation have thus continued to attract a lot of attention [24].

5) INTERIOR-POINT METHODS

Interior point methods (IPM) are a class of optimization techniques that were initially developed as an alternative to the Simplex method for solving linear programs [97], with the introduction of Karmakar's method [98], a polynomial-time linear programming method. Whereas the Simplex method exploits the convexity of the feasible region of the LP problem, searching along the vertices of the polytope that defines the feasible region for the optimal solution to the problem, IPMs take a different approach, attempting to confine the search path within the feasible region, and establishing and following a ‘‘central path’’ towards the optimal solution of the problem. Besides having pseudo-polynomial complexity, IPMs also exhibit some advantages relative to the Simplex method, such as being especially efficient for large-scale problems, and making more rapid convergence towards the optimal point [99]. The successes of IPMs in LP incited research efforts to extend them to general nonlinear problems, and these methods have attractive properties that make them especially suitable for nonlinearly constrained optimization, such as the efficient handling of inequality constraints (which is quite problematic for the classical Newton-based methods), rapid convergence, and not having to start from a strictly feasible initial solution [100].

IPM-based reactive power optimization and voltage control has been presented for example in [101]–[103]. In reference [101], the primal-dual logarithmic barrier interior point algorithm (PDIPM) was applied to the solution of the optimal reactive power dispatch (ORD) problem. It is highlighted in the paper that ORD is a large-scale highly nonlinear, nonconvex optimization problem, and the characteristics of the chosen IPM that make it suitable for application to this problem are the insensitivity of the problem complexity (i.e. number of iterations required to reach to solution) to the problem size, more efficient handling of the nonlinear inequality constraints, and numerical robustness, even ability to handle large-scale, ill-conditioned problems. Reference [103] used a version of the PDIPM method known as predictor-corrector PDIPM (PC-PDIPM) to solve the reactive power optimization/voltage control problem, a method which seeks to improve the search direction at each iteration [104]. The authors focused in the development of their algorithm on computational speed to be suitable for real-time application, reliability to converge even from an initially infeasible starting point, and the effective detection and handling of infeasibility. Comparison with quadratic programming and least squares-based infeasibility handling showed that the developed PDIPM method scaled better with the number of constraints (i.e. increase in number of constraints having less impact on computational speed), and the infeasibility detection and handling approach taken added much less computational burden to the overall optimization process. Interior point methods have thus been found to be very suitable for solving the large-scale, highly nonlinear constrained OPF problems, an example of which is the VVO problem [101].

6) MIXED INTEGER PROGRAMMING AND DECOMPOSITION TECHNIQUES

In all the solution algorithms discussed thus far, only continuous control variables are considered in the formulation of the VVO problem. However, as pointed out in section III, the presence of discrete control variables (e.g. transformer tap positions in ULTC transformers) makes the general formulation of the VVO problem a mixed integer programming problem, implying that integrality constraints have to be enforced on a subset of control variables. The motivation for considering a continuous approximate formulation of the problem has been the very high computational expense associated with the full mixed integer nonlinear programming (MINLP) problem, especially in large-scale systems with possibly thousands of mixed integer and continuous variables, and nonlinear objective and constraint functions [4]. The drawback of this approach is that achieving feasibility of the continuous solution by rounding off the values of the control variables required to be integral to the nearest integer values may in many cases be difficult. Moreover, the objective value of the rounded-off solution may deviate significantly from that of the continuous optimal solution [80]. These considerations, along with the advances in computational capabilities of modern computers that have enhanced the tractability of this class of problems, have encouraged the search for effective algorithms to treat the full MINLP [105].

The most commonly used optimization algorithm for MINLP is the branch-and-bound (B&B) algorithm, developed by Land and Doig [106] to solve integer linear programming problems, which was subsequently extended to the solution of MINLP problems. In the B&B method of solving MIP problems, an indirect approach is taken where firstly a continuous version of the problem is optimized by relaxing the integrality constraints (thus obtaining a continuous optimal solution), then each of the integrality constraints is progressively enforced until an integer optimal solution is found. The key components of the algorithm are branching, where for a given continuous optimal solution the associated integer feasible solutions are evaluated for optimality; and bounding, where the prevailing integer optimal value is used as an upper bound to eliminate from further consideration any alternatives that cannot possibly achieve a better optimal solution. Further details on the algorithm can be found for example in [99].

Examples of MINLP formulations of the VVO problem can be found in [53], [65], [108], [109]. Due to the need to simultaneously treat both continuous and discrete variables, it is common to apply decomposition techniques in solving mixed-integer programming problems. The problem is reformulated into two separable optimization sub-problems, a continuous one and a discrete one. The two sub-problems are then solved alternately, and related together by some decomposition technique, such as Benders decomposition [110], one of the most commonly applied decomposition methods to which the MINLP problem is very amenable. Such an approach has been used in [108], for example, where Benders decomposition is used along with the B&B algorithm to solve the combined reactive power planning and real-time voltage control (or reactive power dispatch) problem formulated as a MINLP problem.

The main advantage of solving the VVO problem as a mixed-integer programming problem is the greater accuracy of problem formulation and resulting optimal solution that can be achieved, enabling the accurate modeling of all control devices involved in the problem, including discrete ones such as shunt capacitors, ULTC transformers, and a variety of FACTS devices [108]. This comes at the cost of greater computational complexity of the problem, however. Improvement of algorithms geared towards this class of problems and such paradigms as parallel computing can enhance results that are achievable using this approach of solving the VVO problem.

B. HEURISTIC/INTELLIGENT SEARCH-BASED METHODS FOR VOLT/VAR OPTIMIZATION

Heuristic/intelligent search-based optimization techniques employ a variety of optimum-seeking strategies that are distinctly different from the approaches taken in conventional optimization algorithms. The search strategies employed in these techniques are meant to overcome many of the deficiencies of the conventional optimization problems, such as the local (rather than global) nature of the search, the limited ability to handle combinatorial problems with discrete decision variables, and the requirement for smoothness of the objective and constraint functions for gradient-based methods, among other factors [22]. Over the past few decades a wide variety of these heuristic optimum-seeking techniques have been developed. A representative sample of them are discussed in this section, as they have been applied to the VVO problem, particularly Genetic Algorithms (GA), Evolutionary Programming (EP), Particle Swarm Optimization (PSO), Fuzzy Set Theory, and Expert Systems (ES). The main distinctive characteristics of each technique are briefly discussed, and a sample of applications is also given.

1) GENETIC ALGORITHM

Genetic algorithm (GA) is a population-based search algorithm that is modelled after the processes of natural selection and natural genetics, combining the features of survival of the fittest in a population of optimal solution candidates, efficient exploitation of historical information, and randomized information exchange among the population candidate solutions so as to evolve the population into a new generation of improved candidate solutions [111]. The development of GAs was inspired by the robustness, efficiency and efficacy through adaptation observed in biological processes, and efforts were made to develop artificial software systems that could mimic and replicate the natural processes responsible for these characteristics, such as selection, crossover and mutation [112], [113]. GAs, though conceptually and computationally simple, constitute an efficient, effective and robust approach to search for optimal solutions to a variety of problems in diverse environments, with no reliance on such limiting assumptions of conventional optimization methods as continuity, existence of derivatives, and unimodality [111]. Once an initial population of candidate solutions is generated, either randomly or heuristically, the population is evolved through the sequential and iterative application of the selection, crossover and mutation operations, into a new generation of improved solution candidates [114].

A number of works have applied GAs to the VVO problem, examples of which can be found in [60], [115]–[119]. In reference [115], the property of GAs of being domain-independent search mechanisms, providing powerful search characteristics for large, complex search spaces without requiring full knowledge of the problem domain is highlighted.

Reference [116] proposes an alternative crossover method and incorporation of stochastic "if-then" rules (akin to expert systems) into the GA applied to reactive power planning, so as to enhance the algorithm's efficiency and effectiveness. A hybridized GA is considered in [117] for the solution of the reactive power operation and planning problem, combining it with successive linear programming (SLP), and using a new population selection and generation method that makes use of Benders' cuts, in an effort to combine the positive characteristics of deterministic and non-deterministic optimization techniques.

Many appealing characteristics have been highlighted in the literature that make the genetic algorithm an effective search mechanism. The effectiveness, however, is also a function of the algorithm design, including choices regarding the selection, crossover and mutation operations, the encoding of the candidate solutions used, and the fitness function [114].

2) EVOLUTIONARY PROGRAMMING

Evolutionary Programming (EP) was conceived and developed by L.J. Fogel in the early 1960s as an alternative approach to realizing artificial intelligence (AI), utilizing the concepts of Darwinian evolution to iteratively generate increasingly appropriate solutions to a given optimization problem [120]. It can be seen as an approach to optimization that makes use of simulated evolution to evolve a set of solutions (or organisms) which exhibit increasing intellect evidenced by ability to make correct predictions, to translate those predictions into suitable actions, and to adapt behaviour so as to meet specific goals in a range of environments [112]. As an evolutionary algorithm, EP employs the key concept of selection-by-fitness, which entails the generation of a population of candidate solutions (to an optimization problem), devising a suitable fitness function with which to evaluate the worth of each candidate solution in light of stated objectives, and application of evolutionary operators such as mutation to evolve the population through generations of ever-improving candidate solutions [121]. The selection of candidate solutions to propagate through to the next generation can be either elitist (the best in each generation are selected to form the next one) or by stochastic tournament (probabilistic selection of next-generation candidate solutions).

EP has been used as the solution algorithm to the VVO problem in [122]–[124]. The global search characteristics of the EP algorithm, and the non-reliance on the smoothness and/or convexity properties of the objective and constraint functions for effective search, are highlighted in [122] as making it suitable for solving the reactive power optimization and voltage control problem, which is highly nonlinear and nonconvex. By maintaining a population of candidate solutions at each iteration, which are propagated through future generations using probabilistic transition rules as a function of their overall merit, with a Gaussian relationship between parents and offspring, the EP algorithm is able to move over hills and valleys of the search space, and therefore arrive at the globally optimal solution. In [123], enhanced evolutionary algorithms (evolutionary programming and evolutionary strategies), the enhancement consisting in use of alternative mutation strategies, have been applied to the solution of the reactive power dispatch problem, demonstrating that enhancements can be made to the standard algorithm to improve its efficiency and effectiveness. The effectiveness of population-based evolutionary algorithms in finding pareto-optimal solutions in multiobjective optimization has been pointed out in [124], where a multiobjective evolutionary algorithm has been developed for the optimal reactive dispatch problem.

The main evolutionary operations used in EP are mutation, competition and reproduction. As with other evolutionary algorithms, parameter selection plays a key role in exploiting the various desirable attributes of the algorithm, and ensuring its efficiency and effectiveness.

3) PARTICLE SWARM OPTIMIZATION

Swarm intelligence is a stream of AI research that got established in the early 1990s, based on the study of the swarm behaviour of natural creatures, in terms of how decision making of the individual is influenced by both the individual's experience and the experiences of others [125]–[127]. Particle swarm optimization (PSO), one variant of swarm intelligence techniques that has become prominent, was developed by Eberhart and Kennedy [128], and is based on the analogy of swarms of birds and fish schooling. The algorithm uses a population of particles exploring the search space in search of the optimal solution to an optimization problem. Associated with each particle is a position and a velocity in a two-dimensional search space, and the change in position of the particles as a function of the current best positions of the individual and of the overall population is what constitutes the population's evolution towards the optimal point. The use of a population of candidate solutions, incorporating randomness and memory, as well as diversification at the beginning, and intensification towards the end of the search, adds greatly to PSO's efficiency as a search mechanism [129].

Having been originally developed to treat nonlinear optimization problems with continuous variables, a number of enhancements to the standard PSO algorithm have been proposed and developed, to improve the algorithm's efficiency, and to extend its applicability to other problems (e.g. combinatorial optimization, and mixed-integer nonlinear programming (MINLP) problems).

Examples of the application of PSO to the solution of the VVO problem can be found in [57], [130]–[134]. In reference [57], a multi-objective Volt/VAR control problem that considers robustness in addition to power loss minimization has been solved using the PSO algorithm, exploiting the ability to structure the algorithm so as to handle multiple objectives [135]. A modified PSO algorithm has been applied to optimal reactive power dispatch in [130], the modification consisting in adding mutation to the standard algorithm in order to improve its global search characteristics and prevent rapid convergence to local optima. Reference [131] follows a different approach to enhancing the global search characteristics of the PSO algorithm, which is to hybridize it with the Tabu Search algorithm, another stochastic search algorithm [136]. In [132], the reactive power and voltage control problem has been solved using the differential evolution (DE) (an evolutionary computation algorithm) [137] and PSO algorithms, and performance comparison of the two methods has been made, particularly in terms of solution quality and convergence characteristics. The authors found the PSO algorithm to slightly outperform the DE algorithm, although exhibiting relatively greater computational effort. Reference [133] takes advantage of the PSO's ability to better handle discrete control variables than the conventional optimization methods, and applies it to the solution of the optimal reactive power dispatch problem considering discrete variables. The focus in [134] is on how PSO-based optimal reactive power dispatch can enhance system security considering the impact of intermittent renewable generation such as wind power generation.

Besides being able to address diverse optimization objectives, as can be deduced from the surveyed literature, the PSO algorithm additionally has the desirable characteristic of being quite simple to implement, in the sense that simple rules governing individual agent behaviour can result in sophisticated swarm behaviour. The model of each individual agent (or particle) is relatively simple, yet can lead to effective and efficient collective behaviour of the whole swarm in terms of searching for the optimal solution in a search space. Hybridization with other methods, and other enhancements to the standard algorithm, are often considered to improve the efficiency and effectiveness of the algorithm [129].

4) FUZZY SET THEORY

Although conventional optimization problems are computed with the assumption of precise information, in reality most real-world data that serves as input to the optimization problems is embedded with uncertainty and imprecision. Power systems are especially prone to a significant amount of uncertainty in operational data, largely due to their large scale, being geographically widely distributed, complexity in operational dynamics, and susceptibility to unexpected events [138]. Fuzzy set theory is a mathematical approach that can be used to capture this uncertainty and imprecision of information, the incorporation into the optimization problem of which can enhance the robustness of the obtained results [97]. Fuzzy set theory enables objective and constraint functions to be represented as fuzzy sets, where the membership to these sets represents the degree of closeness to the optimum (for the objective function) and the degree of enforcement of the constraints (for constraint functions). The maximization of membership functions then implies the simultaneous optimization of the objective function and enforcement of the constraint set, all while taking uncertainties into account. This leads to a better compromised solution, more robust in the sense of being less sensitive to parameter variations [24].

Fuzzy set theory is not actually an optimization technique, and so it is normally used in conjunction with optimization techniques, where it essentially serves as a tool for modeling uncertainty and imprecision in the problem formulation. Applications to reactive power optimization and voltage control have been many over the years, combining with a variety of optimization techniques, examples of which can be found in [58], [59], [139]–[144]. In reference [58], fuzzy set theory was combined with a strength-pareto evolutionary algorithm (SPEA2) to solve the multi-objective reactive power/voltage control problem. References [139]–[141] formulate a fuzzy-linear programming-based reactive power/voltage control, combining the reliability and speed characteristics of linear programming with fuzzy set theory's ability to more efficiently depict the realistic system objective and constraint functions, leading to a more practical solution of the problem. Other examples of hybrid methods incorporating fuzzy set theory or fuzzy logic are the fuzzy-dynamic programming approach presented in [142], and the fuzzy-PSO multi-objective algorithm presented in [143].

The strength of fuzzy set theory that has been exploited in reactive power optimization and voltage control (among other power system applications) is the capability of handling ambiguity, conflicting objectives, and soft constraints in a flexible way that can moreover improve computational complexity of power system optimization problems [138]. By providing the means to effectively model uncertainty and imprecision, and to incorporate the approximate reasoning and subjective judgment of expert operators into the mathematical model [141], fuzzy set-based modeling facilitates the realization of a better compromised solution, where both accuracy and robustness of the solution are taken into account [138].

5) EXPERT SYSTEM

Expert systems (also known as knowledge-based systems) constituted one of the earliest approaches to building AI systems in the 1960s, and were among first successful commercial applications of the then nascent field of artificial intelligence [145]. An expert system (ES) can be defined as an intelligent computer-based system in which representations of human expert knowledge are stored, and it can apply inference procedures and heuristics to this knowledge base to solve complex problems in a manner that a human expert would do. An ES is fashioned after the model of human reasoning, which may be considered to be based on the creation of categories, application of specific (a priori) rules, use of heuristics (i.e. rules-of-thumb, representing conventional wisdom), as well as use of past experience (precedence-based reasoning). Most expert systems make use of rule-based reasoning, the main components of which are the knowledge/rule base containing much of the problem-solving knowledge, a database containing some data of interest to the system, an inference engine generating the decisions, and a user interface providing a means for user interaction with the system [146].

TABLE 3. Summary of main characteristics of conventional optimization techniques.

Technique	Operating principle/main characteristics	Main positive attributes	Main deficiencies
First-order gradient-based methods (RG, CG, GRG)	○ Iterative search based on 1st-order gradient of objective ○ Constraints handled by adding penalty terms to objective function to form Lagrangian ○ GRG uses successive constraint set linearization and slack variables to handle inequality constraints	○ Earliest approaches to algorithmically solve OPF ○ Unsophisticated, reliable, at moderate computational expense ○ Suitable for large-scale application (e.g. CG, due to low memory requirement)	○ Slow convergence rate; RG susceptible to zig-zag behaviour close to optimal point ○ Difficulty handling inequality constraints ○ Need for smoothness of objective function ○ Can only find local optima
Second-order gradient-based methods (Newton, Quasi-Newton)	○ Approach similar to 1st-order methods, with additional incorporation of 2nd-order derivative information of objective function ○ Newton's algorithm is the representative method under the category ○ Quasi-Newton methods approximate the 2nd-order derivative information to reduce computational expense	○ Very effective methods; addition of 2nd-order derivative information significantly improves convergence rate ○ Efficient and robust for a large class of problems, under some mild assumptions (e.g. sufficient accuracy of quadratic approximate model in vicinity of solution)	○ Similar issues as those of 1st-order methods, except having higher convergence rate ○ Convergence requires Hessian matrix to be positive semidefinite ○ Computationally more expensive than 1st-order methods
(Sequential) Quadratic Programming (SQP)	○ Special case of NLP with quadratic objective function and linear constraint set ○ Involves iterative quadratic and linear approximation of objective and constraint functions respectively about the current iterate	○ Achieves good balance between model accuracy and computational expense (accuracy of QP and speed of LP) ○ Efficient solution techniques available that can solve the QP formulation effectively and reliably	○ Has similar drawbacks as those outlined above of gradient-based methods ○ Convergence requires Hessian matrix of quadratic approximation to be positive semidefinite
Linear Programming (LP)	○ Both objective and constraint functions are linear ○ For VVO problem, this entails successive linearization of objective and constraint functions about the current operating point	○ Reliable, good convergence characteristics, even for large-scale problems ○ Fast computational speed ○ Availability of efficient solution techniques (typically Simplex and Interior-Point Methods)	○ Loss of accuracy due to linearization may lead to solution that's not only non-optimal, but perhaps even infeasible for original nonlinear problem ○ For successive linear formulation, solution is only locally optimal
Interior-Point Methods (IPM)	○ Make use of path-following techniques that confine the search path within the feasible region ○ Initially developed for application to LP as alternative to Simplex method; been extended to the treatment of NLP problems	○ Very effective and efficient, especially for large-scale problems, both linear and nonlinear ○ Rapid convergence, more effective constraint handling than classical gradient-based methods ○ Better handling of infeasibility	○ Reliability concerns for particularly difficult problems ○ Combines mathematical rigour with some heuristics; proper parameter selection (e.g. barrier parameters) can be challenging, affecting effectiveness and convergence properties
Mixed-Integer Programming (MIP)	○ Explicitly considers both continuous and discrete variables, thus a more accurate model for the VVO problem ○ Decomposition techniques made use of to treat continuous and discrete portions as subproblems ○ Branch-and-bound the main solution technique for MINLP formulations	○ A more accurate solution is achieved without the need to round off values of discrete variables so as to enforce integrality constraints, which may lead to non-optimality, even infeasibility relative to the exact formulation	○ Computationally more expensive, and requires sophisticated techniques to simultaneously handle both continuous and discrete variables ○ Issues similar to classical gradient-based methods of difficulty of inequality constraint handling, only locally optimal

Expert systems are especially applicable to fields such as power system operation, where a wealth of system operational knowledge and expertise has been accumulated, and can be used to build intelligent decision support systems that can aid system operators in making decisions and taking quick action especially under anomalous conditions, where not only correct action, but also speed of execution can be critical in preventing major emergencies [147].

A number of researchers have built expert systems for optimal reactive power dispatch and voltage control, examples

TABLE 4. Summary of main characteristics of nonconventional/heuristic optimization techniques.

Technique	Operating principle/main characteristics	Main positive attributes	Main deficiencies
Genetic algorithm (GA)	○ Population-based evolutionary stochastic search algorithm, modeled after mechanics of natural genetics, incorporating crossover, mutation and selection ○ Adaptation through use of genetic operators, enabling use of historical information, and randomized information exchange among population candidates, is what characterizes the search for the optimal point	○ Conceptually and computationally simple, yet efficient, effective and robust search mechanism applicable to various problem classes ○ Not limited by the properties of convexity, smoothness, unimodality of objective function, requirements typical of classical techniques ○ Can achieve global convergence and global optimality	○ Computation time can be long, thus limited scalability (although parallelization is possible to improve computational efficiency) ○ Effectiveness a function of design aspects like encoding, choice of fitness function, and other parameters of the algorithm
Evolutionary Programming (EP)	○ Evolutionary stochastic search algorithm using simulated evolution to evolve a population of candidate solutions of "increasing intellect" in search of optimal point ○ Stresses mutation (rather than crossover, opposed to GA)	○ Capable of global convergence and global optimality, by judicious choice of mutation and selection-by-fitness mechanisms (which can be either elitist or by stochastic tournament)	○ Computationally quite expensive for OPF problems, which typically have thousands of variables and constraints
Particle Swarm Optimization (PSO)	○ Modeled after swarm behavior of natural creatures, where an individual makes decisions based on best own experience and best group experience ○ Population of particles evolved towards the optimal point by modifying each particle's position as a function of its current best and group's best position	○ Simple both conceptually and in terms of implementation; simple rules governing individual particle behavior can result in sophisticated swarm behaviour ○ Many enhancements to the standard algorithm are possible, to enable application to a wide variety of problems	○ Convergence properties are highly influenced by parameter selection for the algorithm
Fuzzy Set Theory-based methods	○ Fuzzy set theory used as a tool for modeling uncertainty present in objective, constraint functions, and system parameters ○ Objective, constraint functions are represented as fuzzy sets; membership to these sets represents the degree of closeness to optimum point and degree of enforcement of constraints	○ Provides a better compromised solution, balancing robustness (i.e. insensitivity to parameter variations) and optimality ○ Furnishes capability to handle ambiguity, conflicting objectives, and soft constraints in a flexible way that can improve computational complexity	○ Not actually an optimization problem, only a way to handle uncertainty and imprecision; thus largely needs to be combined with an optimization technique; this may affect overall complexity and effectiveness, depending on problem formulation and optimization algorithm used
Expert System (ES)	○ Uses a computer-based representation of human expert knowledge, in conjunction with an information database and an inference engine to solve complex problems requiring human expertise and experience ○ Main components are the knowledge base, database, inference engine, and interfaces to the user and to other programs needed to execute the system's functionalities	○ Accumulated wealth of knowledge in the field can be exploited to build an intelligent decision support system to assist system operators respond quickly and effectively especially under anomalous conditions ○ Has desirable attributes of efficiency, reproducibility, consistency, and opportunity for expertise consolidation	○ Lacks many natural strengths of a human expert operator, such as common sense, creativity, adaptability, learning ability

of which are to be found in [148]–[153]. In many of these applications of expert systems to the reactive power/voltage control problem, the emphasis is placed on leveraging human expert knowledge and experience, and historical information to build a system that can quickly provide effective remedial action in emergency conditions, when human operator reaction may be too slow, and conventional optimization methods ineffective [148]. In reference [149], an ES is built that applies empirical rules to generate appropriate controls when slight voltage violations occur, whereas mathematical programming software is used to address more severe contingencies. In [150] an ES is developed for reactive power and voltage control based on a sensitivity-tree approach, where the most effective control measures to alleviate abnormal voltage conditions are determined on the basis of the rule base coded into the ES. Scalability and possibility for real-time

TABLE 5. Comparison of conventional with nonconventional/heuristic optimization techniques.

Characteristic	Classical/conventional optimization techniques	Nonconventional/heuristic optimization techniques
Computational speed	Varies among the various techniques, but *generally faster than heuristic techniques*	*Generally slower than conventional techniques*, largely due to dependence on heuristic search, thus a function of parameter selection
Reliability/quality of solution	A function of problem formulation; e.g. LP is generally reliable (as regards convergence), while Newton's algorithm requires sufficient accuracy of quadratic approximate model relative to original model at each iterate	Mainly a function of parameter selection; no theoretical guarantees can be made generally
Accuracy of model/solution	Somehow a trade-off between accuracy and complexity; e.g. LP formulation (of VVO problem) is fast, but only approximate; MINLP is accurate but also computationally expensive	Generally more versatile, able to handle problems of varying detail and accuracy, often not requiring a precise mathematical model; not dependent on such properties as smoothness of functions, etc.
Robustness	Exhibit sensitivity to initial starting point (e.g. Newton's algorithm), degree of nonlinearity, ill-conditioned nature of problem, and other problem parameters	Many techniques employ heuristics that make them robust, i.e. able to handle problems of diverse characteristics and parameters
Complexity of solution technique	Well-grounded theoretically, well-understood and quite straightforward to implement algorithmically	Heuristic nature of the methods generally requires domain expertise; parameter tuning often requires experience and good understanding of the problem
Handling of discrete variables	Not well-suited to handle discrete variables	Many have ability to handle discrete, even mixed-integer problems quite naturally
Convergence properties	Generally a function of problem definition and such factors as initial starting point and system parameters (e.g. nature of Hessian matrix in case of 2^{nd}-order methods)	Can generally achieve global convergence, independent of problem formulation, although computational expense may be a limiting factor
Global optimality properties	All local solvers, global optimality only achievable in the case of convex problems (which VVO problem is not)	Many can achieve global optimality, although computational expense may be high, if not prohibitive

application are highlighted as the main characteristics of the proposed system. A similar sensitivity-based approach has been used in the ES developed in [151] for reactive power control for voltage profile improvement. In [152], historical information and real-time data is exploited to develop an ES for substation voltage and reactive power control. The opportunity to leverage years of operating experience in developing the ES is highlighted as one of the main advantages of this approach. The ES developed in [153] is focused on monitoring and improving power system voltage stability. The ES can use the empirical knowledge in the knowledge base to effectively identify the critical load buses most susceptible to excessive voltage violations, and recommend the most effective remedial actions, as an aid to the system operator.

Expert systems present many advantages as intelligent decision support systems where decisions have to be made to solve complex problems, as in the case of reactive power/voltage control under emergency conditions. Notable among these advantages are the opportunity to combine the knowledge and experience of several human experts, accumulated over a period of time, along with historical information, to build an efficient and effective decision-support system, little reliance on precise mathematical models of the system, thus especially effective under anomalous operating conditions, and others such as reproducibility, consistency, and lack of fatigue (which human operators are very susceptible to). Some obvious disadvantages of expert systems are that they lack the human capabilities of common sense, creativity, and learning. There is also the likelihood of gradual degradation of the system, requiring periodical update of the rule base to remain up to date as the modeled system undergoes any changes [146].

V. COMPARATIVE ANALYSIS OF SOLUTION APPROACHES FOR THE VVO PROBLEM

It is quite evident that the two classes of optimization techniques discussed in the preceding section present diverse characteristics, both in terms of operating principle, as well as strengths and drawbacks when gauged against the desired performance characteristics outlined at the beginning of section IV.

The key characteristic of classical/conventional optimization methods is their implementation of a mathematically rigorous and systematic iterative procedure in the search for the optimal solution to an optimization problem within the feasible space. They do differ, however, in key performance metrics, such as accuracy, speed, reliability, convergence characteristics, and effectiveness of handling inequality constraints and discrete variables, among other criteria. Collectively, the class of conventional optimization methods suffer from a number of significant deficiencies or drawbacks, notably the inherent difficulty of handling discrete variables, the requirement for the (nonlinear) objective and constraint functions to be smooth (i.e. for the gradient-based methods), and the difficulty of handling nonconvexity in nonlinear problems (meaning they can only find local optimal solutions) [21].

Heuristic optimization techniques employ a variety of optimum-seeking strategies that differ conceptually from those employed in conventional optimization methods. By and large, these techniques make use of a population of candidate search points, which, coupled with their stochastic nature, generally gives them global search characteristics (that is, the ability to globally converge to a solution where one exists, independently of the initial point, and to find the globally optimal solution, despite nonconvexity of the objective function and the feasible region). They do suffer some drawbacks, however, when compared with the conventional methods, such as lacking mathematical rigor (by virtue of their heuristic nature), being relatively computationally more expensive, and their effectiveness being very dependent on judicious choice of the algorithm parameters.

Tables 3 to 5 present a succinct summary of the salient characteristics of all the optimization methods that have been discussed in this review. Tables 3 and 4 present details for the conventional and heuristic optimization techniques respectively, and table 5 presents a high-level comparative analysis of the conventional and heuristic techniques. The comparison is made on the basis of some of the key performance characteristics for an optimization technique that have been outlined in section IV, such as computational speed, reliability, robustness, convergence and global optimality properties, among others. The tables provide a general overview of the relative strengths and shortcomings of the two classes of methods, which should prove to be informative to researchers and other practitioners in the field of engineering optimization.

VI. CONCLUSION AND FUTURE WORK

Volt/VAR optimization is one of the key operational tools needed by electric power system operators, and has a significant impact on the security, economy, technical viability and efficiency of system operation. It is also one of the most complex optimization problems to solve, being nonlinear, nonconvex and involving both continuous and discrete variables. This paper has presented a survey of the formulation of the problem (encompassing the objectives, decision variables and constraints), as well as a thorough discussion of the various solution techniques that have been applied to the problem over the years.

The challenge of efficiently and effectively solving the VVO problem is reflected in the diversity of the solution techniques that have been applied to the problem, which exhibit varying characteristics, both in operating principle and how effectively they address the key performance characteristics of the optimization problem. Conventional optimization methods have proven to be efficient, reliable, fast and quite straightforward to algorithmically implement, but suffer from significant drawbacks when applied to the VVO problem, as discussed in section IV.A. Particularly, shortcomings exist in their convergence and global optimality properties, and the difficulty in the handling of inequality constraints and discrete variables. The nonconventional/heuristic optimization techniques present some advantages exactly where the conventional techniques fall short, such as superior global search characteristics, thus having the ability to achieve global convergence and global optimality independently of the problem formulation, and the natural ability to handle discrete variables. Their main drawbacks are that their heuristic nature implies that parameter selection weighs heavily on their efficiency and effectiveness, and they incur relatively greater computational expense.

Multi-faceted approaches are clearly needed to devise novel operational techniques that can deal with the emerging complexities as power systems become ever more complex, due to such developments as deregulation, electricity markets, proliferation of distributed renewable generation, and smart grid initiatives. Therefore, the authors plan to continue their future work in this area by developing and implementing VVO approaches that combine conventional and heuristic methods in order to take advantage of both classes' complementary strengths. Parallel computing will be a significant emphasis of the research, as a way to save time, save money, solve complex problems and leverage remote resources, which will help to increase the scalability of the developed methods and make the optimization problem more computationally tractable. This work will be discussed in more detail in future publications.

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