New Stability Results of the Delay Dynamical System via a Novel Relaxed Condition

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ABSTRACT This paper focuses on the asymptotic stability (AS) problem of the delay dynamical system (DDS). An improved delay-product-type functional (DPTF) strategy is applied to construct a more general Lyapunov-Krasovskii functional (LKF). This method introduces more relevant information and improves the information capacity of LKF. In addition, the definition of the conditions $V_1(x_t)$ and $V_5(x_t)$ are further relaxed, thereby developing new relaxed conditions. Then, based on the appropriate integral inequalities and the reciprocally convex combination lemma (RCCL). Finally, the feasibility of the method in this paper is verified based on two numerical examples.

INDEX TERMS Time-varying delay, delay-product-type functional method, relaxed condition, novel Lyapunov-Krasovskii functional.

I. INTRODUCTION

A dynamic system refers to a system whose state changes over time [1]–[3]. The state of system operation changes with time. Therefore, the current moment of the state can be described by the change of time. In the real world, any system is in constant motion [4], [5]. Only when the state of the system changes significantly over time in motion is a dynamic system. Therefore, dynamic systems are inseparable in our life and production [6]–[8].

However, TD has always existed in the real world. TD appears in some engineering application systems such as communication systems, T-S fuzzy control systems, neural network systems, and power grid systems [9]–[11]. The appearance of TD in the system may cause system performance degradation and affect the normal operation of the system [12]–[14]. Therefore, how to solve this problem has

become one of the critical issues that scholars have paid attention to in recent years.

The proposal of the LKF method has a new direction for dealing with the TD problem in the system. At present, many scholars have conducted in-depth research on this method. They built many different LKFs and provided a variety of construction methods [15]–[19]. In [15], the author studies the sliding mode control of the system based on the LKF method. The authors focused on the problem of linear systems with TVD in [16]. In [17], an improved delay-dependent was proposed. In [18], the authors proposed that the integral inequality of the quadratic function plays an important role in the stability of the system. In [19], Based on the exponential stability of the neural network, the author studied the TVD state of the packet interval. In [28], the author focused on the control problem of the chaotic Lur'e system with TD. In [29]–[31], the author used the T-S fuzzy method to solve some nonlinear problems of the DDSs.

So far, there are many different LKF construction methods. Suppose the cross-sectional area of LKF is increased.

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In that case, this method will not reduce the conservativeness of the result much, and it will also increase the heavy calculation of the computer. Then, how to get a suitable LKF has become the key issue of this paper, which has stimulated the interest of this paper. By comparing a variety of different research methods, this paper proposes an improved DPTF method, which can effectively reduce the constraints of the criterion, thereby achieving a reduction in the conservativeness of the criterion.

On the basis of the above discussion, this paper considers the AS of the DDS. Compared with the methods proposed by existing scholars, the highlights of this article are as follows.

- First, the improved DPTF is applied to the research on the structure of LKF.
- Secondly, new relaxed conditions are proposed, especially the constraints of $V_1(x_t)$ and $V_5(x_t)$ are further relaxed, which promotes reduce the conservativeness of the guidelines and improve the control performance of the criteria.
- Finally, augmented RCCL inequalities and new boundary techniques, a novel optimized criterion is developed.

Notation: $\mathcal{I}_n$ and $0_{m \times n}$ represent the $n \times n$ identity matrix and $m \times n$ zero matrix respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\| \cdot \|$ is the Euclidean norm of a vector. $P > 0 (\geq 0)$ represents that P is a positive definite matrix. $\text{diag}\{D_1, D_2, \ldots, D_N\}$ denotes a block-diagonal (or diagonal) matrix and the diagonal elements are $D_k, k = 1, 2, \ldots, N$. A^{-1} and A^T are the inverse and transpose of matrix A. The symbol $*$ represents symmetric terms in a symmetric matrix (SM).

II. PRELIMINARIES

Consider the following DDS:

$$\begin{cases} \dot{x}(t) = \mathcal{A}x(t) + \mathcal{A}_d x(t - \sigma(t)), \\ x(t) = \psi(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ indicates the state vector, $\psi(t)$ means the initial condition, $\mathcal{A}$ and $\mathcal{A}_d \in \mathbb{R}^{n \times n}$ are system matrices.

The $\sigma(t)$ is a time-varying delay (TVD) differentiable function and it satisfies:

$$0 \leq \sigma(t) \leq \sigma, \quad \mu_1 \leq \dot{\sigma}(t) \leq \mu_2, \quad (2)$$

where σ and μ_1, μ_2 are given constants.

Lemma 1 [25]: There is a matrix $\mathcal{R} > 0$ such that all $\varsigma(s)$ in $[c, d] \longrightarrow \mathbb{R}^n$ satisfy the following inequality:

$$\int_c^d \varsigma^T(s)\mathcal{R}\varsigma(s)\mathrm{d}s \geq \frac{1}{d-c}\left(\int_c^d \varsigma(s)\mathrm{d}s\right)^T \mathcal{R}$$
$$\times \left(\int_c^d \varsigma(s)\mathrm{d}s\right) + \frac{3}{d-c}\Omega^T \mathcal{R}\Omega,$$

where $\Omega = \int_c^d \omega(s)\mathrm{d}s - \frac{2}{d-c}\int_c^d \int_\theta^d \omega(s)\mathrm{d}s\mathrm{d}\theta$.

Lemma 2 [25]: For a real scalar $\beta \in (0, 1)$, SMs $\chi_1 > 0$, $\chi_2 > 0$, any matrices ς_1, ς_2, we get the following result:

$$\begin{bmatrix} \frac{1}{\beta}\chi_1 & 0 \\ 0 & \frac{1}{1-\beta}\chi_2 \end{bmatrix} \geq \begin{bmatrix} \chi_1 + (1-\beta)\daleth_1 & (1-\beta)\varsigma_1 + \beta\varsigma_2 \\ * & \chi_2 + \alpha\daleth_2 \end{bmatrix},$$

where $\daleth_1 = \chi_1 - \varsigma_2 \chi_2^{-1} \varsigma_2^T$, $\daleth_2 = \chi_2 - \varsigma_1^T \chi_1^{-1} \varsigma_1$.

III. MAIN RESULTS

Theorem 1: For given scalars μ_1, μ_2 and σ. System (1) is AS with any TVD $\sigma(t)$ satisfying (2), if there have matrices $\mathcal{R} > 0 \in \mathbb{R}^{2n \times 2n}$ and $\mathcal{T} > 0 \in \mathbb{R}^{2n \times 2n}$. SMs $\mathcal{P} \in \mathbb{R}^{5n \times 5n}$, $\mathcal{Q}_{1a} \in \mathbb{R}^{2n \times 2n}$, $\mathcal{Q}_{2a} \in \mathbb{R}^{2n \times 2n}$, $\mathcal{Q}_{1b} \in \mathbb{R}^{2n \times 2n}$, $\mathcal{Q}_{2b} \in \mathbb{R}^{2n \times 2n}$, $\mathcal{U}_1 \in \mathbb{R}^{2n \times 2n}$, $\mathcal{U}_2 \in \mathbb{R}^{2n \times 2n}$, $\mathcal{M}_1 \in \mathbb{R}^{n \times n}$ and $\mathcal{M}_2 \in \mathbb{R}^{n \times n}$, any matrices $\mathcal{S}_1 \in \mathbb{R}^{4n \times 4n}$, $\mathcal{S}_2 \in \mathbb{R}^{4n \times 4n}$, $\mathcal{S}_3 \in \mathbb{R}^{2n \times 2n}$, $\mathcal{S}_4 \in \mathbb{R}^{2n \times 2n}$, $\mathcal{W}_1 \in \mathbb{R}^{n \times n}$ and $\mathcal{W}_2 \in \mathbb{R}^{n \times n}$ so as to the following inequalities hold for

$$\overline{\mathcal{Q}}_{1,\sigma(t)} \geq 0, \quad \overline{\mathcal{Q}}_{2,\sigma(t)} \geq 0, \quad \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t)) > 0, \quad (3)$$

$$\Psi_{\sigma(t)} > 0, \quad \overline{\Omega}_2(\dot{\sigma}(t)) > 0, \quad (\Gamma^\perp)^T \Sigma_{\sigma(t)|\dot{\sigma}(t)}(\Gamma^\perp) < 0, \quad (4)$$

where other equations are defined in APPENDIX B.

Proof: This paper choose the following LKF for system:

$$V(x_t) = \sum_{i=1}^5 V_i(x_t), \quad (5)$$

where

$$V_1(x_t) = \alpha^T(t)\mathcal{P}\alpha(t),$$

$$V_2(x_t) = \int_{t-\sigma(t)}^t \eta^T(s)\overline{\mathcal{Q}}_{1,\sigma(t)}\eta(s)\mathrm{d}s$$
$$+ \int_{t-\sigma}^{t-\sigma(t)} \eta^T(s)\overline{\mathcal{Q}}_{2,\sigma(t)}\eta(s)\mathrm{d}s,$$

$$V_3(x_t) = \sigma \int_{-\sigma}^0 \int_{t+u}^t \eta^T(s)\mathcal{R}\eta(s)\mathrm{d}u\mathrm{d}s,$$

$$V_4(x_t) = \int_{t-\sigma}^t \int_\theta^t \int_u^t \eta^T(s)\mathcal{T}\eta(s)\mathrm{d}s\mathrm{d}u\mathrm{d}\theta,$$

$$V_5(x_t) = \sigma(t)\beta^T(t)\mathcal{U}_1\beta(t) + (\sigma - \sigma(t))\varepsilon^T(t)\mathcal{U}_2\varepsilon(t).$$

First of all, relaxed the positive definite condition of $V(x(t))$, the $V_1(x_t) + V_5(x_t)$ can be considered together and written as

$$V_1(x_t) + V_5(x_t)$$
$$= \alpha^T(t)\left(\mathcal{P} + \sigma(t)\begin{bmatrix} \overline{e}_1 \\ \frac{\overline{e}_2}{\sigma(t)} \end{bmatrix}^T \mathcal{U}_1 \begin{bmatrix} \overline{e}_1 \\ \frac{\overline{e}_2}{\sigma(t)} \end{bmatrix}\right.$$
$$\left. + (\sigma - \sigma(t))\begin{bmatrix} \overline{e}_1 \\ \frac{\overline{e}_3}{\sigma-\sigma(t)} \end{bmatrix}^T \mathcal{U}_2 \begin{bmatrix} \overline{e}_1 \\ \frac{\overline{e}_3}{\sigma-\sigma(t)} \end{bmatrix}\right)\alpha(t)$$
$$= \alpha(t)\left(\mathcal{P} + \mathcal{F}_1^T(\sigma(t)\mathcal{U}_1 + (\sigma - \sigma(t))\mathcal{U}_2)\mathcal{F}_1\right.$$
$$\left. + \text{Sym}\{\mathcal{F}_2^T\mathcal{U}_1\mathcal{F}_2 + \mathcal{F}_1^T\mathcal{U}_2\mathcal{F}_3\} + \frac{\mathcal{F}_2^T\mathcal{U}_1\mathcal{F}_2}{\sigma(t)} + \frac{\mathcal{F}_3^T\mathcal{U}_2\mathcal{F}_3}{\sigma - \sigma(t)}\right)$$
$$\times \alpha(t). \quad (6)$$

If $\Psi_{\sigma(k)} > 0$ holds, conditions $\mathcal{U}_1 > 0, \mathcal{U}_2 > 0$ and $\mathcal{P} + \mathcal{U}_{\mathcal{F}} - \frac{\sigma(t)}{\sigma^2}\mathcal{F}_3^T\mathcal{S}_3^T\mathcal{U}_1^{-1}\mathcal{S}_3\mathcal{F}_3 - \frac{\sigma - \sigma(t)}{\sigma}\mathcal{F}_2^T\mathcal{S}_4\mathcal{U}_2^{-1}\mathcal{S}_4^T\mathcal{F}_2 > 0$ can be got. Thus, using Lemma 2, for any matrices S_3 and S_4, $\frac{\mathcal{F}_2^T\mathcal{U}_1\mathcal{F}_2}{\sigma(t)} + \frac{\mathcal{F}_3^T\mathcal{U}_2\mathcal{F}_3}{\sigma - \sigma(t)}$ can be estimated as

$$
\begin{aligned}
&\frac{\mathcal{F}_2^T\mathcal{U}_1\mathcal{F}_2}{\sigma(t)} + \frac{\mathcal{F}_3^T\mathcal{U}_2\mathcal{F}_3}{\sigma - \sigma(t)} \\
&= \begin{bmatrix} \mathcal{F}_2 \\ \mathcal{F}_3 \end{bmatrix}^T \begin{bmatrix} \frac{1}{\sigma(t)}\mathcal{U}_1 & 0 \\ 0 & \frac{1}{\sigma - \sigma(t)}\mathcal{U}_2 \end{bmatrix} \begin{bmatrix} \mathcal{F}_2 \\ \mathcal{F}_3 \end{bmatrix} \\
&\geq \frac{2\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{U}_1\mathcal{F}_2 + \frac{\sigma + \sigma(t)}{\sigma^2}\mathcal{F}_3^T\mathcal{U}_2\mathcal{F}_3 \\
&\quad + \mathrm{Sym}\left\{ \frac{\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_3\mathcal{F}_3 + \frac{\sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_4\mathcal{F}_3 \right\} \\
&\quad - \frac{\sigma(t)}{\sigma^2}\mathcal{F}_3^T\mathcal{S}_3^T\mathcal{U}_1^{-1}\mathcal{S}_1\mathcal{F}_3 - \frac{\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_4\mathcal{U}_2^{-1}\mathcal{S}_2^T\mathcal{F}_2.
\end{aligned}
\tag{7}
$$

Thus, $V_1(x_t) + V_5(x_t)$ can be further estimated as

$$
\begin{aligned}
&V_1(x_t) + V_5(x_t) \\
&\geq \alpha^T(t)(\mathcal{P} + \mathcal{F}_1^T(\sigma(t)\mathcal{U}_1 + (\sigma - \sigma(t))\mathcal{U}_2)\mathcal{F}_1 \\
&\quad + \mathrm{Sym}\{\mathcal{F}_1^T\mathcal{U}_1\mathcal{F}_2 + \mathcal{F}_1^T\mathcal{U}_2\mathcal{F}_3\}\frac{2\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{U}_1\mathcal{F}_2 \\
&\quad + \frac{\sigma + \sigma(t)}{\sigma^2}\mathcal{F}_3^T\mathcal{U}_2\mathcal{F}_3 + \mathrm{Sym}\left\{ \frac{\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_3\mathcal{F}_3 \right. \\
&\quad \left. + \frac{\sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_4\mathcal{F}_3 \right\} - \frac{\sigma(t)}{\sigma^2}\mathcal{F}_3^T\mathcal{S}_3^T\mathcal{U}_1^{-1}\mathcal{S}_3\mathcal{F}_3 \\
&\quad - \frac{\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_4\mathcal{U}_2^{-1}\mathcal{S}_4^T\mathcal{F}_2 \bigg) \alpha(t) \\
&= \alpha^T(t)\left(\mathcal{P} + \mathcal{U}_{\mathcal{F}} - \frac{\sigma(t)}{\sigma^2}\mathcal{F}_3^T\mathcal{S}_3^T\mathcal{U}_1^{-1}\mathcal{S}_3\mathcal{F}_3 \right. \\
&\quad \left. - \frac{\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T\mathcal{S}_4\mathcal{U}_2^{-1}\mathcal{S}_4^T\mathcal{F}_2 \right) \alpha(t).
\end{aligned}
\tag{8}
$$

According to the above analysis, using RCCL, $V_1(x_t) + V_5(x_t) > \epsilon \| x(t) \|^2$ can guarantee a small enough one $\epsilon > 0$, if $\Psi_\sigma > 0$ and $\Psi_0 > 0$ hold.
The $\dot{V}_i(x_t)$ as shown below

$$
\dot{V}(x_t) = \sum_{i=1}^{5} \dot{V}_i(x_t).
$$

$$
\begin{aligned}
\dot{V}_1(x_t) &= 2\alpha^T(t)\mathcal{P} \\
&\quad \times \begin{bmatrix} \dot{x}(t) \\ x(t) - (1 - \dot{\sigma}(t))x(t - \sigma(t)) \\ (1 - \dot{\sigma}(t))x(t - \sigma(t)) - x(t - \sigma) \\ \sigma(t)x(t) - (1 - \dot{\sigma}(t))\sigma(t)\Theta_1(t) \\ (1 - \dot{\sigma}(t))(\sigma - \sigma(t))x(t - \sigma(t)) - (\sigma - \sigma(t))\Theta_2(t) \end{bmatrix} \\
&= \xi^T(t)\Xi_1\xi(t).
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
\dot{V}_2(x_t) &= \xi^T(t)\Xi_2\xi(t) \\
&\quad - \dot{\sigma}(t)\int_{t-\sigma(t)}^{t} \eta^T(s)\mathcal{Q}_{1b}\eta(s)\mathrm{d}s \\
&\quad - \dot{\sigma}(t)\int_{t-\sigma}^{t-\sigma(t)} \eta^T(s)\mathcal{Q}_{2b}\eta(s)\mathrm{d}s.
\end{aligned}
\tag{10}
$$

$$
\dot{V}_3(x_t) = \sigma^2\eta^T(t)\mathcal{R}\eta(t) - \sigma\int_{t-\sigma}^{t} \eta^T(s)\mathcal{R}\eta(s)\mathrm{d}s.
\tag{11}
$$

$$
\begin{aligned}
\dot{V}_4(x_t) &= \frac{\sigma^2}{2}\eta^T(t)\mathcal{T}\eta(t) \\
&\quad - \int_{t-\sigma(t)}^{t}\int_{\theta}^{t} \eta^T(s)\mathcal{T}\eta(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad - \int_{t-\sigma}^{t-\sigma(t)}\int_{\theta}^{t-\sigma(t)} \eta^T(s)\mathcal{T}\eta(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad - (\sigma - \sigma(t))\int_{t-\sigma(t)}^{t} \eta^T(s)\mathcal{T}\eta(s)\mathrm{d}s.
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\dot{V}_5(x_t) &= \dot{\sigma}(t)\beta^T(t)\mathcal{U}_1\beta(t) + 2\sigma(t)\beta^T(t)\mathcal{U}_1\dot{\beta}(t) \\
&\quad - \dot{\sigma}(t)\varepsilon^T(t)\mathcal{U}_2\varepsilon(t) + 2(\sigma - \sigma(t))\varepsilon^T(t)\mathcal{U}_2\dot{\varepsilon}(t) \\
&= \dot{\sigma}(t)\beta^T(t)\mathcal{U}_1\beta(t) - \dot{\sigma}(t)\varepsilon^T(t)\mathcal{U}_2\varepsilon(t) \\
&\quad + 2\sigma(t)\beta^T(t)\mathcal{U}_1 \\
&\quad \times \begin{bmatrix} \dot{x}(t) \\ \frac{x(t) - (1 - \dot{\sigma}(t))x(t - \sigma(t)) - \dot{\sigma}(t)\Theta_1(t)}{\sigma(t)} \end{bmatrix} \\
&\quad + 2(\sigma - \sigma(t))\varepsilon^T(t)\mathcal{U}_2 \\
&\quad \times \begin{bmatrix} \dot{x}(t) \\ \frac{(1 - \dot{\sigma}(t))x(t - \sigma(t)) - x(t - \sigma) + \dot{\sigma}(t)\Theta_2(t)}{\sigma - \sigma(t)} \end{bmatrix} \\
&= \xi^T(t)\Xi_4\xi(t).
\end{aligned}
\tag{13}
$$

The following zero equation applies to SMs $\mathcal{M}_1$ and $\mathcal{M}_2$, we have

$$
0 = \sigma\left[x^T(t)\mathcal{M}_1x(t) - x^T(t - \sigma(t))\mathcal{M}_1x(t - \sigma(t)) \right.
$$
$$
\left. - 2\int_{t-\sigma(t)}^{t} x^T(s)\mathcal{M}_1\dot{x}(s)\mathrm{d}s \right],
\tag{14}
$$

$$
0 = \sigma\left[x^T(t - \sigma(t))\mathcal{M}_2x(t - \sigma(t)) - x^T(t - \sigma)\mathcal{M}_2 \right.
$$
$$
\left. \times x(t - \sigma) - 2\int_{t-\sigma}^{t-\sigma(t)} x^T(s)\mathcal{M}_2\dot{x}(s)\mathrm{d}s \right].
\tag{15}
$$

Based on (14) and (15), we have

$$
\begin{aligned}
\Upsilon_1 &= -\sigma\int_{t-\sigma(t)}^{t} \eta^T(s)\left(\mathcal{R} + \frac{\dot{\sigma}(t)}{\sigma}\mathcal{Q}_{1b} \right. \\
&\quad \left. + \frac{\sigma - \sigma(t)}{\sigma}\mathcal{T} + \begin{bmatrix} 0 & \mathcal{M}_1 \\ \mathcal{M}_1 & 0 \end{bmatrix} \right) \eta(s)\mathrm{d}s,
\end{aligned}
$$

$$
\begin{aligned}
\Upsilon_2 &= -\sigma\int_{t-\sigma}^{t-\sigma(t)} \eta^T(s)\left(\mathcal{R} \right. \\
&\quad \left. + \frac{\dot{\sigma}(t)}{\sigma}\mathcal{Q}_{2b} + \begin{bmatrix} 0 & \mathcal{M}_2 \\ \mathcal{M}_2 & 0 \end{bmatrix} \right) \eta(s)\mathrm{d}s.
\end{aligned}
$$

Next utilizing Lemma 1, Υ_1 and Υ_2, can obtain the following inequality.

$$
\begin{aligned}
\Upsilon_1 &\leq -\frac{\sigma}{\sigma(t)}\begin{bmatrix} \int_{t-\sigma(t)}^{t} \eta(s)\mathrm{d}s \\ \int_{t-\sigma(t)}^{t} \eta(s)\mathrm{d}s - \frac{2}{\sigma(t)}\int_{t-\sigma(t)}^{t}\int_{\theta}^{t} \eta(s)\mathrm{d}s\mathrm{d}\theta \end{bmatrix}^T \\
&\quad \times \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\begin{bmatrix} \int_{t-\sigma(t)}^{t} \eta(s)\mathrm{d}s \\ \int_{t-\sigma(t)}^{t} \eta(s)\mathrm{d}s - \frac{2}{\sigma(t)}\int_{t-\sigma(t)}^{t}\int_{\theta}^{t} \eta(s)\mathrm{d}s\mathrm{d}\theta \end{bmatrix} \\
&= -\frac{\sigma}{\sigma(t)}\xi^T(t)(\mathcal{N}_1 + \sigma(t)\mathcal{N}_2)^T\overline{\Omega}_1(\sigma(t), \dot{\sigma}(t)) \\
&\quad \times (\mathcal{N}_1 + \sigma(t)\mathcal{N}_2)\xi(t) \\
&= -\xi^T(t)(\frac{\sigma}{\sigma(t)}\mathcal{N}_1^T\overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\mathcal{N}_1 \\
&\quad + \mathrm{Sym}\{\sigma\mathcal{N}_1^T\overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\mathcal{N}_2\} \\
&\quad + \sigma \cdot \sigma(t)\mathcal{N}_2^T\overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\mathcal{N}_2)\xi(t),
\end{aligned}
\tag{16}
$$

$$\Upsilon_2 \leq -\frac{\sigma}{\sigma - \sigma(t)} \left[\begin{array}{c} \int_{t-\sigma}^{t-\sigma(t)} \eta(s) ds \\ \int_{t-\sigma}^{t-\sigma(t)} \eta(s) ds - \frac{2}{\sigma-\sigma(t)} \int_{t-\sigma}^{t-\sigma(t)} \int_{\theta}^{t-\sigma(t)} \eta(s) ds d\theta \end{array} \right]^T$$
$$\times \overline{\Omega}_2(\dot{\sigma}(t))$$
$$\times \left[\begin{array}{c} \int_{t-\sigma}^{t-\sigma(t)} \eta(s) ds \\ \int_{t-\sigma}^{t-\sigma(t)} \eta(s) ds - \frac{2}{\sigma-\sigma(t)} \int_{t-\sigma}^{t-\sigma(t)} \int_{\theta}^{t-\sigma(t)} \eta(s) ds d\theta \end{array} \right]$$
$$= -\frac{\sigma}{\sigma - \sigma(t)} \xi^T(t) (\mathcal{N}_3 + (\sigma - \sigma(t)) \mathcal{N}_4)^T \overline{\Omega}_2(\dot{\sigma}(t))$$
$$\times (\mathcal{N}_3 + (\sigma - \sigma(t)) \mathcal{N}_4) \xi(t)$$
$$= -\xi^T(t) (\frac{\sigma}{\sigma - \sigma(t)} \mathcal{N}_3^T \overline{\Omega}_2(\dot{\sigma}(t)) \mathcal{N}_3$$
$$+ \text{Sym}\{\sigma \mathcal{N}_3^T \overline{\Omega}_2(\dot{\sigma}(t)) \mathcal{N}_4\}$$
$$+ \sigma(\sigma - \sigma(t)) \mathcal{N}_4^T \overline{\Omega}_2(\dot{\sigma}(t)) \mathcal{N}_4) \xi(t). \tag{17}$$

By using Lemma 2 to conduct $-\frac{\sigma}{\sigma(t)} \mathcal{N}_1^T \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t)) \mathcal{N}_1$ and $-\frac{\sigma}{\sigma - \sigma(t)} \mathcal{N}_3^T \overline{\Omega}_2(\dot{\sigma}(t)) \mathcal{N}_3$ in (13) and (14) together, can get the following inequality.

$$\xi^T(t) (-\frac{\sigma}{\sigma(t)} \mathcal{N}_1^T \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t)) \mathcal{N}_1$$
$$- \frac{\sigma}{\sigma - \sigma(t)} \mathcal{N}_3^T \overline{\Omega}_2(\dot{\sigma}(t)) \mathcal{N}_3) \xi(t)$$
$$= -\xi^T(t) \left[\begin{array}{c} \mathcal{N}_1 \\ \mathcal{N}_3 \end{array} \right]^T \left[\begin{array}{cc} \frac{\sigma}{\sigma(t)} \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t)) & 0 \\ 0 & \frac{\sigma}{\sigma-\sigma(t)} \overline{\Omega}_2(\dot{\sigma}(t)) \end{array} \right] \left[\begin{array}{c} \mathcal{N}_1 \\ \mathcal{N}_3 \end{array} \right] \xi(t)$$
$$\leq \xi^T(t) (\Xi_7 + \Pi) \xi(t), \tag{18}$$

where
$$\Pi = \frac{\sigma - \sigma(t)}{\sigma} \mathcal{N}_1^T \mathcal{S}_2 \overline{\Omega}_2^{-1}(\dot{\sigma}(t)) \mathcal{S}_2^T \mathcal{N}_1$$
$$+ \frac{\sigma(t)}{\sigma} \mathcal{N}_3^T \mathcal{S}_1^T \overline{\Omega}_1^{-1}(\sigma(t), \dot{\sigma}(t)) \mathcal{S}_1 \mathcal{N}_3.$$

Then
$$-\int_{t-\sigma(t)}^{t} \int_{\theta}^{t} \eta^T(s) \mathcal{T} \eta(s) ds d\theta$$
$$-\int_{t-\sigma}^{t-\sigma(t)} \int_{\theta}^{t-\sigma(t)} \eta^T(s) \mathcal{T} \eta(s) ds d\theta$$
$$\leq -\frac{2}{\sigma^2(t)} \left(\int_{t-\sigma(t)}^{t} \int_{\theta}^{t} \eta(s) ds d\theta \right)^T \mathcal{T}$$
$$\times \left(\int_{t-\sigma(t)}^{t} \int_{\theta}^{t} \eta(s) ds d\theta \right) - \frac{2}{(\sigma - \sigma(t))^2}$$
$$\times \left(\int_{t-\sigma}^{t-\sigma(t)} \int_{\theta}^{t-\sigma(t)} \eta^T(s) ds d\theta \right)^T \mathcal{T}$$
$$\times \left(\int_{t-\sigma}^{t-\sigma(t)} \int_{\theta}^{t-\sigma(t)} \eta^T(s) ds d\theta \right)$$
$$= \xi^T(t) \Xi_8 \xi(t). \tag{19}$$

Finally, merging (9) to (19), we get
$$\dot{V}(x_t) \leq \xi^T(t) \Xi \xi(t). \tag{20}$$

Based on the system (1), below zero equation is applicable to any matrix with the proper dimension W_1 and W_2.

$$0 = 2[x^T(t) \mathcal{W}_1 + \dot{x}(t) \mathcal{W}_2][\mathcal{A}^T x(t)$$
$$+ \mathcal{A}_d^T x(t - \sigma(t)) - \dot{x}(t)]$$
$$= \xi^T(t) \text{Sym}\{\mathcal{L}\Gamma\} \xi(t), \tag{21}$$

From inequalities (5)-(21), we can obtain
$$\dot{V}(x_t) \leq \xi^T(t) \Sigma_{\sigma(t)|\dot{\sigma}(t)} \xi(t). \tag{22}$$

Note that $\Gamma \xi(t) = 0$, according to [24] $(\Gamma^{\perp})^T \Sigma_{\sigma(t)|\dot{\sigma}(t)} (\Gamma^{\perp})$ is equivalent to $\xi^T(t) \Sigma_{\sigma(t)|\dot{\sigma}(t)} \xi(t) \leq 0$. Thus, DDS (1) is AS.∎

Remark 1: Based on [25], for a delayed dynamical system, the general approach to the DPTF method is to introduce additional non-integral terms as shown in

$$(\sigma(t) - \underline{\sigma}) \Delta_1^T(t) \varphi_1 \Delta_1(t) + (\underline{\sigma} - \sigma(t)) \Delta_1^T(t) \varphi_2 \Delta_1(t),$$

where φ_1 and φ_2 being SMs, $\sigma(t)$ represents TVD, $\underline{\sigma}$ and $\overline{\sigma}$ being the bound of $\sigma(t)$. When $\sigma(t) = \underline{\sigma}$, we have $(\sigma(t) - \underline{\sigma}) \Delta_1^T(t) \varphi_1 \Delta_1(t) = 0$. Similarly, when $\sigma(t) = \overline{\sigma}$, we have $(\underline{\sigma} - \sigma(t)) \Delta_1^T(t) \varphi_2 \Delta_1(t) = 0$. However, in actual engineering $\sigma \in [\underline{\sigma}, \overline{\sigma}]$, which takes into account the characteristics of DPTF. This method introduces more information, such as TD information and TD derivative information, into LKF. It can better increases the information contained in LKF and helps improve the effectiveness of the criteria. In addition, DPTF relaxes the constraints of the conditions. Before establishing the criteria, we only need weaker constraints, which reduces the conservativeness of the criteria.

Remark 2: Unlike the existing research method [24]–[26], this paper considers the relaxed conditions with DPTF strategy. Therefore, the positive definiteness of $V_1(x_t)$ and $V_5(x_t)$ can be given together. $V_1(x_t) + V_5(x_t)$:

$$V_1(x_t) + V_5(x_t) \geq \alpha^T(t) \Psi_{\sigma(t)} \alpha(t).$$

Based on the results in (19), $\mathcal{P} > 0$ and $\mathcal{U}_i > 0$ can be represented by $\Psi_{\sigma(t)} > 0$. This relaxation method has been studied and used by many scholars. It can reduce the constraints of the free matrix.

Remark 3: Different from existing methods [22], the construction of the integral term

$$\int_{t-\sigma}^{t} \Delta_2^T \overline{\mathcal{Q}}_{1,\sigma(t)} \Delta_2 ds + \int_{t-\overline{\sigma}}^{t-\sigma(t)} \Delta_2^T \overline{\mathcal{Q}}_{2,\sigma(t)} \Delta_2 ds,$$
$$\overline{\mathcal{Q}}_{1,\sigma(t)} = \mathcal{Q}_{1a} - \sigma(t) \mathcal{Q}_{1b},$$
$$\overline{\mathcal{Q}}_{2,\sigma(t)} = \mathcal{Q}_{2a} + (\sigma - \sigma(t)) \mathcal{Q}_{2b}.$$

when $\sigma - \sigma(t) = 0$, $\overline{\mathcal{Q}}_{2,\sigma(t)}$ will degenerate to the constant matrix $\mathcal{Q}_2$ in this paper. This construction method can reduce the free matrix's constraints and make the free matrix contain TD information. When the computer performs large-scale calculations, this method can effectively reduce the memory footprint of the calculation.

IV. NUMERICAL ILLUSTRATIONS

In this section, the proposed method is verified based on two numerical experiments.

Example 1: Research DDS(1), based on the parameters in [11], [12], [14], [20]–[23], [27]

$$\mathcal{A} = \left[\begin{array}{cc} -2 & 0 \\ 0 & -0.9 \end{array} \right], \quad \mathcal{A}_d = \left[\begin{array}{cc} -1 & 0 \\ -1 & -1 \end{array} \right].$$

TABLE 1. The achieved MAUBs for different value μ for Example 1.

Methods	$\mu = 0.2$	$\mu = 0.5$	$\mu = 0.8$
[11]	3.834	2.420	2.137
[12]	4.060	3.055	2.615
[7]	4.140	3.140	2.710
[19]	4.220	3.090	2.660
[20]	4.216	3.233	2.789
[21]	4.199	3.166	2.735
[22]	4.209	3.196	2.759
[26]	4.218	3.221	2.792
Theorem 1	4.473	3.496	2.976

For various μ with $\mu = \mu_2 = -\mu_2$, the Maximum allowable upper bounds (MAUBs) are calculated by Theorem 1, which lists in Table 1. According to the data, the criterion designed in this paper has a lower conservative bound than other documents. A conservative extended convexity matrix inequality is used to replace the general RCCL to deal with the delay-related terms. This paper uses the new boundary technology and the improved DPTF method to obtain larger MAUBs compared to the results in [11], [12], [14], [20]–[23], [27].

Letting $x(0) = [-0.8, 0.8]^T$, $\sigma(t) = 0.2 sint + 4.273$, $x(0) = [-0.8, -0.7]^T$, $\sigma(t) = 0.5 sint + 2.996$ and $x(0) = [0.9, 0.5]^T$, $\sigma(t) = 0.8 sint + 2.176$, the state of the system (1) are shown in the left and right of Figure. 1-3. Through the Figure. 1-3 can be seen that the delayed dynamical system of the zero solution is asymptotically stable. Then it showed that the criterion is feasible and effective.

Example 2: Research DDS(1), based on the parameters in [12], [13], [20], [22], [23], [27]

$$\mathcal{A} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad \mathcal{A}_d = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}.$$
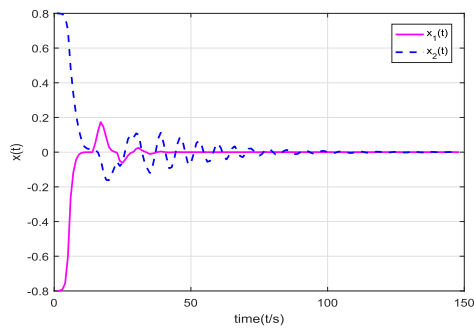

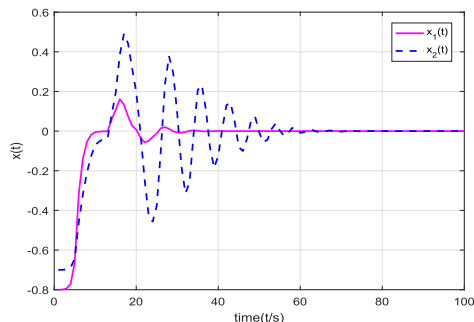**FIGURE 1.** State trajectories for $\mu = 0.2$.
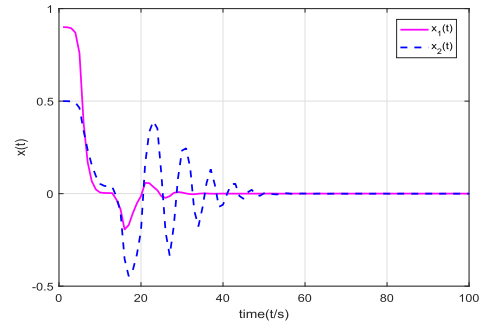

FIGURE 2. State trajectories for $\mu = 0.5$.
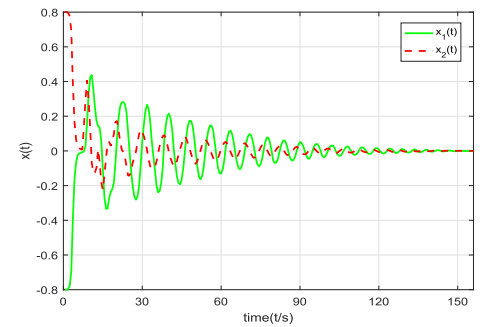

FIGURE 3. State trajectories for $\mu = 0.8$.

FIGURE 4. State trajectories for $\mu = 0.2$.

TABLE 2. The achieved MAUBs for different value μ for Example 2.

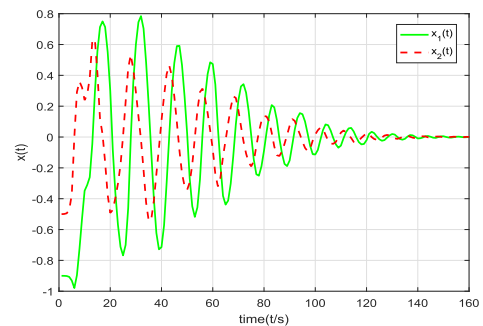| Methods	$\mu = 0.2$	$\mu = 0.5$	$\mu = 0.8$
[12]	3.672	1.411	1.275
[13]	4.466	2.352	1.768
[20]	4.517	2.415	1.838
[22]	4.556	2.509	1.940
[23]	4.543	2.496	1.922
[27]	4.670	2.664	2.072
Theorem 1	4.781	2.793	2.207

FIGURE 5. State trajectories for $\mu = 0.5$.

This example is described by the system (1). For different μ with $\mu = \mu_2 = -\mu_2$, the comparison among the MAUBs obtained by Theorem 1 with other methods in [12], [13], [20], [22], [23], [27] are summarized in Table 2. Comparing the results in Table 2, we know that the criterion designed in this paper can solve larger MAUBs. Obviously, Theorem 1 gives the greater the permissible maximum new integral inequality. Then by employing the DPTF strategy and Wirtinger-Based on integral inequality, the performance is enhanced better.
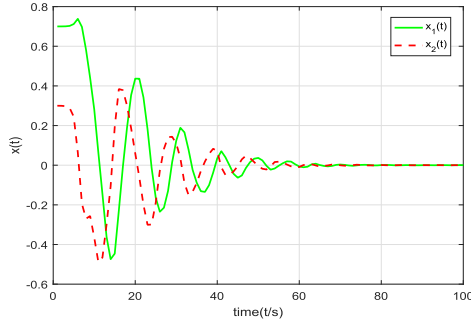
FIGURE 6. State trajectories for $\mu = 0.8$.

The results of this article are better, mainly because of the use of delayed partition technology.

Setting $x(0) = [-0.8, 0.8]^T$, $\sigma(t) = 0.2 sint + 4.581$, $x(0) = [-0.9, -0.5]^T$, $\sigma(t) = 0.5 sint + 2.293$ and $x(0) = [0.7, 0.3]^T$, $\sigma(t) = 0.8 sint + 1.407$, the state trajectories of system (1) are shown in the left and right of Figures. 4-6. The Figures. 4-6 show that the trajectory of the zero solution of the delayed dynamical system is AS, which further highlights the feasibility of the criterion.

V. CONCLUSION

In this article, we solved the AS problem of DDS. An improved DPTF strategy is applied to construct a more effective LKF, which contains more information and reduces the constraints of the free matrix. In addition, the definition of the conditions $V_1(x_t)$ and $V_5(x_t)$ have been further relaxed, thereby developing new relaxed conditions. Then, by using valid integral inequalities, the novel optimized criterion has been developed based on functional analysis theory. Finally, the feasibility of the method in this paper has been verified based on two numerical examples. Future work will improve this new type of LKF and applied to other DDSs. In the future, we will work to extend our method to more practical engineering applications.

APPENDIX A

$$\Theta_1(t) = \int_{t-\sigma(t)}^{t} \frac{x(s)}{\sigma(t)} ds,$$

$$\Theta_2(t) = \int_{t-\sigma}^{t-\sigma(t)} \frac{x(s)}{\sigma - \sigma(t)} ds,$$

$$\Theta_3(t) = \int_{t-\sigma(t)}^{t} \int_{\theta}^{t} \frac{x(s)}{\sigma(t)} ds d\theta,$$

$$\Theta_4(t) = \int_{t-\sigma}^{t-\sigma(t)} \int_{\theta}^{t-\sigma(t)} \frac{x(s)}{\sigma - \sigma(t)} ds d\theta,$$

$$\alpha(t) = [x^T(t), \sigma(t)\Theta_1^T(t), (\sigma - \sigma(t))\Theta_2^T(t),$$
$$\sigma(t)\Theta_3^T(t), (\sigma - \sigma(t))\Theta_4^T(t)]^T,$$

$$\eta(t) = [x^T(t), \dot{x}^T(t)]^T, \quad \beta(t) = [x^T(t), \Theta_1^T(t)]^T$$

$$\varepsilon(t) = [x^T(t), \Theta_2^T(t)]^T, \quad \xi(t) = [x^T(t), x(t - \sigma(t)),$$
$$x(t - \sigma), \dot{x}^T(t), \dot{x}^T(t - \sigma(t)), \dot{x}^T(t - \sigma),$$
$$\Theta_1^T(t), \Theta_2^T(t), \Theta_3^T(t), \Theta_4^T(t)]^T.$$

APPENDIX B

$$\Sigma_{\sigma(t)|\dot{\sigma}(t)}$$
$$= \sum_{i=1}^{8} \Xi_i + \text{Sym}\{\mathcal{L}\Gamma\},$$

$$\Xi_1 = \text{Sym}\{\Phi_1^T(\sigma(t))\mathcal{P}\Phi_2(\sigma(t), \dot{\sigma}(t))\},$$

$$\Xi_2 = \Phi_3^T \mathcal{Q}_{1a}\Phi_3 - (1 - \dot{\sigma}(t))\Phi_4^T \mathcal{Q}_{1a}\Phi_4$$
$$- \sigma(t)(\Phi_3^T \mathcal{Q}_{1b}\Phi_3 - (1 - \dot{\sigma}(t))\Phi_4^T \mathcal{Q}_{1b}\Phi_4)$$
$$+ (1 - \dot{\sigma}(t))\Phi_4^T \mathcal{Q}_{2a}\Phi_4 - \Phi_5^T \mathcal{Q}_{2a}\Phi_5$$
$$+ (\sigma - \sigma(t))((1 - \dot{\sigma}(t))\Phi_4^T \mathcal{Q}_{2b}\Phi_4 - \Phi_5^T \mathcal{Q}_{2b}\Phi_5),$$

$$\Xi_3 = \sigma^2 \Phi_3^T (\mathcal{R} + \frac{1}{2}\mathcal{T})\Phi_3,$$

$$\Xi_4 = \dot{\sigma}(t)\Phi_6^T \mathcal{U}_1 \Phi_6 - \dot{\sigma}(t)\Phi_7^T + \text{Sym}\{\Phi_6^T \mathcal{U}_1$$
$$\times \Phi_8(\sigma(t), \dot{\sigma}(t)) + \Phi_7^T \mathcal{U}_2 \Phi_9(\sigma(t), \dot{\sigma}(t))\},$$

$$\Xi_5 = \sigma(e_1^T \mathcal{M}_1 e_1 - e_2^T \mathcal{M}_2 e_2 + e_2^T \mathcal{M}_2 e_2 - e_3^T \mathcal{M}_2 e_3),$$

$$\Xi_6 = -Sym\{\sigma \mathcal{N}_1^T \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\mathcal{N}_2\} - \sigma \cdot \sigma(t)\mathcal{N}_2^T$$
$$\times \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\mathcal{N}_2 - Sym\{\sigma \mathcal{N}_3^T \overline{\Omega}_2(\dot{\sigma}(t))\mathcal{N}_4\}$$
$$- \sigma(\sigma - \sigma(t))\mathcal{N}_4^T \overline{\Omega}_2(\dot{\sigma}(t))\mathcal{N}_4,$$

$$\Xi_7 = -\frac{2\sigma - \sigma(t)}{\sigma}\mathcal{N}_1^T \overline{\Omega}_1(\sigma(t), \dot{\sigma}(t))\mathcal{N}_1$$
$$- \frac{\sigma + \sigma(t)}{\sigma}\mathcal{N}_3^T \overline{\Omega}_2(\dot{\sigma}(t))\mathcal{N}_3 - \frac{\sigma - \sigma(t)}{\sigma}$$
$$\times \text{Sym}\{\mathcal{N}_1^T \mathcal{S}_1 \mathcal{N}_3\} - \frac{\sigma(t)}{\sigma}\text{Sym}\{\mathcal{N}_1^T \mathcal{S}_2 \mathcal{N}_3\},$$

$$\Xi_8 = -2\Phi_{10}^T \mathcal{T}\Phi_{10} - 2\Phi_{11}^T \mathcal{T}\Phi_{11},$$

$$e_i = [0_{n\times(i-1)n} \quad I_{n\times n} \quad 0_{n\times(10-i)}], \quad i = 1, 2, \cdots, 10,$$

$$\bar{e}_i = [0_{n\times(i-1)n} \quad I_{n\times n} \quad 0_{n\times(5-i)}], \quad i = 1, 2, \cdots, 5,$$

$$\Phi_1(\sigma(t))$$
$$= [e_1^T, \sigma(t)e_7^T, (\sigma - \sigma(t))e_8^T,$$
$$\sigma(t)e_9^T, (\sigma - \sigma(t))e_{10}^T]^T,$$

$$\Phi_2(\sigma(t), \dot{\sigma}(t))$$
$$= [e_4^T, e_1^T - (1 - \dot{\sigma}(t))e_2^T,$$
$$(1 - \dot{\sigma}(t))e_2^T - e_3^T, \sigma(t)e_1^T - (1 - \dot{\sigma}(t))\sigma(t)e_7^T,$$
$$(1 - \dot{\sigma}(t))(\sigma - \sigma(t))e_2^T - (\sigma - \sigma(t))e_8^T]^T,$$

$$\Phi_3 = [e_1^T, e_4^T]^T, \quad \Phi_4 = [e_2^T, e_5^T]^T, \quad \Phi_5 = [e_3^T, e_6^T]^T,$$

$$\Phi_6 = [e_1^T, e_7^T]^T, \quad \Phi_7 = [e_1^T, e_8^T]^T,$$

$$\Phi_8(\sigma(t), \dot{\sigma}(t))$$
$$= [\sigma(t)e_4^T, e_1^T - (1 - \dot{\sigma}(t))e_2^T - \dot{\sigma}(t)e_7^T]^T,$$

$$\Phi_9(\sigma(t), \dot{\sigma}(t))$$
$$= [(\sigma - \sigma(t))e_4^T, (1 - \dot{\sigma}(t))e_2^T - e_3^T + \dot{\sigma}(t)e_8^T]^T,$$

$$\Phi_{10} = [e_9^T, e_1^T - e_7^T]^T, \quad \Phi_{11} = [e_{10}^T, e_2^T - e_8^T]^T,$$

$$N_1 = [0^T, e_1^T - e_2^T, -2e_9^T, -e_1^T - e_2^T + 2e_7^T]^T,$$

$$N_2 = [e_7^T, 0^T, e_7^T, 0^T]^T,$$

$$N_3 = [0^T, e_2^T - e_3^T, -2e_{10}^T, -e_2^T - e_3^T + 2e_8^T]^T,$$

$$N_4 = [e_8^T, 0^T, e_8^T, 0^T]^T,$$

$$\overline{\Omega}_1(\sigma(t), \dot{\sigma}(t)) = diag\{\Lambda_{1,\sigma(t)}, 3\Lambda_{1,\sigma(t)}\},$$

$$\overline{\Omega}_2(\dot{\sigma}(t)) = diag\{\Lambda_{2,\sigma(t)}, 3\Lambda_{2,\sigma(t)}\},$$

$$\mathcal{L} = [e_1^T \mathcal{W}_1 + e_4 \mathcal{W}_2],$$
$$\Gamma = [\mathcal{A}^T e_1 + \mathcal{A}_d^T e_2 - e_4],$$
$$\Lambda_{1,\sigma(t)} = \mathcal{R} + \frac{\dot{\sigma}(t)}{\sigma}\mathcal{Q}_{1b} + \frac{\sigma - \sigma(t)}{\sigma}\mathcal{T} + \begin{bmatrix} 0 & \mathcal{M}_1 \\ \mathcal{M}_1 & 0 \end{bmatrix},$$
$$\Lambda_{2,\sigma(t)} = \mathcal{R} + \frac{\dot{\sigma}(t)}{\sigma}\mathcal{Q}_{2b} + \begin{bmatrix} 0 & \mathcal{M}_2 \\ \mathcal{M}_2 & 0 \end{bmatrix},$$
$$\Psi_{\sigma(t)} = \begin{bmatrix} \mathcal{P} + \mathcal{U}_{\mathcal{F}} & \frac{\sqrt{\sigma(t)}}{\sigma}\mathcal{F}_3^T \mathcal{S}_3^T & \frac{\sqrt{\sigma - \sigma(t)}}{\sigma}\mathcal{F}_2^T \mathcal{S}_4^T \\ * & \mathcal{U}_1 & 0 \\ * & 0 & \mathcal{U}_2 \end{bmatrix},$$
$$\mathcal{U}_{\mathcal{F}} = \mathcal{F}_1^T(\sigma(t)\mathcal{U}_1 + (\sigma - \sigma(t))\mathcal{U}_2)\mathcal{F}_1 + \mathrm{Sym}\{\mathcal{F}_1^T \mathcal{U}_1 \mathcal{F}_2$$
$$+ \mathcal{F}_1^T \mathcal{U}_2 \mathcal{F}_3\} + \frac{2\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T \mathcal{U}_1 \mathcal{F}_2^T$$
$$+ \frac{\sigma + \sigma(t)}{\sigma^2}\mathcal{F}_3^T \mathcal{U}_2 \mathcal{F}_3$$
$$+ \mathrm{Sym}\{\frac{\sigma - \sigma(t)}{\sigma^2}\mathcal{F}_2^T \mathcal{S}_3 \mathcal{F}_3 + \frac{\sigma(t)}{\sigma^2}\mathcal{F}_2^T \mathcal{S}_4 \mathcal{F}_3\},$$
$$\mathcal{F}_1 = [\bar{e}_1^T, 0]^T, \quad \mathcal{F}_2 = [0, \bar{e}_2^T]^T, \quad \mathcal{F}_3 = [0, \bar{e}_3^T]^T,$$
$$\overline{Q}_{1,\sigma(t)} = Q_{1a} - \sigma(t)Q_{1b}, \quad \overline{Q}_{2,\sigma(t)} = Q_{2a} + (\sigma - \sigma(t))Q_{2b}.$$

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