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# A Secure Constellation Design for Polarized Modulation in Wireless Communications

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**ABSTRACT** This paper presents a modulation constellation design technique for polarization modulation communication in wireless communication. Such constellation can be used for confidential polarization modulation communication on the depolarization channel. Information theory is innovatively applied to constellation design, which allows us to take the channel depolarization effects into account during the constellation design process. Specifically, we derive the security capacity of the communication system under depolarized channel conditions and use it as the objective function for secure constellation optimization. We use a heuristic genetic algorithm to solve the above optimization problem. The numerical results show that compared to the quasi-optimal geometry-based constellation, the resulting constellation can have an advantage of secure capacity and have better secure performance.

**INDEX TERMS** Constellation design, polarization modulation, depolarization effects, secure capacity, physical layer security.

## I. INTRODUCTION

Transmission efficiency and security are of great importance to modern communication; new generation wireless networks are required to supply services with high efficiency and enough security for users. Polar modulation, as a vector modulation method, has great potential in improving the efficiency and security of communication systems [1]. Andrews *et al.* [2] have proved out that the polarization of electromagnetic waves can bring additional degrees of freedom in wireless communication systems, thereby increasing channel capacity. Paper [3] proposes the four-dimensional 'hybrid' constellation formed as the direct product of an absolute amplitude/phase state and a polarization state. The authors in paper [4] proposed a polarization-amplitude-phase modulation (PAPM) scheme to improve the channel capacity.

To improve the security of the communication system, paper [5], [6] uses polarization modulation to implement an anti-eavesdropping and anti-deciphering scheme. Specifically, it uses the polarization state to carry confidential information and uses the channel's depolarization effect to further encrypt the polarized constellation. The paper [7] models and analyzes the spatial orientation characteristics of polarization modulation and proposes a physical layer security scheme based on polarization direction modulation. This scheme can reduce the effective eavesdropping range of eavesdroppers and improve the physical layer security.

Although polarization modulation has excellent potential in the efficiency and security of wireless communications, vector characteristic of polarization modulation makes traditional scalar constellation design methods no longer completely applicable. Therefore, some methods for polarized constellation design are proposed, and they can be divided into the following three categories:

- Placing the signals at the vertices of a regular polyhedron inscribe within the Poincare sphere, when this exists [8]. This method is the case for M=4,6,8,12,20 only, which may be close to the optimum, but lacks generality.
- Optimizing the signal constellation using a numerical method based in the maximization of minimum distance among signals. This method has been used in [9], is asymptotically optimum in Gaussian environment, but not in Stokes space.
- Use the optimum approach consisting of placing the signals based on the minimization of the symbol error probability.

However, these constellation design principles are aimed at efficient constellations with little consideration for

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security. In order to solve the above problems, we propose a channel-based constellation design method. We use information theory to establish the link between constellation design and polarization channels. Specifically, the confidential capacity is used as the objective function for secure constellation optimization.

This paper has the following contributions:

- We have designed a security constellation, which can achieve bigger security capacity in the presence of depolarization effects.
- We found a trade off between the security capacity and the channel capacity of the constellation.
- We modeled a channel-based constellation optimization model, deduced the constellation design objective function under different objectives (efficiency or security), and proposed an optimization solution method based on genetic algorithms (GA).

The rest of this paper is organized as follows. In Section II, Polarized channels are modeled and the impact of depolarization effects on constellations are analyzed. In Section III, a method for designing efficient constellations is illustrated. In Section IV, a method for designing secure constellations is illustrated. Numerical results are given in section V. Finally, conclusions are drawn in section VI.

Notations:$(\cdot)^*$ represents complex conjugation. Superscripts $(\cdot)^T$ and $(\cdot)^H$ denote transpose and conjugate transpose, respectively. $det(A)$ is the determinant of matrix $A$ and $vec(A)$ is the operator that forms a vector from successive columns of matrix $A$. $|x|$ is the absolute value (or modulus) of $x$. $E\cdot$ is the expectation operator.

## II. THE DUAL-POLARIZED RAYLEIGH FADING CHANNEL WITH DEPOLARIZATION

### A. THE DUAL-POLARIZED RAYLEIGH FADING CHANNEL WITH DEPOLARIZATION

Consider dual-polarized channel and each channel has additive white Gaussian noise (AWGN). The communication model is given by

$$\begin{bmatrix} E_{rv} \\ E_{rh} \end{bmatrix} = H_C \begin{bmatrix} E_{tv} \\ E_{th} \end{bmatrix} + \begin{bmatrix} n_v \\ n_h \end{bmatrix} \tag{1}$$

where $\begin{bmatrix} E_{tv} \; E_{th} \end{bmatrix}^T$ and $\begin{bmatrix} E_{rv} \; E_{rh} \end{bmatrix}^T$ are the transmit and receive polarized signals in Jones vector notation under the $\{\hat{h}, \hat{v}\}$ polarization basis. The AWGN noise $n_h$ and $n_v$ are assumed to be independent and identically distributed (i.i.d) circularly symmetric complex Gaussian distribution $\mathcal{CN}(0, N_0)$, where $N_0$ is the noise power spectral density. $H_c$ is the dual-polarized Rayleigh channel response matrix.

We model $H_c$ using the analytical method described in [10] as

$$H_c = GH_P = \begin{bmatrix} h_{vv} \; h_{vh} \\ h_{hv} \; h_{hh} \end{bmatrix} \tag{2}$$

where $G$ is a scalar complex Gaussian term representing Rayleigh fading; $H_p$ models the channel depolarization effects including the impacts of correlation and power imbalance; $h_{xy}$ represents a complex channel gain between the transmitted $x$-polarized component and the received $y$-polarized component.

$H_p$ can be characterized by its correlation matrix

$$vec({H_p}^H) = {R_P}^{1/2} vec({H_w}^H) \tag{3}$$

where $H_w$ is a $2 \times 2$ matrix whose four elements are independent circulary symmetric complex exponentials $\alpha_k e^{j\beta_k} (k = 1, \ldots, 4)$ with unit amplitude $|\alpha_k| = 1$ and the angles $\beta_k$ being uniformly distributed over $[0, 2\pi)$. $R_P$ is the polarization correlation matrix of $H_p$ and given by [10]

$$R_{p=} \begin{bmatrix} 1 & \sqrt{\mu\chi}\vartheta^* & \sqrt{\chi}\sigma^* & \sqrt{\mu}\delta_1^* \\ \sqrt{\mu\chi}\vartheta & \mu\chi & \sqrt{\mu}\chi\delta_2^* & \mu\sqrt{\chi}\sigma^* \\ \sqrt{\chi}\sigma & \sqrt{\mu}\chi\delta_2 & \chi & \sqrt{\mu\chi}\vartheta^* \\ \sqrt{\mu}\delta_1 & \mu\sqrt{\chi}\sigma & \sqrt{\mu\chi}\vartheta & \mu \end{bmatrix} \tag{4}$$

where

- $\mu$ and $\chi$ represent the inverse of, respectively, the co-polar ratio (CPR) and the cross-polar discrimination (XPD). The CPR can be explained by the polarization selectivity of reflection and diffraction process and is given by

$$\mu = E[|h_{hh}|^2/|h_{vv}|^2] \tag{5}$$

XPD can be explained by the cross-polar transmission, it can be given by

$$\chi = E[|h_{hv}|^2/|h_{vv}|^2] = E[|h_{vh}|^2/|h_{hh}|^2] \tag{6}$$

CPR and XPD are usually referred to as two kinds of power imbalances, which can have significant effect on the performance of polarization modulation communication.

- $\sigma$ and $\vartheta$ are the receive and transmit correlation coefficients for co-located antennas (i.e. the correlation coefficients between $vv$ and $hv$, $hh$ and $hv$, $vv$ and $vh$ or $hh$ and $vh$). For simplicity, we can consider that the receive and transmit correlation coefficients $\sigma$ and $\vartheta$ are equal to zero, as often observed experimentally [10].
- $\delta_1$ and $\delta_2$ are the co- and anti-polar correlation coefficients. $\delta_1$ is the co-polar correlation coefficients defining the correlation between $hh$ and $vv$; $\delta_2$ is anti-polar correlation coefficients characterizing the correlation between $hv$ and $vh$. In the dual-polarized communication scenario, the antennas are spatially correlated. Thus $|\delta_1| = 1$ and $|\delta_2| = 1$, as found by the ray-tracing simulations [11].

Define a parameter set $\Lambda = \{\mu, \chi, \vartheta, \sigma, \delta_1, \delta_2\}$, which can accurately describe the channel depolarization. In the remainder of this paper, we consider (only for simplicity) $\vartheta = \sigma = 0, \delta_1 = \delta_2 = 1$ according the above analysis. Thus, $\Lambda$ can be re-expressed as $\Lambda = \{\mu, \chi, 0, 0, 1, 1\}$. We mainly consider the effect of channel parameters $\mu$, $\chi$ because they are the main factors leading to the channel depolarization.

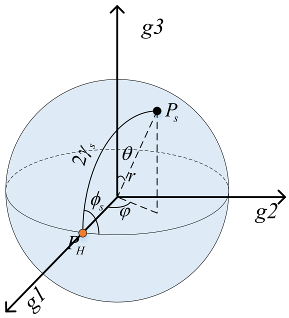


**FIGURE 1.** The Polarization State on the Poincare Sphere.

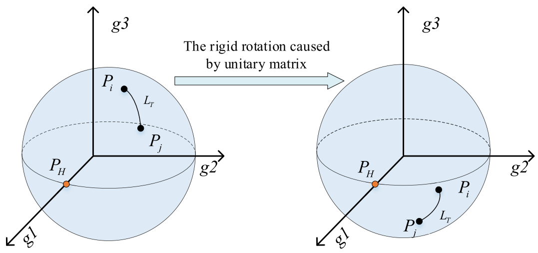


**FIGURE 2.** The Rigid Rotation Caused by Unitary Matrix.

### B. THE IMPACT OF CHANNEL DEPOLARIZATION ON POLARIZATION CONSTELLATION

We introduce the Poincare sphere as a visualizing graphical tool for analysing the impact of channel depolarization on the constellation. Each polarization state can be mapped to an exclusive point on the the surface of the unit Poincare sphere [1]. As shown in Fig.1, $P_s$ is polarization constellation point; $P_H$ denotes the polarization constellation point corresponding to horizontal polarization state; $2\gamma_s$ is the length of the arc from $P_s$ to $P_H$, $\phi_s$ is the angle between such arc and the equator. Besides, each polarization state can alternatively be represented with spherical coordinates $(r, \theta, \varphi)$ where $r$ is the radius of the Poincare sphere, $\theta$ is the elevation angle, and $\varphi$ is the azimuth angle.

Following(2), perform singular value decomposition (SVD) on the channel matrix $H_c$

$$H_c = U\Sigma V = U \begin{bmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{bmatrix} V \tag{7}$$

where $U$ and $V$ are the unitary matrixes by the SVD of $H_c$; $\Sigma$ is the diagonal matrix by the SVD of $H_c$; $\lambda_1$ and $\lambda_2$ ($\lambda_1 \geq \lambda_2 \geq 0$) are the eigenvalues of $\Sigma$.

The impact suffered by receiver's received constellation points is analyzed as follows.

According to equation (7), the channel matrixes can be classified into two categories: unitary matrixes and diagonal matrixes. When multiplying unitary matrix, the polarization constellation points will be rotated as a rigid body on the Poincare sphere. For example, assuming $P_i$ and $P_j$ are two adjacent constellation points (as shown in Fig.2), and their spherical distance is $L_T$. Such distance will keep invariable after undergoing the unitary matrix caused rotation; also the

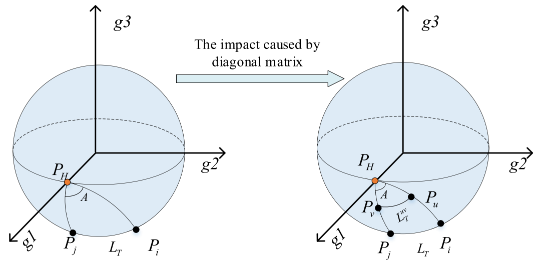


**FIGURE 3.** The Collapse Caused by Diagonal Matrix.

carried information is not lost. However, when multiplying diagonal matrix, the polarization constellation points will move along with the constellation arc. As shown in Fig.3, the constellation points will move toward $P_{\rm H}$, namely $P_i$ and $P_j$ are moved to $P_v$ and $P_u$. Thus, it can be found that the diagonal matrix will result in the polarization constellation distortion; also the signal power is attenuated, and the carried information will be lost.

In practical polarization modulation schemes, the depolarization effect can significantly impair communication reliability and reduce the capacity of the wireless channel. Wu *et al.* [12] had proved that the channel capacity decrease under the impairment of the depolarization effect. Therefore, we seek to alleviate the impairment of the depolarization effect and improve the channel capacity by designing constellations that are not sensitive to channel depolarization effects.

The channel depolarization effect on the constellation distortion can be used for physical layer security. Dong *et al.* [6] has used the difference between the legal channel and eavesdropping channel caused by the depolarization effect to encrypt the wireless communication physical layer. Therefore, by designing a constellation which is sensitive to the depolarization effect, the difference between the legal channel and the eavesdropping channel can be further increased, and the security capacity of the polarization modulation system can be improved.

## III. GENERATING EFFICIENT CONSTELLATION

### A. CONSTELLATION-CONSTRAINED POLARIZATION CHANNEL CAPACITY

Channel capacity, formulated by Shannon [13], is a measure of how fast information can be transmitted reliably over a given channel. Channel capacity is usually limited by the transmission symbol probability distribution and channel noise. However, based on the analysis in section II, for polarized communication systems, channel parameters and constellation diagrams also have an impact on channel capacity. Therefore, we expanded the definition of channel capacity to constellation-constrained polarization channel capacity (CCPCC), which covers the impacts of constellation diagrams and depolarization effects on channel capacity. Therefore, CCPCC can serve as a bridge between constellation design and polarized channels.

The constellation-constrained polarization channel capacity $C_\Gamma$ is given by

$$C_\Gamma = \max_{p(\gamma,\phi)} \sum_{s=1}^{M} p(\gamma_s, \phi_s) I(\theta, \varphi\,|\gamma_s, \phi_s)[bit/symbol] \quad (8)$$

where $\Gamma$ is a polarization modulation constellation set $\Gamma = \{(\gamma_0, \phi_0), (\gamma_1, \phi_1), \cdots, (\gamma_s, \phi_s) \cdots, (\gamma_{M-1}, \phi_{M-1})\}$, $M$ is modulation order, $s$ is the index integer in the range of $[0, M-1]$. $p(\gamma_s, \phi_s)$ is the probability that the transmission polarization state is $(\gamma_s, \phi_s)$. In this paper, we choose a quasi-optimal equal input symbol probability distribution as (9). $I(\theta, \varphi\,|\gamma_s, \phi_s)$ is the mutual information between transmit polarization $(\gamma_s, \phi_s)$ and receive polarization $(\theta, \varphi)$ on the Poincare sphere, and it is given by (10).

$$p(\gamma_0, \phi_0) = \cdots p(\gamma_s, \phi_s) \cdots = p(\gamma_{M-1}, \phi_{M-1}) = \frac{1}{M} \quad (9)$$

$$I(\theta, \varphi\,|\gamma_s, \phi_s) = E_{(\theta,\varphi)}\left[\log_2\left(\frac{f(\theta, \varphi|\gamma_s, \phi_s)}{f(\theta, \varphi)}\right)\right] \quad (10)$$

where $f(\theta, \varphi|\gamma_s, \phi_s)$ is the conditional probability density function (PDF) of the normalized polarization state $(\theta, \varphi)$ given the transmitted state $[\gamma_s, \phi_s]^T$ and can be given by [12]

$$f(\theta, \varphi|\gamma_s, \phi_s) = \frac{\sin\theta}{4\pi} \frac{4\det(R_r)}{\left(\bar{g}_0 - \bar{g}_1\varepsilon_1 - \bar{g}_2\varepsilon_2 - \bar{g}_3\varepsilon_3\right)^2} \quad (11)$$

where $R_r$ is the receive covariance matrix as shown in (12) at the bottom of the page [14]. $\left(\bar{g}_0, \bar{g}_1, \bar{g}_2, \bar{g}_3\right) = (r_{hh} + r_{vv}, r_{hh} - r_{vv}, r_{hv} + r_{vh}, j(r_{hv} - r_{vh}))$ and $(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$.

### B. CONSTELLATION OPTIMIZATION MODEL

Through the above analysis, the polarization channel capacity depends on the channel environment $\Lambda$ and the transmitted symbol set $\Gamma$, i.e, $C(\Lambda, \Gamma)$. If we know the channel depolarization parameters through channel estimation, we can obtain the optimal constellation to achieve better channel capacity. Therefore, the constellation design can be transformed into the following optimization problem

$$\max_{\Gamma}(C_\Gamma) \quad s.t. \begin{cases} \mu = \mu_1 \\ \chi = \chi_1 \\ \vartheta = \sigma = 0 \\ \delta_1 = \delta_2 = 1 \end{cases} \quad (13)$$

**Algorithm 1** Framework of Genetic Algorithm

**Input:** The modulation order, $M$; The channel co-polar ratio (CPR) parameter $\mu$; The cross-polar discrimination (XPD) parameter $\chi$;

**Output:** The best constellation $\Gamma^*$;

1: Group Initialization: Randomly generate an initial population that contains $N_P$ individuals, and the individual is expressed as the genetic code of the chromosome;

2: Fitness value $f$ calculate according to (8), and test whether the optimization criteria are satisfied. If so, go to step 8;

3: Fitness value dynamic linear calibration F according to (16);

4: Selection: select parents based on roulette principle;

5: Crossover: cross to generate new individuals with adaptive cross probability $P_c$, formula (17);

6: Mutation: mutate to generate new individuals with adaptive mutation probability $P_m$, formula (18);

7: Generating new group through crossover and mutation, then back to step 2;

8: **return** Export the best constellation $\Gamma^*$;

Genetic algorithm (GA) [15] is used to solve this new work optimization problem. In fact, many commonly used heuristic optimization algorithms can solve this optimization problem, such as ant colony algorithm (ACO), particle swarm algorithm (PSO), etc. This paper focuses on constructing the objective function and the optimization model, so this article does not go into very details about the optimization method. The steps of GA realization are shown in Algorithm 1. The parameters for GA are shown in Table 1.

We use the matlab platform for optimization. The optimization begins with a group of $N_P$ random constellations in Stokes space. Since we use the polarization phase descriptor to describe the polarization state, the individual of the initial population contains two genes, one gene describes the amplitude ratio information $\gamma$, and one gene describes the phase difference information $\phi$, and the two genes are taken The value ranges are

$$\begin{aligned} \gamma &\in [0, \pi/2] \\ \phi &\in [0, 2\pi) \end{aligned} \quad (14)$$

Based on the value range and quantization accuracy of the gene, we obtain the gene length L of a single constellation

$$R_r = \left\langle \begin{bmatrix} E_{rh} \\ E_{rv} \end{bmatrix} \begin{bmatrix} E_{rh} \\ E_{rv} \end{bmatrix}^H \right\rangle = \begin{bmatrix} r_{hh} & r_{hv} \\ r_{vh} & r_{vv} \end{bmatrix}$$
$$= \begin{bmatrix} c_{vv}\chi + c_{vh}\sqrt{\mu\chi}\vartheta + c_{hv}\sqrt{\mu\chi}\vartheta^* + c_{hh}\mu + N_0 & c_{vv}\sqrt{\chi}\sigma^* + c_{hv}\sqrt{\mu}\delta_1^* + c_{vh}\sqrt{\mu}\chi\delta_2^* + c_{hh}\mu\sqrt{\chi}\sigma^* \\ c_{vv}\sqrt{\chi}\sigma + c_{vh}\sqrt{\mu}\delta_1 + c_{hv}\sqrt{\mu}\chi\delta_2 + c_{hh}\mu\sqrt{\chi}\sigma & c_{vv} + c_{vh}\sqrt{\mu\chi}\vartheta + c_{hv}\sqrt{\mu\chi}\vartheta^* + c_{hh}\mu\chi + N_0 \end{bmatrix}$$
$$where: c_{vv} = \sin^2\gamma_s, c_{vh} = c_{hv}^* = \sin\gamma_s\cos\gamma_s e^{j\phi_s}, c_{hh} = \cos\gamma_s \quad (12)$$

**TABLE 1.** The parameters for GA.

| | |
|---|---|
| The population size | $N_P = 50$ |
| The maximum evolution generation | $N_G = 500$ |
| The maximum crossover probability | $P_{c1} = 0.5$ |
| The minimum crossover probability | $P_{c2} = 0.9$ |
| The maximum mutation probability | $P_{m1} = 0.02$ |
| The minimum mutation probability | $P_{m2} = 0.05$ |
| The discrete precision for $\gamma_s$ | $eps_1 = 0.01$ |
| The discrete precision for $\phi_s$ | $eps_2 = 0.01$ |
| The range for $\gamma_s$ | $\gamma_s \epsilon [0, \pi/2]$ |
| The range for $\phi_s$ | $\phi_s \epsilon [0, 2\pi]$ |
| The initial value for select pressure value | $\xi_0$=2 |
| The Scaling factor for select pressure value | $c = 0.9$ |

point as

$$L = \left\lceil \log_2^{(\pi/2-0)/eps+1} \right\rceil + \left\lceil \log_2^{(2\pi-0)/eps+1} \right\rceil \tag{15}$$

For the constellation set of $M$-order polarization modulation, the length of a single individual gene is $M * L$. The genes are randomly initialized to obtain the initial population. The fitness function is defined as the CCPCC. During the evolution process, the children are modified in an attempt to increase the CCPCC. We made some modifications to the traditional genetic algorithm to solve our optimization problem. In step 3, the fitness value dynamic linear calibration is performed to increase the selection efficiency of GA. In step 4 and 5, adaptive crossover probability and adaptive mutation probability methods are adopted to accelerate convergence rate [16]. In the end, the constellation with the highest fitness within the final population is presented in the output.

$$F = f - f_{\min}^k + \xi^k \tag{16}$$

where $\xi^k = c\xi^{k-1}$, $k$ is the iteration number.

$$P_c = \begin{cases} \dfrac{P_{c1}\left(F_{\max} - F\right)}{\left(F_{\max} - F_{avg}\right)}, F \geq F_{avg} \\ P_{c2}, F < F_{avg} \end{cases} \tag{17}$$

$$P_m = \begin{cases} \dfrac{P_{m1}\left(F_{\max} - F\right)}{\left(F_{\max} - F_{avg}\right)}, F \geq F_{avg} \\ P_{m2}, F < F_{avg} \end{cases} \tag{18}$$

GA does well at finding constellations for the maximum CCPCC. Fig. 4 shows the variation of CCPCC during the optimization process under different channel parameters. It shows GA algorithm can converge well in solving the constellation optimization problem and can find the optimal constellation for max channel capacity.

### C. CALCULATED RESULTS

The resulting constellations are shown next. We present four groups of typical constellation in different channel parameters for $M = 8$ in in Table 2 and Fig.5.

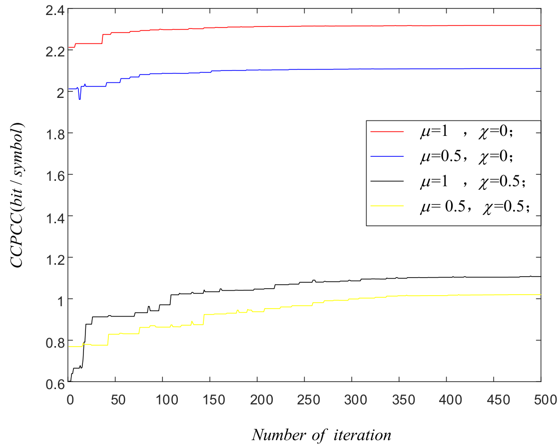


**FIGURE 4.** Optimization process for capacity constellation.

From the optimization results, we can find that the optimal constellation changing with the channel depolarization parameters $\Lambda$. When there is no channel depolarization, i.e., $\Lambda = \{1, 0, 0, 0, 1, 1\}$, the dual-polarized channel is degraded into two independent AWGN channels. The constellation points are evenly distributed on the Poincare sphere(Fig.5.a), which is similar to the traditional polyhedron constellation. When the co-polar ratio $\mu$ is the leading cause of the depolarization effect, i.e., $\Lambda = \{\mu, 0, 0, 0, 1, 1\}$, the constellation tends to move towards the vertical polarization state $P_V$(Fig.5.b), which is consistent with the constellation distortion rule shown in paper [6]. When the cross-polarization discrimination $\chi$ is the leading cause of the depolarization effect, i.e., $\Lambda = \{1, \chi, 0, 0, 1, 1\}$, the constellation tends to move towards the equatorial plane, (Fig.5.c), in which the two antennas have the lowest coupling degree. When there are both cross-polarization discrimination $\chi$ and co-polar ratio $\mu$, i.e., $\Lambda = \{\mu, \chi, 0, 0, 1, 1\}$,the constellation distribution shows a combination of the above two trends.

## IV. GENERATING SECURE CONSTELLATION

### A. SECRECY CAPACITY

The secure transmission system model is shown in Fig.6, which includes the transmitter Alice, the legitimate receiver Bob and the eavesdropper Eve. They all are equipped with dual-polarized antennas. $H_{AB}$ denotes the channel between Alice and Bob. $H_{AE}$ denotes the channel between Alice and Eve. The Time-Division Duplex (TDD) method is adopted, so during the coherence time, Alice and Bob can acquire $H_{AB}$ according to the reciprocity characteristic of the wireless channel. Because wireless channel is location specific, when the distance between Eve and Bob is large than the half wavelength, $H_{AB}$ and $H_{AE}$ is independent.

From an information theoretic point of view, the communication channel involved can be modeled as a broadcast channel, following the wire-tap channel model introduced by Wyner [17]. The secrecy capacity is the maximum achievable perfect secrecy rate. Perfect secrecy is achieved when the

**TABLE 2.** Signal constellations for maximum channel capacity in different channel parameters.

| $\Gamma$ | | $P_0$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ |
|---|---|---|---|---|---|---|---|---|---|
| $\mu = 1, \chi = 0$ | $\gamma_s$ | 0.7885 | 1.1766 | 1.3306 | 0.0123 | 0.7946 | 0.6283 | 0.5914 | 0.8870 |
| | $\phi_s$ | 5.9024 | 4.6556 | 0.9888 | 3.5562 | 1.9593 | 0.8599 | 4.3178 | 3.1447 |
| $\mu = 0.5, \chi = 0$ | $\gamma_s$ | 0.6899 | 0.6838 | 0.8747 | 0.9856 | 1.3737 | 0.6468 | 0.2895 | 0.2464 |
| | $\phi_s$ | 5.6383 | 4.3239 | 0.7862 | 3.0218 | 5.5277 | 1.9347 | 3.2245 | 0.3624 |
| $\mu = 1, \chi = 0.5$ | $\gamma_s$ | 0.0370 | 0.7762 | 1.5708 | 0.8624 | 0.7885 | 0.7885 | 0.7885 | 0.7762 |
| | $\phi_s$ | 0.9213 | 3.1569 | 1.2284 | 3.1569 | 0 | 6.2586 | 0 | 3.1324 |
| $\mu = 0.5, \chi = 0.5$ | $\gamma_s$ | 0.6406 | 0.6098 | 0.0862 | 0.6468 | 0.8501 | 0.5975 | 0.6160 | 0.7269 |
| | $\phi_s$ | 6.2340 | 3.1508 | 2.3646 | 0.0061 | 2.9973 | 6.2709 | 3.1569 | 5.0917 |

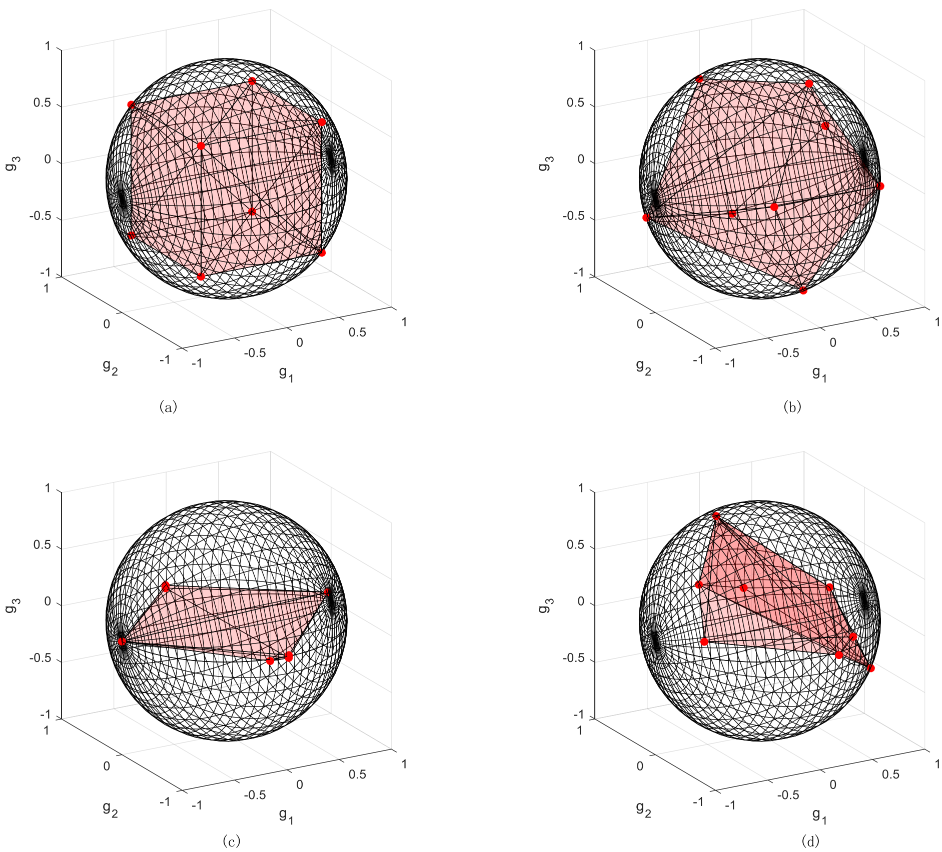


**FIGURE 5.** Resulting constellations for maximum channel capacity: (a) constellation for $\mu = 1$, $\chi = 0$; (b) constellation for $\mu = 0.5$, $\chi = 0$; (c) constellation for $\mu = 1$, $\chi = 0.5$; (d) constellation for $\mu = 0.5$, $\chi = 0.5$.

transmitter and the legitimate receiver can communicate at some positive rate while ensuring that the eavesdropper gets zero bits of information. The secrecy capacity is given by [17]

$$C_s = C_{AB} - C_{AE} \tag{19}$$

where $C_{AB}$ and $C_{AE}$ can be calculated according to (8). When Alice wants to send a message to Bob, they send pilot sequence to each other, then the channel state information (CSI) matrix $H_{AB}$ is acquired by Alice and Bob. After that, Alice sends the confidential message. For Alice, with the perfect CSI, the depolarization effect can be eliminated by Channel pre-Compensation [5]. Therefore, Bob is easy to decode the constellation diagram and get the confidential message and the channel capacity $C_{AB}$ equals to the dual-polarized Rayleigh channel with $\mu = 1, \chi = 0$. However, for Eve, due to the lack of CSI, it's hard to do channel equalization or pre-compensation, Eve suffers the depolarization effect, the channel capacity $C_{AE}$ decrease. The difference of $C_{AB}$ and $C_{AE}$ provides the possibility of secure transmission.

### B. CONSTELLATION OPTIMIZATION

As evidenced in section II, the channel secrecy capacity $C$ depends not only on the dual-polarized depolarization channel parameters but also the constellation sets $\Gamma$. Therefore,

**TABLE 3.** Signal constellation for maximum secure capacity in different channel parameters.

| $\Gamma$ | | $P_0$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ |
|---|---|---|---|---|---|---|---|---|---|
| $\mu = 0.5, \chi = 0$ | $\gamma_s$ | 1.1950 | 1.1150 | 1.4291 | 1.1889 | 0.9425 | 0.7330 | 0.7823 | 0.9425 |
| | $\phi_s$ | 3.0525 | 1.5846 | 0.2088 | 4.5143 | 5.4970 | 2.3278 | 3.8694 | 0.4484 |
| $\mu = 1, \chi = 0.5$ | $\gamma_s$ | 0.4620 | 1.4538 | 0.1725 | 0.3573 | 1.1827 | 0.7885 | 0.7885 | 1.1766 |
| | $\phi_s$ | 1.2100 | 6.0866 | 2.9727 | 5.1101 | 4.3362 | 4.7354 | 2.0760 | 1.5723 |
| $\mu = 0.5, \chi = 0.5$ | $\gamma_s$ | 0.2094 | 0.7946 | 1.2813 | 0.3881 | 0.7823 | 0.9856 | 1.1704 | 1.2505 |
| | $\phi_s$ | 1.6399 | 1.1792 | 3.4088 | 4.7293 | 2.1620 | 4.7293 | 5.7488 | 1.2714 |

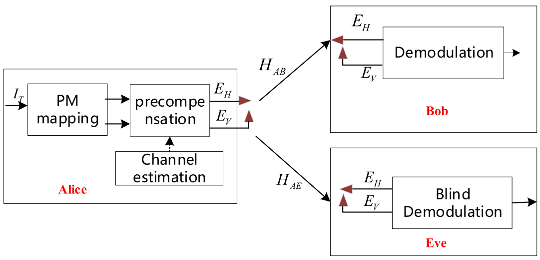


**FIGURE 6.** The secure communication system model.

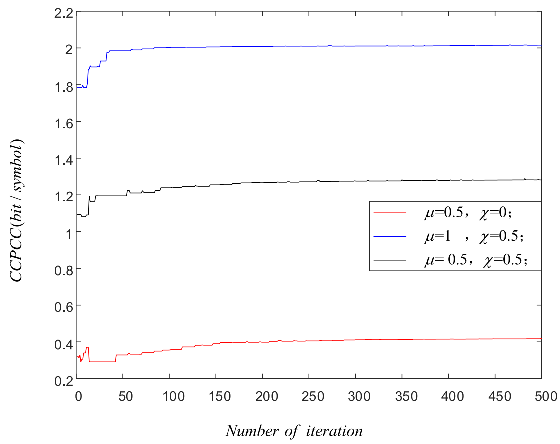


**FIGURE 7.** Optimization process for security constellation.

in this section, we seek to increase the channel secure capacity by optimizing constellation. Through submitting (8) to (19), the goal is to perform the following optimization:

$$\max_{\Gamma} \left( C \left( \Gamma \left| \Lambda_{AB} \right. \right) - C \left( \Gamma \left| \Lambda_{AE} \right. \right) \right) \tag{20}$$

where $\Lambda_{AB} = \{1, 0, 0, 0, 1, 1\}$ and $\Lambda_{AE} = \{\mu, \chi, 0, 0, 1, 1\}$.

Similarly, the GA is used to find constellation that give a maximum channel secure capacity. The only change necessary to the GA from Section III.B involves the objection function. The expression (20) is used in the GA as objective function. For the sake of simplicity of the article, we do not describe the operation of the genetic algorithm in detail. For more details, refer to section III.B. GA does well at finding constellations for the maximum channel secure capacity $C_s$. Fig. 7 shows the variation of $C_s$ during the optimization process under different channel parameters. It shows GA algorithm can converge well in solving the constellation optimization problem and can find the optimal constellation for max channel secure capacity.

### *C. CALCULATED RESULTS*

The resulting constellation for maximum channel secure capacity from the output of the GA are shown next. For the sake of brevity, only two constellation in different channel parameters for $M = 8$ are shown in Table 2 and Fig.8.

From the optimization results, we can find that the optimal constellation for maximum secure capacity is different under different channel parameters. When the channel selectivity(described by $\mu$) is the main cause of the depolarization effect, i.e., $\Lambda = \{\mu, 0, 0, 0, 1, 1\}$, the constellation tends to move towards the horizontal polarization state $P_H$. When the cross polarization (described by $\chi$) is the main cause of the depolarization effect, i.e., $\Lambda = \{1, \chi, 0, 0, 1, 1\}$, the constellation tends to move towards the $g_1og_3$ plane. When there are both cross-polarization discrimination $\chi$ and co-polar ratio $\mu$, i.e., $\Lambda = \{\mu, \chi, 0, 0, 1, 1\}$,the constellation distribution shows a combination of the above two trends.

Those trends are exactly the opposite of the constellation trends for maximum channel capacity in section III.C. The reason for this opposite trend is as follows. When the channel has a depolarization effect, the depolarization effect causes a reduction in channel capacity. In order to achieve the maximum channel capacity, the constellation that is less affected by the depolarization effect wins. However, when the security capacity is targeted, the channel depolarization effect becomes the source of security for the polarization modulation system. In order to achieve the maximum security capacity, the constellation that is greatly affected by the depolarization effect wins. So there is a trade off between security and efficiency.

## V. NUMERICAL RESULTS

In this section, constellation-constrained polarization channel capacity (CCPCC) and secure capacity ($C_s$) in different channel parameters are calculated by numerically integrating (8) and (19) respectively. During the simulation, some restrictions are as follows.

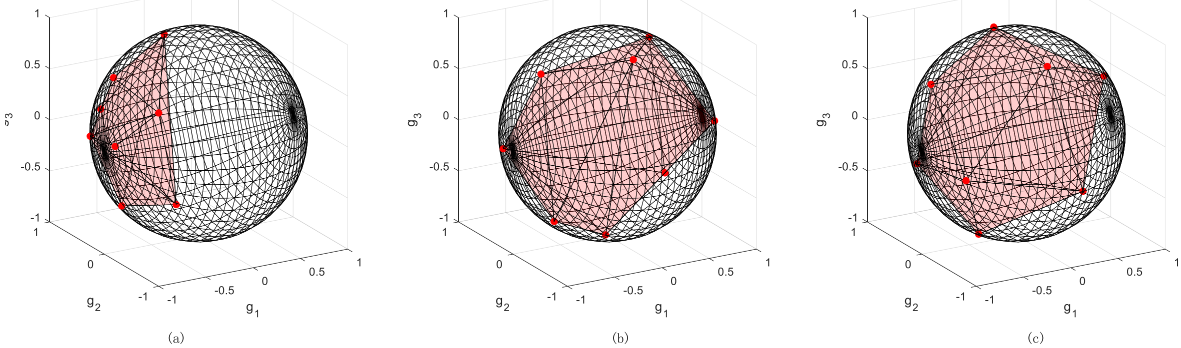


**FIGURE 8.** Resulting constellation for the maximum channel secure capacity: (a) constellation for $\mu = 0.5$, $\chi = 0$; (b) constellation for $\mu = 1$, $\chi = 0.5$; (c) constellation for $\mu = 0.5$, $\chi = 0.5$.

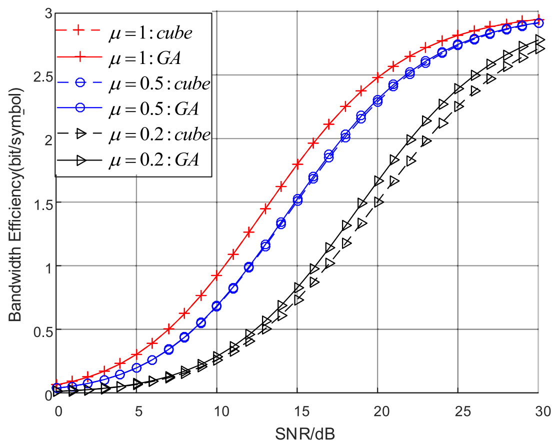


**FIGURE 9.** Channel capacity comparison between cube constellation and optimal GA constellation in different co-polar ratio $\mu = 1, 0.5, 0.2$.

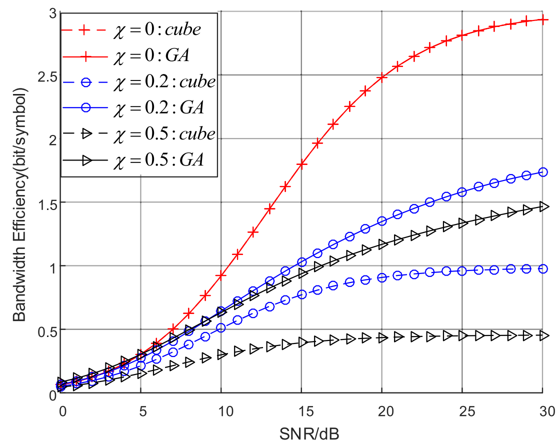


**FIGURE 10.** Channel capacity comparison between cube constellation and optimal GA constellation in different cross-polarization discrimination $\chi = 0, 0.2, 0.5$.

- According to the channel model in section II.A, we focus on the channel depolarization parameters $\mu$, $\chi$, $\Lambda = \{\mu, \chi, 0, 0, 1, 1\}$.
- The input symbols are equal probability distribution.
- The modulation order $M$ is 8 ($M = 8$), which can display the characteristics of the constellation points geometric distribution in an intuitive way. But the method and conclusions can apply to constellation design of other modulation orders.
- The 8-order cube constellation is selected as the comparison group because it has been proved asymptotically optimum both in Euclidean space and Stokes Space.

### A. CONSTELLATION-CONSTRAINED POLARIZATION CHANNEL CAPACITY

Fig.9 illustrates channel capacity comparison between cube constellation and optimal GA constellation in different co-polar ratio $\mu = 1, 0.5, 0.2$. Among them, the dotted lines represent the cube constellation, and the solid lines represent the optimal GA constellation. Both constellation' channel capacity increase with increasing $\mu$ which can be explained by a smaller power imbalance between $vv$ and $hh$ channels. $\mu = 1$ means the channel has no depolarization, in this case, the channel capacity curve of the GA constellation coincides with the curve of the cube constellation and approach $log2(M)$ as the SNR increases. $\mu < 1$ means the channel has depolarization effect, in this case, the optimal GA constellation exhibits a slight increase (maximum 0.2 bit/symbol) in channel capacity compared to the cube constellation.

Fig.10 illustrates channel capacity comparison between cube constellation and optimal GA constellation in different cross-polarization discrimination $\chi = 0, 0.2, 0.5$. Among them, the dotted lines represent the cube constellation, and the solid lines represent the optimal GA constellation. Both constellation' channel capacity decrease with the increasing $\chi$, which can be explained by more severe depolarization, thus stronger power coupling from $H$ to $V$ and from $V$ to $H$. $\chi = 0$ means the channel has no depolarization, in this case, the channel capacity curve of the GA constellation coincides with the curve of the cube constellation. $\chi > 0$ means the channel has depolarization effect, in the case, the optimal constellation exhibits a dramatic increase (maximum 1bit/symbol) in channel capacity compared to the cube constellation.

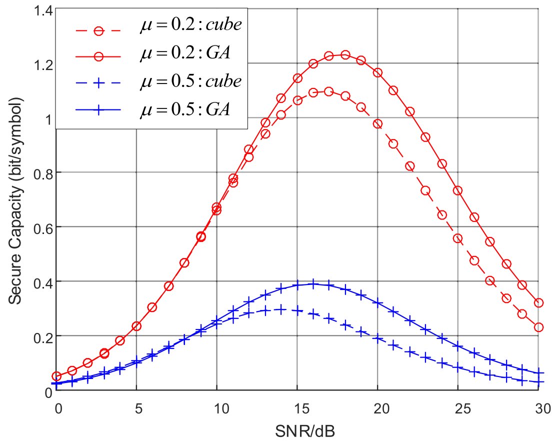


**FIGURE 11.** Secure capacity comparison between cube constellation and optimal GA constellation in different co-polar ratio $\mu = 0.5, 0.2$.

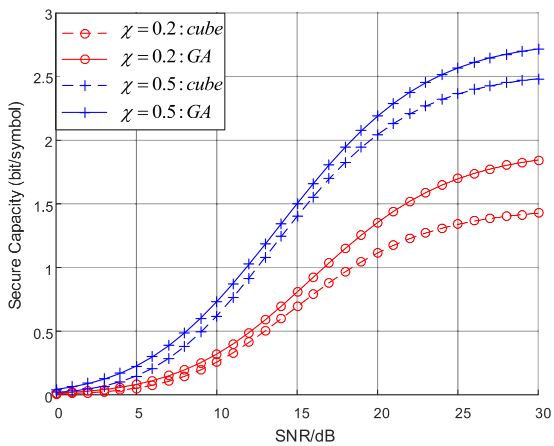


**FIGURE 12.** Secure capacity comparison between cube constellation and optimal GA constellation in different cross-polarization discrimination $\chi = 0.2, 0.5$.

As proved in Fig.9 and Fig.10, GA constellation can increase channel capacity for both cross-polarization discrimination $\chi$ and co-polar ratio $\mu$, but play a more significant role in the $\chi > 0$ situation. This is because $\chi$ has a more significant effect on channel capacity than $\mu$ when PolSK information is carried by the amplitude ratio and relative phase between $H$ and $V$ components, therefore the power coupling effect will be more significant than the power imbalance between $vv$ and $hh$ channels.

### B. SECURE CAPACITY ANALYSIS

Fig.11 shows secure capacity comparison between cube constellation and optimal GA constellation in different co-polar ratio $\mu = 0.5, 0.2$. Among them, the dotted lines represent the cube constellation, and the solid lines represent the optimal GA constellation. Both constellation' secure capacity increase with decreasing $\mu$, which can be explained by bigger difference between the legitimate channel and eavesdropped channel. Besides, the optimal GA constellation shows an increase (maximum 0.5bit/symbol) in secure capacity compared to the cube constellation. The main reason for the increase is GA constellation tends to gather around the horizontal polarization state $P_H$, which makes the GA constellation more vulnerable to co-polar ratio $\mu$.

Fig.12 shows secure capacity comparison between cube constellation and optimal GA constellation in different cross-polarization discrimination $\chi = 0.2, 0.5$. Among them, the dotted lines represent the cube constellation, and the solid lines represent the optimal GA constellation. Both constellation' secure capacity increase with increasing $\chi$, which can be explained by a bigger difference between the legitimate channel and eavesdropped channel. Besides, the optimal GA constellation shows an increase (maximum 0.5bit/symbol) in secure capacity compared to the cube constellation. The main reason for the increase is GA constellation tends to move towards the $g_1og_3$, which makes the GA constellation more vulnerable to cross-polarization discrimination $\chi$.

## VI. CONCLUSION

This paper explores a novel secure constellation design in Stokes space for polarization modulation communication. It is the first time that the concepts of channel secure capacity are applied to the constellation design field. Based on channel capacity and secure capacity, a series of power-efficient constellation and secure constellation for different channel parameters are designed respectively. Numerical results in section V indicate that the new design techniques perform well compared the other existing constellation.

Future work includes the design of the constellation which can balance of channel capacity and secure capacity.

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