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# Semi-Supervised Gated Spectral Convolution on a Directed Signed Network

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**ABSTRACT** A complex network is a powerful tool that enables a complex system in the real world to be represented as a network structure. Due to the heterogeneous edges and nodes implying rich semantic information, network representation has received considerable attention in both research and industrial domain. Over the recent years, the graph convolutional network (GCN) has provided a novel approach for learning network embeddings. However, this primarily supports undirected unsigned networks; that is, it cannot be directly applied to directed signed networks because it is challenging to effectively depict the direction and signs of edges in such models. In this paper, we therefore propose a method for semi-supervised gated spectral convolution in directed signed networks. We first extend the concept of the GCN to directed signed networks, which not only preserves the advantages of the traditional GCN but also properly describes the significance of the directions and signs of the edges. We then innovatively define sign (label) propagation rules in directed signed networks, rendering the networks semi-supervised. Furthermore, we enhance the balance theory to constrain the process of sign propagation to obtain network embedding with better interpretability. To satisfy the needs of large-scale complex networks, we propose a gating mechanism to adaptively forget sign information, which significantly reduces the time–space complexity of the sign propagation process. Finally, we compare the proposed method with state-of-the-art baselines using four real-world data sets for the classical link sign prediction task. Experimental results demonstrate that the proposed method is competitive.

**INDEX TERMS** Directed signed networks, spectral graph convolutional networks, balance theory, gating mechanism, network embedding, link sign prediction.

## I. INTRODUCTION

Acomplex network is a powerful tool that enables a complicated system in the real world to be described as a network structure [1]. These networks can be classified from different perspectives. When a network contains edges with signs (positive or negative) and directions, it is called a directed signed network or a directed polar network. A directed signed network can portray complex real-world systems in more detail, as its directed positive edges can depict positive meanings, such as like, friendship, trust, cooperation, and its directed negative edges can represent negative meanings, such as dislike, hostility, suspicion, competition. This system has been widely used to model many complex networks with dichotomous relationships, such as the relationships between friends and foes in social networks [2], trust networks with credible and suspicious comments [3], protein interactions in biological networks [4], traffic control networks [5], representing support or confrontation in international relation networks [6]. Therefore, the appropriate representation of directed signed networks is critically important and very challenging [7].

The current mainstream method for network representation is network representation learning (NRL), which learns a network representation as a low-dimensional vector. Deep-learning-based NRL has achieved great success in many graph-theoretic data mining tasks of networks, such as node classification [8], [9], link prediction [10]–[12],

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clustering [13], [14], and visualization [15], [16]. One of the most important deep-learning-based NRL methods is the graph convolutional network (GCN), which defines the convolution of a graph structure using spectral theory and introduces the relationship between the Laplacian matrix and the Fourier transform. This not only extends the traditional convolution neural network characteristics but also performs well in undirected and unsigned networks. Therefore, the GCN model has inspired our research work on directed signed networks. However, there are some unsatisfactory aspects of GCNs that need to be addressed. First, they only focus on undirected unsigned networks without considering the direction and polarity of edges. However, both the direction and the polarity of edges have a critically important influence on the formation and evolution of networks. Second, a GCN has a high time–space complexity, as it operates on the global structure of networks. Although some improved GCN methods such as fast GCN [17] reduce the complexity, the computational cost of GCNs is still very high. Third, balance theory, a critical tool in signed network analysis [16], [18]–[21] is not properly combined in GCN-based methods. Overall, none of the existing GCN-based methods possess all of the desired properties: considering the signs and directions of edges, low time–space complexity, and social theoretical interpretation.

To address these problems, we propose a method for an innovative semi-supervised gated spectral convolution in a directed signed network (DS-SGS-GCN). The main contributions of our method are as follows.

- We extend the notion of the GCN to directed signed networks, which not only preserves the advantages of a traditional GCN but also properly describes the significance of the directions and signs of edges. The formation and evolution of networks can therefore be depicted more accurately.
- We innovatively define sign (label) propagation rules in directed signed networks, thus making the networks semi-supervised. Based on the semi-supervised symbol propagation, more sign information is obtained for GCN, which significantly improved the experimental effect.
- We enhance balance theory to constrain the process of sign propagation to obtain network embedding with better interpretability.
- To satisfy the needs of large-scale complex networks, we propose a gating mechanism to adaptively forget some sign information, which not only significantly reduces the time–space complexity of the sign propagation process but compared with that before application, it can maintain or even further improve the effectiveness of the experiment.

Finally, we compare our method with state-of-the-art baselines using four real-world data sets for the classical link sign prediction task. Experimental results demonstrate that our method is competitive.

The rest of the paper is organized as follows. In Section II, we discuss some related works. In Section III, we introduce some related definitions about DS-SGS-GCN method. In Section IV, we explain our proposed model's process in detail. In Section V, we show experiment tasks and the data sets we used, list the comparative experiments, and give the experimental results and parameter analysis. And we conclude our work and discuss some future works in Section VI.

## II. RELATED WORK

Due to the wide range of possible application scenarios for directed signed networks, they have attracted more attention in recent years [22].

The signed network concept was first proposed by sociological researchers in the 1940s. Based on the theory of social psychology, Heider analyzed the interaction between negative and positive relationships in human cognitive triangles [23]. Cartwright and Harary used graph theory to describe the existence of balanced and unbalanced cognitive triangles and put forward the concept of balance theory [24]. Research had been conducted into signed social networks with positive and negative polarity based on balance theory. The researchers aimed to analyze the characteristics and evolution of signed networks, reveal their community structure, and find groups in them. However, these studies were suitable for small-scale social networks and had certain limitations. So researchers subsequently began to pay more attention to signed networks. Leskovec and Kunigis first began to conduct signed network research with positive and negative relationships and established public data sets [2], [25], [26].

With the development of network representation learning (NRL), researchers began to study signed NRL. Kunigis expanded the spectrum analysis method for an unsigned network, reduced the dimension of the the signed network characteristics, and further predicted the links and communities [16]. Wu used the spectrum of the adjacency matrix of the sign graph to divide the communities in k-balanced sign graphs, and further proved that k-balanced sign graphs were separable in the spectral space [27]. Zheng and Skillicorn [22] defined two normalized Laplacian eigenspectrum methods based on a random walk according to Kunigis' signed network spectrum analysis, which represents the nodes as low-dimensional vectors and ensures the first-order nearest-neighbor approximation [28].

In recent years, deep-learning-based network representation methods were proposed. SiNE [18] was a good attempt in the task transition from unsigned networks to signed networks. It analyzed the role of the negative edges in signed networks, and further introduces balance theory in a deep-learning framework. Kim proposes SIDE [7] that combined balance theory with a random walk. It learned the node representation in networks based on the connected sign of edges. Tyler presents SGCN [29] which integrated balance theory with the spatial GCN method. It defined a

loss-function constraint using balance theory, which is more effective than many previous results.

Although the deep learning based representation learning methods for signed networks achieve some success, they ignore the edge direction and sign in the sampling process, thus making the neighbor relationship violate the balance theory of the signed network [24], that is, the learned representation misses part of the original polarity semantic relation of the network. In fact, the key of the representation learning for directed signed network is how to correctly handle the propagation relationship between positive and negative edges of the network, so that the final representation can not only retain the original network sign information but also dig out the unknown or high-dimensional symbol information. This is the goal of this paper.

## III. RELATED DEFINITION

For the convenience of our proposed method's description, we give the related definition about semi-supervised gated spectral convolution on a directed signed network (DS-SGS-GCN) in this section. We first define the directed signed network and describe it using an adjacency matrix, then we explain the signed reachable matrix and the signed Laplacian matrix.

### A. SIGNED NETWORKS

*Definition 1 (Unsigned Network):* An unsigned network can be abstractly defined as $G = (V, E)$, where $V = \{1, 2, \ldots, n\}$ denotes the set of $N$ nodes, and E is the set of edges with $e(i, j) \in E$, $e(i, j) \in \{0, 1\}$, where $e(i, j) = 0$ denotes that there is no edge between i and j, and $e(i, j) = 1$ means that there is an edge between i and j. In an undirected network, $e(i, j)$ is equal to $e(j, i)$, and in a directed network, $e(i, j)$ may not be equal to $e(j, i)$.

*Definition 2 (Signed Network):* The definition of a signed network emphasizes the sign of the edges. It is defined as $G = (V, E, W)$, where $w(i, j) \in W$, $w(i, j) \in \{-1, 0, 1\}$, where $w(j, i) = -1$ indicates that the sign of $e(i, j)$ is negative, $w(j, i) = 0$ means that the sign of $e(i, j)$ is unknown, and $w(j, i) = 1$ shows that the sign of $e(i, j)$ is positive. A signed network $G = (V, E, W)$ can be represented as an adjacency matrix denoted as $A \in R^{n \times n}$.

$$A(i, j) = \begin{cases} 1, & \text{if sign of } e(i, j) \text{ is positive} \\ 0, & \text{if sign of } e(i, j) \text{ is unknown} \\ -1, & \text{if sign of } e(i, j) \text{ is negative} \end{cases} \quad (1)$$

Figure 1 shows a randomly selected local trust network structure from the Epinions data set [30] (It provides the relationship between users' trust and distrust of others' comments. More information about this data set will be covered in V-A2), in which positive relations (trust) are expressed with red edges and negative relations (distrust) with black edges. Its corresponding adjacency matrix is shown in (2). It can be seen that the adjacency matrix is asymmetric and relatively sparse, and the task of link sign prediction is to predict the true

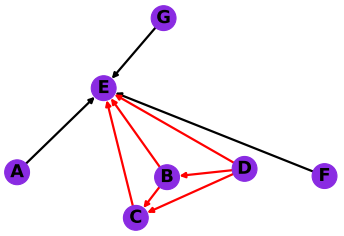



**FIGURE 1.** Simple graph of a signed network from the Epinions data set.

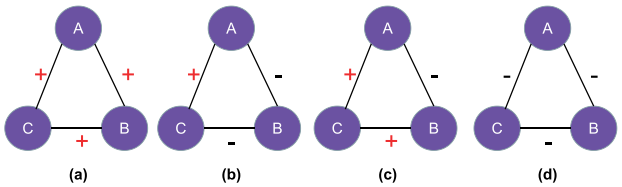



**FIGURE 2.** Balanced and unbalanced triangles in balance theory.

symbol value of elements with zero values in the adjacency matrix.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \quad (2)$$

### B. BALANCE THEORY

Since Heider proposed the theory of social structure balance, it has become the basic theory of symbolic networks. Some basic rules defined in theory have been widely used in the task of link sign prediction in signed networks. The definition of balance theory is described by the structural balance triangles in Figure 2, where a positive relation is expressed with a red "+" and a negative relation as a black "−". In Figure 2, (a) and (b) are balanced triangles, whereas (c) and (d) are unbalanced triangles.

These fundamental balance structures imply the commonsense ideas that "a friend's friend is a friend, a friend's enemy is an enemy, and an enemy's enemy is a friend." Similarly, they can also be applied in trust networks and many other signed networks. Based on these structural balance triangles, we can obtain high-order sign relations through label propagation. Taking Figure 2(a) as an example, when knowing any two edges, the sign of the third edge can be predicted as positive. We call the third edge a "virtual edge". The existence of the virtual edge is the basis of balance theory constraining the process of label propagation in signed networks. Based on this kind of constraint, the symbol propagation processes in signed networks are much improved, and this further embodies the advantages of semi-supervised learning.

It is noted that we only focus on the balanced structural triangles rather than the unbalanced ones to ensure the

correctness of the signed reachable matrix defined below. This is because in the real world, the number of unbalanced triangles is much smaller than the number of balanced triangles [29].

### C. DIRECTED SIGNED NETWORK

As mentioned above, most complex systems in the real world are the best described using directed signed networks. Therefore, we need to handle the representation problem of directed signed networks. For convenience, we first define the propagation matrix, denoted as $A_{sign}$ of such networks.

*Definition 3 (Directed Signed Network Propagation Matrix $A_{sign}$):* $A_{sign}$ is defined in (3).

$$A_{sign} = A + A^T + I \tag{3}$$

Specifically, each element in $A_{sign}$ is calculated with (4).

$$A_{sign}(i, j) = A(i, j) + A(j, i) + \lfloor i/j \rfloor * \lfloor j/i \rfloor \tag{4}$$

It can be seen that $A_{sign}$ not only retains original weights but also filters the conflict of zero-order sign values. This eliminates wrong polarity relationships in signed networks and adds self-connections to pave the way for subsequent sign propagation. Specifically, each element of the propagation matrix $A_{sign}$ is expressed as follows.

$$A_{sign}(i, j) = \begin{cases} 3, & \text{sign of self-connection} \\ 2, & \text{sign of } e(i, j) \text{ is positive} \\ 1, & \text{sign of } e(i, j) \text{ may be positive} \\ 0, & \text{sign of } e(i, j) \text{ is unknown} \\ -1, & \text{sign of } e(i, j) \text{ may be negative} \\ -2, & \text{sign of } e(i, j) \text{ is negative} \end{cases} \tag{5}$$

The propagation matrix $A_{sign}$ is consistent with first-order balance theory. Taking the social network as an example. ( In the real network, self-connection is irrelevant data, but in the sign propagation, self-connection is essential.)

- If user $i$ is user $j$'s friend and $j$ is $i$'s friend, then $i$ and $j$ must be friends with each other, and the value of $A_{sign}(i, j)$ is 2.
- If user $i$ is user $j$'s friend and the relationship from $j$ to $i$ is unknown, then $i$ and $j$ may be friends with each other, so the value of $A_{sign}(i, j)$ is 1.
- If user $i$ is $j$'s friend but $j$ is $i$'s foe, then the relationship between $i$ and $j$ is unknown, so the value of $A_{sign}(i, j)$ is 0.
- The result of the first-order negative relation propagation between user $i$ and user $j$ is still correct.

The propagation adjacency matrix quantifies the expression of first-order balance theory. However, it is noted that the weight of each element of $A_{sign}$ may interfere with the sign propagation based on normal balance triangles. Therefore, we simplify the propagation adjacency matrix to $\tilde{A}_{sign}$ which is called the ''activated propagation adjacency matrix''.

*Definition 4 (Directed Signed Network Activated Propagation Adjacency Matrix $\tilde{A}_{sign}$):* $\tilde{A}_{sign}$ is defined in (6).

$$\tilde{A}_{sign} = sgn(A_{sign}) \tag{6}$$

where sgn(·) is a signum function described in (7).

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases} \tag{7}$$

This means that $\tilde{A}_{sign}$ eliminates the weight but retains the sign information for each element. Therefore, the elements of $\tilde{A}_{sign}$ can be expressed as (8).

$$\tilde{A}_{sign}(i, j) = \begin{cases} 1, & \text{for positive information} \\ 0, & \text{none valid information} \\ -1, & \text{for negative information} \end{cases} \tag{8}$$

We take $\tilde{A}_{sign}$ as the first-order signed reachable matrix (we will visual this later) to realize the high-order interpretation and sign propagation task.

### D. DIRECTED SIGNED REACHABLE MATRIX

Based on the definitions above, we define the directed signed network reachable matrix to realize a semi-supervised sign propagation process in our DS-SGS-GCN model using enhanced balance theory.

*Definition 5: (Directed Signed Network m-Step Reachable Matrix $M_m$):* The reachable matrix of a directed signed network $G = (V, E, W)$ is defined in (5).

$$M_m(i, j) = \begin{cases} 1, & \text{for positive reachability} \\ 0, & \text{unreachable} \\ -1, & \text{for negative reachability} \end{cases} \tag{9}$$

*Theorem 1:* The directed signed network m-step reachable matrix $M_m \in R^{N \times N}$ makes sense.

Here, we prove theorem 1 with a constructive method:

**Step 1.** Zero-order and first-order reachable matrices make sense.

We define the zero-order and first-order reachable matrices in (10a) and (10b).

$$M_0 = I \tag{10a}$$

$$M_1 = \tilde{A}_{sign} \tag{10b}$$

where $M_0$ is the zero-order reachable matrix and $I$ is a unit matrix. $M_0$ implies that, each node in the directed signed network $G$ has a positive relation with itself. This is the most essential relationship, as it affects the subsequent sign propagation process from low-order to high-order. $M_1$ in (10b) is the first-order reachable matrix. As mentioned in the previous subsection III-C, this is just the activated propagation adjacency matrix.

**Step 2.** The second-order signed network reachable matrix $M_2$ makes sense.

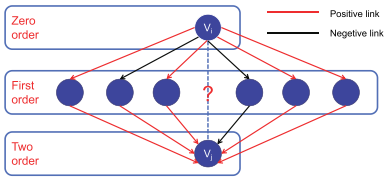



**FIGURE 3.** Second-order sign propagation diagram.

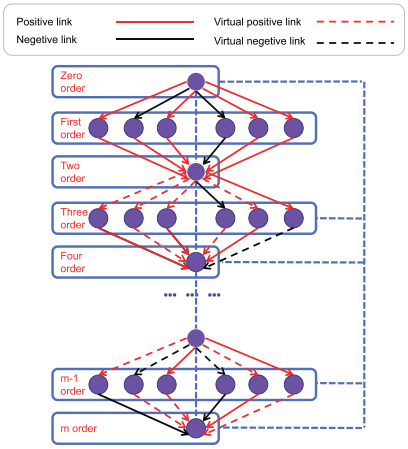



**FIGURE 4.** m-order sign propagation diagram.

The second-order signed network reachable matrix $M_2$ is expressed in (11).

$$M_2 = \mathrm{sgn}(M_1 \cdot M_1) = \mathrm{sgn}(\sum_{k=1}^{n} M_1(i,k) * M_1(k,j)) \quad (11)$$

The meaning of $M_2$ is as follows. For any two nodes $(i, j)$ in the directed signed network, we find all the common neighbors (k) based on first-order reachability. If the signs of edge $e(i, k)$ and edge $e(k, j)$ are determined, then the sign of the edge $e(i, j)$ can be predicted using balance theory. Otherwise, there is no valid sign propagation from node $i$ to node $j$. As mentioned in Definition 4, because the low-order sign information needs to be propagated continuously to the high-order using balance theory, $M_2$ should be activated with the signum function.

**Step 3.** If the (m-1)-order reachable matrix $M_{m-1}$ makes sense, then the m-order reachable matrix $M_m$ also makes sense, because it can be expressed as in (12).

$$M_m = \mathrm{sgn}(M_{m-1} \cdot M_1) = \mathrm{sgn}(\sum_{k=1}^{n} M_{m-1}(i,k) * M_1(k,j)) \quad (12)$$

The result of $M_m$ can be regarded as the activated weighted sum of symbol information from node $i$ to node $j$ through different paths in m-steps (we will visual this in subsection III-E). Similar to $M_2$, $M_m$ needs to be activated with the signum function to eliminate the bias caused by the weight.

Proof completed. ■

The intuitive meaning of $M_2$ is shown in Figure 3. It can be seen from this that both the sign prediction of the virtual edge and the second-order sign propagation are based on the known first-order information and the added self-connection.

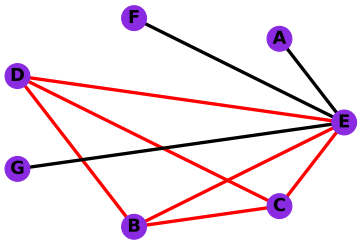



**FIGURE 5.** Example of visualizing the first-order reachable matrix.

Similarly, we get the intuitive meaning of $M_m$ as shown in Figure 4.

From the above diagram, we find that the virtual link plays an important role in the transmission of signs in the subsequent propagation process, which embodies the advantage of graph-based semi-supervised learning in a directed signed network.

Now, we take the network in Figure 1 as a toy example to explain the significance of the directed signed reachable matrix. The first-order reachable matrix is visualized in Figure 5, and its formal expression is shown in (13).

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{pmatrix} \quad (13)$$

In Figure 5, a red edge indicates a positive link between two points, and a black edge indicates a negative link. It can be seen that the result of Figure 5 is exactly the same as the meaning of $M_1$.

Based on the first-order reachable matrix, the second-order reachable matrix is visualized in Figure 6, and its formal expression is described in (14). In Figure 6, solid lines represent low-order (original) sign information, and dashed lines represent high-order (virtual) sign information.

$$M_2 = \begin{pmatrix} 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 \end{pmatrix} \quad (14)$$

It is noted that, with the increase of the order, the amount of information contained in the reachable matrix increases sharply, and even tends to be saturated (complete graph). Because of this, the cost of the subsequent training will increase. We will tackle this problem in IV-B2.

### *E. SIGNED LAPLACIAN MATRIX*

In this subsection, we will define the Laplacian matrix for directed signed networks, and further derive the

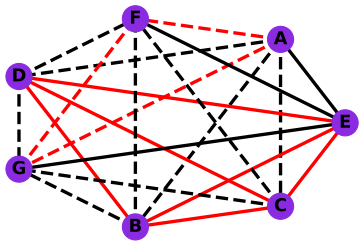



**FIGURE 6.** Example of visualizing the second-order reachable matrix.

spectral-domain convolution operation for directed signed networks.

The Laplacian matrix is the basis of spectral-domain GCNs. Its excellent properties, such as being a semi-positive definite and symmetric matrix, support the traditional Fourier transform and the spectral convolution of undirected unsigned networks. The normal Laplacian matrix [31] of an undirected unsigned network, denoted as $L$, is defined in (15).

$$L = D - A \tag{15}$$

where $A$ is the adjacency matrix of a network and $D$ is the diagonal degree matrix of $A$. As the Laplacian matrix is only suitable for undirected unsigned networks, we should define an effective Laplacian matrix for directed signed networks and apply it to our DS-SGS-GCN model.

*Definition 6 (The Directed Signed Laplacian Matrix $L_{sign}$):* The Laplacian matrix of directed signed networks $L_{sign} \in R^{N\times N}$ is defined in (16).

$$L_{sign} = \overline{D} - \tilde{A}_{sign} = \overline{D} - \mathrm{sgn}(A + A^T + I) \tag{16}$$

where $\overline{D}$ is the diagonal degree matrix defined by (17) and $\tilde{A}_{sign}$ is the activated propagation adjacency matrix of directed signed networks in Definition 4.

$$\overline{D} = diag(\sum_{j=0}^{n} \left|\tilde{A}_{sign}(i,j)\right|) \quad (i = 1, 2, \ldots, n) \tag{17}$$

Now we carry out spectral decomposition to $L_{sign}$ and obtain the expression in (18).

$$L_{sign} = U \Lambda U \tag{18}$$

where $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$ is the diagonal eigenvalue matrix and $U = (\overrightarrow{u_1}, \overrightarrow{u_2}, \ldots, \overrightarrow{u_n})$ is its corresponding eigenvector matrix. By using $U$ as the basis of the Fourier transform, we define the rules of forward and inverse Fourier transformation as shown in (19a) and (19b).

$$\widehat{P} = U^T P \tag{19a}$$

$$P = U\widehat{P} \tag{19b}$$

where $P$ represents any node vector in the directed signed network. According to these two formulas, the representation of a directed signed network can be converted from the spatial domain to the spectral domain ($U^T P$), and the convolution kernel can also be converted to the spectral domain ($U^T X$). Then, the convolution operation between the directed signed

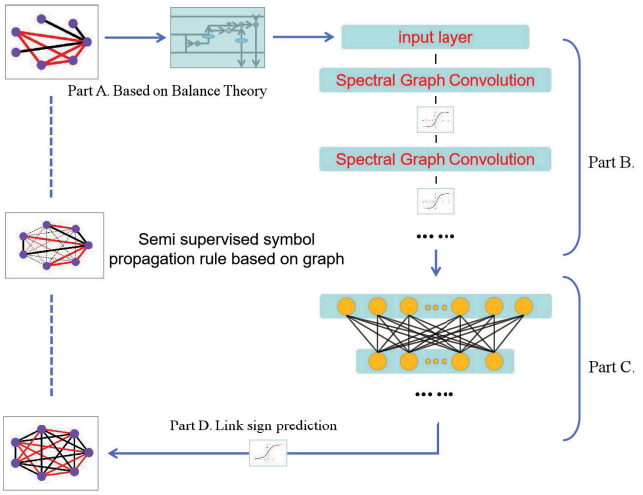



**FIGURE 7.** Frame of our DS-SGS-GCN model.

Laplacian matrix and the convolution kernel is fulfilled, which is described by (20).

$$(X \otimes P) \leftarrow U((U^T X) \cdot (U^T P)) \tag{20}$$

where $\otimes$ represents the convolution rule of the directed signed network, and $X$ is the convolution kernel matrix.

## IV. DS-SGS-GCN

This section describes the DS-SGS-GCN method, including the model framework, methods, and the algorithm description.

### A. MODEL

The model architecture of the DS-SGS-GCN is shown in Figure 7, which consists of four parts as follows.

In Part A, we first process the signed network based on traditional theory and compute its adjacent matrix, reachable matrix, and Laplacian matrix. Then, these are fed to the gating mechanism. The output, that is, the m-order reachable matrix or the m-order forget-gate-controlled reachable matrix of the directed signed network, is the input layer of Part B.

In Part B, there is a series of spectral graph convolution layers that are used to encode the directed signed networks to learn the node vector representation. In order to introduce non-linear transformations, we add non-linear activation function layers between the spectral graph convolution layers. The non-linear activation function used here is a hyperbolic tangent function (tanh), as this can transfer the information of the nearest neighbors better than ReLU or some other operators, which makes the training process more efficient. After passing through these layers, we get the node representation, which is the input for Part C.

Part C decodes the output from Part B. The decoding strategy is an open problem; the method could also use deep learning. However, in this paper, we use the simple inner product decoder and sigmoid activity function to get the result of the link sign classification. The original directed signed network is then reconstructed.

In Part D, we obtain the reconstructed adjacency matrix to verify the effectiveness of our model in link sign prediction tasks.

### B. METHODS

In the framework shown in Figure 7, the following methods are employed. First, the spectral convolution theory of unsigned networks is extended and applied to directed signed networks. Secondly, social balance theory is introduced into the spectral convolution, and the constraints of label propagation are defined. Thirdly, the gating mechanism is designed to complete the encapsulation of label propagation in directed signed networks. Among which, the forget gate adaptively retains high-order constraints and forgets low-order constraints. Finally, an encoder–decoder network is designed to implement the link sign prediction task for directed signed networks. This will be explored in detail.

#### 1) SEMI-SUPERVISED LEARNING

In a directed signed network, there is a large amount of unlabeled data and a small amount of labeled data. If these two kinds of data are both fed to our DS-SGS-GCN model, it should gain better performance. Therefore, we introduce balance theory to determine the basic regulation of sign propagation, so as to comprehensively consider the positive and negative sign propagation rules and find the relationships between labeled and unlabeled data. Inspired by graph-based semi-supervised learning [31], we define the sign transfer matrix $S$ in (21).

$$\begin{aligned} S_m(i,j) = \ & \mathrm{sgn}(\alpha S_{m-1}(i,j) + (1-\alpha) \\ & * \mathrm{sgn}(\sum_{k=1}^{n} S_{m-1}(i,k) * S_1(k,j))) \\ & \text{with } S_1 = \tilde{A}_{sign} \end{aligned} \tag{21}$$

It can be seen that the first-order sign propagation depends not only on the information from neighbors but also on the existing sign states. The parameter $\alpha$ adjusts the effect of balance theory and the known low-order sign state. It can be seen that when the value of $\alpha$ is set to 0, the label transfer matrix is just the reachable matrix of the directed signed network, as shown in (22).

$$S_m(i,j) = sgn(\sum_{k=1}^{n} S_{m-1}(i,k) * S_1(k,j)) \text{ with } \alpha = 0 \tag{22}$$

#### 2) GATING MECHANISM

As mentioned above, in the process of sign propagation, a large amount of unlabeled data obtains labels. However, this causes a sharp increase in the amount of information in the reachable matrix, which in turn increases the time–space complexity of the model. To address this problem, inspired by the gating mechanism of LSTM networks [32] and GRU [33], we define the corresponding update gate, reset gate, and forget gate for the acquisition of the m-order reachable matrix.

A gated reachable matrix can selectively ignore low-order sign information and retain high-order sign information, thus making the matrix sparsity and the model validity balanced. It is noted that the gating units in our model are not the same as those that are most commonly applied. They are not used in the convolutional network but only encapsulate the iterative propagation of sign information. Therefore, the convolution kernel parameters do not need to be trained. Our gating mechanism is shown in Figure 8.

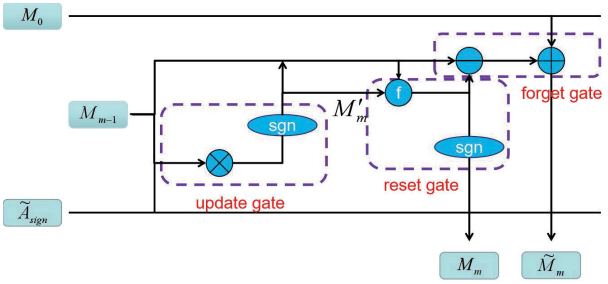



**FIGURE 8.** Flow diagram of our gating mechanism.

There are three kinds of gates in the gating mechanism: update gate, reset gate, and forget gate. The input of the $m$-th gating unit is the zero-order signed reachable matrix $M_0$, the activated propagation adjacency matrix $\tilde{A}_{sign}$, and the output value from the previous reset gate $M_{m-1}$. The output is the $m$-order reachable matrix $M_m$ or an $m$-order forget-gate-controlled reachable matrix $\widetilde{M}_m$.

The update gate updates the output from the previous layer according to the sign propagation rules defined by balance theory to obtain the quasi-output result from this layer. Its formal representation is as follows.

$$\text{update gate: } M'_m = sgn(M_{m-1} \cdot \tilde{A}_{sign}) \tag{23}$$

The reset gate summarizes the original reachable sign information and the quasi-reachable sign information of this layer. Its formal representation is shown in (24).

$$\text{reset gate: } M_m = sgn(\alpha M_{m-1} + (1-\alpha)M'_m) \tag{24}$$

The forget gate is used to forget some information in the lower order, so it adopts the principle of coexistence at the same order. In other words, the information transmitted by the sign is not randomly forgotten, but the gate selectively forgets the low-order symbol reachable information while retaining the symbol reachability information of this iteration and the zero-order symbol reachability information. Its formal expression is as follows.

$$\text{forget gate: } \widetilde{M}_m = M_m - M_{m-1} + M_0 \text{ with } M_0 = I \tag{25}$$

Therefore, the sign operations in Figure 8 represent matrix multiplication, matrix subtraction, and matrix addition, respectively. The function $f(x,y) = (\alpha x + (1-\alpha)y)$ is the basic rule of sign propagation of balance theory in graph-based semi-supervised learning, where $\alpha$ is the hyper-parameter $\alpha \in [0,1]$.

The gating mechanism therefore encapsulates the recursive process of the high-order signed reachable matrix, and further implements the semi-supervised sign propagation algorithm

in directed signed networks. In addition, the forget gate makes an effective initialization for application to large networks.

### C. ALGORITHM

According to the above series of processes, our DS-SGS-GCN method is described as Algorithm 1.

**Algorithm 1** Semi-Supervised Gated Spectral Convolution on a Directed Signed Network

**Input**: Directed Signed network adjacency matrix $\mathbf{A}$; Graph $\mathbf{G}$ based on signed networks; The order of reachable Matrix $\mathbf{m}$; The number of hidden layers $\mathbf{L}$; Epochs $\mathbf{E}$.

**Output**: Node repression matrix $\mathbf{H^{(L)}}$; Reconstructed $\tilde{\mathbf{A}}_\mathbf{r}$;

1 $\mathbf{A_{sign}} \leftarrow A + A^T + I$
2 $\tilde{\mathbf{A}}_\mathbf{sign} \leftarrow \mathrm{sgn}(A_{sign})$
3 $\widetilde{\mathbf{D}} \leftarrow diag\{\sum_{i=1}^{n} \left|\tilde{A}_{sign}(i,j)\right|\}$
4 $\mathbf{L_{sign}} \leftarrow \widetilde{D} - \tilde{A}_{sign}$
5 $\mathbf{M_0} \leftarrow I$
6 $\mathbf{M_1} \leftarrow \tilde{A}_{sign}$
7 **for** $i \in \{2, \ldots, m\}$ **do**
8  $\mathbf{M_i'} \leftarrow \mathrm{sgn}(M_{i-1} \cdot \tilde{A}_{sign})$
9  $\mathbf{M_i} \leftarrow \mathrm{sgn}(\alpha M_{i-1} + (1-\alpha)M_i')$
10  **if** *use forget gate* **then**
11  $\widetilde{\mathbf{M}}_\mathbf{i} \leftarrow M_i - M_{i-1} + M_0$
12  **end**
13 **end**
14 **if** *use forget gate* **then**
15  $\mathbf{H^{(0)}} \leftarrow \widetilde{M}_m$
16 **else**
17  $\mathbf{H^{(0)}} \leftarrow M_m$
18 **end**
19 **for** *epoch* $\in \{1, \ldots, E\}$ **do**
20  **for** $l \in \{1, \ldots, L\}$ **do**
21  $\mathbf{H}^{(l)} \leftarrow \sigma(\widetilde{D}^{-1/2}\tilde{A}_{sign}\widetilde{D}^{-1/2}H^{(l-1)}W)$
22  $\mathbf{Z} \leftarrow H^{(l)}$
23  $\mathbf{Q}_\mathbf{r} \leftarrow \sigma(ZZ^T)$
24  Update parameters $(W)$ with $\tilde{A}_r$
25  **end**
26 **end**

From line 1 to line 6, we obtain the propagation adjacency matrix, the activated propagation adjacency matrix, and its corresponding degree matrix based on the initial adjacency matrix. Then, the signed Laplacian matrix and the low-order signed reachable matrix are determined.

From line 7 to line 13, the gating mechanism is implemented iteratively. The update gate and the reset gate must be used, but the forget gate is optional.

Therefore, at line 14, it is determined whether the input of the model is an m-order reachable matrix or an m-order forget-gate-controlled reachable matrix.

From line 19 to the end its encoder adopts semi-supervised gated spectral convolution, its decoder is the inner product, and the loss function is weighted cross entropy with logits. Therefore, unlike a non-probabilistic GAE [34], we use our proposed methods as the encoder. The specific formula is as follows.

$$\tilde{A}_r = \sigma(ZZ^T), \text{ with } Z = \text{SGS-GCN}(M_m, A) \quad (26)$$

where $\tilde{A}_r$ is the reconstructed adjacency matrix. During the training process, we add the option of $L2$ regulation to prevent over-fitting.

## V. EXPERIMENTAL ANALYSIS

In this section, we compare our methods with state-of-the-art methods on four data sets to evaluate their performance. Experimental results show the improved effects of our DS-SGS-GCN model.

### A. EXPERIMENTAL SETUP

Before we show the results of our experiment, it's necessary to introduce the experiment setting briefly.

#### 1) BASELINE METHODS

We regard four excellent algorithms involved in signed networks as baselines.

- Signed spectral embedding (SSE) [16]: This proposes two normalized spectral analysis methods for signed graphs and uses the spectral clustering algorithm to get a good graph embedding representation.
- Signed network embedding in social media (SiNE) [18]: This presents a deep-learning framework SiNE for signed network embedding. Balance theory is applied in the model, and the validity of this method is proved.
- Representation learning in signed directed networks (SIDE) [7]: This gives a general network embedding method that represents the sign and direction of edges in the embedding space. The SIDE method is mainly based on a random walk and balance theory. This has achieved excellent results in some real data sets.
- Signed graph convolutional network [29]: This applies balance theory and GCNs to signed networks. The appropriate application of spatial convolution and balance theory makes the model excellent.

#### 2) DATA SETS

Four real-world signed network data sets have been chosen for our experiments. Their details are as follows [30].

- The Bitcoin-Alpha and Bitcoin-OTC data sets are trust networks for bitcoin transactions from two different bitcoin trading platforms. Since Bitcoin users are anonymous, the platform records the trust scores of both parties to maintain a record of users' reputations. Therefore, applying the link sign prediction task to this kind of data set can help users avoid transactions with fraudulent and risky users.

**TABLE 1.** The summary of four real world signed networks.

| Data | Bitcoin-Alpha | Bitcoin-OTC | Slashdot | Epinions |
|---|---|---|---|---|
| # node | 3774 | 6005 | 32259 | 15216 |
| # Links | 24180 | 35592 | 423702 | 597179 |
| # Positive Links | 22645 | 32029 | 322861 | 525204 |
| # Negative Links | 1532 | 3563 | 100841 | 71975 |

- Slashdot is a technology-related news website. Each user of this website can tag another user as a friend (positive) or a foe (negative). They may have different opinions, ideas, and even factions, and the link relationship predictions will have a positive impact on the analysis of a certain point of view.
- Epinions is a general consumer review site. Members of the website can decide whether to trust others' reviews. Trust-signed networks are made from these relationships to determine which reviews are shown to the user.

The specific information in the experimental data sets is summarized in Table 1. It can be seen that positive and negative linkes are imbalanced in these datasets.

In the experiment, for large networks, i.e., Slashdot and Epinions, we used a breadth-first search algorithm to select users and find a suitable and complete network randomly.

#### 3) PARAMETER SETTINGS

In all experiments, we set two layers of spectral graph convolutional, with dimensions of 64 and 32. The hyperparameter $\alpha$ is set to 0. For the convolution kernel parameter, we used the Xavier initialization [35]. And the parameters were optimized using Adam with a fixed learning rate of 0.01. The hyperparameter m was tuned using matrix sparsity and grid search on the validation set. In general, the second-order signed reachable matrix is used to achieve the best effect when the forget gate is not applied. However, when it is applied, the high-order forget-gate-controlled reachable matrix of a 6th-order could achieve better results.

Our baselines' parameters were set as those in the original paper. The node embedding representations obtained were used as the encoding layer to participate in the evaluation of the subsequent link sign prediction. If the corresponding source code was not published, we adopted the best-known experimental results for these data sets [29].

### B. EVALUATION CRITERIA

We used two final evaluation standards: the area under the receiver operating characteristic curve (AUC) and the F1 score. The usual definitions of these accuracy measures are as follows.

Consider a binary classification prediction problem in which the outcomes are labeled either as positive (p) or negative (n). There are four possible outcomes from a binary classifier:

- True positive: The outcome from a prediction is p, and the actual value is also p.

**TABLE 2.** AUC scores for link sign prediction.

| Method | Bitcoin-Alpha | Bitcoin-OTC | Slashdot | Epinions |
|---|---|---|---|---|
| SSE | 0.764 | 0.803 | 0.769 | 0.822 |
| SiNE | 0.778 | 0.814 | 0.792 | 0.849 |
| SIDE | 0.630 | 0.618 | 0.547 | 0.571 |
| SGCN | 0.796 | 0.823 | 0.804 | 0.864 |
| DS-SGS-GCN | 0.838 | **0.909** | 0.851 | **0.869** |
| DS-SGS-GCN+ | **0.840** | **0.909** | **0.860** | 0.861 |

**TABLE 3.** F1 scores for link sign prediction.

| Method | Bitcoin-Alpha | Bitcoin-OTC | Slashdot | Epinions |
|---|---|---|---|---|
| SSE | 0.898 | 0.923 | 0.820 | 0.901 |
| SiNE | 0.888 | 0.878 | 0.854 | 0.914 |
| SIDE | 0.738 | 0.750 | 0.646 | 0.711 |
| SGCN | 0.917 | 0.925 | 0.865 | 0.933 |
| DS-SGS-GCN | **0.973** | **0.971** | **0.906** | 0.952 |
| DS-SGS-GCN+ | 0.970 | 0.970 | **0.906** | **0.953** |

- False positive: The outcome from a prediction is p whereas the actual value is n.
- True negative: Both the prediction outcome and the actual value are n.
- False negative: The prediction outcome is n whereas the actual value is p.

The definition of the true positive rate (TPR) describes how sensitive the model is to the positive case category, and the mean of the false positive rate (FPR) represents how sensitive the model is to the negative category. The mathematical expressions of TPR and FPR correspond to the following formulas (27) and (28).

$$TPR = \frac{TP}{TP + FN} \tag{27}$$

$$FPR = \frac{FP}{FP + TN} \tag{28}$$

Based on the value of TPR and FPR, we can obtain the receiver operating characteristic (ROC) curve, which is used to reflect the changing relationship between TPR and FPR. The AUC value is the area under the ROC curve, which is created by plotting the TPR against the FPR at various threshold settings. The larger the AUC is, the better the discrimination ability of the model.

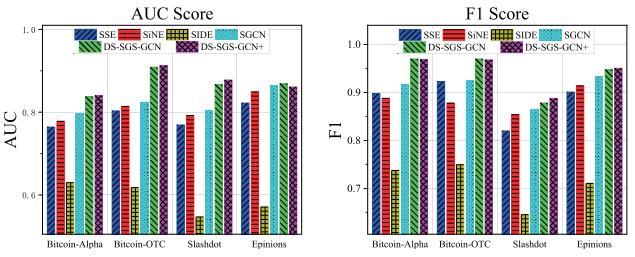
The F1 score is the harmonic mean of precision and recall, which is equivalent to the comprehensive evaluation index of precision and recall.

$$F1 = \frac{2 * precision * recall}{precision + recall} \tag{29}$$

with $precision = TP/(TP + FN)$ and $recall = TPR$

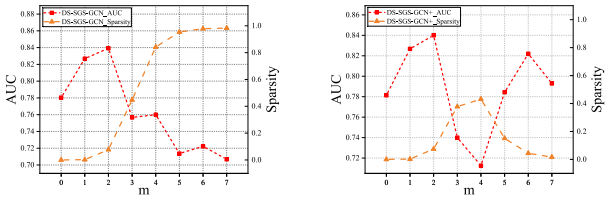
### C. EXPERIMENTAL RESULTS

The comparison results in terms of the AUC and F1 scores are summarized in the charts 2, 3 and Figure 9.



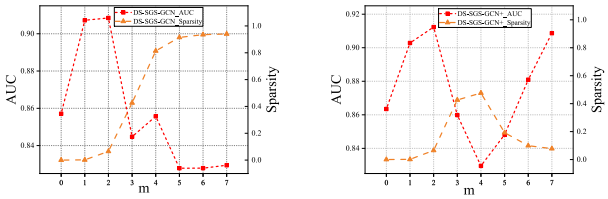

(a) AUC comparison results (b) F1 comparison results

**FIGURE 9.** Left, AUC comparison results and right, F1 comparison results.



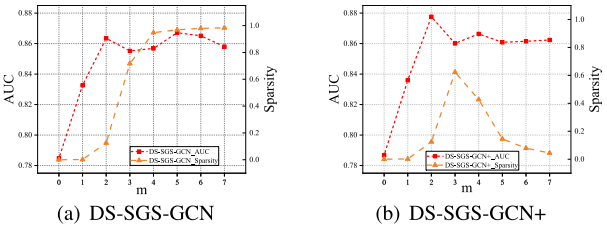

(a) DS-SGC-GCN (b) DS-SGC-GCN+

**FIGURE 10.** The comparative experiments result on Bitcoin-Alpha data set.



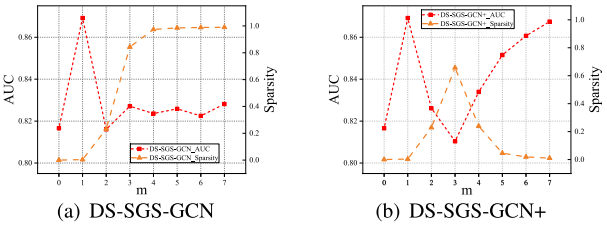

(a) DS-SGS-GCN (b) DS-SGS-GCN+

**FIGURE 11.** The comparative experiments result on Bitcoin-OTC data set.



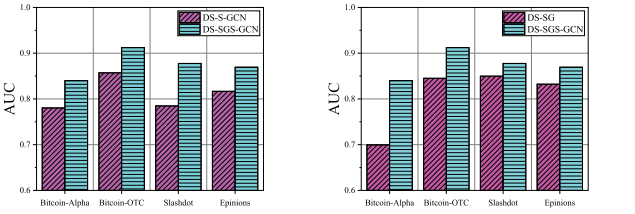

(a) DS-SGS-GCN (b) DS-SGS-GCN+

**FIGURE 12.** The comparative experiments result on Slashdot data set.




(a) DS-SGS-GCN (b) DS-SGS-GCN+

**FIGURE 13.** The comparative experiments result on Epinions data set.




(a) Effect of the gating mechanism. (b) Effect of the spectral convolution

**FIGURE 14.** The effect of the methods of "gating mechanism" and "spectral convolution".

- DS-SGS-GCN: This directly utilizes the m-order signed reachable matrix as the constraint information and input layer.
- DS-SGS-GCN+: This uses a forget gate based on DS-SGS-GCN and uses an m-order signed forget-gate-controlled reachable matrix as the input layer for the model.

It can be seen that the network representation effect of our method is better than for the other algorithms. More importantly, the DS-SGS-GCN+ model did not reduce the prediction of the model by forgetting the low-order information and even performed slightly better than the normal DS-SGS-GCN on some data sets. This may be due to the inclusion of higher-order information for low-order information and the redundancy expressed by low-order information and the signed Laplacian matrices. In addition, the time–space complexity of DS-SGS-GCN+ using high-order information is significantly reduced due to the setting of the forget gate. We will cover this in the next subsection.

### D. RESULT ANALYSIS

For the standard DS-SGS-GCN model, the low-order reachable matrix has a good effect on each data set. However, if the scalability of the model is to be increased to facilitate subsequent research or application, learning high-order, signed, reachable information is an indispensable step. This also explains the reason why the forget-gate-controlled matrix is used in the higher order when the effect of the second-order signed reachable matrix is good. Taking the Bitcoin-Alpha data set as an example, the time–space complexity will be expressed by the sparsity of the matrix [36] as shown in Figure 10(a).

We then try to control the amount of constraint information in the signed reachable matrix, i.e., the sparsity of the matrix, and encapsulate the series of processes into the forget gate in the gating mechanism for experimental processing. We then obtained the results shown in Figure 10(b). It can be seen that when the sparsity s of the signed reachable matrix is between 0.0 and 0.1, the effect of the model is relatively good, some even better than the low-order's performance. The results of our comparative experiments on the other three data sets are shown in (11) to (13). Above all, we can find the forget-gate-controlled reachable matrix's good performance on these data sets. Not only decrease the time-space complexity to a certain degree but maintain the signed prediction effect. Therefore to better observe the effect of gating mechanism (graph-based semi-supervised learning is encapsulated

in it) and spectral convolution on link sign prediction task,we have made an additional comparative experiment, as shown in Figures 14(a) and 14(b), which demonstrate the validity of our proposed method.

## VI. CONCLUSION

This paper combines balance theory, sign propagation theory, the semi-supervised learning method in an unsigned network, and the spectral-domain convolution method to propose a semi-supervised gated spectral convolution for a directed signed network. We first extended the notion of GCN to directed signed networks, which not only preserves the advantages of traditional GCN but also properly describes the significance of the directions and signs of edges. Then, we innovatively defined sign (label) propagation rules in directed signed networks, thus making the networks semi-supervised. Thirdly, we enhanced balance theory to constrain the process of sign propagation to obtain network embedding with better interpretability. Moreover, to satisfy the needs of large-scale complex networks, we proposed a gating mechanism to adaptively forget sign information, which significantly reduces the time–space complexity of the sign propagation process. Finally, we compared our method with state-of-the-art baselines using four real-world data sets for the classical link sign prediction task. Experimental results demonstrate that our method is competitive.

We have three plans for future work. The first is to integrate the gating mechanism with spectral convolution and add a forget gate between the convolution layer and the activation layer to reduce the representation similarity of each node caused by over-convolution. The second is to increase the spectral convolution depth to increase the effect of the model. The third is to introduce a multi-granularity gating mechanism to construct a mixed symbolic reachable matrix to obtain a better model effect.

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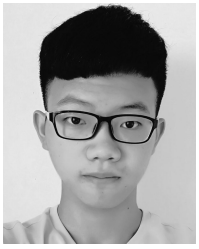

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