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The Influence of Nonlinear Stiffness of Novel Flexible Road Wheel on Ride Comfort of Tracked Vehicle Traversing Random Uneven Road

YAOJI DENG, YOUQUN ZHAO, WEI PI, YUHAO LI, SHILIN FENG, AND YIYAN DU
College of Energy and Power Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

Corresponding author: Youqun Zhao (yqzhao@nuaa.edu.cn)

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ABSTRACT The linear wheel model is the most widely used in ride comfort analysis of tracked vehicles. However, as the speed of the vehicle increases and the application of new materials, and the road wheel becomes lighter and softer, its nonlinear characteristics become more and more obvious and can't be ignored in vehicle dynamics simulation. This paper aims to study the effect of nonlinear stiffness of a novel flexible road wheel (FRW) with unique suspension bearing structure on the ride comfort of a tracked vehicle traversing random uneven road. The linear and nonlinear models of FRW were established by fitting the load-deflection data obtained from the static loading test of the physical prototype. The established linear and nonlinear models of FRW were added to a half-vehicle model of a tracked vehicle which has been proved by published test results. The ride comfort of the half-vehicle model of the tracked vehicle with linear and nonlinear models of FRW on random uneven road surface was studied in detail. The study results show that, compared with the linear wheel model, the nonlinear model has a tremendous influence on the dynamic response of the tracked vehicle, effectively suppressing the vibration, especially for the high frequency excitation. In addition, the nonlinear factor has a greater impact on the dynamic performance of the wheel than on the suspension and the body. The research results enrich the study of nonlinear dynamics of tracked vehicles, and provide reference for nonlinear modeling of other pneumatic tires and non-inflatable wheels.

INDEX TERMS Ride comfort, flexible road wheel, wheel stiffness test, nonlinear wheel model, Matlab/Simulink.

I. INTRODUCTION

For general military tracked vehicles, the crawler propulsion device includes the crawler, driving wheel, induction wheel, tensioning device, road wheel and supporting belt wheel. Its basic function is to support the vehicles; transform the torque output into the traction force to push the vehicle forward through the driving wheel and crawler, transmit the ground braking force to realize effective braking of tracked vehicles [1]. As an important part of the propulsion device of tracked vehicle, the performance of the road wheels directly affects the riding comfort, combat performance and maneuverability of tracked vehicles.

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Tracked vehicles usually have a high degree of off-road mobility, and they are often required to pass through uneven fields, trenches, low walls and soft ground, which will bring strong vibration [2]. In the operation process of tracked vehicles, the harm brought by the vibration has a great impact on the environment and the health of passengers. In addition, the violent vibration of the car body also limits the increase in the vehicle speed, hindering the high-speed development trend of tracked vehicles. Currently, most of the road wheels for tracked vehicles is a rigid solid structure, and the vibration damping effect is not satisfactory [3]. Besides, this structure also has the disadvantages of low carrying efficiency and large mass. Therefore, it is urgent to develop a new type of road wheel to improve the overall performance of tracked vehicles, cooperating with suspension system design.

Aiming at the above-mentioned shortcomings of traditional rigid road wheel, a large number of researchers have invented some road wheels with novel structure and material to improve the performance of the road wheel in some specific aspects. For example, Yan *et al.* [4] introduced the damping layer into the road wheel structure to improve the vibration damping performance. Verma *et al.* [5] utilized carbon fiber reinforced polymer composites to develop a lightweight road wheel with a high strength and stiffness. Abeyratne [6] invented a lightweight rim for idler and road wheels, which greatly reduces the vehicle weight and increase the vehicle speed. Simula *et al.* [7] developed a hollow-shell road wheel with a circumferential cavity, increasing the strength and reduces the weight. Although these new road wheels have achieved some beneficial effects, they have not fundamentally breakthrough solid rigid structure, and the original inefficient bearing mechanism does not change. Under this background, this paper introduced a new kind of flexible road wheel with a composite structure, which has a unique suspension-bearing mechanism [8-11]. This structure has great potential to improve the lightweight level, loading efficiency and ride comfort of tracked vehicles.

In the dynamic analysis of tracked vehicles, the road wheel is often modeled as a linear model. For example, Ata and Oyadiji [12] and Ata and Salem [13] developed a linear point contact wheel model for APC M113 tracked vehicle and studied the influence of suspension configurations on the vehicle dynamic performance. Dhir and Sankar [14] summarized various analytical wheel models including point contact model, rigid tread band model, fixed foot-print model, adaptive foot-print model for ride comfort simulation of tracked vehicles. Davis [15] developed a radial-spring tire model which envelop irregular features of a rigid road and reflect both the elevation and slope characteristic of the road contacting the tyre. These wheel models are linear and focus on wheel/track-terrain interaction modeling without considering the nonlinear wheel stiffness. However, as the increase in speed of modern tracked vehicles, and the road wheel becomes lighter and softer, the nonlinearity of the road wheel has an increasing influence on vehicle dynamic performance and can't be ignored in the actual vehicle performance research [16]–[19].

In the past, most studies focus on the nonlinear dynamics of passenger cars rather than tracked vehicles. Meanwhile, the nonlinear factors only consider the nonlinearity of the suspension system [20]–[22]. For example, Solomon and Padmanabhan [23], [24] established a nonlinear model of a hydro-gas suspension system for studying the effect of suspension parameters on the ride comfort of a tracked vehicle. Zhu *et al.* [25] developed a nonlinear mechanical model of the air spring which consists of three split force branches, asymmetrical hysteresis and frequency dependency. Demir *et al.* [26] derived an analytical nonlinear vehicle model including nonlinear tire and suspension stiffness for a hybrid fuzzy logic suspension control approach. Jin and Luo [27] studied the stochastic optimal active control of a half-vehicle model considering the nonlinear suspension damping and stiffness. Yıldız *et al.* [28] developed a nonlinear model of the damper to predict the damping force based on voltage input, internal state and velocity input. Liang *et al.* [29] established a nonlinear systems of tracked vehicle half-car suspensions considering the nonlinear damping of springs. Kilicaslan [30] develop a suspension system considering nonlinear actuator dynamics for active suspension control.

From the above literature review, it can be found that the research on nonlinear dynamics of tracked vehicles is uncommon, and the nonlinear stiffness of road wheels is rarely considered. In fact, as the speed increases, the linear model can't accurately reflect the actual dynamic performance of tracked vehicle. Developing a more detail nonlinear dynamic model has better practical value and guiding significance for ride comfort analysis, the design and control of the tracked vehicle system. The aim of this study is to study the influence of nonlinear stiffness of novel flexible road wheel (FRW) on ride comfort of tracked vehicle traversing random uneven road. The linear and nonlinear models of FRW are established based on the experimental data obtained from wheel static loading test. They are added to a half-vehicle dynamic model of tracked vehicle verified by the published data to establish a complete nonlinear dynamic model of the tracked vehicle. The ride comfort of half-vehicle nonlinear dynamic model of tracked vehicle on random rough road is simulated by using MATLAB/Simulink software. The dynamic responses of tracked vehicle with linear and nonlinear FRW models in time and frequency domain were compared and analyzed.

The specific chapters of this paper are organized as follows: Section 2 introduces the structure and operation mechanism of FRW; The modeling method of the linear and nonlinear model of flexible road wheels is described in Section 3; Section 4 presents the validated half-vehicle dynamic model of tracked vehicle and road excitation model; The results of simulation analysis is shown in Section 5; Section 6 summarizes and prospects the full paper.

II. STRUCTURE AND OPERATION MECHANISM OF FRW

The rigid road wheel (RRW) is generally composed of rubber, rim and wheel body. The rim and wheel body are usually cast into a whole, and the outer rubber band is bonded to the rim with cementing compound and finally vulcanized. Different from the rigid road wheel (RRW), the components of the flexible road wheel (FRW) include: elastic ring, combination card, hinge group, hub and rubber, as shown in Fig. 1. A plurality of elastic rings are fixed by the combination card, constituting an integrated structure. The rubber layer is wrapped outwardly against the surface of the integrated structure. After vulcanized and glued, the elastic outer ring is formed. The elastic outer ring and the hub are connected by several hinge groups evenly distributed along the circumferential direction.

Fig. 2 shows the bearing mechanism of the rigid road wheel (RRW) and flexible road wheel (FRW). As illustrated in Fig. 2(a), only the bottom part of RRW contacted with the track bears the load, so the loading efficiency is low according

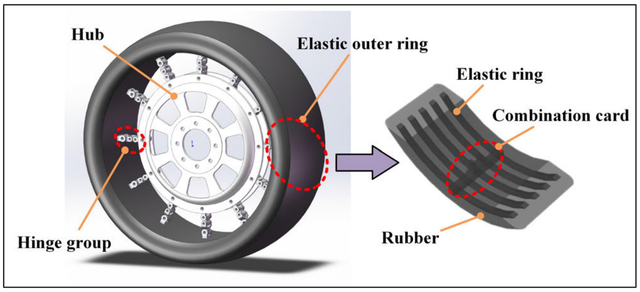

FIGURE 1. Structure of FRW.

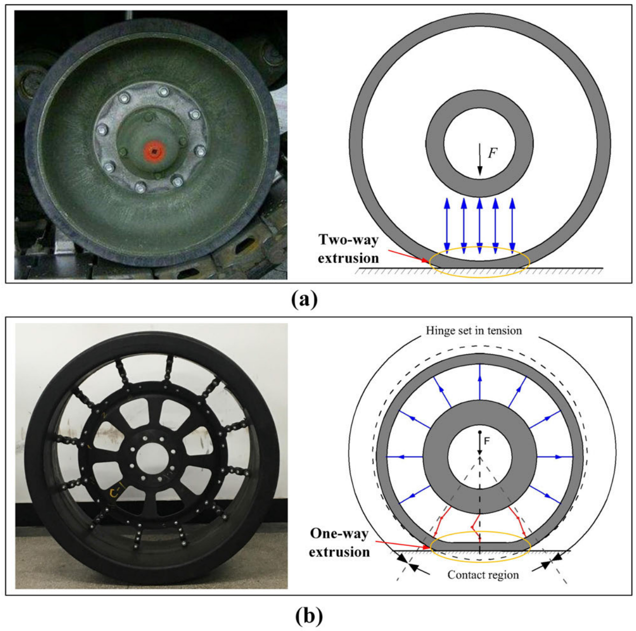

FIGURE 2. Static bearing mechanism of RRW (a) and FRW (b).

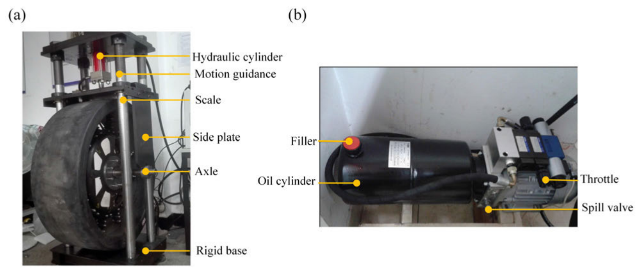

FIGURE 3. Wheel stiffness experiment: (a) Mechanical characteristics test-bed; (b) Hydraulic pressure station.

to the plate and shell theory [31]. Meanwhile, the rubber layer is subjected to the two-way extrusion between the rigid track and rim. Thus the deformation amount of rubber is large, leading to heat up and causing rubber damage. Different from the bearing mechanism of RRW, FRW has a unique suspended load-bearing mechanism, as shown in Fig. 2(b). When the FRW is subjected to the load, the elastic outer ring undergoes elliptical deformation, and the hinge group in the contact area will be free to bend without stretching and compression, the other hinge groups in non-contact area are all in tension. This structure is more evenly stressed and has a higher load-bearing efficiency. The rubber at the bottom is only subjected to one-way extrusion from the rigid track, and the inner surface of the rubber allows greater deformation in the radial direction, thus the heat generation is reduced and the life is increased. These potential advantages of FRW are theoretically feasible, but further numerical and theoretical studies are also needed.

III. NONLINEAR AND LINEAR MODEL OF FRW

A. STATIC LOADING EXPERIMENT

In order to study the nonlinear radial stiffness of FRW, the wheel static loading experiment is carried out on a self-made wheel mechanical characteristics test-bed, as shown in Fig. 3. The test-bed mainly includes the rigid base, manual/electric servo device, data acquisition and processing device, control device and hydraulic pressure station. The function of the electric servo device is to apply vertical load to the wheel quickly and efficiently through the bearing side plate. The motion guiding device is to ensure the direction of the radial force applied to the wheel and measure the displacement. The specific test procedure is as follows: firstly, the pressure plate is lifted for a certain distance by the controller; then the FRW is rolled under the pressure plate, so that the side plate groove catches the wheel axle and make it symmetrical in left and right. Next, the pressure plate is slowly moved to contact the wheel and record the guide column scale x_1. Continue to slowly move the plate to the desired position, record the guide column scale x_2, and the hydraulic pressure p. After the test is completed, the data is processed. The force F applied to FRW can be defined as:

$$F = pA + G_b \tag{1}$$

where A is the cross-sectional area of the piston, G_b is the weight of the pressure plate and the side plates.

B. NONLINEAR AND LINEAR MODEL

The linear wheel model was first used in the analysis of vehicle dynamics, but it has been proved that this assumption is inaccurate [16]. The stiffness of the wheel are affected by many factors such as wheel deformation, vibration speed and excitation frequency. Therefore, it is necessary to develop a nonlinear model of the road model based on the linear model. According to the reference [32], in engineering applications, the elastic force and damping force of the road wheel are given precisely enough by the equation (2) and (3):

$$F_w = K_1 x + K_2 x^2 \tag{2}$$

$$F_c = Cx \tag{3}$$

where K_1 and K_2 are the stiffness factors, and C is the damping coefficient. This model is simple and clear which meet the requirements of the vehicle ride comfort analysis. Therefore, this model will be used to develop the nonlinear model of FRW.

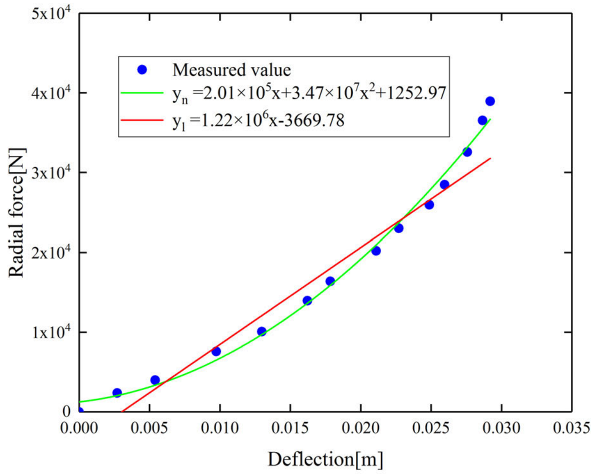

FIGURE 4. Relationship between deflection and radial force of FRW.

TABLE 1. Linear and polynomial fitting results of experimental data.

Fitting parameters	Linear regression analysis	Polynomial regression analysis
Stiffness coefficient	K=1.22E6	K_1=2.01E5 K_2=3.47E7
Reduced Chi-Sqr	4.88564E6	1.47484E6
Adj. R-square	0.95558	0.99245

From the static loading experiment described in Section 3.1, we can get the relationship between the deflection of FRW and radial load. Then linear and polynomial regression analysis of the existing experimental data were carried out, respectively, and the results are shown in Fig. 4. For a linear model, the function expression without the intercept is:

$$y_l = 1.22 \times 10^6 x \tag{4}$$

For a nonlinear model, the function expression without the intercept is:

$$y_n = 2.01 \times 10^5 x + 3.47 \times 10^7 x^2 \tag{5}$$

To evaluate the goodness of fit of the deflection-load curve, two indexes, the Reduced Chi-Sqr and Adj. R-square, are extracted from the linear and polynomial regression equations. The smaller the Reduced Chi-Sqr is, the better the fitting degree is. The closer Adj. R-square is to 1, the better the regression equation fits the observed values. The parameters of the fitting regression equations are shown in Table 1. It can be seen from Table 1 that compared with linear regression analysis, the Reduced Chi-Sqr of the polynomial regression analysis is smaller and the Adj. R-square is closer to 1, indicating that the polynomial fitting effect of the deflection-load curve of FRW is better than the linear regression analysis.

IV. HALF VEHICLE MODEL OF TRACKED VEHICLE WITH FRWS

A. HALF-VEHICLE DYNAMIC MODEL

The dynamic model of a tracked vehicle is the basis for system design and performance analysis. As a coupled nonlinear vibration system with complex multi-degree-of-freedom,

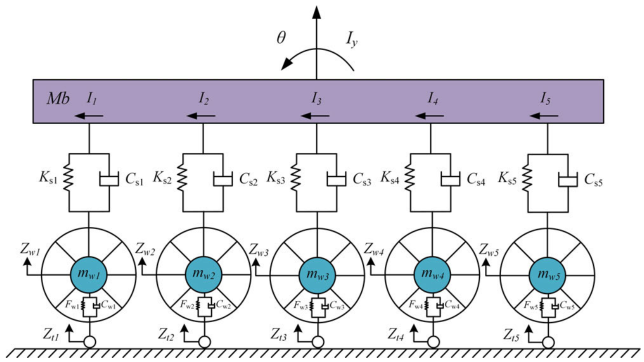

FIGURE 5. Half-vehicle dynamics model of tracked vehicle with FRWs.

it is very complicated and difficult to precisely describe and analyze the dynamic characteristics. A large number of researches and related data show that, according to the research purpose and focus, the dynamic model can be simplified accordingly, which can facilitate the analysis and calculation, and ensure the accuracy and reliability of the results [33]. In this paper, the system dynamic model is proposed based on the following reasonable assumptions.

Assumption 1: It is considered that the road surface through which the wheels on both sides of the vehicle pass is the same.

Assumption 2: The center of mass of the car body is symmetrical about the longitudinal axis. Only the vertical and pitch vibrations of the vehicle are considered.

Assumption 3: The suspension form is an independent linear suspension, and the damping characteristics of each damper are the same.

Assumption 4: The elastic force and gravity of the road wheel are applied to the center of gravity of the road wheel, and each of the road wheel has the same nonlinear elastic characteristics.

Based on the above assumptions, the half-vehicle model of a M113-A3 tracked vehicle was established in the Matlab/Simulink, in which half of the body mass is supported by five FRWs, as shown in Fig. 5. This model is asymmetrical about the vertical axes going through the center of mass of the vehicle body. It has seven degrees of freedom, two degrees of freedom (vertical bounce and pitch) are related to the mass of the vehicle, and five degrees of freedom (vertical bounce) are associated with the five FRWs. The suspension system is represented by a parallel combination of linear damper and spring stiffness, and FRW is modeled as a nonlinear spring stiffness and linear damper. Based on the Lagrangian equation, the dynamic equation of the seven degree-of-freedom half-vehicle model of tracked vehicle can be obtained as follows:

$$m_b\ddot{z}_b + \sum_{i=1}^{5} k_{si}(z_{bi} + l_i\theta - z_{wi}) + \sum_{i=1}^{5} c_{si}(\dot{z}_{bi} + l_i\dot{\theta} - \dot{z}_{wi}) = 0 \tag{6}$$

$$I\ddot{\theta} + \sum_{i=1}^{5} k_{si}l_i(z_{bi} + l_i\theta - z_{wi}) + \sum_{i=1}^{5} c_{si}l_i(\dot{z}_{bi} + l_i\dot{\theta} - \dot{z}_{wi}) = 0 \tag{7}$$

TABLE 2. Half-vehicle model parameters for tracked vehicle [34-35].

Symbol	Description	values
m_b	Body mass (kg)	5109
I	Body inertia (kg/m2)	128,56
m_{wi}	Wheel mass (kg)	113.5
k_1	Stiffness coefficient 1 (N/m)	2.01E5
k_2	Stiffness coefficient 2 (N/m)	3.47E7
c_{wi}	Wheel damping (Ns/m)	300
c_{si}	Suspension damping (Ns/m)	22,520
k_{si}	Suspension stiffness (N/m)	104,000
z_b	Body vertical displacement	NA
z_{wi}	Wheel displacement	NA
z_{ti}	Road displacement	NA
θ	Pitch angle	NA
l_1	First FRW center[a] (m)	1.35
l_2	Second FRW center (m)	0.69
l_3	Third FRW center (m)	0.02
l_4	Fourth FRW center (m)	-0.66[b]
l_5	Fifth FRW center (m)	-1.32

Notes: N.A, not applicable. [a] The horizontal distance between the body centre and FRW centre. [b] Negative values mean that the distance is behind of the body centre.

TABLE 3. Prediction and measured undamped natural frequencies of tracked vehicle system.

Mode no.	Natural frequency (Hz)				Vibration mode
	Current work		Reference [35]		
	Laden	Unladen	Laden	Unladen	
1	1.45	1.76	1.65	1.85	Hull bounce
2	1.15	1.28	1.23	1.31	Hull pitch
3	12.75	10.78	11.39	11.32	Wheel 1st bounce
4	12.75	11.78	12.20	11.08	Wheel 2nd bounce
5	12.75	11.76	12.26	12.05	Wheel 3rd bounce
6	12.75	11.75	12.34	12.01	Wheel 4th bounce
7	12.76	11.82	11.97	11.41	Wheel 5th bounce

TABLE 4. Classification standard for road roughness.

level	$Gq(n_0)\times10^{-6}$/m3	level	$Gq(n_0)\times10^{-6}$/m3
A	16	E	4096
B	64	F	16384
C	256	G	65536
D	1024	H	262144

$$m_{wi}\ddot{z}_{wi} - k_{si}(z_b + l_i\theta - z_{wi}) - c_{si}(\dot{z}_b + l_i\dot{\theta} - \dot{z}_{wi}) + c_{si}(\dot{z}_{wi} - \dot{z}_{ti}) + k_1(z_{wi} - z_{ti}) + k_2(z_{wi} - z_{ti})^2 = 0 \tag{8}$$

Table 2 explains all the variables and parameters in above equations, and lists their typical values used in the ride comfort simulations of tracked vehicles.

B. MODEL VALIDATION

In order to verify the established half-vehicle dynamic model of M113-A3 tracked vehicle, the predicted natural frequencies of the undamped tracked vehicle was compared with the experimental data. The system matrix of the state space equation is first calculated based on the established tracked half-vehicle vibration model. Then, the system state space model is transformed into the transfer function form, and at the same time the system natural frequency is obtained. The system predicted natural frequency calculated by the MATLAB program is compared with the measured data of the actual tracked vehicle. The natural frequencies of laden and unladen body mass are both predicted. For easy comparison, the road wheel parameters used are the same as in the reference [35]. Table 3 compares the predicted and measured natural frequencies of laden and unladen tracked vehicle. The results show that the predicted natural frequency is close to the measured results, so the validity of the half-vehicle dynamics model of the tracked vehicle established in MATLAB/Simulink is verified.

C. ROAD EXCITATION MODEL

When the tracked vehicle is driving on the road, although the engine, the drive train, etc. may cause the vehicle to vibrate, the main cause of the vibration is caused by the unevenness of the road surface. The working environment of tracked vehicles is generally in the wild, and the road input has a strong randomness. Therefore, in this paper, the road surface excitation model is established by random road input.

The road surface excitation model is divided into a frequency domain model and a time domain model [36]. However, the frequency domain model can't be used for nonlinear system, and multi degree-of-freedom dynamics model. Besides, the range of results sought is generally in the time domain, and the theoretical results obtained will not be proved in practice in the frequency domain model. Therefore, the time domain model is used to establish the road excitation model in this paper.

The power spectral density of road roughness is generally chosen to describe the random road input. Combined with the relevant international standard documents [37], the spectral density of road roughness is fitted by the following expression:

$$G_q(n) = G_q(n_0)(\frac{n}{n_0})^{-w} \tag{9}$$

where: n is the frequency of the spatial description; n_0 is the reference spatial frequency; $G_q(n_0)$ is the unevenness excitation coefficient of the road surface; w is the index of the relevant spatial frequency.

According to the difference of power spectral density, the road roughness is divided into 8 levels, and the grading standards of the national standard GB7031-86 is listed in Table 4.

The method of filtering white noise is to use the ideal unit white noise as an input, and transform it into an excitation as an output method after a first-order filter change [38]. Based on the filtered white noise method, the time domain description of the road surface excitation corresponding to

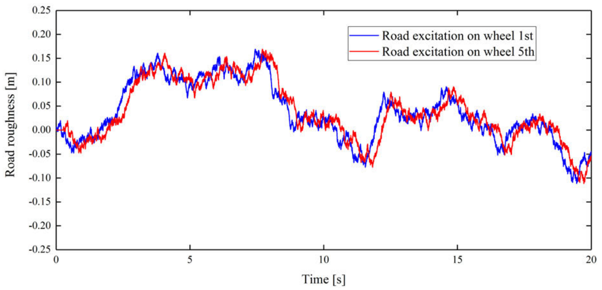

FIGURE 6. Road excitation of tracked vehicle.

equation (9) can be expressed as:

$$\dot{z}_i = 2\pi f_0 z_i + 2\pi\sqrt{G_0 u \omega(t)} \tag{10}$$

where f_0 is the lower cutoff frequency; G_0 is the road roughness; $\omega(t)$ is the white Gaussian noise when the mean is zero.

In this paper, it is assumed that the vehicle is traveling at a constant speed of 10m/s on a random uneven road surface. The uneven road surface is simulated by filter white noise method, and the road surface unevenness level is D level. Finally, the simulation result of road surface is shown in Fig 6. Note that five FRWs are subjected to the same road excitations with constant time delay, which is consistent with the actual operating conditions of tracked vehicles.

V. RESULTS ANALYSIS AND DISCUSSION

Matlab is a powerful software program developed by Math Works company, in which the Simulink toolkit is used to simulate system models in real time. In this paper, Simulink tool kit was used to simulate the half-vehicle dynamic model of tracked vehicle including linear and nonlinear models of FRWs, and the dynamic responses of the linear and nonlinear system including bounce and pitch motions of vehicle body, the dynamic deflection of suspension system, and the dynamic wheel load were compared and analyzed in detail. All simulation cases are carried out on a class D random uneven road at a speed of 10m/s.

A. EFFECT OF NONLINEAR FRW MODEL ON HULL DYNAMIC RESPONSE

Fig. 7 shows the comparison of bounce acceleration of the vehicle body between the linear and nonlinear model in the time (a) and frequency (b) domains. Compared with the linear model, the nonlinear model of FRW has a significant inhibiting effect on the bounce acceleration of the vehicle body in the time domain, as shown in Fig. 7(a). The maximum value of the bounce acceleration for the nonlinear model is greatly reduced.

In Fig. 7(b), it can be clearly found that there are three bounce resonance peaks for the linear model in the frequency domain, which occur at frequencies of 1.45HZ, 3.85HZ, and 13.09HZ, respectively. However, the peak value of the resonance amplitude decreases as the frequency increases. Compared with the linear model, the frequency of the nonlinear model corresponding to the resonance peak is slightly reduced, and the peak value of the resonance amplitude is greatly reduced. In addition, it can be also found that the nonlinear model suppresses the high-frequency vibration more obviously.

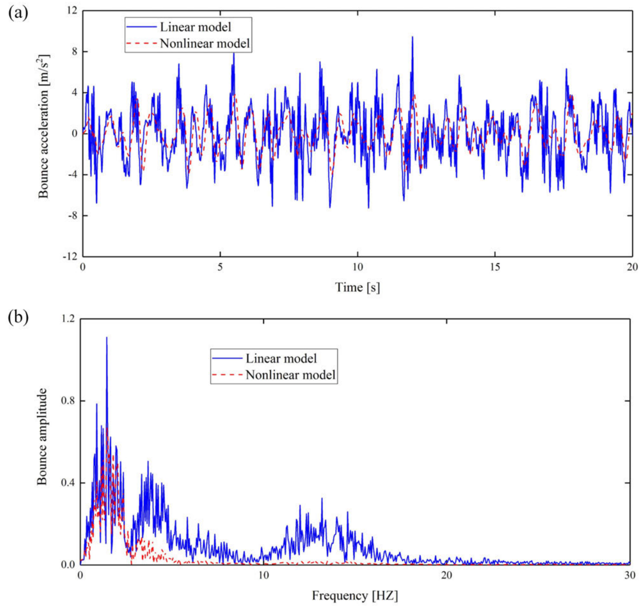

FIGURE 7. Comparison of bounce acceleration of vehicle body between linear model and nonlinear model in time (a) domain and frequency (b) domain.

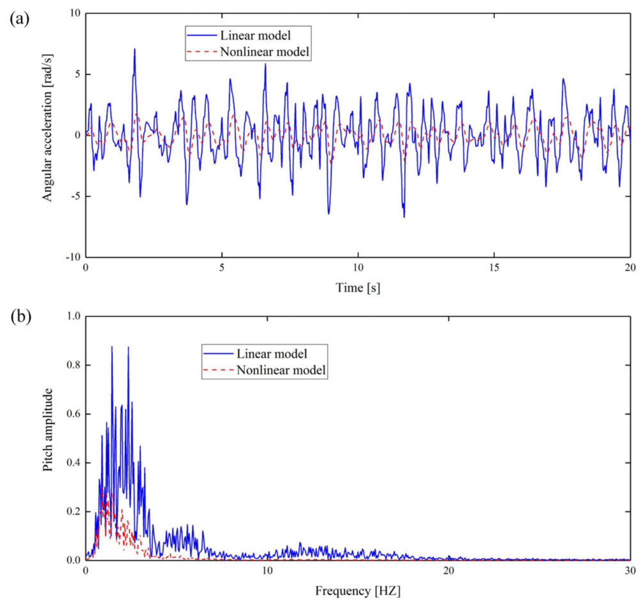

FIGURE 8. Comparison of angular acceleration of vehicle body between linear model and nonlinear model in time (a) domain and frequency (b) domain.

Fig. 8 shows the comparison of the angular acceleration of vehicle body between the linear and nonlinear model in the time (a) domain and frequency (b) domain.In Fig. 8(a), it could be found that the effect of the nonlinear model on angular acceleration is similar to the effect on the bounce acceleration. Compared with the linear model, the nonlinear

TABLE 5. Maximum and RMS values of various evaluation indexes for vehicle body vibration.

Evaluation index	Maximum value			RMS value		
	LM	NM	Variation	LM	NM	Variation
BA (m/s^2)	9.46	4.13	-56.34%	0.81	0.46	-43.21%
AA (rad/s^2)	7.09	2.12	-70.10%	0.63	0.21	-66.67%
BD (m)	0.28	0.14	-50.00%	0.087	0.051	-41.38%
PA (rad)	0.0834	0.0596	-28.54%	0.0156	0.0127	-18.59%

Notes: BA, Bounce acceleration; AA, Angular acceleration; BD, Bounce displacement; PA, Pitch angle; LM, Linear model; NM, Nonlinear model.

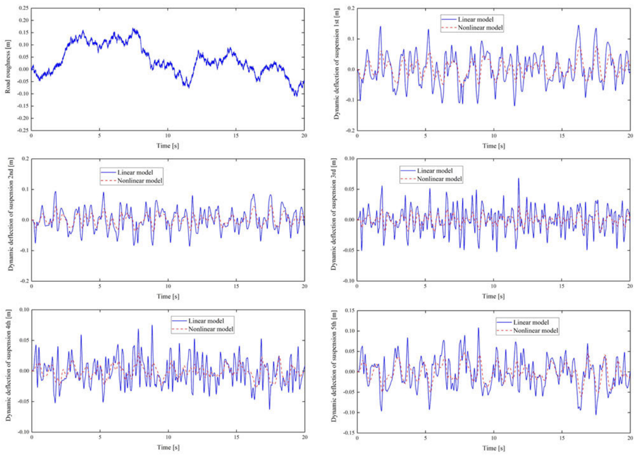

FIGURE 9. Comparison of dynamic deflection of suspension between linear model and nonlinear model in time domain.

model significantly suppresses the angular acceleration of the vehicle body in the time domain.

As shown in Fig. 8(b), for the linear model, it is observed that there are two distinct resonance peaks of angular acceleration in the frequency domain, and the corresponding frequencies are 1.36HZ and 5.89HZ respectively. Compared with the linear model, the nonlinear model greatly suppresses the resonance amplitude of angular acceleration and acts more prominently in the high frequency region.

To make the comparison more intuitive, Table 5 compares the maximum and root-mean-square (RMS) values of bounce acceleration (BA), angular acceleration (AA), bounce displacement (BD), and pitch angle (PA) of vehicle body between the linear model and nonlinear model. It can be clearly seen from the table 5 that, the nonlinear model significantly reduces the maximum and RMS values of the BA, AA, BD, PA, compared with the linear model. Furthermore, the nonlinear model has the greatest influence on the maximum value of AA with about 70 percent reduction and the least influence on the maximum value of PA with about 30 percent reduction. The effect of the nonlinear model on the RMS value of these indexes is less than the effect on the maximum value of those. It can be concluded that the bounce and pitch motion of the vehicle body is greatly suppressed after considering the nonlinear stiffness of FRW.

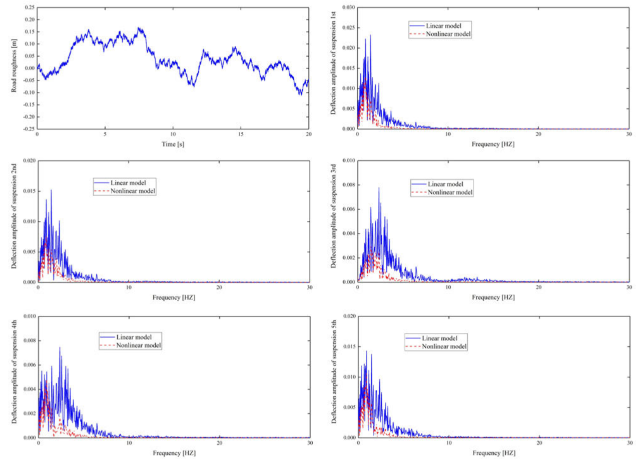

FIGURE 10. Comparison of dynamic deflection amplitude of suspension between the linear model and nonlinear model in frequency domain.

B. EFFECT OF NONLINEAR FRW MODEL ON SUSPENSION DYNAMIC RESPONSE

Fig. 9 show the comparison of the dynamic deflections of five suspension systems between the linear and nonlinear model in the time domain. As shown in Fig. 9, compared with the linear model, the nonlinear model significantly suppresses the dynamic deflection of five suspension systems in the time domain. However, the influence of the nonlinear model on the dynamic deflection of suspensions at different locations is also different and the specific difference is shown in Table 4.

Fig. 10 shows the comparison of the dynamic deflections of five suspension systems between the linear and nonlinear model in the frequency domain. It can be found that compared with the linear model, the nonlinear model greatly suppresses resonance amplitude of the suspension dynamic deflection, and also reduce the resonance frequency. Notice that there is only one resonant frequency of suspension dynamic deflection.

To make the comparison more intuitive, Table 6 compares the maximum and root mean square (RMS) values of the suspension dynamic deflection between the linear and nonlinear model. It can be seen from Table 6 that the nonlinear model dramatically reduces the maximum and RMS values of the suspension dynamic deflection. Furthermore, the dynamic deflection of the first suspension, fifth suspension, second suspension, fourth suspension and third suspension are arranged from largest to smallest, which is consistent with the actual situation of tracked vehicles. The influence of the nonlinear model on the dynamic deflection of suspension system is in the same order. It can be concluded that the nonlinear model has the greatest influence on the dynamic deflection of the suspension in the middle and decreases toward both sides.

C. EFFECT OF NONLINEAR FRW MODEL ON WHEEL DYNAMIC RESPONSE

Fig. 11 shows the comparison of dynamic FRW load between the linear model and nonlinear model in the time domain.

TABLE 6. Maximum and RMS values of suspension dynamic deflection.

Evaluation index	Maximum value (m)			RMS value		
	LM	NM	Variation	LM	NM	Variation
DD of S 1st	0.144	0.076	-47.22%	0.0172	0.0094	-45.35%
DD of S 2nd	0.093	0.049	-47.31%	0.0111	0.0056	-49.55%
DD of S 3rd	0.069	0.023	-66.67%	0.0056	0.0021	-62.50%
DD of S 4th	0.074	0.031	-58.11%	0.0055	0.0035	-36.36%
DD of S 5th	0.109	0.061	-44.04%	0.0107	0.0075	-29.91%

Notes: DD, Dynamic deflection; S, Suspension; LM, Linear model; NM, Nonlinear model.

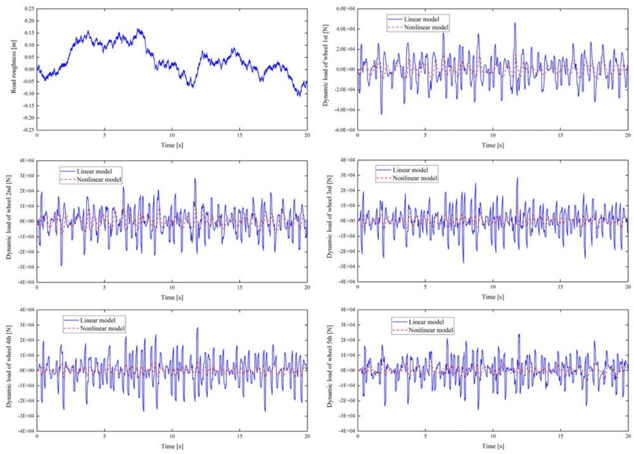

FIGURE 11. Comparison of dynamic FRW load between the linear model and nonlinear model in time domain.

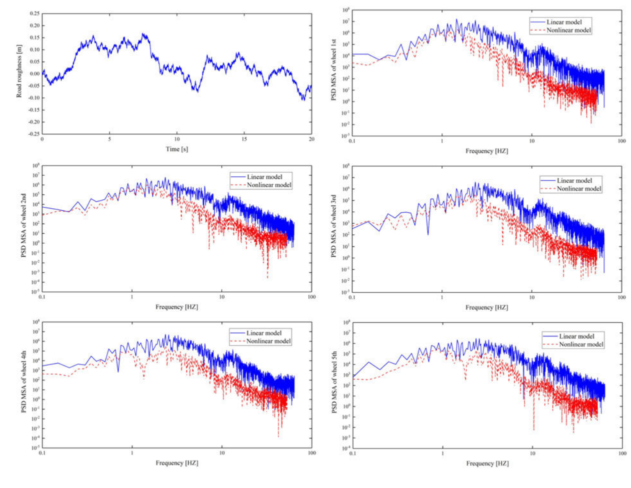

FIGURE 12. Comparison of PSD MSA of dynamic FRW load between the linear and nonlinear model in frequency domain.

As shown in Fig. 11, compared with the linear model, the nonlinear model greatly reduces the dynamic load of five FRWs in the time domain. It can also be seen that the nonlinear model has different effects on the dynamic load of FRW at different positions.

Fig. 12 shows the comparison of PSD (power spectral density) MSA (mean squared amplitude) of the dynamic FRW load between the linear model and nonlinear model in the frequency domain. It can be seen that as the frequency increases, the PSD MSA of dynamic FRW load appears to increase

TABLE 7. Maximum and RMS values of wheel dynamic load.

Evaluation index	Maximum value (KN)			RMS value		
	LM	NM	Variation	LM	NM	Variation
DL of W lst	4.66	1.54	-66.95%	0.41	0.14	-65.85%
DL of W 2nd	2.95	1.39	-52.88%	0.25	0.09	-64.00%
DL of W 3rd	2.85	0.95	-66.67%	0.21	0.07	-66.67%
DL of W 4th	2.91	0.71	-75.60%	0.23	0.05	-78.26%
DL of W 5th	2.65	0.75	-71.70%	0.17	0.05	-70.59%

Notes: DL, Dynamic load; W, Wheel; LM, Linear model; NM, Nonlinear model.

first and then decrease. Compared with the linear model, the nonlinear model reduces the PSD MSA of dynamic FRW load. In addition, the nonlinear model has less influence on the PSD MSA of dynamic FRW load at low-frequency range. However, as the frequency increases, the influence of nonlinear model on PSD MSA of dynamic FRW load becomes larger.

To make the comparison more intuitive, Table 7 compares the maximum and root-mean-square (RMS) values of dynamic FRW load between the linear and nonlinear model. It can be found from Table 7 that compared with the linear model, the nonlinear model greatly reduces the maximum and RMS values of dynamic FRW load, and the most decline is more than 60%. In addition, the first FRW has the largest dynamic load and as the position goes further, the dynamic load has a downward trend, which is consistent with the actual situation of tracked vehicles.

VI. CONCLUSION AND PROSPECT

In the research about the ride comfort of tracked vehicles, the road wheel models are almost linear model, but the nonlinear effect of the road wheel becomes more prominent as the vehicle speed increases and the road wheel becomes lighter and softer. Therefore, it is necessary to study the influence of the nonlinear stiffness of the road wheel on the ride comfort of the tracked vehicle.

This paper introduces a new type of road wheel with a composite flexible structure, which has great potential in improving the lightweight level, maneuverability and ride comfort of tracked vehicles. To study the influence of the nonlinear stiffness of FRW on the ride comfort of a typical tracked vehicle, a succinct squared stiffness model of FRW was established by fitting the load-deflection curve obtained from the wheel static loading experiment. Then this nonlinear model was substituted into a half-vehicle model of the tracked vehicle which has been verified by published experimental data. Finally, the dynamic response of tracked vehicles with linear and nonlinear FRW models on random uneven road in the time domain and frequency domain were analyzed and compared in detail.

Based on the experimental and simulation results, the following conclusions can be drawn. The established nonlinear model of FRW based on the radial stiffness test is simple and effective. It can correctly reflect the nonlinear characteristics of FRW and can be used for the ride comfort simulation of the tracked vehicle without increasing too much computing costs. The nonlinear model of FRW has a great influence on the dynamic response of the tracked vehicle on random uneven road. It greatly inhibits the vibration of the vehicle body, suspension system and FRWs, especially the high-frequency vibration. It is also found that the nonlinear FRW model has a greater impact on the bounce motion of the vehicle body than the pitch motion. The effect of nonlinear FRW model on the dynamic response of the wheel is also greater than the effect on the suspension and vehicle body. Therefore, it can be concluded that the nonlinear factor of the FRW can't be ignored in the ride comfort simulation and suspension system control of track vehicles. The research results in this paper enrich the study of nonlinear dynamics of tracked vehicles, and provide reference for nonlinear modeling of other pneumatic tires and non-inflatable wheels.

However, only the nonlinear effects of the road wheel are considered, and the influence of the combined nonlinearity of the suspension and road wheel on the ride comfort of a track vehicle will be studied in the future.

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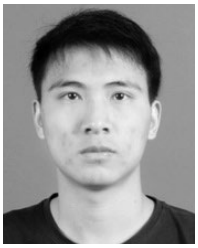
YAOJI DENG is currently pursuing the Ph.D. degree in Vehicle Engineering with the Nanjing University of Aeronautics and Astronautics, Nanjing, China. His current research interests include tire dynamic, vibration analysis, and vehicle dynamic simulation and control.

YOUQUN ZHAO received the B.S., M.S., and Ph.D. degrees from Jilin University, Changchun, China, in 1990, 1993, and 1998, respectively. He is currently a Professor in vehicle engineering with NUAA. His current research interests include tire dynamic, vibration analysis, and vehicle dynamic simulation and control.



WEI PI is currently pursuing the M.S. degree in vehicle engineering with the Nanjing University of Aeronautics and Astronautics, Nanjing, China. His current research interests include tire dynamic, and vehicle dynamic simulation and control.

YUHAO LI is currently pursuing the M.S. degree in vehicle engineering with the Nanjing University of Aeronautics and Astronautics, Nanjing, China. His current research interests include tire dynamic, and vehicle dynamic simulation and control.

SHILIN FENG is currently pursuing the M.S. degree in vehicle engineering with the Nanjing University of Aeronautics and Astronautics, Nanjing, China. His current research interests include tire dynamic and vibration analysis.

YIYAN DU is currently pursuing the M.S. degree in vehicle engineering with the Nanjing University of Aeronautics and Astronautics, Nanjing, China. Her current research interests include vehicle dynamic simulation and control.