# Interference Neutralization With Partial CSIT for Full-Duplex Cellular Networks 

WEIMIN DU ${ }^{\text {® }}$, ZUJUN LIU ${ }^{\text {® }}$, (Member, IEEE), AND FAN LI²<br>${ }^{1}$ State Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China<br>${ }^{2}$ Qingdao Topscomm Communication Co., Ltd., Qingdao 266109, China<br>Corresponding author: Zujun Liu (liuzujun@mail.xidian.edu.cn)

This work was supported in part by the National Natural Science Foundation of China under Grant 61571340, and in part by the Fundamental Research Funds for the Central Universities.


#### Abstract

Compared to half-duplex (HD) communications, full-duplex (FD) communications have the potential to double the throughput of the cellular networks. However, the inter-user interference (IUI) from the uplink (UL) users to the downlink (DL) users severely hinders the improvement of the throughput. In this paper, we propose an interference neutralization (IN) scheme to eliminate the IUI with the help of partial channel state information at the transmitter (CSIT). Before the UL signals are transmitted, we rotate their phases through a set of scalars known by the base station (BS), which endows the conventional receive antennas of the BS with the reconfigurability. The BS assists the IUI management by precoding and forwarding the UL signals to the DL users. Consequently, the UL signals forwarded by the BS locate in the opposite direction of the IUI so as to neutralize it. After the IUI is neutralized, all the symbols of the last time slots can be recovered. The DoF region and the corresponding feasible conditions are analyzed. The analysis and numerical results demonstrate that 1.5 -fold DoF over that of the HD systems can be achieved via the proposed IN scheme with only partial CSIT and 2 symbol extensions. It shows that the proposed scheme is a low complexity interference management scheme for the FD cellular networks.


INDEX TERMS Full-duplex, inter-user interference (IUI), CSIT, Degrees of Freedom (DoF).

## I. INTRODUCTION

As a potential candidate technology for the next generation of mobile communications [1]-[3], full-duplex (FD) communication has a tendency to double the network throughput by enabling the transmission and the reception at the same time and frequency compared with the conventional halfduplex (HD) systems. Operating in FD mode, the base station (BS) may receive the strong interference arising from its transmitting side i.e., self-interference (SI) which results in the performance degradation. More explicitly, the radiated power of the downlink (DL) interferes with its desired received signals from the uplink (UL) [4], [5]. Fortunately, a certain amount of work [3], [6]-[9] has been done to suppress the SI within an acceptable range by utilizing the combination of propagation-domain, analogue-circuit domain and digital-domain approaches. It is reported in [10] and [11] that the SI can be suppressed as much as 110 dB , which makes the FD BS practicable.

[^0]Although the SI of the FD BS can be well suppressed, the inter-user interference (IUI) from the UL users to the DL users is the major bottleneck for advancing the spectral efficiency of the FD cellular networks. There are a large number of papers on the IUI problem. To the best of our knowledge, there are three main approaches that deal with the IUI in the FD cellular networks, i.e., the interference alignment (IA), the reconfigurable antenna (RA) and the user pairing.

The IA technique, by consolidating the interference into a minimized sub-space, has been extensively used in the interference channels [12]-[14]. By the means of the IA technique, it is demonstrated in [15]-[20] that deploying the FD BS is capable of doubling the Degrees of Freedom (DoF) of the networks compared to the HD systems. Besides, these papers give the DoF region and the feasible conditions for the FD cellular networks. However, doubling the DoF requires the perfect full channel state information at transmitter (CSIT) and infinite time slot extensions [12], which means the doubling is not easy to be achieved in practical systems.

The RA technique is another popular way to handle the IUI. Jafar [21] have proved that even without CSIT and infinite time extensions, blind IA can align the interference efficiently in some scenarios where the channel coefficients display some special staggered block coherence structures. On this basis, the antenna switching is utilized to create such special structures in dealing with the interference issue of MIMO broadcast channels [22]. Yang et al. extend the RA technique to the FD cellular networks and characterize the DoF of the networks for both no CSIT and partial CSIT in [23]. In the absence of full CSIT and infinite time slot extensions, the maximum sum DoF achieved via the RA is still $2 M$ where $M$ is the number of the BS antennas, but the reconfigurability results in an extra hardware cost. As pointed out in [24] and [25], the extra hardware cost can be reflected in the complexity of the antenna structure, the power consumption of active components, the generation of harmonics and inter modulation products, and the need for fast tuning in the antenna radiation characteristics.

The user pairing technique has been extensively discussed in [26]-[33] as a practical technique to deal with the interference in the FD cellular networks. However, the user pairing technique is sensitive to the distance between the UL users and the DL users [30]. It is hard to obtain the closedform solutions due to the process of maximizing the sum rate through selecting users and designing beamformers. Moreover, the optimization model will be replaced and recalculated as long as the configurations of the users and the antennas are changed.

Interference neutralization (IN) is first introduced in [34] and [35] as a new transmission technique to remove (decrease) the interference in a network. It refers to the distributed zero-forcing scheme where the interfering signal passes through multiple nodes before arriving at the undesired destination in the multi-hop relay networks [36]-[40]. For a $K$-user two-hop network with $L$ relays, the condition for IN i.e., $L \geqslant K(K-1)+1$ is given in [37]. In [38], the authors show that, by using IN, 2 DoF can be achieved under the twouser two-hop network with two FD relays. In [39] and [40], the IN scheme based on the cyclic IA framework is investigated for the FD relay-interference channel. The IN for the non-layered two-unicast wireless FD relay networks has been studied in [41].

In this paper, motivated by the idea of IN, we propose an IN scheme which neutralizes the IUI through the BS amplifying and forwarding the UL signals to the DL users. There are three fundamental differences between the conventional IN in the relay networks and the proposed IN scheme in the FD cellular networks. First of all, the relays usually have no own messages to transmit while the BS needs to transmit its desired signals to the DL users. Moreover, the number of the required relays for IN is more than one while there is only one BS in a cellular network. Secondly, in the relay networks, the paths between the sources and destinations can be neglected for shadowing or too large separation [42], [43]. On the contrary, the IUI from the UL users to the DL users
is the major interference which is required to be canceled in the FD cellular networks. Thirdly, the interference in the relay networks is neutralized with full CSI in [38]-[40] while the interference is neutralized with partial CSIT in this paper. In fact, the BS can not only amplify and forward but also detect and forward the UL signals to the DL users. But it takes $T$ time slots for the BS to decode the UL signals in the proposed scheme, which implies the detect and forward method would decrease the throughput of the network. Hence, we only consider the amplification and forward strategy throughout this paper.

The main contributions of this paper are:

1) We propose a novel IN scheme for the FD cellular networks with partial CSIT. In the proposed IN scheme, over $T$ time slots, $K_{u}$ UL users repetitively transmit their messages to the BS with $M$ antennas and the BS acts as a relay to amplify and forward the received signals. Different from the relays in FD relay networks, the BS in the FD cellular networks needs to transmit its own message to the DL users. With IUI perfectly neutralized, the intended signal spaces of the BS are preserved. We endow the conventional receive antennas of the BS with the reconfigurability, by rotating the phases of the UL signals through a set of scalars.
2) We derive the corresponding feasible conditions for the proposed IN scheme. The results show that under some network configurations, the DoF regions are consistent with that of [16], which shows the optimality. By analysis and numerical results, considering all available network configurations, we demonstrate the proposed IN scheme can achieve at most 1.5 -fold DoF compared to the HD systems. Moreover, the DoF is achieved by the conventional antennas and partial CSIT.
3) Through the analysis and comparison, we show the advantage of the proposed IN scheme over the IA and the RA schemes in terms of complexity. It can be taken as an optional scheme of low complexity for the interference management in the FD cellular networks.
The rest of this paper is organized as follows. In Section II, the system model of the FD cellular network is described. In Section III, the IN scheme is proposed. In Section IV, the DoF, the feasible conditions, the maximum sum DoF and the DoF regions of the proposed scheme are analyzed. In Section V, the numerical results and comparison are provided. Conclusions are provided in Section VI.

Throughout this paper, we denote $\mathbf{A}^{T}, \mathbf{A}^{\dagger}, \operatorname{rank}(\mathbf{A})$ and $r_{k}^{\prime}(\mathbf{A})$ as the transpose, the conjugate transpose, the rank and the Kruskal-rank of the matrix $\mathbf{A}$, respectively. We denote $\mathbf{A} \otimes \mathbf{B}$ and $\mathbf{A} \odot \mathbf{B}$ as the Kronecker product and the Khatri-Rao product of the two matrices $\mathbf{A}$ and $\mathbf{B}$. The operation vec (A) stands for vectoring matrix $\mathbf{A}$ by stacking its columns. diag $\{\mathbf{a}\}$ represents a diagonal matrix whose diagonal elements are from $\mathbf{a} . \mathbb{E}\{ \}$ is used to denote the expectation. For the positive integers $m$ and $n, \mathbf{I}_{m}$ and $\mathbf{0}_{m \times n}$ mean the $m \times m$ identity matrix and the $m \times n$ zero matrix. For the real number $a$, we denote $\lfloor a\rfloor$ as the largest integer that is not
larger than $a . \mathcal{C N}(0,1)$ represents the circularly-symmetric complex Gaussian random distribution with zero mean and unit variance.

## II. SYSTEM MODEL

We consider an FD cellular network where the BS communicates with $K_{u}$ UL users and $K_{d}$ DL users simultaneously, as shown in Fig. 1. The BS equipped with $M$ antennas operates in FD mode while each user equipped with a single antenna operates in HD mode. Considering the SI can be well suppressed [3], [6]-[9], we refer to the scenarios in [16]-[20], [23] and then also assume that the SI of the FD BS is suppressed perfectly.


FIGURE 1. The system model of the FD cellular networks.
The definition of the partial CSIT is that the CSIT is only available at the BS, which means the BS has the knowledge of both the transmit and receive side CSI, all the DL users have the knowledge of their receive side CSI and none of the UL users has any CSI. Throughout this paper, only the one-tap channel is considered. For the multi-path channel, we can apply the Orthogonal Frequency Division Multiplexing (OFDM) technique to divide the channel into several sub-channels, and then the one-tap channel is sufficient to represent each sub-channel. Moreover, all the channels are assumed to be block fading, i.e., all the channel coefficients remain constant within every consecutive $T$ time slots [23].

For the BS, the received signal at time $t$ can be expressed as

$$
\begin{equation*}
\mathbf{y}_{B S}(t)=\sum_{i=1}^{K_{u}} \mathbf{f}_{i} x_{i}^{U}(t)+\mathbf{z}^{U}(t) \tag{1}
\end{equation*}
$$

where $\mathbf{f}_{i} \in \mathbb{C}^{M \times 1}$ is the UL channel from the $i^{\text {th }}$ UL user to the BS and $x_{i}^{U}(t) \in \mathbb{C}^{1 \times 1}$ is the transmitted signal from the $i^{\text {th }}$ UL user to the BS. The noise vector of the BS is denoted by $\mathbf{z}^{U}(t) \in \mathbb{C}^{M \times 1}$ with the element $z_{i}^{U} \sim \mathcal{C N}(0,1)$.

For the $j^{\text {th }}$ DL user, the received signal at time $t$ is given by

$$
\begin{equation*}
y_{j}^{D}(t)=\mathbf{g}_{j}^{T} \mathbf{x}^{D}(t)+\sum_{i=1}^{K_{u}} h_{j i} x_{i}^{U}(t)+z_{j}^{D}(t) \tag{2}
\end{equation*}
$$

TABLE 1. Notations.

| Notation | Description |
| :---: | :---: |
| M | The number of the BS antennas. |
| $K_{u}$ | The number of the UL users. |
| $K_{d}$ | The number of the DL users. |
| $T$ | The number of time slots. |
| $\mathbf{f}_{i} \in \mathbb{C}^{M \times 1}$ | The UL channel vector from the $i^{\text {th }}$ UL user to the BS. |
| $\underline{\mathbf{F}}_{\text {rotated }} \in \mathbb{C}^{T M \times K_{u}}$ | The effective UL channel matrix which includes the phase rotation factor $V_{i}^{U}$ over $T$ time slots. |
| $\widetilde{\mathbf{F}}(t) \in \mathbb{C}^{M \times K_{u}}$ | The effective UL channel matrix which includes the phase rotation factor $V_{i}^{U}$ at time $t$. |
| $x_{i}^{U}(t)$ | The transmitted signal from the $i^{\text {th }}$ UL user to the BS at time $t$. |
| $V_{i}^{U}(t)$ | The phase rotation factor of the $i^{\text {th }}$ UL user at time $t$. |
|  | The effective transmitted signal of the $i^{\text {th }}$ |
|  | UL user which is repetitively transmit- |
| $s_{i}^{U}$ | ted over $T$ time slots in the proposed IN scheme, i.e., $\forall t \in[1: T], x_{i}^{U}(t)=$ |
|  | $V_{i}^{U}(t) s_{i}^{U}$ |
| $\underline{\mathbf{s}}^{U} \in \mathbb{C}^{K_{u} \times 1}$ | The effective transmitted signal vector of the $i^{\text {th }}$ UL user over $T$ time slots in the proposed IN scheme i.e., $\underline{\mathbf{s}}^{U}=$ $\left[s_{1}^{U}, s_{2}^{U}, \cdots, s_{K_{u}}^{U}\right]^{T}$. |
| $\mathbf{z}^{U}(t) \in \mathbb{C}^{M \times 1}$ | The noise vector of the BS at time $t$. |
| $\underline{\mathbf{z}}^{U} \in \mathbb{C}^{T M \times 1}$ | The noise vector of the BS over $T$ time slots. |
| $\mathbf{y}_{B S}(t) \in \mathbb{C}^{M \times 1}$ | The received signal vector of the BS at time $t$. |
| $\underline{\mathbf{y}}_{B S} \in \mathbb{C}^{T M \times 1}$ | The received signal vector of the BS over $T$ time slots. |
| $h_{j i}$ | The IUI channel from the $i^{\text {th }}$ UL user to the $j^{\text {th }}$ DL user. |
| $\widetilde{\mathbf{h}}_{j}(t) \in \mathbb{C}^{K_{u} \times 1}$ | The effective interference channel vector of the $j^{\text {th }} \mathrm{DL}$ user at time $t$. |
| $\mathbf{V}^{\mathrm{AF}} \in \mathbb{C}^{M \times M}$ | The precoding matrix for the forwarded signal from the UL channel. |
| $\mathbf{g}_{j}^{T} \in \mathbb{C}^{1 \times M}$ | The DL channel from the BS to the $j^{\text {th }}$ DL user. |
| $\mathbf{d} \in \mathbb{C}^{n K_{d} \times 1}$ | The $n K_{d}$ desired messages for the DL users, which are not precoded at the BS, over $T$ time slots. |
| $\mathbf{R} \in \mathbb{C}^{n K_{d} \times T M}$ | The effective receive channel of the DL users over $T$ time slots. |
| $\mathbf{W} \in \mathbb{C}^{T M \times n K_{d}}$ | The precoding matrix for the desired signal at the BS, i.e., $\mathbf{W}=\mathbf{R}^{\dagger}\left(\mathbf{R R}^{\dagger}\right)^{-1}$. |
| $\mathbf{x}^{D}(t) \in \mathbb{C}^{M \times 1}$ | The desired signal for the DL users, which is precoded, at time $t$. |
| $\underline{\mathbf{x}}^{D} \in \mathbb{C}^{T M \times 1}$ | The desired signal for the DL users, which is precoded over $T$ time slots i.e., $\underline{\mathbf{x}}^{D}=$ Wd. |
| $\mathbf{x}_{B S}(t) \in \mathbb{C}^{M \times 1}$ | The superposed signal of the precoded desired signal of the BS and the amplified and forwarded signal by the BS. |
| $z_{j}^{D}(t)$ | The noise of the $j^{\text {th }}$ DL user at time $t$. |
| $\widetilde{z}_{j}^{D}(t)$ | The effective noise of the $j^{\text {th }}$ DL user at time $t$. |
| $\mathbf{z}_{j}^{D} \in \mathbb{C}^{T \times 1}$ | The effective noise of the $j^{\text {th }}$ DL user over $T$ time slots. |
| $\mathcal{D}_{\left(M, K_{d}, K_{u}, T\right)}$ | The DoF region under the network configurations ( $M, K_{d}, K_{u}, T$ ). |
| $D^{*}\left(M, K_{d}, K_{u}, T\right)$ | The achieved maximum sum DoF under the network configurations ( $M, K_{d}, K_{u}, T$ ). |
| $D^{*}(M)$ | The achieved maximum sum DoF over all the network configurations. |

where $h_{j i} \in \mathbb{C}^{1 \times 1}, \mathbf{g}_{j}^{T} \in \mathbb{C}^{1 \times M}$ and $\mathbf{x}^{D}(t) \in \mathbb{C}^{M \times 1}$ are the interference channel from the $i^{\text {th }}$ UL user to the $j^{\text {th }} \mathrm{DL}$ user, the DL channel from the BS to the $j^{\text {th }}$ DL user and


FIGURE 2. The schematic of the proposed IN which enables $K_{u}$ UL users transmit $K_{u}$ messages to the BS and the BS transmit $\boldsymbol{n} K_{\boldsymbol{d}}$ messages to the DL users over $\boldsymbol{T}$ time slots.
the transmitted signal of the BS, respectively. In this paper, we focus on the DoF analysis and only Rayleigh fading is considered for simplicity. We assume that all the elements of the noise vectors and all the channel vectors are independent and identically distributed (i.i.d.) circularly-symmetric complex Gaussian random variables with zero mean and unit variance [13], [14], [16]-[19]. The transmit power constraints listed below must be satisfied for all the transmitters.

$$
\begin{align*}
\mathbb{E}\left\{\left|x_{i}^{U}(t)\right|^{2}\right\} & \leqslant P, \forall i \in\left\{1, \ldots, K_{u}\right\}, \\
\mathbb{E}\left\{\mathbf{x}^{D}(t)^{\dagger} \mathbf{x}^{D}(t)\right\} & \leqslant P . \tag{3}
\end{align*}
$$

We assume that the codewords span $N$ channel uses. For the BS, the messages for the $K_{d}$ DL users $\left(W_{1}^{D}, \cdots, W_{K_{d}}^{D}\right)$ drawn uniformly from the index set $\left[1: 2^{N R_{i}^{D}}\right]$ are encoded to $\mathbf{x}^{D}(t)$. For the $j^{\text {th }}$ UL user, the message for the BS $W_{i}^{U}$ drawn uniformly from the index set $\left[1: 2^{N R_{i}^{U}}\right]$ are encoded to $x_{i}^{U}(t)$.

When the BS or the DL users receive signals, the reverse procedure is processed. At the BS, the messages $\left(W_{1}^{U}, \cdots, W_{K_{u}}^{U}\right)$ are decoded from the received signal $\mathbf{y}_{B S}$. At the $i^{\text {th }}$ DL user, the message $W_{i}^{D}$ is decoded from the received signal $\mathbf{y}^{D}$. If there exists a rate tuple $\left(R_{1}^{D}, \cdots, R_{K_{d}}^{D}, R_{1}^{U}, \cdots, R_{K_{u}}^{U}\right)$ that makes probability $\operatorname{Pr}\left(\hat{W}_{i}^{U} \neq W_{i}^{U}\right)$ and $\operatorname{Pr}\left(\hat{W}_{j}^{D} \neq W_{j}^{D}\right)$ tends to 0 when $N$ tends to infinity $\forall i \in\left[1: K_{u}\right]$ and $\forall j \in\left[1: K_{d}\right]$, that the rate tuple is said to be achievable. The DoF of the UL and the DL are defined by (4) and (5), respectively.

$$
\begin{align*}
& \mathrm{DoF}_{\mathrm{u}}=\lim _{P \rightarrow+\infty} \frac{\sum_{i=1}^{K_{u}} R_{i}^{U}}{\log _{2} P} .  \tag{4}\\
& \mathrm{DoF}_{\mathrm{d}}=\lim _{P \rightarrow+\infty} \frac{\sum_{j=1}^{K_{d}} R_{j}^{D}}{\log _{2} P} . \tag{5}
\end{align*}
$$

The DoF of the FD cellular network is defined as the sum of the UL DoF and the DL DoF, i.e.,

$$
\begin{equation*}
\mathrm{DoF}_{\text {sum }}=\mathrm{DoF}_{\mathrm{u}}+\mathrm{DoF}_{\mathrm{d}} . \tag{6}
\end{equation*}
$$

## III. INTERFERENCE NEUTRALIZATION

 WITH PARTIAL CSITIn this section, we design an IN scheme for the FD cellular networks with partial CSIT i.e., the CSIT is not available to the UL users. As illustrated in Fig. 2, the signal processing of the proposed IN scheme can be divided into three stages: the UL transmission, the DL transmission, and the interference neutralization. For the UL transmission, all the UL users repetitively transmit their signals with different rotated phases known by the BS over $T$ time slots. For the DL transmission, the BS amplifies and forwards the received signals from the UL users to the DL users. For the $i^{\text {th }}$ DL user as well as sends its desired signals to the DL users, it linearly combines its received mixed signals to recover its desired signals.

## A. THE UL TRANSMISSION

Lacking CSIT, interference cannot be consolidated into a smaller sub-space by the beamforming at the UL users. But the blind IA, which can be realized by the reconfigurable antenna technique, can align multiple interfering signals into the same signal space at each receiver without any CSIT. From [22], the key to the blind IA is the following: the channel of the desired user changes while that of all undesired users remains fixed over $T$ symbols.

In [23], the UL channels are changed each time slot through the reconfigurable receive antennas at the BS while the IUI channels remain fixed under the block fading assumption. Following the idea of the blind IA, by constructing the phase rotation factors, we change the UL channels as if the conventional transmit antennas are reconfigured each time slot.


FIGURE 3. The UL transmission where $K_{u}$ users repetitively transmit their respective signals whose phases has been rotated by a set of $\left\{V_{i}^{U}(t)\right\}_{i \in\left[1: K_{u}\right], t \in[1: T]}$ which are known to the BS. We can regard the rotation on the transmitted signals as the rotation on the UL and interference channels.

As shown in Fig. 3, the $i^{\text {th }}$ UL user $\left(i \in\left[1: K_{u}\right]\right)$ repetitively sends the message $s_{i}^{U}$ to the BS throughout $T$ time slots.

Before $s_{i}^{U}$ is transmitted, its phase is rotated by $\left\{V_{i}^{U}(t)\right\}_{i \in\left[1: K_{u}\right], t \in[1: T]}$ to assist the BS to decode the data symbols. Substituting the transmitted signal of the $i^{\text {th }}$ UL user i.e., $x_{i}^{U}(t)=V_{i}^{U}(t) s_{i}^{U}$ into (1), we have

$$
\begin{equation*}
\mathbf{y}_{B S}(t)=\sum_{i=1}^{K_{u}} \mathbf{f}_{i} V_{i}^{U}(t) s_{i}^{U}+\mathbf{z}^{U}(t) t \in[1: T] \tag{7}
\end{equation*}
$$

Here, we construct the rotation phases as

$$
\begin{equation*}
V_{i}^{U}(t)=e^{j \frac{2 \pi t}{T+i}} \tag{8}
\end{equation*}
$$

where $i \in\left[1: K_{u}\right]$.
For $i^{\text {th }}$ UL user, the product $\mathbf{f}_{i} V_{i}^{U}(t)$ and the signal $s_{i}^{U}$ can be viewed as its effective UL channel and its effective transmitted signal at the $t^{\text {th }}$ time slot. Thus, the received signal of the BS over $T$ time slots can be written by

$$
\begin{equation*}
\underline{\mathbf{y}}_{B S}=\underline{\mathbf{F}}_{\text {rotated }} \cdot \underline{\mathbf{s}}^{U}+\underline{\mathbf{z}}^{U} \tag{9}
\end{equation*}
$$

where $\underline{\mathbf{s}}^{U} \in \mathbb{C}^{K_{u} \times 1}$ and $\underline{\mathbf{F}}_{\text {rotated }} \in \mathbb{C}^{T M \times K_{u}}$ are (10) and (11), respectively.

$$
\begin{align*}
\underline{\mathbf{s}}^{U} & =\left[\begin{array}{llll}
s_{1}^{U} & s_{2}^{U} & \cdots & s_{K_{u}}^{U}
\end{array}\right]^{T} .  \tag{10}\\
\underline{\mathbf{F}}_{\text {rotated }} & =\left[\begin{array}{cccc}
\mathbf{f}_{1} V_{1}^{U}(1) & \mathbf{f}_{2} V_{2}^{U}(1) & \cdots & \mathbf{f}_{K_{u}} V_{K_{u}}^{U}(1) \\
\mathbf{f}_{1} V_{1}^{U}(2) & \mathbf{f}_{2} V_{2}^{U}(2) & \cdots & \mathbf{f}_{K_{u}} V_{K_{u}}^{U}(2) \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{f}_{1} V_{1}^{U}(T) & \mathbf{f}_{2} V_{2}^{U}(T) & \cdots & \mathbf{f}_{K_{u}} V_{K_{u}}^{U}(T)
\end{array}\right] . \tag{11}
\end{align*}
$$

For the sake of the IN analysis, we denote the effective channels of the $K_{u}$ UL users at the $t^{\text {th }}$ time slot by

$$
\widetilde{\mathbf{F}}(t)=\left[\begin{array}{llll}
\mathbf{f}_{1} V_{1}^{U}(t) & \mathbf{f}_{2} V_{2}^{U}(t) & \cdots & \mathbf{f}_{K_{u}} V_{K_{u}}^{U}(t) \tag{12}
\end{array}\right]
$$

where $\widetilde{\mathbf{F}}(t) \in \mathbb{C}^{M \times K_{u}}$. Similarly, the effective interference channel vector of the $j^{\text {th }}$ DL user at the $t^{\text {th }}$ time slot can be written by

$$
\widetilde{\mathbf{h}}_{j}(t)=\left[\begin{array}{llll}
h_{j 1} V_{1}^{U}(t) & h_{j 2} V_{2}^{U}(t) & \cdots & h_{j K_{u}} V_{K_{u}}^{U}(t) \tag{13}
\end{array}\right]^{T} .
$$

Proposition 1: For $T M \geqslant K_{u},{ }^{1} \underline{\mathbf{F}}_{\text {rotated }} \in \mathbb{C}^{T M \times K_{u}}$ in (9) has full column rank when the channel is block fading over $T$ time slots.

Proof: Please see Appendix A.
From (9) and the Proposition 1, the BS can decode $K_{u}$ data streams at most. Therefore, the DoF of the UL is

$$
\begin{equation*}
\operatorname{DoF}_{\mathrm{u}}=\min \left\{M, \frac{K_{u}}{T}\right\} \tag{14}
\end{equation*}
$$

where $M$ is the upper bound of the UL DoF.
If the phase of $s_{i}^{U}$ is not rotated by $V_{i}^{U}, \underline{\mathbf{F}}_{\text {rotated }}$ becomes $\underline{\mathbf{F}}$ of which rank is $M$.

$$
\underline{\mathbf{F}}=\left[\begin{array}{cccc}
\mathbf{f}_{1} & \mathbf{f}_{2} & \cdots & \mathbf{f}_{K_{u}}  \tag{15}\\
\mathbf{f}_{1} & \mathbf{f}_{2} & \cdots & \mathbf{f}_{K_{u}} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{f}_{1} & \mathbf{f}_{2} & \cdots & \mathbf{f}_{K_{u}}
\end{array}\right]
$$

For this case, the BS can only decode $M$ data streams over $T$ time slots. However, in the proposed IN scheme, we construct the $V_{i}^{U}(t)$ which can make $\underline{\mathbf{F}}_{\text {rotated }}$ be of full column rank. Thus, the BS can decode $K_{u}$ data streams over $T$ time slots. Also, the DoF of the UL is $\frac{K_{u}}{T}$. Especially, for the case of $K_{u}=T M$, the DoF of the UL can be $M$.

## B. THE DL TRANSMISSION

During the DL transmission, the BS plays a significant role in canceling the multi-user interference among the DL users and assisting the DL users to neutralize the IUI from the UL users.

Supposing that there are $n K_{d}$ encoded messages in total which are needed to transmit to the $K_{d}$ DL users over $T$ time slots, we denote them as

$$
\mathbf{d}=\left[\begin{array}{llll}
d_{1} & d_{2} & \cdots & d_{n K_{d}} \tag{16}
\end{array}\right] .
$$

We can adopt the zero-forcing (ZF) precoder to cancel the multi-user interference. The ZF precoder is given by

$$
\begin{equation*}
\mathbf{W}=\mathbf{R}^{\dagger}\left(\mathbf{R} \mathbf{R}^{\dagger}\right)^{-1} \tag{17}
\end{equation*}
$$

where $\mathbf{R} \in \mathbb{C}^{n K_{d} \times T M}$ is the effective receive channel matrix in which the IUI has been neutralized and its specific form will be given in Section III-C. Thus, the precoded desired signal for the DL users, over $T$ time slots is

$$
\begin{equation*}
\underline{\mathbf{x}}^{D}=\mathbf{W d} \tag{18}
\end{equation*}
$$

where $\underline{\mathbf{x}}^{D}=\left[\mathbf{x}^{D}(1)^{T}, \mathbf{x}^{D}(2)^{T}, \cdots, \mathbf{x}^{D}(T)^{T}\right]^{T} \in \mathbb{C}^{T M \times 1}$ and $\mathbf{x}^{D}(t) \in \mathbb{C}^{M \times 1}$ is the precoded desired signal of the BS at time $t$.

[^1]

FIGURE 4. The DL transmission where the DL signals of the BS can be divided into two parts: the signals for the DL users denoted by $\mathbf{x}^{D}(t)$ and the signals used for the IN denoted by $v^{A F}(t) y_{B S}(t-1)$. $v^{A F}(t)$ is an $\boldsymbol{M} \times \boldsymbol{M}$ matrix and the superscript $\boldsymbol{A F}$ is short for amplify and forward.

As shown in Fig. 4, the DL signals are the superposition of the UL signals forwarded by the BS and the ZF precoded desired signals. The DL users will neutralize the IUI with these signals to recover the desired signals.

As shown in (19), the BS merely transmits $\mathbf{x}^{D}$ (1) to the DL users at the first time slot. At the subsequent time slot, i.e., $t \geqslant 2$, the BS transmits the superposition of its desired signal $\mathbf{x}^{D}(t)$ and the precoded signal $\mathbf{V}^{A F}(t) \mathbf{y}_{B S}(t-1)$ where $\mathbf{V}^{A F} \in \mathbb{C}^{M \times M}$ is the precoding matrix to the DL users.

$$
\mathbf{x}_{B S}(t)= \begin{cases}\mathbf{x}^{D}(1) & t=1  \tag{19}\\ \mathbf{x}^{D}(t)+\mathbf{V}^{A F}(t) \mathbf{y}_{B S}(t-1) & t \geqslant 2\end{cases}
$$

With the facts of (2), (7) and (19), the received signal of the $j^{\text {th }}$ DL user is given by

$$
\begin{align*}
y_{j}^{D}(t)= & \mathbf{g}_{j}^{T} \mathbf{x}_{B S}+\sum_{i=1}^{K_{u}} h_{j i} V_{i}^{U}(t) s_{i}^{U}+z_{j}^{D}(t) \\
= & \mathbf{g}_{j}^{T} \mathbf{x}^{D}(t)+\mathbf{g}_{j}^{T} \mathbf{V}^{A F}(t) \sum_{i=1}^{K_{u}} \mathbf{f}_{i} V_{i}^{U}(t-1) s_{i}^{U} \\
& +\sum_{i=1}^{K_{u}} h_{j i} V_{i}^{U}(t) s_{i}^{U}+\widetilde{z}_{j}^{D}(t) \tag{20}
\end{align*}
$$

where $\widetilde{z}_{j}^{D}(t)=z_{j}^{D}(t)+\mathbf{g}_{j}^{T} \mathbf{V}^{A F}(t) \mathbf{z}^{U}(t-1)$ and $V_{i}^{U}(-1)$ is assumped to be 0 .

## C. THE INTERFERENCE NEUTRALIZATION

The DL users need to eliminate the superposed signals which include the IUI from the UL users and the signals amplified and forwarded by the BS. As shown in Fig. 5, we control the synthesis of the first $n K_{u}$ superposed signals through a set of scalars $\left[u_{1}, u_{2}, \cdots, u_{n K_{u}}\right.$ ] to make sure that it can zero-force the synthetic vector of any last $(T-n)$ superposed signals.


FIGURE 5. The process of the $\operatorname{IN}$ where $\mathbf{v}_{\boldsymbol{i}}$ represents the superposed signals of the IUI and the amplified signals forwarded by the BS.

We combine the received signals at the $j^{\text {th }}$ DL user over $T$ time slots as

$$
\mathbf{y}_{j}^{D}=\left[\begin{array}{ll}
\mathbf{p} & \mathbf{Q}
\end{array}\right]_{T \times\left(K_{u}+1\right)}\left[\begin{array}{l}
1  \tag{21}\\
\mathbf{s}
\end{array}\right]_{\left(K_{u}+1\right) \times 1}+\mathbf{z}_{j}^{D}
$$

where

$$
\begin{aligned}
\mathbf{p} & =\left[\begin{array}{llll}
\mathbf{g}_{j}^{T} \mathbf{x}^{D}(1) & \mathbf{g}_{j}^{T} \mathbf{x}^{D}(2) & \cdots & \mathbf{g}_{j}^{T} \mathbf{x}^{D}(T)
\end{array}\right]^{T}, \\
\mathbf{z}_{j}^{D}= & {\left[\begin{array}{lll}
\widetilde{z}_{j}^{D}(1) & \widetilde{z}_{j}^{D}(2) & \cdots \\
\widetilde{z}_{j}^{D}(T)
\end{array}\right]^{T}, } \\
\mathbf{Q} & =\left[\begin{array}{c}
\widetilde{\mathbf{h}}_{j}^{T}(1) \\
\mathbf{g}_{j}^{T} \mathbf{V}^{A F} \\
(2) \\
\widetilde{\mathbf{F}}^{T}(1)+\widetilde{\mathbf{h}}_{j}^{T}(2) \\
\vdots \\
\mathbf{g}_{j}^{T} \mathbf{V}^{A F}(T) \widetilde{\mathbf{F}}(T-1)+\widetilde{\mathbf{h}}_{j}^{T}(T)
\end{array}\right] .
\end{aligned}
$$

The $j^{\text {th }}$ DL user linearly combines its received signals over $T$ time slots through the matrix $\mathbf{U}_{j} \in \mathbb{C}^{n \times T}$.

$$
\mathbf{U}_{j} \mathbf{y}_{j}^{D}=\left[\begin{array}{ll}
\mathbf{U}_{j} \mathbf{p} & \left.\mathbf{U}_{j} \mathbf{Q}\right]_{\mathbf{s}}^{1}+\mathbf{U}_{j} \mathbf{z}_{j}^{D}, \tag{22}
\end{array}\right.
$$

where $\mathbf{U}_{j}$ satisfies

$$
\begin{equation*}
\mathbf{U}_{j} \mathbf{Q}=\mathbf{0} \tag{23}
\end{equation*}
$$

The IUI can be neutralized completely due to (23). Without loss of generality, we can define the matrix $\mathbf{U}_{j}$ as

$$
\mathbf{U}_{j}=\left[\begin{array}{ll}
\mathbf{U}_{j}^{e q} & -\mathbf{I}_{n}, \tag{24}
\end{array}\right]
$$

where $\mathbf{U}_{j}^{e q} \in \mathbb{C}^{n \times(T-n)}$.
Combining all the received signal over $T$ time slots into a matrix, we obtain

$$
\mathbf{y}_{d}^{e q}=\underbrace{\left[\begin{array}{cc}
\mathbf{U}_{1}^{e q} \mathbf{G}_{1}^{T-n} & -\mathbf{G}_{1}^{n}  \tag{25}\\
\mathbf{U}_{2}^{e q} \mathbf{G}_{2}^{T-n} & -\mathbf{G}_{2}^{n} \\
\vdots & \vdots \\
\mathbf{U}_{K_{d}}^{e q} \mathbf{G}_{K_{d}}^{T-n} & -\mathbf{G}_{K_{d}}^{n}
\end{array}\right]}_{\mathbf{R}}\left[\begin{array}{c}
\mathbf{x}^{D}(1) \\
\mathbf{x}^{D}(2) \\
\vdots \\
\mathbf{x}^{D}(T)
\end{array}\right]+\left[\begin{array}{c}
\mathbf{U}_{1} \mathbf{z}_{1}^{D} \\
\mathbf{U}_{2} \mathbf{z}_{2}^{D} \\
\vdots \\
\mathbf{U}_{K_{d}} \mathbf{z}_{K_{d}}^{D}
\end{array}\right],
$$

where $\mathbf{R} \in \mathbb{C}^{n K_{d} \times T M}, \mathbf{G}_{j}^{T-n}=\operatorname{diag}\left(\mathbf{g}_{j}^{T}, \cdots, \mathbf{g}_{j}^{T}\right) \in$ $\mathbb{C}^{(T-n) \times(T-n) M}$ and $\mathbf{G}_{j}^{n}=\operatorname{diag}\left(\mathbf{g}_{j}^{T}, \cdots, \mathbf{g}_{j}^{T}\right) \in \mathbb{C}^{n \times n M}$.

We denote the matrix $\mathbf{R}$ as the effective receive channel of the DL users, which is used for the ZF precoding in the DL transmission. Because all the channel coefficients are drawn from i.i.d. circularly-symmetric complex Gaussian distributions [12], [19], $\mathbf{R}$ is a full rank matrix almost sure. Therefore, when $n K_{d} \leqslant T M$, the DL users can decode $n K_{d}$ data streams over $T$ time slots. The DoF of the DL is given by

$$
\begin{equation*}
\operatorname{DoF}_{\mathrm{d}}=\min \left\{M, \frac{n K_{d}}{T}\right\} \tag{26}
\end{equation*}
$$

where $M$ is the upper bound of the DL DoF.
During the whole process of the IN scheme, the UL users just rotate the phases of their transmission signals by a set of scalars designed in advance and do not have to access any CSI. The IN is assisted by the BS and completed at the DL users.

## D. EXAMPLES

In this subsection, we give two examples to illustrate the idea of the proposed IN scheme.

Example 1: We consider the FD cellular network shown in Fig. 1 and assume the network configuration ( $M, K_{d}, K_{u}, T$ ) is ( $1,2,1,2$ ). The UL users transmit two symbols $\left(s_{1}^{U}, s_{2}^{U}\right)$ rotated by $\left\{V_{i}^{U}(t)\right\}_{i \in[1: 2], t \in[1: 2]}$ over two time slots. Under this configuration, the received signal of the BS is

$$
\mathbf{y}_{B S}=\underbrace{\left[\begin{array}{ll}
f_{1} e^{i \frac{2 \pi}{3}} & f_{2} e^{i \frac{\pi}{2}}  \tag{27}\\
f_{1} e^{i \frac{\pi \pi}{3}} & f_{2} e^{j \pi}
\end{array}\right]}_{\underline{\mathbf{F}}_{\text {rotated }}}\left[\begin{array}{l}
s_{1} \\
s_{2}
\end{array}\right]+\left[\begin{array}{l}
z_{U}^{U}(1) \\
z_{2}^{U}(2)
\end{array}\right],
$$

where $f_{1}$ and $f_{2}$ are the UL channels from the $1^{\text {st }}$ and $2^{\text {nd }}$ UL users to the BS, respectively. We have demonstrated that the matrix $\underline{\mathbf{F}}_{\text {rotated }}$ has full column rank in Proposition 1. According to (27), the BS can decode $s_{1}^{U}$ and $s_{2}^{U}$ by many methods such as the zero-forcing. Thus, the DoF of the UL is 1. Initially, the BS transmits $x^{D}(1)$ to the DL user and it transmits the superposed signal of $x^{D}(1)$ and $V^{A F} \mathbf{y}_{B S}$ (1) to the DL user at the subsequent time slot. The received signal of the DL user is

$$
\mathbf{y}_{1}^{D}=\left[\begin{array}{cc}
g_{1} x^{D}(1) & \widetilde{\mathbf{h}}_{1}^{T}(1)  \tag{28}\\
g_{1} x^{D}(2) & g_{1} V^{A F} \widetilde{\mathbf{F}}(1)+\widetilde{\mathbf{h}}_{1}^{T}(2)
\end{array}\right]\left[\begin{array}{c}
1 \\
\mathbf{s}^{U}
\end{array}\right]+\mathbf{z}^{D},
$$

where $\mathbf{U}_{1}, V^{A F}, \widetilde{\mathbf{F}}(1), \widetilde{\mathbf{h}}_{1}(1)$ and $\widetilde{\mathbf{h}}_{1}(2)$ should satisfy (23). The DL user left multiplies matrix $\mathbf{U}_{1}=\left[U_{1}^{\text {eq }},-1\right] \in \mathbb{C}^{1 \times 2}$ to neutralize its IUI and recover one symbol over two time slots.

Finally, three symbols (two from the UL channels and one from the DL channels) are transmitted during the two time slots. According to the definition of DoF, the sum DoF under the configuration $(1,2,1,2)$ is 1.5 .

Example 2: In this example, we consider the network configuration where ( $M, K_{d}, K_{u}, T$ ) is ( $1,1,2,2$ ). The UL user transmits one symbol $s_{1}^{U}$ rotated by $\left\{V_{1}^{U}(t)\right\}_{t \in[1: 2]}$ over two
time slots. The received signal of the BS is

$$
\mathbf{y}_{B S}=\left[\begin{array}{l}
f_{1} e^{j \frac{2 \pi}{3}}  \tag{29}\\
f_{1} e^{j \frac{4 \pi}{3}}
\end{array}\right]\left[s_{1}^{U}\right]+\left[\begin{array}{ll}
z_{1}^{U}(1) \\
z_{2}^{U} & (2)
\end{array}\right] .
$$

Obviously, the BS can recover $s_{1}^{U}$ through (29). Initially the BS transmits the ZF precoded vector $\mathbf{x}^{D}(1)$ to the two DL users and it transmits the superposed signal of $\mathbf{x}^{D}(2)$ and $V^{A F} \mathbf{y}_{B S}$ (1) to the two DL users at the subsequent time slot. The received signal of the $j^{\text {th }}(j \in\{1,2\})$ DL user is

$$
\mathbf{y}_{j}^{D}=\left[\begin{array}{cc}
g_{j} D^{D}(1) & \widetilde{h}_{j}(1)  \tag{30}\\
g_{j} D^{D}(2) & g_{j} V^{A F}(1)+\widetilde{h}_{j}(2)
\end{array}\right]\left[\begin{array}{c}
1 \\
s_{1}^{U}
\end{array}\right]+\mathbf{z}_{j}^{D} .
$$

Each DL user left multiplies the matrix $\mathbf{U}_{j}=\left[U_{j}^{e q},-1\right] \in$ $\mathbb{C}^{1 \times 2}$ to neutralize the IUI, i.e.,

$$
\mathbf{U}_{j} \mathbf{y}_{j}^{D}=\left[w_{j 1}, w_{j 2}\right]\left[\begin{array}{c}
1  \tag{31}\\
s_{1}^{U}
\end{array}\right]+\mathbf{z}_{j}^{D},
$$

where

$$
\begin{align*}
& w_{j 1}=U_{j}^{e q} g_{j} x^{D}(1)-g_{j} x^{D}(2),  \tag{32}\\
& w_{j 2}=U_{j}^{e q} \widetilde{h}_{j}(1)-g_{j} V^{A F}(1)-\widetilde{h}_{j}(2) . \tag{33}
\end{align*}
$$

If $w_{j 2}=0(\forall j \in\{1,2\})$, the IUI can be neutralized. We can set up the equations

$$
\left\{\begin{array}{l}
U_{1}^{e q} \widetilde{h}_{1}(1)-g_{1} V^{A F}(1)=\widetilde{h}_{1}(2),  \tag{34}\\
U_{2}^{e q} \widetilde{h}_{2}(1)-g_{2} V^{A F}(1)=\widetilde{h}_{2}(2),
\end{array}\right.
$$

where $U_{1}^{e q}, U_{2}^{e q}$ and $V^{A F}$ are the unknown variables. (34) can be combined into the matrix form as

$$
\underbrace{\left[\begin{array}{ccc}
\widetilde{h}_{1}(1) & 0 & g_{1}  \tag{35}\\
0 & \widetilde{h}_{2}(1) & g_{2}
\end{array}\right]}_{\mathbf{G}}\left[\begin{array}{c}
U_{1}^{e q} \\
U_{e q}^{e q} \\
V^{A F}
\end{array}\right]=\left[\begin{array}{c}
\widetilde{h}_{1}(2) \\
\widetilde{h}_{2}(2)
\end{array}\right] .
$$

The matrix $\mathbf{G}$ is of full row rank. So we can acquire $U_{1}^{\text {eq }}, U_{2}^{\text {eq }}$ and $V^{A F}$ from (35). After neutralizing the IUI, we combine the received signal of the two DL users over two time slots into

$$
\mathbf{y}_{d}^{e q}=\underbrace{\left[\begin{array}{ll}
U_{1}^{e q} g_{1} & -g_{1}  \tag{36}\\
U_{2}^{U e} g_{2} & -g_{2}
\end{array}\right]}_{\mathbf{R}}\left[\begin{array}{l}
x^{D}(1) \\
x^{D}(2)
\end{array}\right]+\left[\begin{array}{l}
\mathbf{U}_{1} \mathbf{z}_{1}^{D} \\
\mathbf{U}_{2} \mathbf{z}_{2}^{D}
\end{array}\right] .
$$

In Appendix D, we prove that

$$
\begin{equation*}
\|\mathbf{R}\| \neq 0 \tag{37}
\end{equation*}
$$

Therefore, the two DL users can decode two symbols out of two time slots. The sum DoF of our scheme under the configuration $(1,1,2,2)$ is 1.5 . For the configuration of two UL users and one DL user, the DoF of the UL is 1 and the DoF of the DL is 0.5 whereas for the configuration of one UL user and two DL users, the DoF of the DL is 0.5 and the DoF of the UL is 1 . The IN scheme is well fit for this symmetric configuration.

## IV. FEASIBLE CONDITIONS AND THE ACHIEVABLE DOF

In this section, we analyze the achievable DoF and the feasible conditions for the proposed IN scheme. In order to illustrate the achievable DoF, we investigate two kinds of the maximum achieved sum DoF, $D^{*}\left(M, K_{d}, K_{u}, T\right)$ and $D^{*}(M)$. The former represents the maximum achieved sum DoF over a given network configuration $\left(M, K_{d}, K_{u}, T\right)$, as shown in (38), while the latter denotes the maximum achieved sum DoF over all available network configurations, as shown in (39). They are both expressed as a multiple of $M$ to illustrate the DoF gains over the HD systems [16], [23].

$$
\begin{align*}
D^{*}\left(M, K_{d}, K_{u}, T\right) & =\max _{\operatorname{DoF}_{\mathrm{u}}, \operatorname{DoF}_{\mathrm{d}} \in \mathcal{D}_{\left(M, K_{d}, K_{u}, T\right)}} \mathrm{DoF}_{\mathrm{u}}+\mathrm{DoF}_{\mathrm{d}} .  \tag{38}\\
D^{*}(M) & =\max _{K_{d}, K_{u}, T} D^{*}\left(M, K_{d}, K_{u}, T\right) \tag{39}
\end{align*}
$$

## A. FEASIBLE CONDITIONS

Proposition 2: The IN for the FD cellular networks with partial CSIT is feasible if the conditions below are satisfied.

$$
\begin{align*}
& K_{d} K_{u} \leq K_{d}+M^{2}  \tag{40a}\\
& (T-1) K_{d} \leqslant T M  \tag{40b}\\
& K_{u} \leqslant T M  \tag{40c}\\
& T \geqslant 2  \tag{40d}\\
& K_{d}, K_{u}, M, T \in \mathbb{Z}^{+} \tag{40e}
\end{align*}
$$

Proof: Please see Appendix B.
The feasibility of (23) is determined by the condition (40a) which limits the maximum achievable sum DoF of the IN scheme. (40b) is the feasible condition of the transmission zero-forcing precoding at the BS. (40c) comes from our assumption that if $K_{u}>T M$, we shut down $\left(K_{u}-T M\right)$ UL users.

## B. MAXIMUM SUM DoF $D^{*}(M)$

If the feasible conditions in Proposition 2 are satisfied, the achievable sum DoF through our proposed scheme for the FD cellular networks with partial CSIT is

$$
\begin{equation*}
\mathrm{DoF}_{\mathrm{sum}}=\frac{K_{u}}{T}+\frac{(T-1) K_{d}}{T} \tag{41}
\end{equation*}
$$

Next, we discuss the upper bound of the DoF gains over the HD systems based on the feasible conditions and the sum DoF of the IN scheme. Due to the maximum achieved sum DoF is a multiple of $M$, we normalize (41) by $M$, i.e.,

$$
\begin{equation*}
\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Norm}}=\frac{\mathrm{DoF}_{\text {sum }}}{M}, \tag{42}
\end{equation*}
$$

which denotes the DoF gain over the HD systems.
For the given $M$ and $T(T \geqslant 2)$, we formulate the following optimization problem $\mathcal{P} 1$ to obtain the maximum achievable DoF.

$$
\begin{align*}
\mathcal{P} 1: & \underset{K_{d}, K_{u}}{\operatorname{maximize}} \frac{K_{u}}{M T}+\frac{(T-1) K_{d}}{M T}  \tag{43a}\\
& \text { subject to } K_{d} K_{u} \leqslant K_{d}+M^{2} \tag{43b}
\end{align*}
$$

$$
\begin{align*}
& (T-1) K_{d} \leqslant T M  \tag{43c}\\
& K_{u} \leqslant T M  \tag{43d}\\
& K_{d}, K_{u} \in \mathbb{Z}^{+} \tag{43e}
\end{align*}
$$

Proposition 3: Over all available network configurations, the maximum normalized sum DoF via our proposed IN scheme is $\mathrm{DoF}_{\text {sum }}^{\mathrm{Norm}}=1.5$, i.e., the maximum sum DoF $D^{*}(M)$ is $1.5 M$.

Proof: We prove Proposition 3 for $M=1$ and $M \geqslant 2$, respectively.

Case I: For $M=1$, we list all the feasible points as below. When $T>2$, the only feasible point of $\mathcal{P} 1$ is $\left(K_{d}=1, K_{u}=\right.$ 2). The sum DoF is

$$
\begin{equation*}
\mathrm{DoF}_{\text {sum }}=\frac{T+1}{T} \leqslant 1.5 \tag{44}
\end{equation*}
$$

When $T=2$, the feasible points of $\mathcal{P} 1$ are $\left(K_{d}=1, K_{u}=2\right)$ and ( $K_{d}=2, K_{u}=1$ ). Either one can achieve 1.5 DoF .

Case II: For $M \geqslant 2$, the problem $\mathcal{P} 1$ is an integer programming problem which has no efficient general algorithm for its solutions [44]. We relax the constrains $K_{d}, K_{u} \in \mathbb{Z}^{+}$, i.e., allowing $K_{d}$ and $K_{u}$ to take on non-integer but nonnegative values, which yields $\mathcal{P} 2$ as

$$
\begin{align*}
\mathcal{P} 2: \underset{K_{d}, K_{u}}{\operatorname{maximize}} & \frac{K_{u}}{M T}+\frac{(T-1) K_{d}}{M T}  \tag{45a}\\
\text { subject to } & K_{d} K_{u} \leqslant K_{d}+M^{2},  \tag{45b}\\
& (T-1) K_{d} \leqslant T M,  \tag{45c}\\
& K_{u} \leqslant T M,  \tag{45~d}\\
& K_{d}, K_{u}>0 \tag{45e}
\end{align*}
$$

$\mathcal{P} 2$ can be solved by the Graphical Solution Method [44]. As long as the obtained $K_{d}^{*}$ and $K_{u}^{*}$ are positive integers, they are optimal. The shaded area in Fig. 6 is the feasible region of $\mathcal{P} 2$. The intercept of the Line 3 on the $K_{u}$-axis reflects DoF $\mathrm{F}_{\text {sum }}^{\text {Norm }}$.


FIGURE 6. The Graphical Solution method for the problem $\mathcal{P} \mathbf{2}$ where the Line 1 , the Line 2 , the Line 3 , the dash line 1 , and the Curve 1 represent the function $K_{\boldsymbol{d}}=\frac{T M}{\boldsymbol{T}-1}$, the function $K_{\boldsymbol{u}}=M T$, the function $\frac{(T-1) K_{d}}{M T}+\frac{K_{u}}{M T}=\operatorname{DoF}_{\text {sum }}^{\text {Norm }}$, the function $K_{d}=1$, and the function $K_{u}=1+\frac{M^{2}}{K_{d}}$, respectively. The shaded area is the feasible region of the problem $\mathcal{P} \mathbf{2}$. The points $A$ and $B$ are the intersection of the Line 1 and the Curve 1 and the intersection of the Line 2 and the Curve 1 . The intercept of the Line 3 on the $K_{u}$-axis reflects DoF ${ }_{\text {sum }}^{\text {Norm }}$ in $\mathcal{P} 2$. (a) $K_{u} \leqslant \boldsymbol{T M}$. (b) $K_{u}>T M$.

Especially, the vertices A and B of the feasible region in Fig. 6 are the optimal points that maximize $\mathcal{P} 2$.

Next, we obtain the upper bound of the maximum achievable normalized DoF of the proposed IN scheme based on the two vertices A and B in Fig. 6.

For the point A whose $\left(K_{d}, K_{u}\right)$ is $\left(\frac{M^{2}}{T M-1}, T M\right)$, the normalized sum DoF is

$$
\begin{equation*}
\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Norm}}=1+\frac{M(T-1)}{(T M-1) T} \stackrel{(a)}{\leqslant} 1.5 \tag{46}
\end{equation*}
$$

where the proof of $(a)$ is in Appendix VI. The upper bound of the normalized DoF under this circumstance is 1.5 for $\mathcal{P} 2$ as well as the original problem $\mathcal{P} 1$.

For the point B whose $\left(K_{d}, K_{u}\right)$ is $\left(\frac{T M}{T-1}, 1+\frac{(T-1) M}{T}\right)$, the normalized sum DoF is

$$
\begin{equation*}
\mathrm{DoF}_{\mathrm{sum}}^{\mathrm{Norm}}=\frac{1}{M T}+\frac{T-1}{T^{2}}+1 \tag{47}
\end{equation*}
$$

which decreases with $T$ increasing if $T \geqslant 2$. We set $T=2$ to obtain the maximum normalized DoF which yields

$$
\begin{equation*}
\operatorname{DoF}_{\mathrm{sum}}^{\mathrm{Norm}}=\frac{1}{2 M}+\frac{1}{4}+1 \leqslant 1.5 \quad \text { when } \quad M \geqslant 2 \tag{48}
\end{equation*}
$$

where the equality holds when $M=2$. For the original problem $\mathcal{P} 1$, the upper bound of the normalized DoF is also 1.5 when $M \geqslant 2$. The upper bound can be achieved via the configuration ( $M=2, K_{d}=2, K_{u}=4, T=2$ ).

In summary, the maximum normalized DoF that the proposed IN scheme can achieve is 1.5 . Also, the maximum achieved sum $\operatorname{DoF} D^{*}(M)$ is $1.5 M$.

## C. DoF REGIONS $\mathcal{D}_{\left(M, K_{d}, K_{u}, T\right)}$

In this subsection, we discuss the DoF regions $\mathcal{D}_{\left(M, K_{d}, K_{u}, T\right)}$ based on the points A and B in Fig. 6. In the point A, the DoF of the UL is assumed to be $M$, the same as the DoF of the DL in the point $B$.

The sum DoF under the assumption of the UL being $M$ is

$$
\begin{equation*}
\mathrm{DoF}_{\text {sum }}=M+\frac{(T-1) K_{d}}{T} \tag{49}
\end{equation*}
$$

According to (40a) and (40e), $K_{u}$ satisfying (50) indicates that the DoF of the DL in HD mode is $K_{d}$ and the maximum sum $\operatorname{DoF} D^{*}\left(M, K_{d}, K_{u}, T\right)$ of the FD cellular network is $K_{d}+M$ under the condition of no IUI.

$$
\begin{equation*}
K_{d} \leqslant\left\lfloor\frac{M^{2}}{T M-1}\right\rfloor \stackrel{(a)}{\leqslant} M \stackrel{(b)}{\leqslant}\left\lfloor\frac{T M}{T-1}\right\rfloor \tag{50}
\end{equation*}
$$

where (a) holds from

$$
\begin{equation*}
(T-1) M^{2} \geqslant M \Rightarrow \frac{M^{2}}{T M-1} \leqslant M \tag{51}
\end{equation*}
$$

and (b) holds from $T \geqslant 2$. The network configuration of Example 1 given in Section II exactly satisfies this assumption and the sum DoF is $\frac{K_{d}}{2}+M$. The DoF region is shown in Fig. 7.

The sum DoF under the assumption of the UL being $M$ is

$$
\begin{equation*}
\mathrm{DoF}_{\mathrm{sum}}=\frac{K_{u}}{T}+M \tag{52}
\end{equation*}
$$



FIGURE 7. The DoF region for the point $A$.
According to constraints (40a) and (40e), $K_{u}$ satisfies

$$
\begin{equation*}
K_{u} \leqslant 1+\left\lfloor\frac{(T-1) M}{T}\right\rfloor \tag{53}
\end{equation*}
$$

The network configuration of Example 2 given in Section II exactly satisfies this assumption and the sum $\operatorname{DoF}$ is $\frac{K_{u}}{2}+M$. For $K_{u} \leqslant M$ and $K_{u}>M$, we have the two DoF regions shown in Fig. 8(a) and 8(b), respectively.


FIGURE 8. The DoF regions for the point B. (a) $K_{u} \leqslant 1+\left\lfloor\frac{(T-1) M}{T}\right\rfloor \leqslant M$ and $K_{d}=\frac{T M}{T-1}$. (b) $M<K_{u} \leqslant 1+\left\lfloor\frac{(T-1) M}{T}\right\rfloor$ and $K_{d}=\frac{T M}{T-1}$.

In the next section, we will demonstrate that the DoF regions in Fig. 7 and 8 (a) are the same with that via the IA scheme [16] while the DoF region in Fig. 8(b) is smaller than that via the IA scheme.

## V. NUMERICAL RESULTS AND COMPARISON

In this section, we will provide the numerical results of the proposed IN scheme and compare it with the IA scheme with full CSIT [16] and the RA scheme [23] for the FD BS cellular systems, from the perspective of DoF, sum rates and complexity.

## A. DOF

In this subsection, we show that the proposed IN scheme under the conditions of Fig. 7(a) and 8(a) is optimal from the perspective of the DoF region. In addition, we illustrate the maximum normalized sum DoF of the proposed IN scheme is 1.5 by numerical results.

For the conditions of Fig. 7(a) i.e., $K_{u}=T M$ and $K_{d} \leqslant\left\lfloor\frac{M^{2}}{T M-1}\right\rfloor$, from Theorem 1 in [16], the achievable UL and DL DoF region $\left(\mathrm{DoF}_{\mathrm{u}}, \mathrm{DoF}_{\mathrm{d}}\right)$ through the IA with full CSIT has the following corner points:

$$
\text { i) }(M, 0) \text { ii) }\left(0, K_{d}\right) \text { iii) }\left(M, \frac{T-1}{T} K_{d}\right) \text { iv) }\left(0, K_{d}\right) \text {. }
$$

which are exactly the same with the corner points in Fig. 7.
For the conditions of Fig. 8(a) i.e., $K_{u} \leqslant 1+\left\lfloor\frac{(T-1) M}{T}\right\rfloor \leqslant$ $M$ and $K_{d}=\frac{T M}{T-1}$, the same procedure may be easily done to obtain corner points of the DoF region below. Also, these corner points coincide exactly with that in Fig. 8.

$$
\text { i) }\left(K_{u}, 0\right) \text { ii) }(0, M) \text { iii }\left(K_{u}, 0\right) \text { iv) }\left(\frac{K_{u}}{T}, M\right)
$$

From (38), the same DoF region $\mathcal{D}_{\left(M, K_{d}, K_{u}, T\right)}$ means the same maximum sum $\operatorname{DoF} D^{*}\left(M, K_{d}, K_{u}, T\right)$. Also, the maximum achieved sum $\operatorname{DoF} D^{*}\left(M, K_{d}, K_{u}, T\right)$ under conditions of 7(a) and 8(a) is optimal.

However, for the network configuration of Fig. 8(b), its corner points are
i) $(M, 0)$
ii) $(0, M)$
iii) $\left(M, \min \left(\frac{K_{d}}{K_{u}}\left(K_{u}-M\right)^{+}, M\right)\right)$
iv) $\left(\frac{K_{u}}{T}, M\right)$.

For this case, the DoF region of the proposed IN scheme is smaller than that of the IA scheme with full CSIT.

In Section IV, we have demonstrated that the maximum DoF $D^{*}(M)$ via the proposed IN scheme is $1.5 M$. It is illustrated that $1.5 M$ DoF can be achieved in Fig. 9, when $M=1$ and $T=2$. Substituting them into $K_{u}^{*}=T M, K_{d}^{*}=$ $\left\lfloor\frac{M^{2}}{T M-1}\right\rfloor$, i.e., the Point A in Fig. 6, we have the network configuration ( $1,1,2,2$ ) that maximizes the normalized sum DoF. By the same procedure, we can obtain the network configurations (1, 2, 1, 2) and (2, 4, 2, 2). These results coincide with the results in Section IV.

For some given network configurations ( $M, K_{d}, K_{u}, T$ ), the maximum sum $\operatorname{DoF} D^{*}\left(M, K_{d}, K_{u}, T\right)$ achieved via the proposed IN scheme is consistent with [16], which shows the optimality.

Both the RA technique and the proposed IN scheme assume the block fading channels. In [23], the UL channels are changed each time slot by the receive reconfigurable antennas at the BS and the IUI channels remain fixed. In the proposed IN scheme, the UL channels are altered by the phase rotation factors, but the IUI channels are also changed. The varying IUI channels result in 0.5 M DoF loss compared with the RA technique.

## B. SUM RATES

We consider an FD cellular network where $M=1, K_{d}=1$ and $K_{u}=2$. All the channel coefficients and noise are drawn from i.i.d. circularly-symmetric complex Gaussian distributions. In comparison, the rate of the HD system is modeled as that of a point to point channel, which has only 1 DoF .


FIGURE 9. Normalized sum DoF with respect to $\boldsymbol{M}$ where the zigzag line is caused by the constraints of $K_{d}, K_{u} \in \mathbb{Z}^{+}$. (a) $K_{u}^{*}=T M, K_{d}^{*}=\left\lfloor\frac{M^{2}}{T M-1}\right\rfloor$. (b) $K_{u}^{*}=1+\left\lfloor\frac{(T-1) M}{T}\right\rfloor, K_{d}^{*}=\left\lfloor\frac{T M}{T-1}\right\rfloor$.

Under the configuration, the numerical results are illustrated in Fig. 10. All the three schemes i.e., the IA [16], the RA [23] and the proposed IN schemes, almost have the same performance in terms of sum rates. As shown in Fig. 10, when $\mathrm{SNR}=80 \mathrm{~dB}$, the rate of the HD system is $25 \mathrm{bps} / \mathrm{Hz}$ while the rates of the three schemes are all around $37.5 \mathrm{bps} / \mathrm{Hz}$. From the definition of the DoF, they all achieve 1.5-fold gains over the HD one.

## C. COMPLEXITY

We compare the complexity of the three schemes from the perspective of the CSI requirement, the computation complexity and the hardware complexity.

Table 2 provides the CSI requirement of the RA, the IA and the proposed IN schemes. It is necessary for the UL users to access their own CSIT in the IA scheme. But it is hard for the UL users to directly acquire their own CSIT. Usually, the CSIT is measured by the DL users and then fed back to


FIGURE 10. Sum rates with respect to SNR in the FD cellular networks where $M=1, K_{d}=1$, and $K_{u}=2$. RA, IA, and HD represent for the RA scheme, the IA scheme, and the HD system, respectively.

TABLE 2. CSI Requirement.

| Scheme | Location | Required CSI |
| :---: | :---: | :---: |
| The IN | The BS | UL channels (1 time over $T$ time slots), DL channels ( 1 time over $T$ time slots), IUI channels (1 time over $T$ time slots). |
|  | UL users | None. |
|  | DL users | None. |
| The IA | The BS | UL channels ( $T$ times over $T$ time slots), DL channels ( $T$ times over $T$ time slots). |
|  | UL users | IUI Channels ( $T$ times over $T$ time slots). |
|  | DL users | IUI Channels ( $T$ times over $T$ time slots). |
| The RA | The BS | UL channels ( $L_{u}$ times over $T$ time slots), DL channels ( $L_{d}$ times over $T$ time slots), where the BS is equipped with $M_{u}$ and $M_{d}$ transmit and receive modes, $T=L_{u} L_{d}$, $L_{u}=\min \left\{K_{d}, M_{d}\right\}, L_{d}=\min \left\{K_{u}, M_{u}\right\}$. |
|  | UL users | None. |
|  | DL users | None. |

the UL users. Due to the fact, the feedback overhead would be severe and the throughput of the networks would decrease sharply. Moreover, for the IA technique, the requirement on CSI is $T$ times over $T$ time slots, since its implementation of the IA technique requires time-varying channels. The CSIT received by the UL users may be outdated. In the proposed IN scheme, the BS requires CSI on the IUI channels. But the BS has to access its own CSIT for $L_{d}$ times over $T$ time slots, since the transmit modes of the BS must be changing to cancel the IUI. Table 2 shows that the IA scheme has the highest complexity in terms of the requirement on CSI.

The IA scheme also has the highest computation complexity and its computation complexity is $O\left(\left(\min \left(K_{u}, K_{d}\right)\right)^{4}\right)$ [46]. When $n_{d}\left(n_{d} \in\left[1: L_{u}\right]\right)$ messages are transmitted by the BS and $n_{u}\left(n_{u} \in\left[1: L_{d}\right]\right)$ messages are transmitted by the UL users in the RA scheme, an $L_{d} n_{d} \times L_{d} L_{u}$ matrix inversion and an $L_{d} L_{u} \times K_{u} n_{u}$ matrix inversion are required for the UL and DL, respectively. The computation complexity of the RA can be expressed as $O\left(\left(\min \left(L_{d} n_{d}, L_{d} L_{u}, K_{u} n_{u}\right)\right)^{3}\right)$. The proposed IN scheme requires a $T M \times K_{u}$ matrix inversion to distinguish
the UL messages to the BS, a $(T-1) K_{d} K_{u} \times\left((T-1) K_{d}+\right.$ $(T-1) M^{2}$ ) matrix inversion to solve a linear system and a $(T-1) K_{d} \times T M$ matrix inversion to distinguish the DL messages from the BS. When considering the feasible condition of the IN scheme, we can denote the computation complexity of the proposed IN scheme by $O\left(\left((T-1) K_{d} K_{u}\right)^{3}\right)$. As shown in Table 3, both the RA and the proposed IN schemes are in the same level in terms of computation complexity.

TABLE 3. Computation Complexity Comparison.

| Schemes | Complexity |
| :---: | :---: |
| The IA | $O\left(\left(\min \left(K_{u}, K_{d}\right)\right)^{4}\right)$ |
| The RA | $O\left(\left(\min \left(L_{d} n_{d}, L_{d} L_{u}, K_{u} n_{u}\right)\right)^{3}\right)$ |
| The proposed IN | $O\left(\left((T-1) K_{d} K_{u}\right)^{3}\right)$ |

TABLE 4. Comparison of Reconfigurable Antenna Techniques.

| Electrical Property | RF MEMS | PIN DIODE | OPTICAL |
| :---: | :---: | :---: | :---: |
| Voltage (V) | $20-100$ | $3-55$ | $1.8-1.9$ |
| Current (mA) | 0 | $3-20$ | $0-87$ |
| Power Consumption | $0.05-0.1 \mathrm{~mW}$ | $5-100 \mathrm{~mW}$ | $0-50 \mathrm{~mW}$ |
| Switches Speed | $1-200 \mu \mathrm{~s}$ | $1-100 \mathrm{~ns}$ | $3-9 \mu \mathrm{~s}$ |
| Isolation $(1-10 \mathrm{GHz})$ | Very High | High | High |
| Loss $(1-10 \mathrm{GHz})(\mathrm{dB})$ | $0.05-0.2$ | $0.3-1.2$ | $0.5-1.5$ |

The major issue of the RA scheme is the requirement on antenna techniques. While the RA techniques can facilitate the interference management, its deployment might bring hardware complexity. As shown in Table 4, the RA techniques can be divided into four categories: the RF MEMS [47], the PIN diodes [48] and the optical switch [49]. The RF MEMS and the optical switch techniques have a relative lower switch speed (at the microsecond level), which may be inappropriate for the interference management of the FD cellular networks. This is because the modes of the antennas are required to keep varying during the process of eliminating the IUI. Besides, the RF MEMS technique requires high actuation voltages (20-100 V). For the PIN diode technique, the power consumption is high $(5-100 \mathrm{~mW})$. Moreover, the power consumption would increase with the increase of the number of the antennas. There is no extra power consumption for the conventional passive antennas.

TABLE 5. Complexity Comparison of the Three Schemes.

| Scheme | The proposed IN | The IA | The RA |
| :---: | :---: | :---: | :---: |
| CSI Requirement | Low | Highest | Low |
| Computation Complexity | Low | Highest | Low |
| Hardware Complexity | Low | Low | Highest |

As summarized in Table 5, the proposed IN scheme has the lowest complexity for the interference management in the FD cellular networks. The rigorous requirement on the CSI makes the IA technique hard to be directly implemented into the real systems. On the other hand, the application of the RA scheme requires the development of the antenna techniques. At the cost of $0.5 M$ DoF loss, the proposed IN scheme can
handle the IUI with the help of partial CSIT under block fading channels.

## VI. CONCLUSION

In this paper, we proposed an IN scheme for the FD cellular networks with partial CSIT. We endow the conventional receive antennas of the BS with reconfigurability through a set of scalars. With the BS amplifying and forwarding signals from the UL users, the DL users can cancel the IUI. We analyzed the DoF region achieved by the proposed IN scheme and the feasible conditions.
Considering all available network configurations, we show that the proposed scheme can achieve at most 1.5 times DoF gains with partial CSIT and with 2 symbol extensions compared to the HD systems. Although the proposed IN scheme is not superior in the maximum sum $\operatorname{DoF} D^{*}\left(M, K_{d}, K_{u}, T\right)$, it has an advantage of complexity over the IA scheme and the RA scheme. Under some network configurations, the proposed IN scheme can also achieve the same DoF region with the IA and the RA schemes, which shows the optimality. The proposed IN scheme can be taken as an optional scheme of low complexity for the interference management of the FD cellular networks.

## APPENDIX A

## PROOF OF PROPOSITION 1

In the case of $T M \geqslant K_{u}$, the matrix $\underline{\mathbf{F}}_{\text {rotated }} \in \mathbb{C}^{T M \times K_{u}}$ of (9) can be represented by the Khatri-Rao product of two partitioned matrices $\mathbf{V}^{\prime} \in \mathbb{C}^{T \times K_{u}}$ and $\mathbf{F}^{\prime} \in \mathbb{C}^{M \times K_{u}}$, i.e.,

$$
\begin{equation*}
\underline{\mathbf{F}}_{\text {rotated }}=\mathbf{V}^{\prime} \odot \mathbf{F}^{\prime} \tag{54}
\end{equation*}
$$

where $\mathbf{V}^{\prime}=\left[\begin{array}{llll}\mathbf{v}_{1}^{U} & \mathbf{v}_{2}^{U} & \cdots & \mathbf{v}_{K_{u}}^{U}\end{array}\right]$ and $\mathbf{F}^{\prime}=\left[\begin{array}{llll}\mathbf{f}_{1} & \mathbf{f}_{2} & \cdots & \mathbf{f}_{K_{u}}\end{array}\right]$. Here, we give the definition of the Kruskal-rank and use it to complete the proof.

Definition 1: The Kruskal rank or $k$-rank of a matrix $\mathbf{A}$, denoted by $r_{k}^{\prime}(A)$, is the maximal number $r$ such that any set of $r$ columns of $A$ is linearly independent [50].
With the fact that $\mathbf{f}_{1}, \mathbf{f}_{2}, \cdots, \mathbf{f}_{K_{u}}$ are drawn from i.i.d. circularly-symmetric complex Gaussian random variables with zero mean and unit variance, the column vectors of the matrix $\mathbf{F}^{\prime}$ are linearly independent and the Kruskal-rank of $\mathbf{F}^{\prime}$ is $K_{u}$.

In the proposed IN scheme, for any UL user $i \in\left[1: K_{u}\right]$, its rotation phase vector $\mathbf{v}_{i}^{U}$ over $T$ time slots is written by

$$
\mathbf{v}_{i}^{U}=\left[\begin{array}{lll}
e^{j \frac{2 \pi}{T+i}} & e^{j \frac{4 \pi}{T+i}} & \cdots e^{j \frac{2 T \pi}{T+i}} \tag{55}
\end{array}\right]^{T}
$$

Apparently, $\mathbf{v}_{1}^{U}, \mathbf{v}_{2}^{U}, \cdots, \mathbf{v}_{K_{u}}^{U}$ are linearly independent and the Kruskal-rank of $\mathbf{V}^{\prime}$ is also $K_{u}$.

According to Lemma 3.1 in [50], $T M \geqslant K_{u}$ and $r_{k}^{\prime}\left(\mathbf{V}^{\prime}\right)=$ $r_{k}^{\prime}\left(\mathbf{F}^{\prime}\right)=K_{u}$, we have

$$
\begin{equation*}
r_{k}^{\prime}\left(\mathbf{V}^{\prime} \odot \mathbf{F}^{\prime}\right)=\min \left\{r_{k}^{\prime}\left(\mathbf{V}^{\prime}\right)+r_{k}^{\prime}\left(\mathbf{F}^{\prime}\right)-1, K_{u}\right\}=K_{u} \tag{56}
\end{equation*}
$$

which means that any set of $K_{u}$ columns of $\underline{\mathbf{F}}_{\text {rotated }}$ are linearly independent. Since the row number is not smaller than the column number, $\underline{\mathbf{F}}_{\text {rotated }}$ has full column rank.

## APPENDIX B

## PROOF OF PROPOSITION 2

As we state in Section III, (40b), (40c), (40d) and (40e) are the feasible conditions of the proposed IN scheme.

Next, we prove the condition (40a). According to the process of the proposed IN scheme, we substitute (24) into (23) to get

$$
\begin{equation*}
\mathbf{U}_{j}^{e q} \widetilde{\mathbf{A}}_{j}(n)=\widetilde{\mathbf{B}}_{j}(n) \tag{57}
\end{equation*}
$$

where
$\widetilde{\mathbf{A}}_{j}(n)=\left[\begin{array}{c}\widetilde{\mathbf{h}}_{j}^{T}(1) \\ \cdots \\ \mathbf{g}_{j}^{T} \mathbf{V}^{A F}(T-n) \widetilde{\mathbf{F}}(T-n-1)+\widetilde{\mathbf{h}}_{j}^{T}(T-n)\end{array}\right]$,
$\widetilde{\mathbf{B}}_{j}(n)=\left[\begin{array}{c}\mathbf{g}_{j}^{T} \mathbf{V}^{A F}(T-n+1) \widetilde{\mathbf{F}}(T-n)+\widetilde{\mathbf{h}}_{j}^{T}(T-n+1) \\ \cdots \\ \mathbf{g}_{j}^{T} \mathbf{V}^{A F}(T) \widetilde{\mathbf{F}}(T-1)+\widetilde{\mathbf{h}}_{j}^{T}(T)\end{array}\right]$.
Based on the DoF of the UL, we set $n$ to be $(T-1)$ to obtain the maximum DoF. (57) can be simplified to

$$
\mathbf{U}_{j}^{e q} \widetilde{\mathbf{h}}_{j}^{T}(1)=\left[\begin{array}{c}
\mathbf{g}_{j}^{T} \mathbf{V}^{A F}(2) \widetilde{\mathbf{F}}(1)+\widetilde{\mathbf{h}}_{j}^{T}(2)  \tag{58}\\
\cdots \\
\mathbf{g}_{j}^{T} \mathbf{V}^{A F}(T) \widetilde{\mathbf{F}}(T-1)+\widetilde{\mathbf{h}}_{j}^{T}(T)
\end{array}\right]
$$

By utilizing the property of Kronecker product [51]

$$
\begin{equation*}
\mathbf{A X B}=\mathbf{C} \Leftrightarrow\left(\mathbf{B}^{T} \otimes \mathbf{A}\right) \operatorname{vec}(\mathbf{X})=\operatorname{vec}(\mathbf{C}) \tag{59}
\end{equation*}
$$

(58) can be rewritten by

$$
\left[\begin{array}{ll}
\mathbf{A}_{j} & \mathbf{B}_{j}
\end{array}\right]\left[\begin{array}{c}
\mathbf{c}_{j}  \tag{60}\\
\mathbf{d}
\end{array}\right]=\mathbf{e}_{j}
$$

where

$$
\begin{aligned}
\mathbf{A}_{j} & =\widetilde{\mathbf{h}}_{j}(1) \otimes \mathbf{I}_{T-1} \in \mathbb{C}^{(T-1) K_{u} \times(T-1)}, \\
\mathbf{c}_{j} & =\operatorname{vec}\left(\mathbf{U}_{j}^{e q}\right) \in \mathbb{C}^{(T-1) \times 1}, \\
\mathbf{d} & =\left[\begin{array}{c}
\operatorname{vec}\left(\mathbf{V}^{A F}(2)\right) \\
\cdots \\
\operatorname{vec}\left(\mathbf{V}^{A F}(T)\right)
\end{array}\right] \in \mathbb{C}^{(T-1) M^{2} \times 1}, \\
\mathbf{e}_{j}= & {\left[\begin{array}{c}
\operatorname{vec}\left(\widetilde{\mathbf{h}}_{j}^{T}(2)\right) \\
\cdots \\
\operatorname{vec}\left(\widetilde{\mathbf{h}}_{j}^{T}(T)\right)
\end{array}\right] \in \mathbb{C}^{(T-1) K_{u} \times 1}, } \\
\mathbf{B}_{j}= & {\left[\begin{array}{c}
-(\widetilde{\mathbf{F}}(1))^{T} \otimes \mathbf{g}_{j}^{T} \\
\\
\end{array} \quad \begin{array}{l}
\mathbf{B}_{j} \in \mathbb{C}^{(T-1) K_{u} \times(T-1) M^{2}} .
\end{array}\right.}
\end{aligned}
$$

Considering the $K_{d}$ DL users, we can obtain

$$
\left[\begin{array}{ll}
\mathbf{A}_{a d d} & \left.\mathbf{B}_{a d d}\right] \mathbf{x}=\mathbf{E}, \tag{61}
\end{array}\right.
$$

where

$$
\begin{aligned}
\mathbf{A}_{\mathrm{add}} & =\operatorname{diag}\left(\mathbf{A}_{1}, \cdots \mathbf{A}_{K_{d}}\right) \in \mathbb{C}^{(T-1) K_{d} K_{u} \times(T-1) K_{d}}, \\
\mathbf{B}_{\mathrm{add}} & =\left(\mathbf{B}_{1}^{T}, \cdots, \mathbf{B}_{K_{d}}^{T}\right)^{T} \in \mathbb{C}^{(T-1) K_{d} K_{u} \times(T-1) M^{2}}, \\
\mathbf{x} & =\left(\mathbf{c}_{1}^{T}, \cdots, \mathbf{c}_{K_{d}}^{T}, \mathbf{d}^{T}\right)^{T} \in \mathbb{C}^{(T-1) K_{d} K_{u} \times 1} \\
\mathbf{E} & =\left(\mathbf{e}_{1}^{T}, \cdots, \mathbf{e}_{K_{d}}^{T}\right)^{T} \in \mathbb{C}^{(T-1) K_{d} K_{u} \times 1}
\end{aligned}
$$

The linear system of (61) has solutions if and only if
$\operatorname{rank}\left(\left[\begin{array}{ll}\mathbf{A}_{\text {add }} & \mathbf{B}_{\text {add }}\end{array}\right]\right)=\operatorname{rank}\left(\left[\begin{array}{lll}\mathbf{A}_{\text {add }} & \mathbf{B}_{\text {add }} & \mathbf{E}\end{array}\right]\right) \leqslant m$,
where we denote $m$ as the number of the unknown variables and $m=(T-1) K_{d}+(T-1) M^{2}$.

Since $\left[\mathbf{A}_{a d d} \mathbf{B}_{a d d}\right]$ is a full rank matrix almost surely and its rank cannot be greater than its number of rows $m,\left[\mathbf{A}_{\text {add }} \mathbf{B}_{\text {add }}\right]$ is a column full rank matrix. Therefore, $\operatorname{rank}\left(\left[\mathbf{A}_{\text {add }} \mathbf{B}_{\text {add }}\right]\right)=\operatorname{rank}\left(\left[\mathbf{A}_{\text {add }} \mathbf{B}_{\text {add }} \mathbf{E}\right]\right)$. The IUI from the UL users to the DL users can be eliminated if $(T-1) K_{d} K_{u} \leqslant(T-1) K_{d}+(T-1) M^{2}$ which can be reduced to the first condition, i.e.,

$$
\begin{equation*}
K_{u} K_{d} \leqslant K_{d}+M^{2} \tag{63}
\end{equation*}
$$

The proof is completed.

## APPENDIX C

## PROOF OF INEQUALITY (46)

For $T \geqslant 2$ and $M \geqslant 2$, we have

$$
\begin{equation*}
\frac{1}{2}(T-1)(T-2) \geqslant 0 \tag{64}
\end{equation*}
$$

Multiply both sides by $M$ and expand it to

$$
\begin{equation*}
\frac{1}{2} T^{2} M-\frac{1}{2} T M-(T-1) M \geqslant 0 \tag{65}
\end{equation*}
$$

Rearrange (65) to

$$
\begin{equation*}
\frac{M(T-1)}{T^{2} M-T} \stackrel{(a)}{\leqslant} \frac{M(T-1)}{T^{2} M-T M} \leqslant \frac{1}{2} \tag{66}
\end{equation*}
$$

where (a) holds from $M, T \geqslant 0$. Adding 1 to both the RHS and the LHS, we have

$$
\begin{equation*}
\frac{M(T-1)}{T^{2} M-T}+1 \leqslant \frac{3}{2} . \tag{67}
\end{equation*}
$$

## APPENDIX D

## PROOF OF INEQUALITY OF (37)

In this section, we prove that inequality of (37), i.e., $\|\mathbf{R}\| \neq 0$. (34) can be rewritten by

$$
\left\{\begin{array}{l}
U_{1}^{e q}=\left(\widetilde{h}_{1}(2)+g_{1} V^{A F}(1)\right) / \widetilde{h}_{1}(1),  \tag{68}\\
\left.U_{2}^{e q}=\widetilde{h}_{2}(2)+g_{2} V^{A F}(1)\right) / \widetilde{h}_{2}(1) .
\end{array}\right.
$$

Since all the channel coefficients are drawn independently from a continuous distribution, $U_{1}^{e q}$ and $U_{2}^{e q}$ are of inequality almost surely [12]. Then, we have

$$
\begin{equation*}
\|\mathbf{R}\|=\left(U_{2}^{e q}-U_{1}^{e q}\right) g_{1} g_{2} \neq 0 \tag{69}
\end{equation*}
$$

The proof is completed.

## REFERENCES

[1] J. G. Andrews et al., "What will 5G be?" IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1065-1082, Jun. 2014.
[2] S. Hong et al., "Applications of self-interference cancellation in 5G and beyond," IEEE Commun. Mag., vol. 52, no. 2, pp. 114-121, Feb. 2014.
[3] A. Sabharwal, P. Schniter, D. Guo, D. W. Bliss, S. Rangarajan, and R. Wichman, "In-band full-duplex wireless: Challenges and opportunities," IEEE J. Sel. Areas Commun., vol. 32, no. 9, pp. 1637-1652, Sep. 2014.
[4] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in Proc. Conf. Rec. 40th Asilomar Conf. Signals Syst. Comput. (ASILOMAR), Nov. 2010, pp. 1558-1562.
[5] D. Nguyen, L. N. Tran, P. Pirinen, and M. Latva-Aho, "Transmission strategies for full duplex multiuser MIMO systems," in Proc. IEEE Int. Conf. Commun. (ICC), Jun. 2012, pp. 6825-6829.
[6] D. W. Bliss, P. A. Parker, and A. R. Margetts, "Simultaneous transmission and reception for improved wireless network performance," in Proc. IEEE/SP 14th Workshop Statist. Signal Process., Aug. 2007, pp. 478-482.
[7] A. K. Khandani, "Two-way (true full-duplex) wireless," in Proc. 13th Can. Workshop Inf. Theory (CWIT), Toronto, ON, Canada, Jun. 2013, pp. 33-38.
[8] S. Li and R. D. Murch, "An investigation into baseband techniques for single-channel full-duplex wireless communication systems," IEEE Trans. Wireless Commun., vol. 13, no. 9, pp. 4794-4806, Sep. 2014.
[9] B. Debaillie et al., "Analog/RF solutions enabling compact full-duplex radios," IEEE J. Sel. Areas Commun., vol. 32, no. 9, pp. 1662-1673, Sep. 2014.
[10] M. E. Knox, "Single antenna full duplex communications using a common carrier," in Proc. 13th Annu. Int. Conf. Wireless Microw. Technol. Conf. (WAMICON), Apr. 2012, pp. 1-6.
[11] M. Duarte et al., "Design and characterization of a full-duplex multiantenna system for WiFi networks," IEEE Trans. Veh. Technol., vol. 63, no. 3, pp. 1160-1177, Mar. 2014.
[12] V. R. Cadambe and S. A. Jafar, "Interference alignment and spatial degrees of freedom for the K user interference channel," in Proc. IEEE Int. Conf. Comтии., May 2008, pp. 971-975.
[13] V. R. Cadambe and S. A. Jafar, "Interference alignment and the degrees of freedom of wireless $X$ networks," IEEE Trans. Inf. Theory, vol. 55, no. 9, pp. 3893-3908, Sep. 2009.
[14] D. Castanheira, A. Silva, and A. Gameiro, "Retrospective interference alignment for the $K$-User $M \times N$ MIMO interference channel," IEEE Trans. Wireless Commun., vol. 15, no. 12, pp. 8368-8379, Dec. 2016.
[15] A. Sahai, S. Diggavi, and A. Sabharwal, "On degrees-of-freedom of full-duplex uplink/downlink channel," in Proc. IEEE Inf. Theory Workshop (ITW), Seville, Spain, Sep. 2013, pp. 1-5.
[16] J. Bai, S. Diggavi, and A. Sabharwal, "On degrees-of-freedom of multi-user MIMO full-duplex network," in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Hong Kong, pp. 864-868, Jun. 2015
[17] S. W. Jeon and C. Suh, "Degrees of freedom of uplink-downlink multiantenna cellular networks," in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Jun. 2014, pp. 1593-1597.
[18] S. W. Jeon and C. Suh, "Degrees of freedom of uplink-downlink multiantenna cellular networks," Trans. Inf. Theory, vol. 62, no. 8, pp. 4589-4603, Aug. 2016.
[19] S. H. Chae, S. H. Lim, and S. W. Jeon, "Degrees of freedom of full-duplex multiantenna cellular networks," IEEE Trans. Wireless Commun., vol. 17, no. 2, pp. 982-995, Feb. 2018.
[20] W. Shin, J. B. Lim, H.-H. Choi, J. Lee, and H. V. Poor, "Cyclic interference alignment for full-duplex multi-antenna cellular networks," IEEE Trans. Commun., vol. 65, no. 6, pp. 2657-2671, Jun. 2017.
[21] S. A. Jafar, "Exploiting channel correlations-Simple interference alignment schemes with no CSIT," in Proc. IEEE Global Telecommun. Conf., Dec. 2010, pp. 1-5.
[22] T. Gou, C. Wang, and S. A. Jafar, "Aiming perfectly in the dark-blind interference alignment through staggered antenna switching," IEEE Trans. Signal Process., vol. 59, no. 6, pp. 2734-2744, Jun. 2011.
[23] M. Yang, S.-W. Jeon, and D. K. Kim, "Degrees of freedom of full-duplex cellular networks with reconfigurable antennas at base station," IEEE Trans. Wireless Commun., vol. 16, no. 4, pp. 2314-2326, Apr. 2017.
[24] C. G. Christodoulou, Y. Tawk, S. A. Lane, and S. R. Erwin, "Reconfigurable antennas for wireless and space applications," Proc. IEEE, vol. 100, no. 7, pp. 2250-2261, Jul. 2012.
[25] W. Lin and H. Wong, "Polarization reconfigurable aperture-fed patch antenna and array," IEEE Access, vol. 4, pp. 1510-1517, 2016.
[26] Z. Liu, Y. Liu, and F. Liu, "Joint resource scheduling for full-duplex cellular system," in Proc. 22nd Int. Conf. Telecommun. (ICT), Apr. 2015, pp. 85-90.
[27] R. Sultan, L. Song, K. G. Seddik, Y. Li, and Z. Han, "Mode selection, user pairing, subcarrier allocation and power control in full-duplex OFDMA HetNets," in Proc. IEEE Int. Conf. Commun. Workshop (ICCW), Jun. 2015, pp. 210-215.
[28] B. Di et al., "Joint user pairing, subchannel, and power allocation in fullduplex multi-user OFDMA networks," IEEE Trans. Wireless Commun., vol. 15, no. 12, pp. 8260-8272, Dec. 2016.
[29] G. C. Alexandropoulos, M. Kountouris, and I. Atzeni, "User scheduling and optimal power allocation for full-duplex cellular networks," in Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC), Jul. 2016, pp. 1-6.
[30] D. Nguyen, L.-N. Tran, P. Pirinen, and M. Latva-Aho, "On the spectral efficiency of full-duplex small cell wireless systems," IEEE Trans. Wireless Commun., vol. 13, no. 9, pp. 4896-4910, Sep. 2014.
[31] H. H. Choi, "On the design of user pairing algorithms in full duplexing wireless cellular networks," in Proc. Int. Conf. Inf. Commun. Technol. Converg. (ICTC), Oct. 2014, pp. 490-495.
[32] V.-D. Nguyen, H. V. Nguyen, C. T. Nguyen, and O.-S. Shin, "Spectral efficiency of full-duplex multi-user system: Beamforming design, user grouping, and time allocation," IEEE Access, vol. 5, pp. 5785-5797, Mar. 2017.
[33] S. Goyal, P. Liu, and S. S. Panwar, "User selection and power allocation in full-duplex multicell networks," IEEE Trans. Veh. Technol., vol. 66, no. 3, pp. 2408-2422, Mar. 2017.
[34] S. Mohajer, S. N. Diggavi, C. Fragouli, and D. Tse, "Transmission techniques for relay-interference networks," in Proc. 46th Annu. Allerton Conf. Commun., Control, Comput., Sep. 2008, pp. 467-474.
[35] S. Mohajer, S. N. Diggavi, C. Fragouli, and D. N. C. Tse, "Approximate capacity of a class of Gaussian interference-relay networks," IEEE Trans. Inf. Theory, vol. 57, no. 5, pp. 2837-2864, May 2011.
[36] S.-W. Jeon and M. Gastpar, "A survey on interference networks: Interference alignment and neutralization," Entropy, vol. 14, no. 10, pp. 1842-1863, Sep. 2012.
[37] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," IEEE J. Sel. Areas Commun., vol. 25, no. 2, pp. 379-389, Feb. 2007.
[38] T. Gou, S. A. Jafar, C. Wang, S.-W. Jeon, and S.-Y. Chung, "Aligned interference neutralization and the degrees of freedom of the $2 \times 2 \times 2$ interference channel," IEEE Trans. Inf. Theory, vol. 58, no. 7, pp. 4381-4395, Jul. 2012.
[39] H. Maier and R. Mathar, "Cyclic interference neutralization on the $2 \times$ $2 \times 2$ full-duplex two-way relay-interference channel," in Proc. IEEE Inf. Theory Workshop (ITW), Seville, Spain, Sep. 2013, pp. 1-5.
[40] H. Maier and R. Mathar, "Cyclic interference neutralization on the fullduplex relay-interference channel," in Proc. Proc. IEEE Int. Symp. Inf. Theory, Jul. 2013, pp. 2309-2313.
[41] T. Gou, C. Wang, and S. A. Jafar, "Toward full-duplex multihop multiflow-A study of non-layered two unicast wireless networks," IEEE J. Sel. Areas Commun., vol. 32, no. 9, pp. 1738-1751, Sep. 2014.
[42] B. Schein and R. G. Gallager, "The Gaussian parallel relay network," in Proc. IEEE Int. Symp. Inf. Theory, Sorrento, Italy, Jun. 2000, p. 22.
[43] H. Bólcksei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," IEEE Trans. Wireless Commun., vol. 5, no. 6, pp. 1433-1444, Jun. 2006.
[44] F. Eisenbrand, "Algorithms for integer programming," EATCS Bulletin, Tech. Rep. 96, 2008, pp. 46-57.
[45] J. A. Lawrence and B. A. Pasternack, Applied Management Science: Modeling, Spreadsheet Analysis, and Communication for Decision Making. Hoboken, NJ, USA: Wiley, 2002.
[46] M. Razaviyayn, M. Sanjabi, and Z.-Q. Luo, "Linear transceiver design for interference alignment: Complexity and computation," IEEE Trans. Inf. Theory, vol. 58, no. 5, pp. 2896-2910, May 2012.
[47] G. M. Rebeiz, "RF MEMS switches: Status of the technology," in 12th Int. Conf. Solid-State Sensors, Actuat. Microsystems. Dig. Tech. Papers, vol. 2, Jun. 2003, pp. 1726-1729.
[48] J. T. Aberle, S.-H. Oh, D. T. Auckland, and S. D. Rogers, "Reconfigurable antennas for wireless devices," IEEE Antennas Propag. Mag., vol. 45, no. 6, pp. 148-154, Dec. 2003.
[49] H. Su, I. Shoaib, X. Chen, and T. Kreouzis, "Optically tuned polarisation reconfigurable antenna," in Proc. IEEE Asia-Pacific Conf. Antennas Propag., Aug. 2012, pp. 265-266.
[50] L. D. Lathauwer, "Decompositions of a higher-order tensor in block termsİpart i: Lemmas for partitioned matrices," SIAM J. Matrix Anal. Appl., vol. 30, no. 3, pp. 1022-1032, 2008.
[51] C. R. Johnson, Topics Matrix Analysis. Cambridge, U.K.: Cambridge Univ. Press, 1991.


WEIMIN DU received the B.S. degree from the Shandong University of Science and Technology, Qingdao, China, in 2013, and the M.S. degree from Xidian University, Xi'an, China, in 2016, where he is currently pursuing the Ph.D. degree. From 2016 to 2017, he was a Hardware Engineer with Qingdao Topscomm Communication Co., Ltd. His research interests mainly include interference management and large scale MIMO.


ZUJUN LIU received the B.S., M.S., and Ph.D. degrees from Xi'an Jiaotong University, Xi'an, China, in 1998, 2001, and 2006, respectively. From 2001 to 2002, he was a Hardware Engineer with ZTE Ltd. He joined Xidian University, China, in 2006, as a Lecturer, where he is currently a Professor with the State Key Laboratory of Integrated Services Networks. His current research interests are interference management for wireless communications, radio resource allocation, and the Internet of Things.


FAN LI received the B.S. and M.S. degrees from Xidian University, Xi' an, China, in 2014 and 2017, respectively. She is currently an Algorithm Engineer with the Xi'an Branch, Qingdao Topscomm Communication Co., Ltd. Her research interests include interference management and relay communications during her Master study.


[^0]:    The associate editor coordinating the review of this manuscript and approving it for publication was Ebrahim Bedeer.

[^1]:    ${ }^{1}$ Whereas when $T M<K_{u}$, we may shut down $\left(K_{u}-T M\right)$ UL users which result in $T M=K_{u}$, the DoF of the UL is still maximum. In other words, $T M \geqslant K_{u}$ is always satisfied in our proposed scheme.

