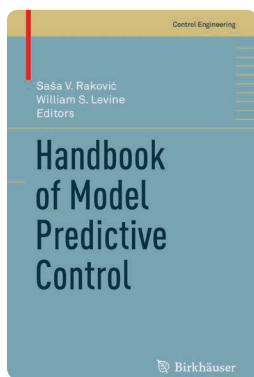


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## HANDBOOK OF MODEL PREDICTIVE CONTROL

by SAŠA V. RAKOVIĆ and WILLIAM S. LEVINE (Editors)

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**M**odel predictive control (MPC), also called receding horizon control, refers to a wide class of optimization-based control methods and algorithms based on a very simple and powerful idea. A dynamical model of the

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system to be controlled is first used to make a finite-horizon prediction of the state evolution as a function of the input sequence. Next, the optimal input sequence is computed by maximizing some performance measure satisfying given state/input constraints. From the optimal solution, only the first action of the optimal control sequence is actually implemented. At the next decision time, the whole procedure is repeated by considering the new current state of the system. MPC algorithms originated in the 1970s to address the increasing economic optimization needs and challenges of the refinery industries [1], [2]. Although initially ignored by academic researchers (who at that time were mainly focused on multivariable techniques and adaptive control), from the early 1990s to present, MPC witnessed a tremendous explosion of research contributions [3], [4] and industrial applications [5].

MPC algorithms were initially implemented using discrete-time linear convolution models identified from data, until later research results and computational software were specifically developed for MPC based on state-space models. These developments broadened the applicability of MPC to a plethora of applications where MPC was once

considered inapplicable due to its computational demands. Over the ensuing years, the theory of linear and nonlinear MPC grew steadily and reached a high level of maturity [6], [7], as demonstrated by its ubiquity in university research groups and advanced control courses. Furthermore, significant advances in stochastic modeling, numerical optimization, dynamical system analysis, and hardware implementations have created new opportunities for MPC. As a result, students and researchers in control now require access to an extensive and up-to-date reference on all aspects of MPC.

*Handbook of Model Predictive Control* contains 27 chapters, coherently organized into three parts. Part 1 is on theory and comprises 12 chapters, ranging from basic MPC theory to advanced studies and MPC formulations. Part 2, on computation, includes eight chapters and covers numerical implementation of MPC-related optimization algorithms. Part 3 discusses applications of MPC in numerous fields, such as automotive, power and energy systems, health care, and finance.

## CONTENTS

### Part 1: Theory

The first chapter introduces the basics of MPC, presenting the intuitive idea behind receding horizon control and tracing the history of MPC from linear quadratic Gaussian regulation theory. A general formulation of linear MPC is then presented, including a discussion of state and output constraints, closed-loop stability and robustness, reference tracking, and an application to an inverted pendulum problem.

The second chapter provides a theoretical analysis of infinite horizon closed-loop performance and the stability of closed-loop trajectories with MPC. Starting with general definitions of finite-horizon and infinite-horizon optimal control problems, bounds on the closed-loop performance are obtained by dynamic programming (DP) techniques. Next, a thorough analysis of the stability conditions for traditional tracking MPC formulations, with or without terminal conditions, is presented. Performance analysis of recent economic MPC formulations (further discussed in Chapter 7) is also covered.

The third chapter provides an analysis of set-valued and Lyapunov methods for MPC, which allow one to characterize how the set of solutions depends on its constitutive parameters and how these closed-loop solutions evolve. A characterization of the open-loop optimal control problem

and its solution is given, followed by discussions of asymptotic stability, recursive feasibility, and robustness analyses using Lyapunov arguments. Set-valued control systems, which arise due to discontinuous control actions and uncertainties, are also presented and analyzed.

The fourth chapter presents the foundations of stochastic MPC algorithms, in which disturbances are accounted for in terms of their statistical properties. A stochastic MPC with chance constraints is initially discussed, and the following three routes are presented to overcome the curse of dimensionality of stochastic DP: 1) a scenario tree-based MPC approach, in which extremal realizations of the uncertainty over the horizon are considered to generate a tree of open-loop evolutions, 2) a polynomial chaos-based expansion to propagate the uncertainty along the trajectory, and 3) an approach based on stochastic tube-based MPC, extending a technique initially developed for robust MPC.

The fifth chapter reviews moving horizon estimation (MHE) formulations, which compute an estimate of state and disturbance trajectories using a batch of output measurements by solving a constrained finite-horizon optimization problem. The chapter discusses recent robust asymptotic stability results and presents a chemical reaction example in which physically unrealistic state estimates are obtained using extended and unscented Kalman filters (whereas MHE provides consistent state estimates).

Chapter 6 further elaborates on stochastic MPC and addresses “dual control,” where the control signal is computed not only to solve a stochastic optimal control problem but also in a way to minimize the uncertainty in reconstructing the system state. The chapter contains three examples of dualized stochastic control, and it concludes with an overview of methods to improve the numerical tractability of dual stochastic MPC formulations.

The seventh chapter presents a detailed description and analysis of economic MPC, in which the cost function is arbitrary and the closed-loop behavior may or may not be asymptotically stabilizing around an equilibrium. Various formulations are presented for deterministic systems, including those with and without terminal ingredients and those with average constraints, where average performance is optimized. Finally, a tube-based economic MPC able to handle uncertainties is outlined.

Chapter 8 focuses on tracking approaches for MPC. This chapter starts with the standard problem of fixed setpoint stabilization and then discusses how trajectory tracking and path following can be achieved in the presence of external disturbances. These cases are exemplified by means of a robotic application, and the chapter ends with a sketch of a formulation that includes an economic objective term.

A number of hybrid MPC problems and formulations are discussed in the ninth chapter. The analysis covers different problems, including piecewise affine state dynamics, mixed (continuous and discrete) state dynamics, systems

**Set-valued control systems, which arise due to discontinuous control actions and uncertainties, are also presented and analyzed.**

with memory and logic variables, and various periodic hybrid systems. Particular attention is given to the type of underlining optimization problem to be solved, including mixed-integer, linear, and quadratic programming. This chapter ends with a general analysis of robustness issues arising in hybrid MPC due to possible discontinuities in the control law.

Chapter 10 discusses MPC theory and implementation for nonlinear systems described by polynomial discrete-time functions in which the objective function and constraints are polynomials. It then shows how suitable semidefinite relaxations, based on sum of squares, can be devised to generate an efficient online or offline explicit control solution. The chapter ends with a discussion on how nonpolynomial systems can be approximated by polynomial ones using Taylor’s expansion.

The 11th chapter discusses implementation strategies of MPC for large-scale systems by means of decentralized/distributed architectures. The chapter begins by describing methods for subdividing the overall system into overlapping or nonoverlapping subsystems. It then describes decentralized MPC (in which interactions among subsystems are disregarded) and distributed cooperative MPC (in which interactions are considered to achieve global optimization and stability).

The 12th chapter further elaborates on MPC large-scale scalable systems with a specific focus on the possibility of adding and removing subsystems, a problem highly motivated by the recent Industry 4.0 paradigm of cyberphysical systems [8]. A decentralized MPC framework is discussed, where stability guarantees are achieved by treating subsystem interactions as unknown disturbances and devising suitable tube-based robust MPC strategies. Conditions for plug-and-play addition and removal are analyzed, and the chapter ends with two examples of power systems and smart grids.

## **Part 2: Computations**

The first chapter of Part 2 reviews numerical methods for linear MPC problems, which require the solution of a convex quadratic programming (QP) problem at each decision, given the current system state. Optimality conditions and efficient interior-point strategies implemented via Riccati-like recursions are then discussed. Next, the parametric

nature of the QP is analyzed to discuss how the optimal set of active constraints evolve and how information obtained at previous decision times can be exploited within an active set QP solver. The chapter ends with a brief overview of available software.

The second chapter addresses numerical methods to solve general (nonconvex) MPC problems, which arise due to nonconvex cost functions, constraints, or nonlinear dynamics. Starting with discretized continuous-time dynamics, the problem is posed as a parametric nonlinear programming (NLP) problem, whose optimality conditions are analyzed. An overview of sequential quadratic programming and interior-point methods for NLP is then provided. Topics including discretization strategies, parametric aspects of the NLP, warm start and convergence, and the matrix structure of the problem are also presented.

The third chapter analyzes techniques for converting nonconvex optimization problems into convex approximations that facilitate real-time application. The chapter first addresses the case of linear dynamics with nonconvex constraints and recent iterative methods based on successive convexification. Convergence analysis and implementation aspects for onboard real-time application are then provided.

The fourth chapter focuses on linear MPC problems and analyzes parametric linear programming (LP) and parametric QP formulations. Starting from the optimality conditions, the piecewise affine structure of the solution is revealed and so-called explicit MPC algorithms are obtained. The case in which some integer variables occur, leading to parametric mixed-integer LP or QP problems, is then treated. The chapter ends with a discussion of computational complexity associated with both offline solution generation and online computation.

The fifth chapter describes the real-time implementation of explicit MPC by analyzing several complexity-reduction approaches. The main goal is to reduce the number of regions that defines the piecewise affine explicit MPC law to alleviate both memory storage and online evaluation time, with or without approximation of the optimal solution. After analyzing the linear MPC case, the chapter addresses the nonlinear MPC case, where a parametric QP problem is solved at each decision time with its parametric components updated to obtain stability guarantees.

The sixth chapter addresses numerical methods to solve robust MPC problems. Starting with a general case where the model dynamics are subject to unknown disturbances, a min-max robust MPC problem is derived, and the associated numerical challenges are identified. Approximated solution strategies are then discussed, starting with uncertain linear systems and extending to uncertain nonlinear systems. For the latter case, the chapter reviews robust DP methods and then covers more tractable scenario tree and tube-based approaches. The chapter ends with a detailed analysis of numerical methods for tube-based robust MPC and set-valued computing.

The seventh chapter discusses scenario optimization methods for stochastic MPC. Starting with uncertain linear systems, a probabilistic optimization problem is posed, and sufficient conditions for probability violation are derived. Constraint removal methods that reduce computational complexity and a numerical example to clarify the tradeoff between scenario removal and performance are then presented.

Chapter 8, which ends Part 2, presents a general and detailed discussion of numerical methods for nonlinear tracking and economic MPC. After defining the structure of the parametric NLP problem to be solved at each decision time and establishing its optimality conditions, the problem is relaxed by adding slack variables to state constraints, leading to a number of nice robust stability properties. Economic MPC approaches in which regularization terms are included to induce closed-loop stability or where explicit stabilizing constraints are added to enforce convergence to the equilibrium point are covered. The chapter ends with two process control case studies.

### **Part 3: Applications**

The first chapter of Part 3 describes different automotive applications of MPC. The chapter presents various dynamic models for powertrains and energy management and highlights specific challenges of each control problem. An overview of the current limitations and future perspectives of nonlinear MPC applications in the automotive field is also provided.

The second chapter presents a comprehensive overview of MPC applications in health care, where the problems are characterized by multivariable systems, nonlinearities, and constraints. Applications to ambulance scheduling, joint movement, automatic anesthesia, insulin-dependent diabetes, human immunodeficiency virus, cancer, and inflammation treatments are given. The chapter ends by clarifying the main obstacles and challenges that must be overcome to enable the widespread use of MPC in health care.

The third chapter describes the application of MPC in power electronics, where the problems are characterized by switched dynamics, multivariable systems, and short computation times. Two different MPC formulations are considered: one where the power converter input can be modulated through an inverter and one where a finite control set is available. For both problems, an explicit MPC approach is used to obtain the solution map offline. Stability, performance analysis, and efficient implementation of the finite control set MPC are also discussed.

The fourth chapter presents two applications of receding horizon control and estimation methods for the control of a ground robot and a tricopter unmanned aerial vehicle. After a brief overview of fast nonlinear MPC and MHE formulations, the chapter proceeds by describing the system dynamics and the output feedback controller implementation for each robot. The architecture includes bidirectional

data exchange over the network to improve performance through onboard parameter updates computed remotely by learning techniques.

The fifth chapter addresses the application of MPC to heating, ventilation, and air-conditioning (HVAC) systems, with the goals of taking advantage of electricity pricing and ambient temperature predictions. The chapter describes the main modeling, design, and implementation aspects of MPC for HVAC systems, including hierarchical and distributed system decomposition, load forecasting, and feedback.

The sixth chapter is focused on the application of MPC algorithms to electric power systems. The chapter begins by describing the complex spatial and temporal interdependencies that arise in power networks. A multilayer MPC structure is then proposed, and temporal and spatial lifting techniques are described to aggregate different agents and timescales. The chapter ends by presenting results of four examples.

The seventh chapter presents the application of MPC to two financial problems. The chapter begins by describing models of account value dynamics, which are inherently stochastic due to highly varying stock prices. Next, the problem of portfolio optimization for maximizing the account value at a future time is posed as an MPC problem to account for diversification and turnover constraints. Dynamic option hedging problems are also addressed, where the option holder is interested in trading the stocks over time so that the account matches the payoff value at the expiration time. Different MPC approaches for these problems are reviewed, and the chapter ends with an overview of future directions.

## AUDIENCE

This handbook is designed for a wide audience. It is an excellent reference for graduate students, researchers, and practitioners in the field of control systems and numerical optimization who want to understand the potential, challenges, and benefits of MPC and its applications. Alternately, it is an up-to-date reference for MPC research experts (both in academia and industry). For this audience, the book helps experts address new MPC-related problems and research directions.

## SUMMARY

This book provides a thorough and comprehensive reference of the underlying theory, implementation, and applications of MPC. The content of the book, contributed by various experts in the field, is well written and suitably organized into three parts. Furthermore, this book does an excellent job meeting several competing goals: clarity of communication to a diversified audience, formal rigor, and

a self-contained presentation of the topics in each chapter. This handbook enables the reader to gain a panoramic viewpoint of MPC theory and practice as well as provides a state-of-the-art overview of new and exciting areas of application at the forefront of MPC research.

## REVIEWER INFORMATION

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