Introduction to the Issue on Compressive Sensing

THE young field of compressive sensing presents a fundamentally new way of going about important tasks in signal processing. Instead of the usual approach of acquiring a finely sampled version of a signal, computing a full set of transform coefficients, and then discarding most of them, we can directly measure a compressed representation of the signal. What makes this possible is that most signals that arise in nature or from human activity are *sparse*; an accurate digital representation requires relatively few nonzero coefficients. Compressive sensing exploits this sparsity, by allowing a digital signal to be reconstructed from far fewer linear measurements than the size of the signal. As long as the measurements satisfy reasonable conditions, their number only needs to be commensurate with the *complexity* of the signal. Such measurements preserve the information content of the signal, while being much smaller than the signal, effectively constituting a compression.

The impact of this approach goes far beyond compression. Whenever acquiring data is difficult, dangerous, or expensive, we can make do with much less data than previously thought possible. For example, in medical imaging, the high radiation dose of an X-ray CT scan can be replaced by relatively few planar radiographs. Better yet, the much safer MRI can often be substituted instead, with the barrier of greater cost being overcome by the ability to use a much shorter scanning protocol. This is just one of a panoply of applications with the potential to revolutionize the way we acquire and process information.

The task of recovering a sparse signal from relatively few measurements becomes the problem of finding the sparsest solution of a severely underdetermined linear system of equations. This problem is NP-hard (in the worst case), yet it can be solved by efficient algorithms in many, useful circumstances. This discovery, just five years old, has led to an explosion of research worldwide in related areas of applied mathematics, engineering, and computer science.

It is not hard to understand the appeal of this field. There are many interesting research avenues, which can be roughly grouped into four topics, all well represented in this issue.

Applications: Linear systems are everywhere, and the ability to solve underdetermined systems means that signals (including images, network states, or any other discretely representable quantities) can be recovered from much less data. The potential for enticing applications is enormous, throughout signal processing and beyond. This issue features papers that consider applications to synthetic aperture radar, multicarrier/OFDM systems, speech recognition, and MRI. The algorithms and methods developed in other papers also have the potential to one day find themselves implemented in hardware.

Algorithms: The original, linear programming approach to solving the sparse recovery problem was remarkable, in that it demonstrated that an NP-hard problem could be solved tractably under reasonable conditions. However, the demands of many real-world applications mean that more efficient algorithms are necessary. Other considerations include the ability to obtain reconstructions from even less data, and better tolerance to noise and signal nonsparsity. Optimization, analysis, and extensive computational testing are used in this issue to develop novel algorithms and numerical implementations that are faster, more robust, and more suitable for large problems.

Theory: Ideally, the empirical performance of algorithms will be accompanied by rigorous guarantees, for only then can one have complete confidence that an algorithm will be successful for a given application. This often leads to interesting and challenging mathematical questions, in areas such as harmonic analysis, high-dimensional geometry, linear algebra, convex analysis, optimization, probability, and random matrix theory. Papers in this issue consider convergence analysis, conditions under which successful reconstruction can be guaranteed, noise robustness bounds, and the construction of measurement systems for which reconstruction is highly probable.

New Models: The original compressive sensing framework reconstructed a sparse signal by solving an optimization problem, minimizing a sparsity-inducing penalty term subject to a data constraint. Subsequent work relaxed this constraint to allow inexact measurements, or replaced the optimization approach with iterative greedy algorithms. Since then, the compressive sensing framework has expanded far beyond the original context. Examples in this issue include modifications for dealing with wideband analog signals, measurements or signals with impulse noise, an adaptive filter approach to reconstruction, reconstruction methods that adapt as new measurements are obtained, and processing compressive measurements without reconstructing at all.

Of course, many papers fall into multiple categories, such as papers concerning algorithm development that also consider theoretical issues and the algorithm's potential applications. Compressive sensing is fundamentally interdisciplinary, with the interplay between applied mathematics, pure mathematics, and engineering serving to fertilize new research. The frontiers of compressive sensing continue to expand, as shown by the papers in this issue.

We dedicate this special issue to the memory of Dr. Dennis Healy, professor of mathematics at the University of Maryland and program manager at the Defense Advanced Research Projects Agency, who passed away on 3 September 2009 after a brief and courageous fight with cancer. In large part, Dennis was personally responsible for the genesis of the compressive sensing community that has produced this special issue. From his pioneering research on sparse wavelet coding for MRI imaging and wavelet filtering/thresholding in the 1990s to his leadership of the Integrated Sensing and Processing, Montage, and Analog-to-Information programs at DARPA in the 2000s, he personally and with great energy fostered the development of new sparsity-based signal and image processing theory and algorithms and new signal acquisition devices that optimize the

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sampling/compression tradeoff. We will miss Dennis greatly for his broad vision, keen insights, and strong leadership.

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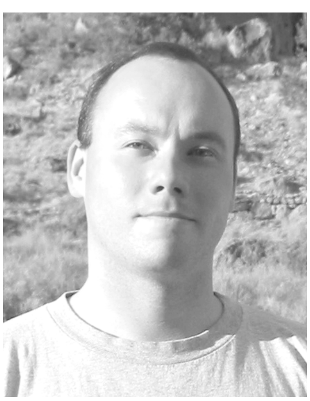

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