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Fractional sliding mode control design for robust synchronization and anti-synchronization of fractional order nonlinear chaotic systems in finite time

Arash Pourhashemi†, Amin Ramezani *, and Mehdi Siahi**

ABSTRACT

This paper deals with the problem of synchronization (anti-synchronization) of fractional nonlinear systems. Here, due to the advantages of fractional calculus and sliding mode control, we provide a new fractional order sliding mode control for synchronization (anti-synchronization) problems. So, in this paper a novel sliding surface is introduced and with and without the existence of uncertainties and external disturbances, finite-time synchronization is achieved by designing a new fractional sliding mode control. This method applied to the class of fractional order nonlinear systems and sufficient conditions for achieving synchronization/anti-synchronization are derived by the use of fractional Lyapunov theory. To show the effectiveness and robustness of the proposal, we applied our method on two identical fractional order financial system to verify the efficacy.

Article Info

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fractional-order nonlinear system, chaos synchronization; anti-synchronization, Lyapunov Stability, uncertainty, disturbance.

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I. INTRODUCTION

Chaotic system is a nonlinear deterministic system with complex and unpredictable behavior. Chaotic behavior as is known to all, is a prevalent phenomenon which can be appeared in nonlinear systems. It has been also seen in a variety of real system in laboratory such as electrical circuits, chemical reactions and fluid dynamics and so forth [1]. Based on chaos theory, the prominent features of chaotic systems are that the highly sensitivity to initial conditions. Chaos synchronization is a phenomenon that may happen when two, or more, dissipative chaotic systems are coupled. Moreover, Synchronization control is one of the important research area in chaos theory and it is simply means that things occur at the same time. The main problem related to the synchronization of two chaotic systems is that the time which complete synchronization will happen is not specified [2].The original synchronization technique investigated and developed by Pecorra and Carroll [3]. Synchronization is a contemporary topic in nonlinear science because of the broad applications in various fields such as automatic control [4], secure communication [5] and signal processing [6]. Therefore, especially during the past decades, synchronization of chaotic systems have been attracted attentions of many researchers in various field of sciences. Different kinds of control methodologies have been applied for synchronization of chaotic systems like adaptive control [7], linear and nonlinear active control [8], back stepping control [9] and sliding mode control [10]. One other interesting phenomena in scholars' view is anti-synchronization of chaotic systems which is noticeable in periodic oscillators.

Fractional calculus is a mathematical tool and has a long history when Leibniz wrote a letter to L'Hôpital, raising the possibility of generalizing the meaning of derivatives from integer order to fractional order (FO) derivatives. But, from then, its applications to physics and engineering have attracted much more attention just in recent years [11]. It has

* Corresponding Author: ramezani@modares.ac.ir
* Department of Electrical Engineering, Central Tehran Branch, Islamic Azad University Tehran, Iran (Department of Electrical and Computer Engineering, Tarbiat Modares University)
†** Department of Electrical Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran
arash.pourhashemi@mshdiau.ac.ir, ramezani@modares.ac.ir, mahdi_siahi@yahoo.com

been recently found that many real phenomena can be more accurately model with FO systems comparing to integer order systems. It is obvious that the advantages of FO system is the number of degree of freedom in modelling of nonlinear phenomena as well as memory which included in. In recent years, many complex systems can be described using fractional integrals and derivatives such as FO Chen system [12], FO Duffing-Holmes [13] , FO Chua system [14], Arneodo systems[15] and so on. Thereafter, different kinds of FO control methodologies are studied like FO PID controller[16], and FO optimal controllers [17].

Among different kinds of control methodologies for synchronization and anti-synchronization which mentioned above sliding mode control owing to its benefits such as fast response, robustness, and low sensitivity to external noise as well as easy realization are considered.

Synchronization problem of FO chaotic systems was first reported by Deng and Li [18]. Finite time synchronization of two different chaotic nonlinear integer order with unknown parameters are investigated in [19]. The issues of Synchronization of a coupled Hodgkin–Huxley neurons were investigated via high order sliding-mode feedback[20]. A novel FO terminal sliding mode control is designed for control and synchronization of FO non autonomous chaotic and hyper chaotic systems in finite time [21]. A modified sliding mode synchronization for a typical three-dimensional FO chaotic systems as well as a modified projective synchronization of FO chaotic systems based on active sliding mode control are considered in[22]. Also, lots of researches have been done for anti-synchronization of FO chaotic systems. For instance, Synchronization and anti-synchronization of two identical chaotic system is investigated[3]. In[23] anti-synchronization of FO chaotic and hyper chaotic systems with unknown parameters by the method of modified adaptive control is investigated.

In this paper, owing to the above advantages of fractional calculus and sliding mode control and combinations of them which are applied in different case studies and some of are mentioned in the literature, we suggest and introduce a novel fractional sliding mode controller for a class of nonlinear systems. In other words, we first propose a novel sliding surface and then design a new fractional order controller for synchronization and anti-synchronization.

The main contributions of this paper can be presented in brief as follows: (1) Synchronization and anti-synchronization of a class of nonlinear fractional order system is discussed in terms of FO sliding mode strategy with a really simple methodology. (2) It is proved that, error dynamics are converged to zero in finite time as well as sliding motion occurs in finite time. That is to say, to show the effectiveness of the method and finite time stability we theoretically investigate stability and reaching time and add the method on different examples for confirmation. (3) Fractional order sign function in our controller design when s(t) is large is able to push the state to converge to the switching manifold faster. (4) The fractional sliding mode methodology is really effective and simple for fast synchronization and anti-synchronization.

Therefore, this paper is organized as follows: A brief review of fractional calculus are presented in section 2. System descriptions are investigated in section 3. Synchronization (anti-synchronization) problem are considered as well as the controller design scheme and the stability analysis of the closed-loop system are included in section 4. In Section 5, simulations results are shown. The conclusions are drawn in Section 6.

II. Some Notations and Definitions

There are different types of definitions for fractional derivatives which among them three of are more commonly used in researcher's work which are called Riemann–Liouville, Grunwald–Letnikov, and Caputo definitions. The initial conditions for Caputo fractional differential operator is the same as integer order one, so, the Caputo fractional order derivative is selected in our research. We also give some definitions which are used in our analyses as well as for completion.

Definition 1 [24]**.** Let m - 1 < α < m, $m \in N$ the Riemann-Liouville fractional derivative of order α of any function $f(t)$ is defined as follows:

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)d\tau}{(t-\tau)^{\alpha+1-n}} \tag{1}$$

Where n is the first integer and is greater than α and Γ is the Gamma function.

$$\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} dt. \tag{2}$$

Definition 2 [24]**.** Let $f \in C_{-1}^{m}, m \in N$ Then (left sided) Caputo fractional differential equation of $f(x)$ is defined by:

$$D_t^{\alpha} f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} f^{m}(\tau) d\tau & m-1 \prec \alpha \prec m \\ \frac{d^m u(x,t)}{dt^m} & \alpha = m \in N \end{cases} \tag{3}$$

Where, m, similar to n is the smallest integer number, larger than α.

Remark [25]**:** When $\alpha \in (0,1)$, the Caputo system ${}_{t_0}^{c}D_t^{\alpha} = f(x,t)$ has the same equilibrium points as the integer order system $\dot{x}(t) = f(x,t)$.

III. Problem Statement

Considering a class of fractional-order Master-Slave systems with uncertainties and external disturbances as (4) and (5) respectively:

$$\begin{cases} D^{\alpha}x_1 = F_1(x) + \Delta f_1(x,t) + d_1^m(t) \\ D^{\alpha}x_2 = F_2(x) + \Delta f_2(x) + d_2^m(t) \\ \quad \vdots \\ D^{\alpha}x_n = F_n(x) + \Delta f_n(x) + d_n^m(t) \end{cases} \tag{4}$$

Where $\alpha \in (0,1)$ is the order of the sys $X(t) = [x1, x2, \ldots, xn]^T = [x, x^{(\alpha)}, \ldots, x^{((n-1))\alpha}]^T \in R^n$ is the state vector, $F(X,t) \in R$ is a given nonlinear function of X and t, $\Delta f_i(x,t)$ and $d_i^m(t)$ represents an unknown uncertainty and external disturbance respectively and

$$\begin{cases} D^{\alpha}y_1 = G_1(x) + \Delta g_1(x,t) + d_1^s(t) + u_1(x,y,t) \\ D^{\alpha}y_2 = G_2(x) + \Delta g_2(x) + d_2^s(t) + u_2(x,y,t) \\ \quad \vdots \\ D^{\alpha}y_n = G_n(x) + \Delta g_n(x) + d_n^s(t) + u_n(x,y,t) \end{cases} \tag{5}$$

Where again similar to the master system, $\alpha \in (0,1)$ is the order of the system, $Y(t) = [y1, y2, \ldots, yn]^T = [y, y^{(\alpha)}, \ldots, y^{((n-1))\alpha}]^T \in R^n$ is the state vector, $G(Y,t) \in R$ is a given nonlinear function of Y and t, $\Delta g_i(y,t)$ and $d_i^s(t)$ represents an unknown uncertainty and external disturbance respectively and finally $u_i(x,y,t)$ are control inputs.

Assumption: The uncertainty terms ($\Delta f_i(x,t)$, $\Delta g_i(x,t)$) and external disturbances terms ($d_i^m(t)$, $d_i^s(t)$) are assumed to be bounded by[26]:

$$\begin{cases} \left| D^{1-\alpha}(\Delta f(x,t)) \right| \le \gamma_f \\ \left| D^{1-\alpha}(d) \right| \le \gamma_d \end{cases} \tag{6}$$

Where γ_f, γ_d are known positive constants.
Synchronization error is defined as follow:

$$e_i = y_i - x_i \tag{7}$$

Where if taking α order fractional derivative from (7):

$$D^{\alpha}e_i = D^{\alpha}y_i - D^{\alpha}x_i \tag{8}$$

Substituting from (4) and (5) into (8) the fractional synchronization error is obtained as follow:

$$\begin{cases} D^q e_1 = B_1 y_1 - A_1 x_1 + g_1(y) - f_1(x) + \ldots \\ \Delta g_1(y,t) - \Delta f_1(x,t) + d_1^s - d_1^m + u_1(x,y,t) \\ D^q e_2 = B_2 y_2 - A_2 x_2 + g_2(y) - f_2(x) + \ldots \\ \Delta g_2(y,t) - \Delta f_2(x,t) + d_2^s - d_2^m + u_2(x,y,t) \\ \vdots \\ D^q e_n = B_n y_n - A_n x_n + g_n(y) - f_n(x) + \ldots \\ \Delta g_n(y,t) - \Delta f_n(x,t) + d_n^s - d_n^m + u_n(x,y,t) \end{cases} \tag{9}$$

If we take $g_i(y) - f_i(x) = \Gamma_i(x,y)$ and rewriting (9) based on error synchronization we have:

$$\begin{cases} D^q e_1 = B_1 e_1 + \Gamma_1(x,y) + \Delta g_1(y,t) - \Delta f_1(x,t) + \\ ..d_1^s - d_1^m + u_1(x,y,t) - x_1(B_1 - A_1) \\ D^q e_2 = B_2 e_2 + \Gamma_2(x,y) + \Delta g_2(y,t) - \Delta f_2(x,t) + \\ ..d_2^s - d_2^m + u_2(x,y,t) - x_2(B_2 - A_2) \\ \vdots \\ D^q e_n = B_n e_n + \Gamma_n(x,y) + \Delta g_n(y,t) - \Delta f_n(x,t) + \\ ..d_n^s - d_n^m + u_n(x,y,t) - x_n(B_n - A_n) \end{cases} \tag{10}$$

IV. Main Result

Now the aim of this paper is introducing a novel fractional sliding mode surface and design a new fractional sliding mode control for synchronization and anti-synchronization of fractional order chaotic system. So, to achieve the goal, a simple novel sliding surface is suggested and based on the sliding manifold fractional sliding mode controller is designed.

V. Sliding Surface Design

Here we introduce the below switching surface as:

$$S_i = e_i + (\mu + \upsilon) D^{-1} sign(e_i) \tag{11}$$

Where $\mu > 0$ and υ is a small positive number.
The following equations will satisfy when the trajectories reaches the sliding surface:

$S = 0$ and $D^{\alpha}S = 0$

Take the fractional derivative from the sliding surface we have:

$$D^{\alpha}S_i = D^{\alpha}e_i + (\mu+\upsilon)D^{\alpha-1}sign(e_i) \quad (12)$$

Therefore, if $D^{\alpha}S_i = 0$ then:

$$D^{\alpha}e_i = -(\mu+\upsilon)D^{\alpha-1}sign(e_i) \quad (13)$$

To design the sliding mode controller, we consider the following fractional order reaching law:

$$D^{\alpha}S = -D^{\alpha-1}(qsign(s) + rs) \quad (14)$$

Where r and q are positive constants. From (12) and (14)

$$D^{\alpha}e_i + (\mu+\upsilon)D^{\alpha-1}sign(e_i) = \\ \ldots - D^{\alpha-1}(qsign(S_i) + rS_i) \quad (15)$$

Substitute from (10) into the (15)

$$B_i e_i + \Gamma_i(x,y) + u_i(x,y,t) - x_i(B_i - A_i) + \ldots \\ (\mu+\upsilon)D^{\alpha-1}sign(e_i) = -D^{\alpha-1}(qsign(S_i) + rS_i) \quad (16)$$

Which controller is designed by the above equation.

VI. Controller Design

Now, the robust fractional sliding mode controller is designed based on the switching sliding surface. From (16) controller can be obtained and the following theorem provides sufficient conditions for robust synchronization of fractional chaotic systems at a pre-specified time.

Theorem 1. The sliding mode dynamic (12) is asymptotic stable to the equilibrium $e_i = 0$ in a finite time.

Proof. Here, we select Lyapunov function $V_e = |e_i|$ and its derivative along the trajectory (13):

$$\dot{V}_e = sign(e_i)\dot{e}_i = sign(e_i).D^{1-\alpha}(D^{\alpha}e_i) \quad (17)$$

Substitute from (13):

$$sign(e_i).D^{1-\alpha}(-(\mu+\upsilon)D^{\alpha-1}sign(e_i)) = \\ \ldots sign(e_i).(-(\mu+\upsilon)sign(e_i)) = -(\mu+\upsilon) \quad (18)$$

Therefore, since the parameters μ, υ are positive $\dot{V}_e = -(\mu+\upsilon) \prec 0$

Which means error dynamical system converge to zero asymptotically.

So as to prove convergence of all errors to zero in finite time from the above equation:

$$\dot{V}_e = \left(\frac{d|e_i|}{dt}\right) = -(\mu+\upsilon) \rightarrow dt = \left(-\frac{d|e_i|}{(\mu+\upsilon)}\right)$$

Integration from both sides of the above equation:

$$t_2 - t_1 = -\frac{1}{(\mu+\upsilon)}[e_i(t_2) - e_i(t_1)] \quad (19)$$

Then,

$$t_2 = t_1 - \frac{e_i(t_1)}{(\mu+\upsilon)} \quad (20)$$

Which means state trajectories of e_i converges to zero in finite time.

Theorem 2. For the controlled error system (10), if the sliding control scheme is designed as (21), the system trajectories will converge to the sliding surface s = 0 in finite time.

$$u_i = -B_i e_i - \Gamma_i(x,y) + x_i(B_i - A_i) - \ldots \\ (\mu+\upsilon)D^{\alpha-1}sign(e_i) - D^{\alpha-1}(qsign(S_i) + rS_i) \quad (21)$$

To check and prove the stability of the controller design, again the following Lyapunov function is considered for sliding surface as:

$$V_s = |S_i| \quad (22)$$

Derivative from (23)

$$\dot{V}_s = sign(S_i).\dot{S}_i = sign(S_i).D^{1-\alpha}(D^{\alpha}S_i) = \\ sign(S_i).D^{1-\alpha}(-D^{\alpha-1}(qsign(S_i) + rS_i)) = \\ -sign(S_i).(qsign(S_i) + rS_i) = -q - r|S_i| \prec 0$$

The state trajectories of the error dynamical system (10) will converge from the reaching phase to the sliding phase and fractional systems (4) and (5) are globally asymptotically synchronized. It has been seen that sliding motion occurs in finite time $t_2 = t_1 - \frac{1}{r}\ln(q + r|s|)$.

For anti-synchronization, the method is the same and error is defined as bellow:

$$e_i = y_i + x_i \quad (23)$$

In this section, the method have been applied on different examples for synchronization and anti-synchronization to illustrate the effectiveness of the proposal. Therefore, we first want to depict the capability of the proposed controller in synchronization/anti-synchronization of fractional chaotic systems which for this, different fractional order chaotic systems with and without uncertainty and disturbances are considered as drive and response systems.

VII. Simulation Results

Example. In this example, we consider two identical fractional financial systems for synchronization and anti-synchronization. The drive system is as follow:

$$\begin{cases} \dfrac{d^{\alpha}x_1}{dt^{\alpha}} = z_1 + (y_1 - a)x_1 + m_1 w_1 \\ \dfrac{d^{\alpha}y_1}{dt^{\alpha}} = 1 - by_1 - x_1^2 + m_2 w_1 \\ \dfrac{d^{\alpha}z_1}{dt^{\alpha}} = -x_1 - cz_1 + m_3 w_1 \\ \dfrac{d^{\alpha}w_1}{dt^{\alpha}} = -x_1 y_1 z_1 \end{cases} \tag{24}$$

And the response system are selected as:

$$\begin{cases} \dfrac{d^{\alpha}x_2}{dt^{\alpha}} = z_2 + (y_2 - a)x_2 + m_1 w_2 + u_1(t) \\ \dfrac{d^{\alpha}y_2}{dt^{\alpha}} = 1 - by_2 - x_2^2 + m_2 w_2 + u_2(t) \\ \dfrac{d^{\alpha}z_2}{dt^{\alpha}} = -x_2 - cz_2 + m_3 w_2 + u_3(t) \\ \dfrac{d^{\alpha}w_2}{dt^{\alpha}} = -x_2 y_2 z_2 + u_4(t) \end{cases} \tag{25}$$

Which again, $U=[u_1, u_2, u_3, u_4]$ are control inputs. In this case, we compare our method with [27] to show synchronization. Error functions for synchronization are defined as:

$e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, $e_4 = w_2 - w_1$

Subtracting drive and response lead the bellow error system:

$$\begin{cases} \dfrac{d^{\alpha}e_1}{dt^{\alpha}} = ae_1 + e_3 + m_1 e_4 + x_2 y_2 + x_1 y_1 + u_1(t) \\ \dfrac{d^{\alpha}e_2}{dt^{\alpha}} = -be_2 + m_2 e_4 + x_1^2 - x_2^2 + u_2(t) \\ \dfrac{d^{\alpha}e_3}{dt^{\alpha}} = -e_1 - ce_3 + m_3 e_4 + u_3(t) \\ \dfrac{d^{\alpha}e_4}{dt^{\alpha}} = x_1 y_1 z_1 - x_2 y_2 z_2 + u_4(t) \end{cases} \tag{26}$$

The parameters of fractional financial chaotic system are taken as q = 0.95, a = 2.1, b = 0.01, c = 2.6, m1 = 8.4, m2 = 6.4, m3 = 2.2 and the initial conditions of the drive and response system are taken as (x1(0), y1(0), z1(0), ω1(0)) = (1, 5, 4, 3) and (x2(0), y2(0), z2(0), ω2(0)) = (10, -4, 6, -2), respectively. Parameter values of input controllers are set as $\mu = 0.1$, $\upsilon = 1$, $q = 0.1$, $r = 0.01$. Due to comparing our results with[27], simulations are depicted in Fig.1-6.

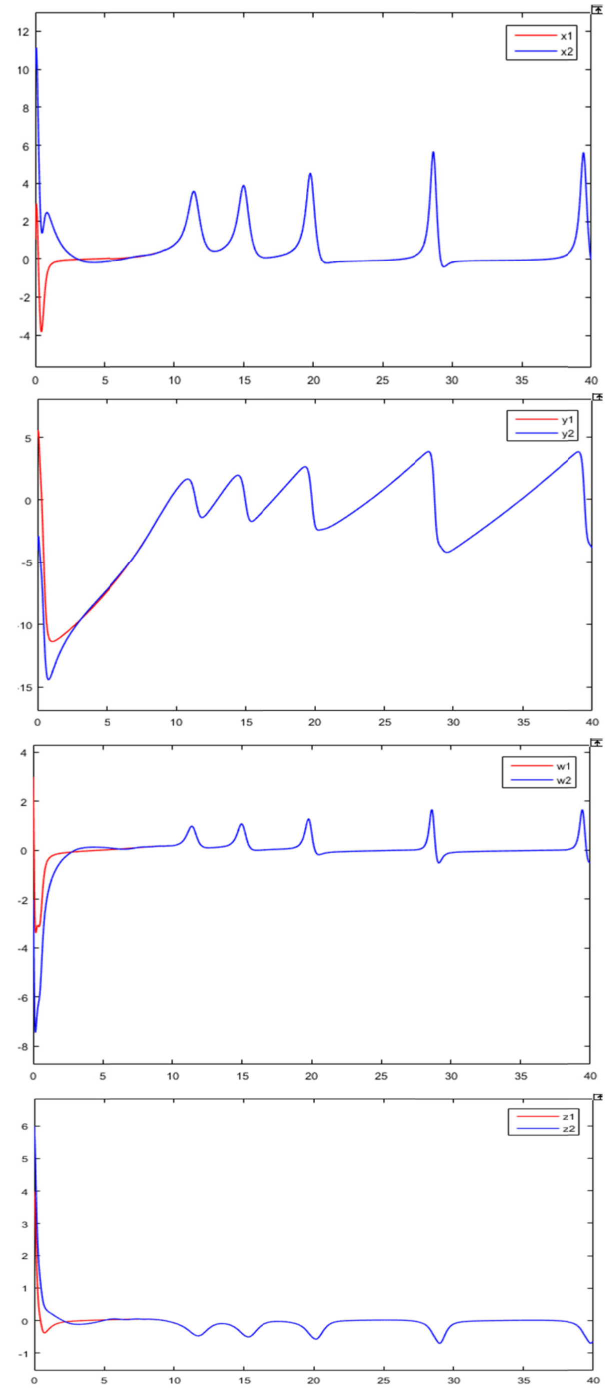

Fig. 1. The error state curves between drive system (24) and response system (25) with $q = 0.95$.

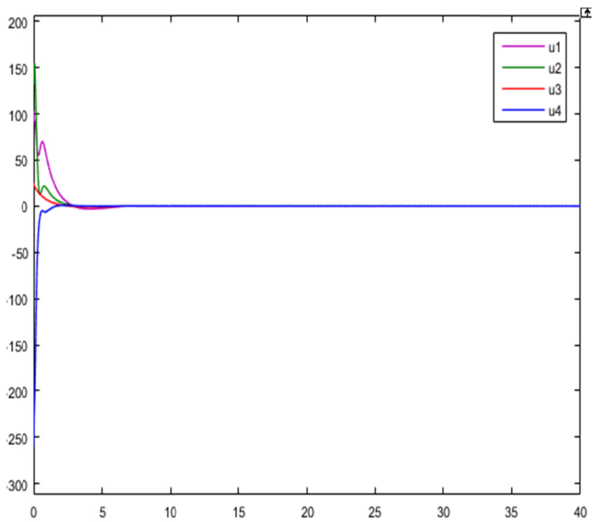

Fig. 2. Control Signals.

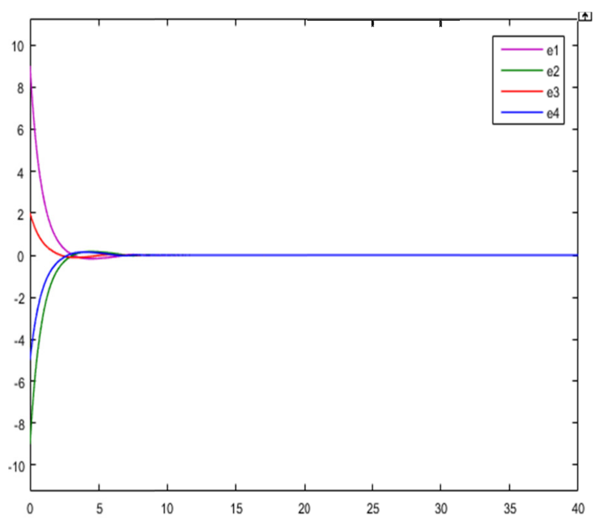

Fig. 3. The error states with order q = 0.95.

In Fig.1 synchronizations are shown. Control signals and error states are illustrated in Fig.2 and Fig.3, respectively. Fig.4 shows the synchronization of drive-response as well as in Fig.5 and Fig.6 control signals and error states of [27] are depicted which are shown faster synchronization and less control efforts.

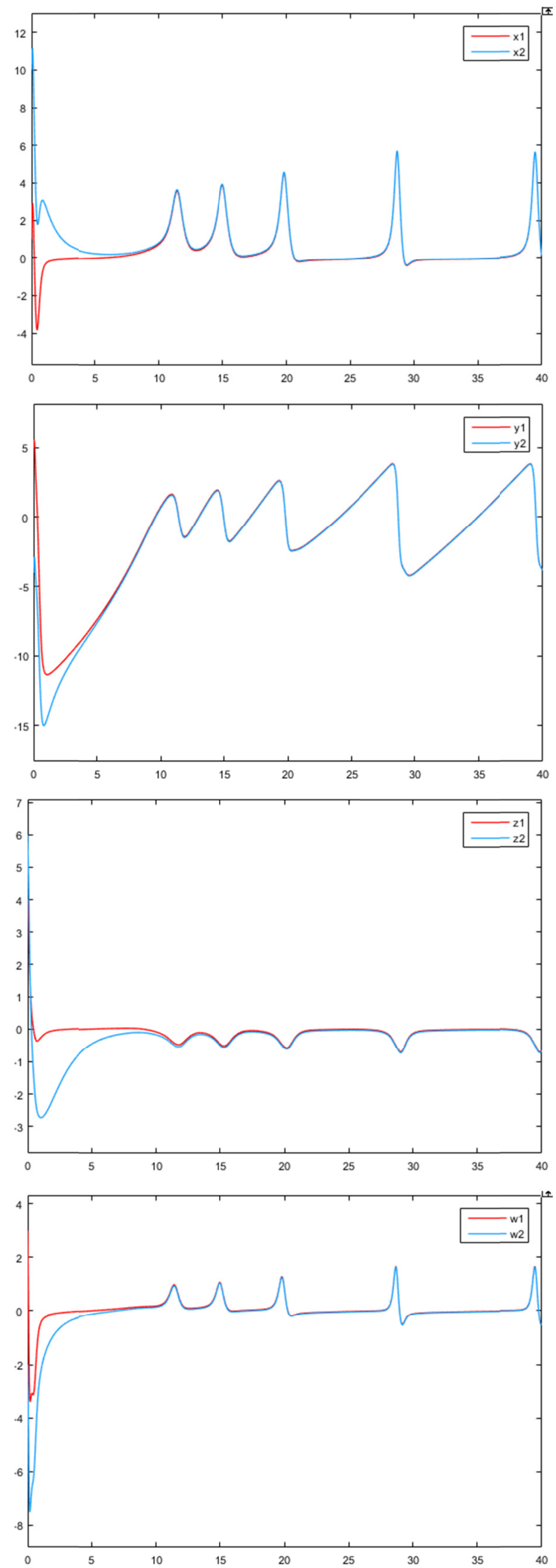

Fig. 4. The error state curves between drive system (24) and response system (25) with $q = 0.95$ [27].

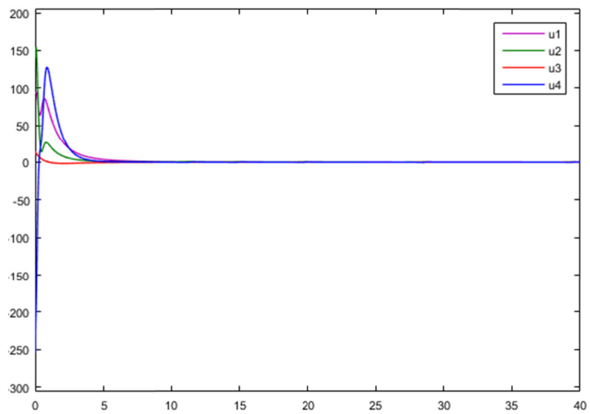

Fig. 5. Control Signals [27].

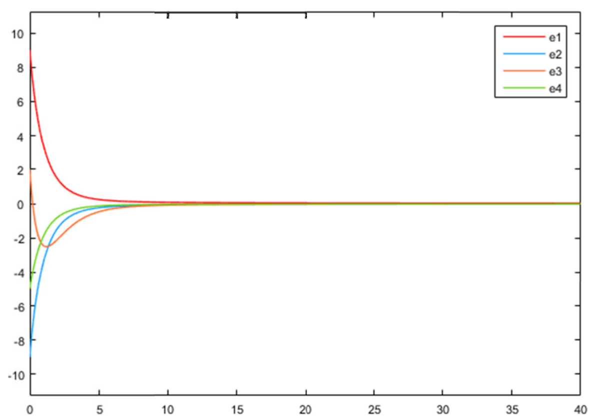

FIG.6. THE ERROR STATES WITH ORDER Q = 0.95 [27].

For anti-synchronization of the systems (24) and (25), we define the error functions as:

$$e_1 = x_2 + x_1,\ e_2 = y_2 + y_1,\ e_3 = z_2 + z_1,\ e_4 = w_2 + w_1$$

The corresponding error dynamics system with the above definitions are:

$$\begin{cases} \dfrac{d^{\alpha} e_1}{dt^{\alpha}} = ae_1 + e_3 + m_1 e_4 + x_2 y_2 + x_1 y_1 + u_1(t) \\ \dfrac{d^{\alpha} e_2}{dt^{\alpha}} = -be_2 + m_2 e_4 + x_1^2 - x_2^2 + u_2(t) \\ \dfrac{d^{\alpha} e_3}{dt^{\alpha}} = -e_1 - ce_3 + m_3 e_4 + u_3(t) \\ \dfrac{d^{\alpha} e_4}{dt^{\alpha}} = x_1 y_1 z_1 - x_2 y_2 z_2 + u_4(t) \end{cases} \quad (27)$$

Fig .7 shows the anti-synchronization and error states are pictured in Fig.8.

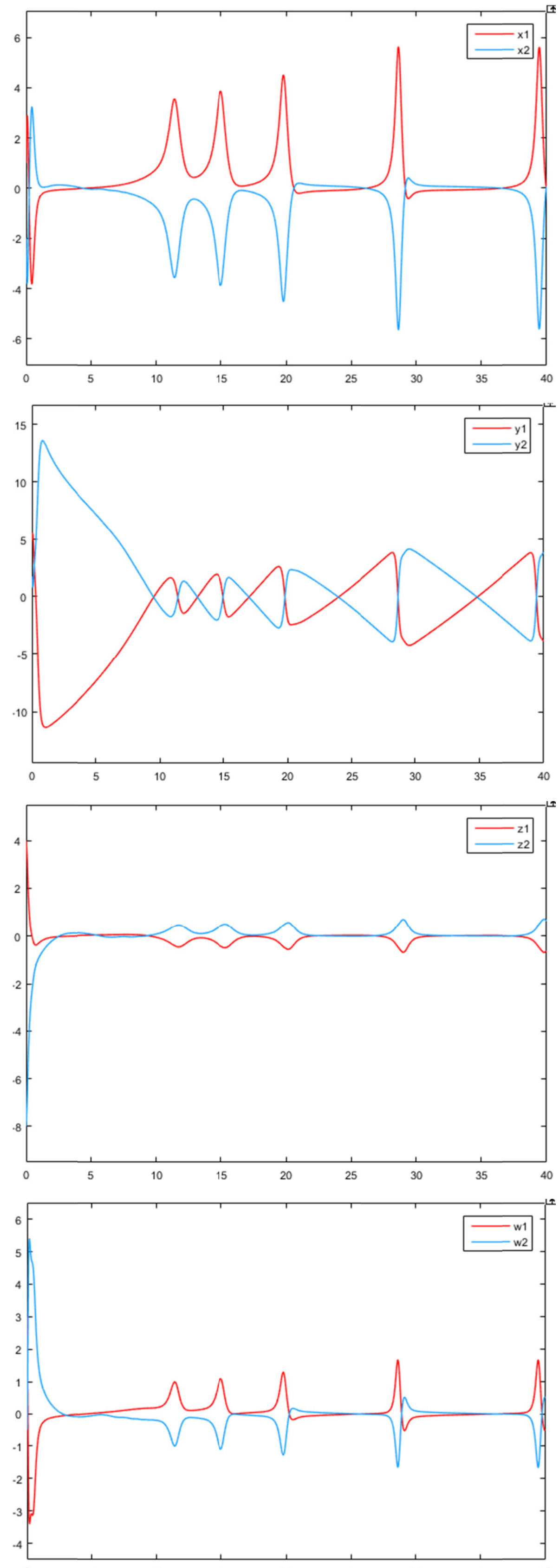

Fig. 7. Anti-synchronization between drive (32) and response system (33) with $q = 0.95$

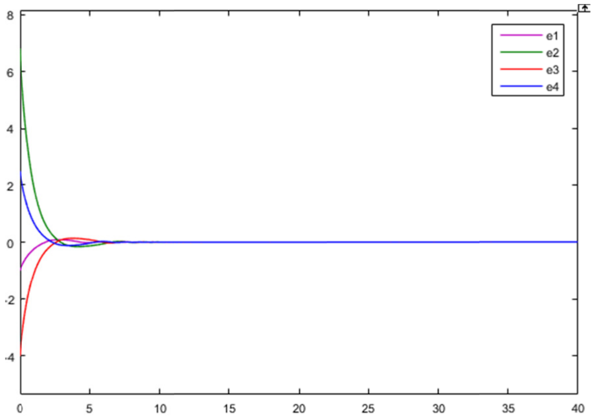

Fig.8. the error states with order q = 0.95.

VIII. CONCLUSIONS

In this paper, the main focus is on designing fractional order sliding mode control for synchronization/anti-synchronization of the class of fractional order nonlinear system. It can be seen that the chaotic behavior of the fractional model in the certain range can be appeared. Due to the significance of synchronization/anti synchronization of fractional order chaotic system in real world and scholar's view, we introduce a novel sliding manifold and new controller for synchronization/anti-synchronization of a class of fractional order system. To see the analytic investigation of the method, Lyapunov stability have been done to ensure the powerfulness of the method. Also, the method is applied on two identical fractional financial systems for more confirmation and effectiveness of the proposal.

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Arash Pourhashemi received the B.Sc. degree in Electrical Engineering from Islamic Azad University ,Gonabad ,IRAN 2007, the M.Sc. degree in Biomedical Engineering from Islamic Azad University , Mashhad ,IRAN, in 2013. He is now a Ph.D. student in Islamic Azad University of Central Tehran branch. His research interests concern the Optimization, Development of Control Strategies, Variable Structure Controls, Control of Complex Systems and Fractional Calculus.

Amin Ramezani (IEEE Member 2005) was born in Khuzestan, Iran, in 1979. He received the B.E. degree in electrical engineering from the Shahid Beheshti University of Tehran, Tehran, Iran, in 2001, and the M.sc in Control systems for Sharif University of Technology, Tehran, Iran, in 2004 and Ph.D. degrees in electrical engineering from the University of Tehran, Tehran, Iran, in 2011. In 2012, he joined the Department of Electrical Engineering, Tarbiat Modares University, Tehran, Iran, as a Lecturer. He is the chair of Advanced Control Lab and cooperates as the PC members of some conferences and special editor and reviewer of international journals in the field of control systems. His current research interests include Fault Tolerant Control Systems, Model-Based Predictive Control Systems, Stochastic Control Systems, Hybrid Systems. Dr. Ramezani is a member of IEEE since 2005 and a member of Iranian Community Instrument and Control Engineers since 2012. He has contributed in more than 30 journal papers and 70 conference papers.

Mehdi Siahi received the B.Sc. degree in Electrical Engineering From Yazd University, Yazd IRAN in 2001 and M.Sc. degree in Control Engineering from the Shahrood University of Technology ,Shahrood ,IRAN, in 2003. He obtained the Ph.D. degree in Control Engineering from Shahrood University of Technology, Shahrood IRAN, in 2008. He is now an associate Professor and has been with Faculty of Electrical Engineering, Islamic Azad University, IRAN from 2004. His current research is on Fault Tolerant Control Systems, Robust Control and Nonlinear Systems.

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