The learning process simulation based on differential equations of fractional orders

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Abstract

This article is an integrated study conducted to develop a learning model which would make it possible to identify the students' changes of knowledge, abilities and skills acquisition over time as well as the formation of special features of their individual background. Authors have justified the application of the cybernetic model based on fractional equations for the description and evaluation of the student's learning process. Learning is dealt as a transformation of young people's knowledge, abilities and skills into a complex background, which envisages its implementation in the future professional activity. The advantage of the suggested model is better approximation characteristics which allow the consideration of a wide range of factors affecting the learning process including the youth's neurodynamic and psychological nature. The research has employed both mathematical modeling methods and psychodiagnostic techniques (surveys, questionnaires). As a result of the findings, students who assimilate the content of teaching information and form personal experience in different ways have compiled different groups; the learning curve constructed on the basis of the heterogeneous differential equation of second order with integer powers has been compared with the set of models with equations of fractional order of aperiodic and fractional power components. The prospect of the issue to explore is the improvement of the suggested model considering special characteristics of cognitive processes aimed at the formation of an individual path of the student's learning.

Keywords

learning process, learning simulation, cybernetic model, differential equations of fractional order, teaching

It will lead to a paradox, from which one day many useful consequences will be drawn.
– Leibniz on fractional derivatives in his letter to l'Hospital, September 30, 1695

1. Introduction

Changes in higher education in recent years focus on providing the system of high quality training of stress resistant and creative specialists who acquire a complex of competencies and social skills and are able to respond quickly to modern social and economic challenges. The research will make it possible to track online the changes over time of knowledge, abilities and

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skills acquisition as well as to reveal and suggest the best way to high quality education of future professionals.

Modern information systems provide a means of development of models which allow us to monitor the learning process, detect upturns and downturns of the learning activeness; all this eventually affects the final result – knowledge, abilities and skills acquisition [1]. The purpose of the research is to generate and describe a cybernetic model of learning which is based on differential equations of fractional order.

Dmitriy A. Novikov has described the facilities of the learning model. Investigating the iterative learning, which is the simplest type, the researcher has proved that the quantification of the learning process can be demonstrated through curves and graphs if external impacts are permanent. The result is meant to be a level of learning which might be measured by time, speed and informational criteria and accuracy of the assignments [2].

The search for means of modelling is presented in works by Vsevolod V. Vasilyev and Lilija A. Symak [3], Yong Zhou, Clara Ionescu and J. A. Tenreiro Machado [4], Devendra Kumar and Dumitru Baleanu [5]. These researchers made a comparative analysis of fractional calculus and classical mathematical analysis and revealed the possibility to use fractional calculus in different scientific spheres as a means for description and modelling of the systems which change over time. The advantage of this approach lies in the flexible transformation from one type of the equation to another, change of the task's physical parameters and the type of initial and final conditions. However, the researchers think that the usage of these equations might be restricted by the complication of the models having two independent variables.

There is an idea grounded in current studies that the obtained equations could be correlated with the models of a blood system [6] and distribution of neural patterns' signals in the nervous tissues of biological objects [7, 8]. Thus, this research has used differential equations of fractional order which consider the learning subjects' nature in the most efficient way and allow us to build an appropriate model that would disclose the change of knowledge and skills formation over time being the basis of the gained experience.

2. Synthesis of the learning process simulation based on differential equations of fractional orders

Having analysed the survey of 130 students of the Institute of Electromechanics, Energy Saving, and Automatic Control Systems of Kremenchuk Mykhailo Ostrohradskyi National University it has become evident that a large number of boys and girls are incapable of their own learning strategy development, they struggle to master teaching information and decide on the best learning method, etc. Tutorials as a means of individualising of learning do not contribute to the task solution. For instance, it has been found out that 30% of students are hesitant about attending tutorials as they think they are a waste of time, 52.3% are confident they can cope with the problem on their own, 17.7% have troubles comprehending the subject's content and fail to formulate questions.

A similar situation is with the results of students' independent work. Despite the individual approach to the development of tasks for independent work (entry testing, potential mathematical expertise evaluation, general and specific skills assessment) it should be acknowledged that their accomplishment is merely pro forma for obtaining a mark and getting the ratings up, which do not contribute to the main goal achievement – readiness for working for an enterprise, comprehension of operating processes and making managerial decisions. In order to actualise the need, virtual laboratory complexes (VLC) have been evolved.

They allow us to use a variety of virtual devices, measurement systems and software and hardware complexes created with the help of diverse software tools of powerful modern computer equipment. These systems are flexible and adaptable to a number of tasks, for example, they facilitate the formation of engineering competencies trough planning skills training and conducting engineering experiments with further analysis of their results. Furthermore, VLC can be exploited as simulators that, due to their mathematical model, assist in studying properties of electrical objects, as software tools to simulate and study the system modes (usual conditions, pre-emergency, emergency), and as hardware and software tools for computer-assisted research [1]. Moreover, VLC serve to record studying results and construct curves of learning.

Using the method of mathematical modelling we attempted to develop a model which would clearly describe the pilot testing results and make it possible to reveal both specific features of teaching information acquisition and formation of students' individual background. Since the obtained data form a curve similar to an exponent, the authors have chosen a cybernetic model for the approximation. It is worth noting that starting from H. Ebbinghaus onwards many researchers point at the exponential nature of memorisation and forgetting [4]. The exploration of recent scientific papers [3, 4, 8, 9] places on record an appreciable quantity of suggested cybernetic models with the second-order equation which would describe any process [2].

The suggested model of the rate of information flow assimilation expressed by a second-order heterogeneous differential equation is an example of cybernetic approach:

$$m\frac{d^2S}{dt^2} + r\frac{dS}{dt} + (\alpha - c)\,S = H, \tag{1}$$

where S is the flow of digestible information as a function of time, t; r – coefficient of resistance to the learning process; α; c – coefficients of forgetting and inference; H – the flow of initial information as a function of time, t; m – inertia value.

In [10] the average values are indicated: coefficient of resistance to the process, $r = 0.5$; coefficient of forgetting $\alpha = 0.3$; inference coefficient $c = 0.25$; inertia value $m = 0.65$.

To calculate the coefficients of differential equations of the information assimilation (1) there has been conducted a pilot testing among the third-year students of Kremenchuk Mykhailo Ostrohradskyi National University during studying the discipline "Theory of Electric Drive". One hundred and thirty eight students who participated in the experiment took a test containing 20 multiple-choice questions with 5-6 options after each lecture. In total they had 14 tests. The average values of students' academic performance according to testing are presented in figure 1.

To process the data, a shift along ordinate axis has been performed so that the "transition process" begins at zero initial conditions ($\Delta_0 = 0.341$). The mean-square error of the approximation is $Std.err. = 0.0306597$, the steady-state value is 0.434615. The final level of knowledge corresponds to the value $Q = k + \Delta_0 = 0.776$.

In modern science the number of applications of fractional calculus in various fields of science and technology that use mathematical methods and computer simulation tools is increasing rapidly.

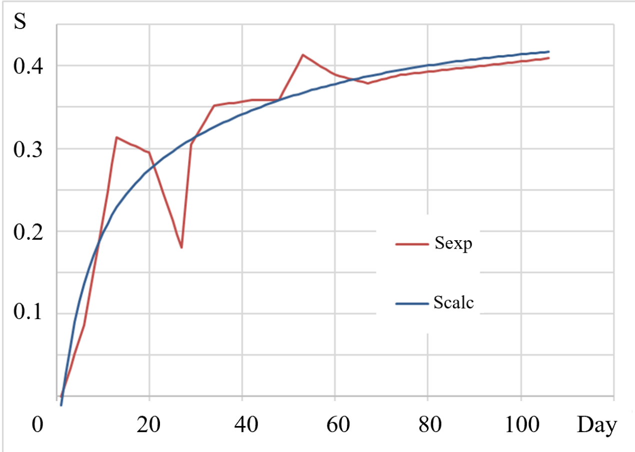


Figure 1: Experimental data and their approximation by a second-order differential equation (1).

For more than three centuries specialists in theoretical mathematics have had no doubts about existence of fractional derivatives. At the same time experts dealing with practical issues have been looking for physical meaning of fractional derivatives.

Nevertheless, the following example clearly demonstrates the understanding of the introduction of fractional orders in degrees of equations. Svante Westerlund has proposed a generalisation of Newton's second law and demonstrated that Hooke's law in elasticity theory ($F = kx$), Newtonian fluid model ($F = kx'$) and Newton's second law ($F = kx''$) can be regarded as the isolated cases of more general relation of the type: $F = kx^{(\mu)}$, where the order of derivative μ can be any real number. Of course, this generalisation cannot be called a conclusion; most likely it is an interpolation between models of processes which are described by whole-order derivatives.

From this point of view let us also consider the learning process the results of which have been demonstrated above. Analysis of the curve construction (figure 1) on the basis of model (1) does not allow us to assert unequivocally that the best solution has been found. Form the mathematical viewpoint, the algorithm based on the minimum mean-square errors method has provided the best possible approximation to the experimental data. Taking into account that the learning process is connected with the information assimilation, short-term and long-term memory, forgetting and inference, the nature of the model, the learning curve may also differ from the classical ones [9]. Starting from the first classical works William Love Bryan and Noble Harter [11] and Edward C. Tolman [12, 13] which studied the learning curve onwards, it is noted that in the beginning the learning process goes quite fast and then starts to slow down [4, 9]. The resulting dependence resembles a curve corresponding to an aperiodic first-order unit or to series connection of two first-order units. Deviations from the experimental curve were attributed to dissimilarity of the students' physiological parameters, mutual influence in the group, and individual abilities to remember and restore information which are stipulated by the nervous system features. More importantly, scholars in the field of pedagogy did not have mathematical methods to use fractional degree equations. The curve obtained from equation

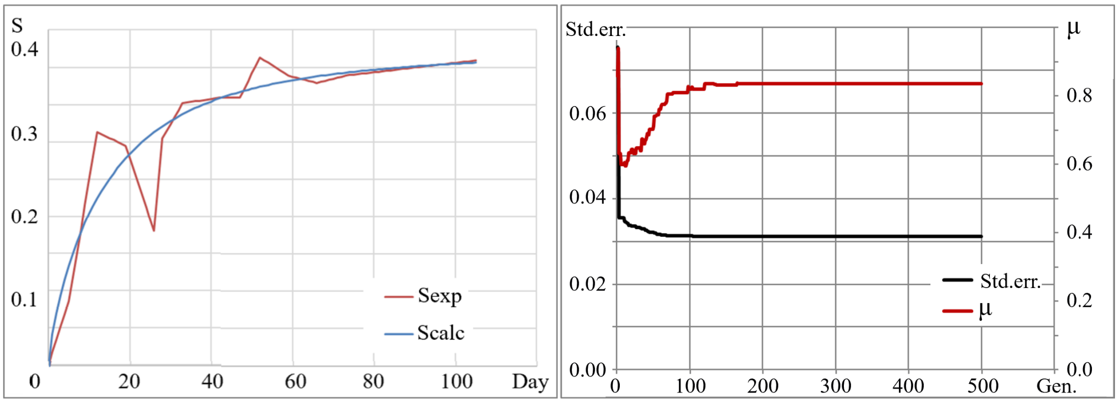


Figure 2: Experimental data and their approximation by model (2): a) initial data and approximating function changing over time; b) iterative process of finding a solution.

(1) will be approximately the same for all groups of students, which makes differentiation by psychophysiological properties impossible. However, the works [3, 4, 9, 14] demonstrate the possibility to use fractional calculus in various fields of science and technology and even to describe both the processes of information assimilation and mastery and the process of decision-making in a group. Scientists prove conclusively that the models and simulations of the processes changing over time, which the processes of information accumulation are, go beyond the framework of equations with derivatives of integer order.

Let us explore the models based on the fractional degree differential equations according to the learning results.

To process the data at a constant step of one day a piecewise linear interpolation between the known modules has been performed. Consider several models whose structure is similar.

Approximation by a fractional aperiodic unit:

$$H\left(p\right)=\frac{k}{a_0p^{\mu}+1}. \tag{2}$$

Parameter $a_0^{(1/\mu)} = 17.672$ corresponds to the physical time constant being measured in days. The graphs on the figure 2 show the initial data and the approximating function changing over time; the graphs on the right display the changes in the mean square error and the order of fractional component in the iterative search for a solution. It confirms the convergence of this process. Also, the attention should be paid to the best agreement between the calculated and experimental graphs in the middle and final parts ($t > 20$).

We also perform the approximation by the fractional aperiodic unit of order $1+\mu$ (figure 3) that can be expressed in two forms:

$$H\left(p\right)=\frac{k}{a_1p^{\mu+1}+a_0p^{\mu}+1}=\frac{k}{T_0^{\mu}p^{\mu}\left(T_1p+1\right)+1}. \tag{3}$$

Physically, this transfer function is the inertial and fractional-integrating units with negative feedback. Parameter $a_0^{(1/\mu)} = T_0 = 21.78$ corresponds to the physical time constant. If $a_1 = T_1T_0$ than $T_1 = 0.8723$ days.

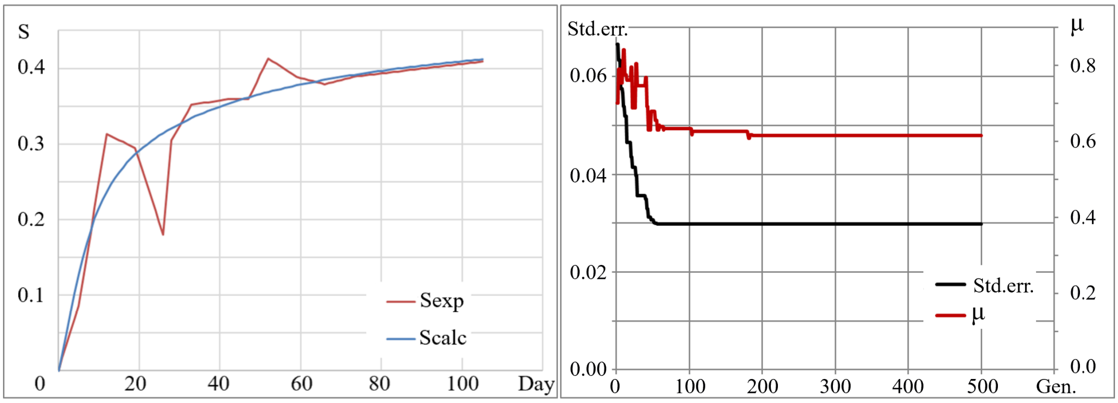


Figure 3: Experimental data and their approximation by model (3): a) approximating function changing over time; b) iterative process of finding a solution.

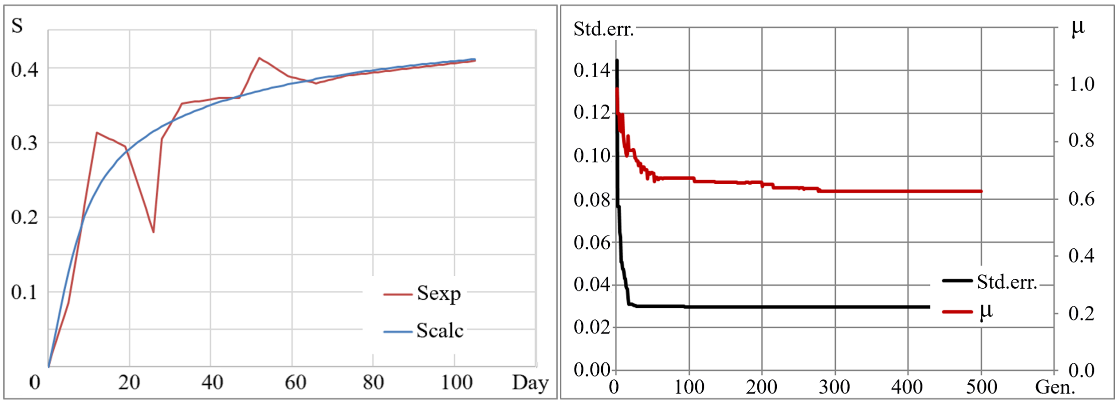


Figure 4: Experimental data and their approximation by model (4): a) approximating function changing over time; b) iterative process of finding a solution.

Approximation by inertial and fractional aperiodic units (figure 4):

$$H\left(p\right)=\frac{k}{\left(Tp+1\right)\left(a_0p^{\mu}+1\right)}. \tag{4}$$

Parameter $a_0^{(1/\mu)}=T_0=21.09$ corresponds to the time constant which together with the order of fractional aperiodic component corresponds to the physical time constant and is as close as possible to the level gained previously. Moreover, it is clearly seen that T_0 is close to the break point between the initial and final stages.

Finally we perform the approximation by the unit (figure 5):

$$H\left(p\right)=\frac{k}{\left(Tp+1\right)\left(a_1p^{\mu+1}+a_0p^{\mu}+1\right)}. \tag{5}$$

Although this function is the most complex, its similitude to the almost the same solutions at repeated starts, as in the previous cases, is worth noting. Here the inertial components

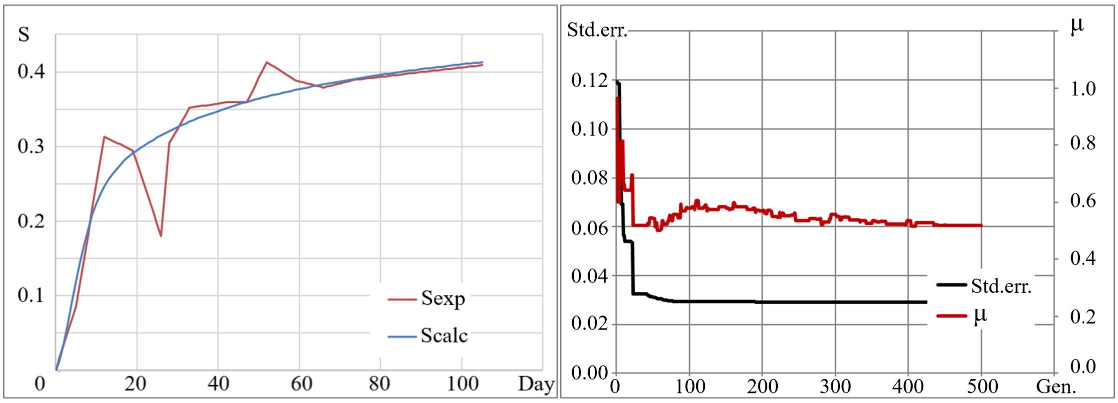


Figure 5: Experimental data and their approximation by model (5): a) initial data and approximating function changing over time; b) iterative process of finding a solution.

$T = 1.865...1.971$ and $a_0^{(1/\mu)} = T_0 = 28.14...29.46$ correspond to the physical time constant which is close to the values obtained in the previous case. Therewith, the additional time constant is calculated: $T_1 = 0.3826...0.4027$.

Summarized data of the models are given in table 1.

The obtained results are quite significant. Firstly, the accuracy of the approximation comparing with the classical model (1) has increased almost by 10%. Secondly, the fractional order in the transfer functions of models (2) – (4) is close to 0.5 which coincides with the description of diffusion processes, turbulent and laminar flows, penetration of nonviscous liquids into porous media, which also allows us to illustrate a concept by an analogy with the signal transmission from axons to dendrites [2], [15].

3. Analisys of the personal data of students

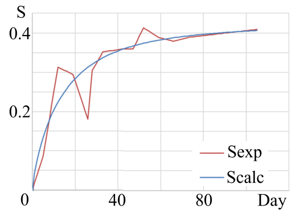
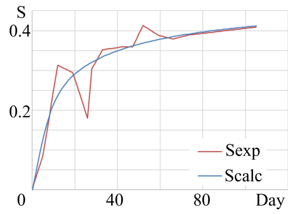
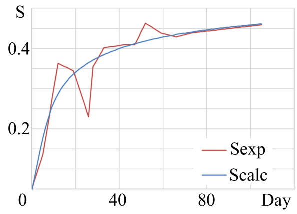
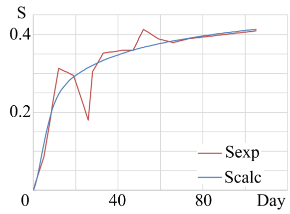
To study the learning of individual students we adopt model (4) which is based on inertial and fractional aperiodic components because it contains only two time constants and one exponent, so, taking into account modern ideas about the learning process, it will be possible to give physical meaning to these parameters.

In further studies the comparison of models' parameters with information transfer processes in the brain will allow the usage of more complex models, for example (5).

Analysing the obtained data, we can specify the elements which are characteristic of the identified groups of students.

I. Students who quickly reached the maximum level of assimilation of information. The speed of the content mastery and the achievement of high and medium levels are characteristic of students of a strong type of nervous system which determines the speed of all cognitive processes and flexibility. They work hard for a long period of time, respond flexibly to questions of varying complexity, are able to use the gained knowledge in new situations, can easily update the necessary information, correlate it with the new; compare, draw analogies, align with their own experience, etc. Furthermore, their resistance allows them to confront negative phenomena

Table 1
Model summary

Model	$T_0 = a_0^{(1/\mu)}$, days	T, T_1, days	Graphics
$H(p) = \frac{k}{a_0 p^{\mu}+1}$	17.67		
$H(p) = \frac{k}{a_1 p^{\mu+1}+a_0 p^{\mu}+1} = \frac{k}{T_0^{\mu} p^{\mu}(T_1 p+1)+1}$	21.78	0.87	
$H(p) = \frac{k}{(Tp+1)(a_0 p^{\mu}+1)}$	21.09	2.73	
$H(p) = \frac{k}{(Tp+1)(a_1 p^{\mu+1}+a_0 p^{\mu}+1)}$	28.80	$T = 1.92$, $T_1 = 0.39$	

which arise during the learning activities (influence of classmates and the teacher, adverse conditions of the organization of training, etc.).

Boys and girls fearlessly take progress tests and work efficiently under conditions of limited time. High level of motivation, awareness of the value of the future profession and the necessity of gradual development as well as intellectual and special aptitudes allow them to adsorb teaching information quickly and effectively. Typical diagram for this group is presented on figure 6 and numerical values of their parameters are $k = 0.3642$, $T = 4.2012$, $a_0 = 3.086$, $T_0 = 5.597$, $\mu = 0.6543$, $\Delta_0 = 0.400$.

II. Students who quickly reached the intermediate level and then slowly improved it. The second group includes students with an inert type of nervous system; however, they are motivated to study and conscious about the value and necessity of their own development and mastery of the future profession. The same scheme of learning is true for boys and girls

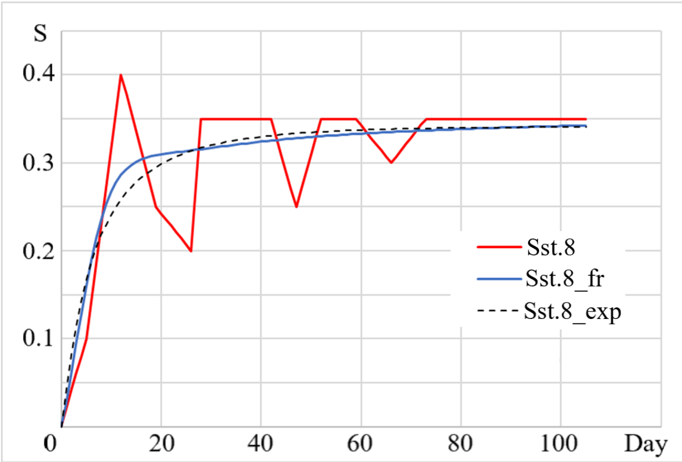


Figure 6: Data of Student No 8.

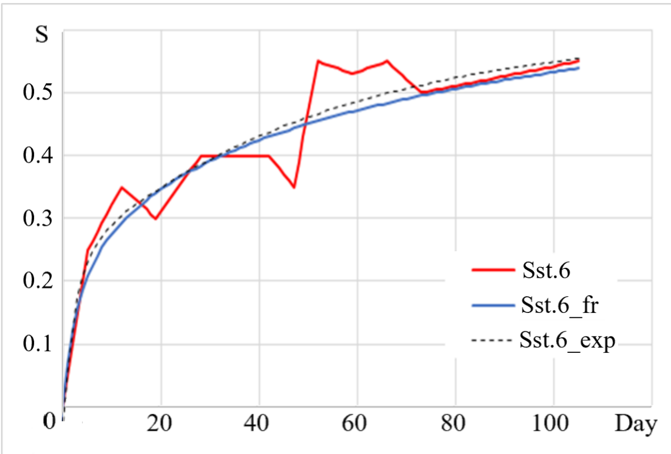


Figure 7: Data of Student No 6.

with a strong nervous system yet lacking motivation, possessing weak intellectual and special (professional) aptitudes (figure 7). Their parameters are $k = 0.9735$, $T = 0.8759$, $a_0 = 8.523$, $T_0 = 116.6$, $\mu = 0.4503$, $\Delta_0 = 0.250$.

III. Students who slowly master the teaching material are of weak and inert types of nervous system. They are fatigued by continuous, hard and responsible work. Even if the classroom climate is favourable and the teacher is reserved and sensible, unexpected questions and false answers significantly affect the final result of the content acquisition. In addition, progress tests and work under conditions of limited time inhibit the progress significantly. Although the students are well motivated and conscious about such values as development, freedom, etc., the slowness of the cognitive processes exerts an impact.

Boys and girls with an inert type of nervous system with parametes $k = 0.3855$, $T = 5.937$, $a_0 = 12.263$, $T_0 = 21.24$, $\mu = 0.8202$, $\Delta_0 = 0.350$ and typical diagram on figure 8 can more

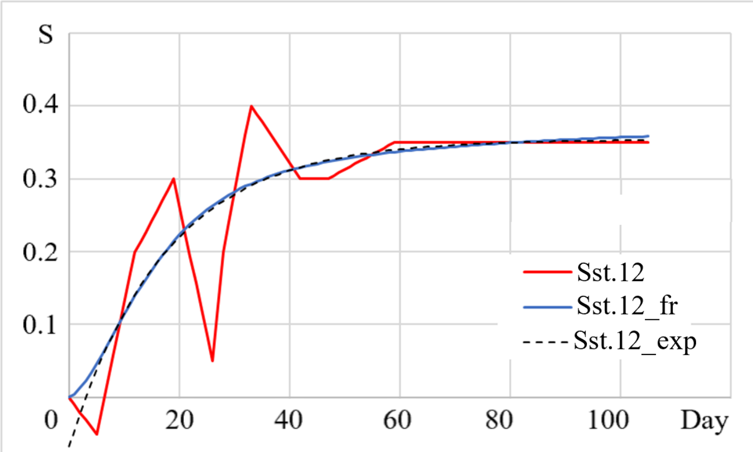


Figure 8: Data of Student No 12.

easily withstand the difficult conditions of persistent and responsible work; nevertheless, a lack of preliminary training and motivation, low level of awareness of values and imperfection of special aptitudes determine the special features of the learning process which are characteristic of the students of Group III.

Thus, the neurodynamic and psychological characteristics of a personality, which impact the effectiveness of learning, can be taken into account by the cybernetic model using equations of fractional order.

4. Conclusions

The developed cybernetic model of learning based on differential equations of second order with fractional degrees allows us to describe the process of learning more accurately, quickly responds to the changes in students' acquisition of the information, increases the efficiency of the content assimilation, enables us to guide the formation of an individual path of the student's development through the improvement and optimization of the class schedule, teaching methods and the system of control measures. The proposed simulation of the learning process facilitates in flexible and timely adjustments to the teaching process; the decay in knowledge and skills acquisition can be altered by a methodically correct presentation with due regard to the neurodynamic and psychological traits of students. So, to work with students of the second group it is necessary to apply methods of motivational and value based stimulation emphasizing the practical value of the information received, outlining the ways of its practical application, analyzing examples from real enterprises, investigating the causes of possible pre-emergency and emergency situations. The diversification of control methods, questioning aimed at stress reduction and creation of a cooperative and emotionally favorable climate contribute to the effectiveness of learning. Work with students of Group III requires additional diagnostic assessment of the level of training, formation of general and special skills, motivational in order to decide on the methods of learning activation.

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