

# Sensitivity of Outgoing Longwave Radiation to Variations of Underlying Surface Emissivity Coefficient

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**Abstract.** We study the sensitivity of outgoing longwave radiation to variations of underlying surface emissivity  $\varepsilon_c(\nu)$ . The approach to solving the problem under consideration is based on the functional theory of sensitivity. Using the analytical result obtained in the work on the differential sensitivity coefficient of the outgoing radiation as well as the computational complex created on the basis of the LBLRTM model, it is shown that the maximum sensitivity of the outgoing longwave radiation to the variations of  $\varepsilon_c(\nu)$  is observed in the ranges of 780–1000  $\text{cm}^{-1}$  and 1015–1200  $\text{cm}^{-1}$ .

**Keywords:** outgoing longwave radiation, sensitivity, emissivity, underlying surface, LBLRTM.

## 1 Introduction

Outgoing longwave radiation is one of the key components of the Earth radiation balance [1,2]. It characterizes the amount of energy generated by the radiation of the underlying surface of the Earth and the ascending radiation of the atmosphere, which goes into space from the “underlying surface – atmosphere” system.

Let the temperature, pressure and the emissivity of the underlying surface be denoted as  $T_s$ ,  $P_s$  and  $\varepsilon_s(\nu)$ , the Planck function will be denoted as  $B(\nu, T_s)$ , and by  $\tau(\nu, P \rightarrow 0; \theta)$  we shall mean transmission function of the atmospheric radiation with the frequency  $\nu$  on the path “the atmospheric level with pressure  $P$  – satellite”. In this case the spectral intensity  $L_{CLR}(\nu, \theta)$  of the outgoing from the cloudless nonscattering atmosphere under a zenith angle  $\theta$  radiation can be represented as [3]

$$L_{CLR}(\nu, \theta) = \varepsilon_s(\nu)B(\nu, T_s)\tau(\nu, P_s \rightarrow 0; \theta) + \int_{P_s}^0 B(\nu, T(P)) \frac{d\tau(\nu, P \rightarrow 0; \theta)}{d \ln(P)} d \ln(P). \quad (1)$$

It should be noted that in this work we neglect the contributions of solar radiation and the rescattering of descending radiation by underlying surface into the spectral intensity (1) of outgoing longwave radiation (OLR).

It can also be shown that if the fraction  $\alpha$  of a pixel is covered by cloud at pressure level  $P_c$ , whose emissivity is  $\varepsilon_c(\nu)$  and the temperature at the upper edge is  $T_c$ , then the spectral intensity of the radiation leaving the cloudy atmosphere is described by the expression

$$L(\nu, \theta) = (1 - \alpha\varepsilon_c(\nu))L_{CLR} + \alpha\varepsilon_c(\nu)L_{CLD}(P_c), \quad (2)$$

where

$$L_{CLD}(P_c) = B(\nu, T_c)\tau(\nu, P_c \rightarrow 0; \theta) + \int_{P_c}^0 B(\nu, T(P)) \frac{d\tau(\nu, P \rightarrow 0; \theta)}{d \ln(P)} d \ln(P)$$

is the intensity of radiation emanating from an opaque cloud at cloud top pressure  $P_c$ .

The flux of the outgoing longwave radiation  $F$  was found by integrating the intensity (2) with respect to angles and frequency. In the case of azimuthal symmetry of the outgoing radiation, the OLR flux is determined by the equation

$$F = 2\pi \int_0^{\pi/2} d\theta \int_0^{\infty} L(\nu, \theta) \sin \theta \cos \theta d\nu.$$

In this paper, the integral of the spectral OLR with respect to angle  $\theta$  was calculated in the framework of the “effective angle approximation”, which is determined by the expression [4,5]

$$F_\nu = 2\pi L(\nu, \theta_{ef}(\nu)) \int_0^{\pi/2} \sin \theta \cos \theta d\theta = \pi L(\nu, \theta_{ef}(\nu)).$$

Using this approach for the flux  $F$ , we obtain:

$$F = \pi \int_0^{\infty} L(\nu, \theta_{ef}(\nu)) d\nu. \quad (3)$$

For the range 2–2750 cm<sup>-1</sup>, the values of effective angles obtained in [4] are given in table 1.

**Table 1.** Dependence of the effective angle  $\theta_{ef}$  on frequency [4].

$\Delta\nu$ , cm <sup>-1</sup>	$\theta_{ef}$ , degree	$\Delta\nu$ , cm <sup>-1</sup>	$\theta_{ef}$ , degree	$\Delta\nu$ , cm <sup>-1</sup>	$\theta_{ef}$ , degree
2–150	50.36	650–800	52.94	1500–1800	50.36
150–250	50.64	800–950	55.86	1800–2100	52.51
250–350	51.62	950–1100	53.08	2100–2400	52.74
350–500	52.57	1100–1250	55.21	2400–2750	55.08
500–650	52.53	1250–1500	51.64		

The equations (2)–(3) show that the OLR flux is determined by the emissivity and the temperature of the underlying surface, temperature and humidity profiles, cloud properties, as well as concentrations of greenhouse gases and aerosols in the atmosphere. Due to the presence of nonlinear relationships between these characteristics of the system and the OLR, the interpretation of experimental results obtained by satellite devices requires data on the OLR sensitivity to system characteristic variations.

To solve such problems, in our work [6], we propose an approach based on the functional theory of sensitivity [7]. The coefficients of the differential sensitivity of the infrared spaceborne hyperspectrometer's readings to variations of the gas composition of the atmosphere have been obtained. It is shown that this coefficient is expressed in terms of the mass absorption coefficient of the studied gas and the universal function determined by the intensity of the outgoing radiation for the undisturbed atmosphere.

In this paper, this approach is used to analyze the effect of variations in the emissivity on the flux of longwave radiation leaving the atmosphere. The relevance of this study is due to the need to estimate how the OLR is affected by the changes in the structure of underlying surface caused by both climate changes in global and regional scales and landuse, to take into account errors in the dependence of the emissivity on the wavenumber during the interpretation of satellite data, and to verify the correctness of the  $\varepsilon_s(\nu)$  definition in climate models.

## 2 The sensitivity of the outgoing radiation flux to variations of the underlying surface

Following [6], the variation of the flux  $\Delta F$ ,

$$\Delta F(\nu(\cdot) \rightarrow \nu'(\cdot)) = F(\nu'(\cdot)) - F(\nu(\cdot)),$$

which is due to change of emissivity  $\varepsilon_s(\nu) \rightarrow \varepsilon'_s(\nu) = \varepsilon_s(\nu) + \Delta\varepsilon_s(\nu)$ , will be presented in the form

$$\Delta F = \int_0^{\infty} \frac{\delta F(\varepsilon_s(\cdot))}{\delta \varepsilon_s(\nu_0) d\nu_0} \Delta\varepsilon_s(\nu_0) d\nu_0. \quad (4)$$

The first functional derivative in (4) is conventionally called the differential sensitivity coefficient (see [6,7]). Function

$$S = \frac{\varepsilon_s(\nu_0)}{F} \frac{\delta F(\varepsilon_s(\cdot))}{\delta \varepsilon_s(\nu_0) d\nu_0}$$

describes a percentage variation  $F$  caused by a change of  $\varepsilon_s$  in a unit interval around  $\nu_0$  by 1%.

Calculating the variational derivative, we find the coefficient of differential sensitivity and the variation of the OLR flux:

$$\frac{\delta F(\varepsilon_s(\cdot))}{\delta \varepsilon_s(\nu_0) d\nu_0} = \pi(1 - \alpha\varepsilon_c(\nu_0))B(\nu_0, T_s)\tau(\nu_0, P_s \rightarrow 0; \theta_{ef}(\nu_0)), \quad (5)$$

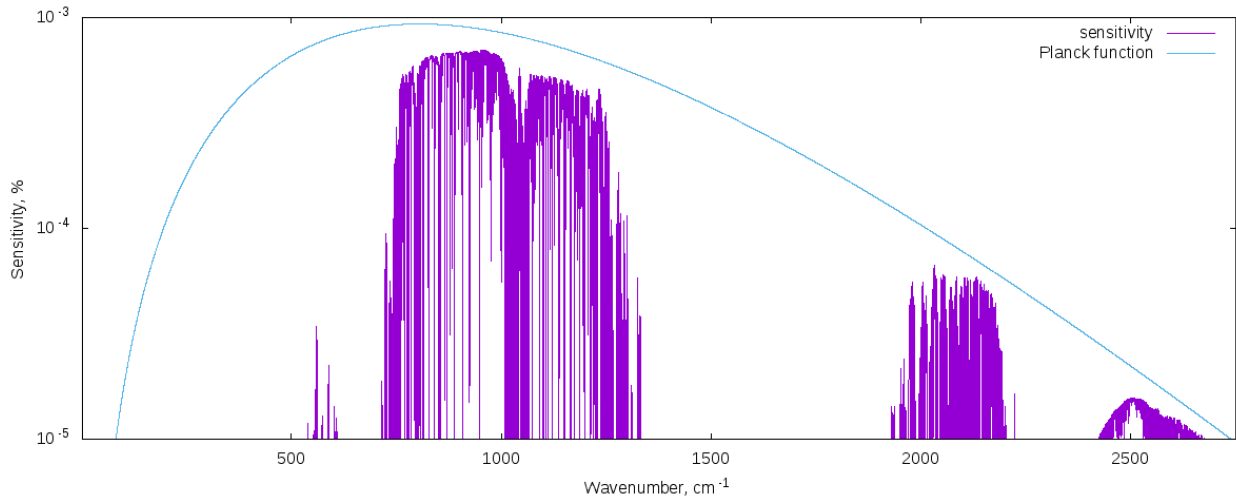
$$\Delta F = \pi \int_0^{\infty} (1 - \alpha\varepsilon_c(\nu_0))B(\nu_0, T_s)\tau(\nu_0, P_s \rightarrow 0; \theta_{ef}(\nu_0)) \Delta\varepsilon_s(\nu_0) d\nu_0. \quad (6)$$

## 3 Results

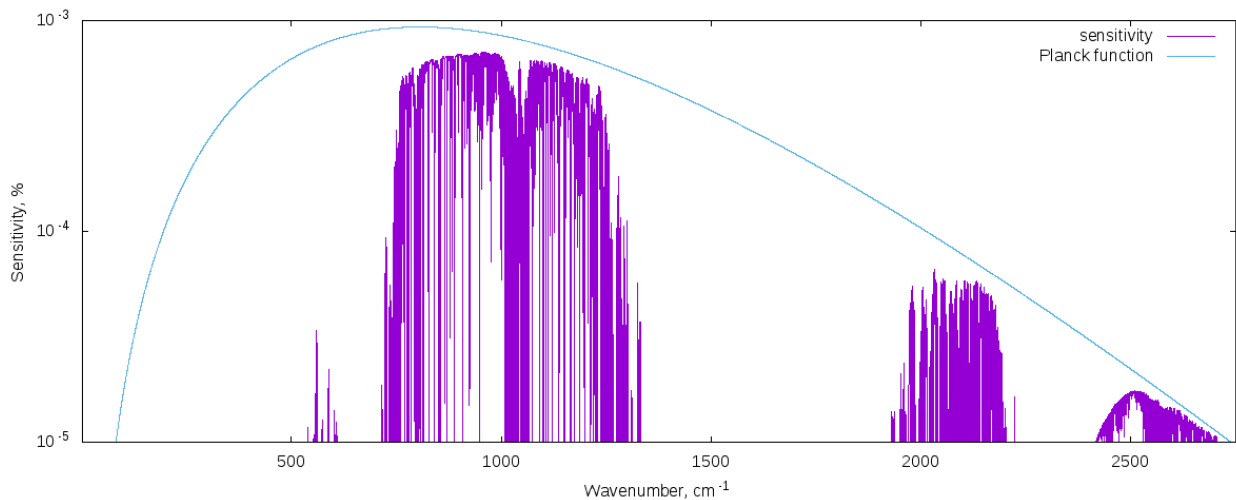
Calculations of the atmospheric transmission function  $\tau(\nu, P_s \rightarrow 0; \theta_{ef})$ , the OLR flux and the differential sensitivity coefficient were performed using the LBLRTM (Line-By-Line Radioactive Transfer Model) [8]. Preparing the necessary characteristics of the atmosphere and the underlying surface for the model as well as generating a configuration file for LBLRTM run was carried out using the program developed by the authors. Atmospheric and

surface data were extracted from the research product AIRS version 6, which contains measurements at 100 atmospheric levels with the altitude range from the surface up to ~60 km. To set the emissivity of the underlying surface we use data from the MODIS UCSB Emissivity Library of the MODIS LST group at University of California, Santa Barbara (UCSB) (<http://www.ices.ucsb.edu/modis/EMIS/html/em.html>).

Figures 1 and 2 show the dependence of flux sensitivity  $S\Delta\nu$  on frequency in a cloudless atmosphere for a sandy surface and for a surface covered with vegetation. It is easy to see that the sensitivity peaks are in the ranges of 780–1000  $\text{cm}^{-1}$  and 1015–1200  $\text{cm}^{-1}$ . Significantly lower sensitivity is observed in the ranges 2100–2200  $\text{cm}^{-1}$  and 2480–2550  $\text{cm}^{-1}$ .



**Figure 1.** The sensitivity of the flux  $S\Delta\nu$  to the variation of the emissivity  $\varepsilon_\nu$  by 1% in the interval  $\Delta\nu = \nu/1200$  for a sandy surface.



**Figure 2.** The sensitivity of the flux  $S\Delta\nu$  to the variation of the emissivity  $\varepsilon_\nu$  by 1% in the interval  $\Delta\nu = \nu/1200$  for a surface covered with vegetation.

## 4 Conclusion

In this paper, the effect of variations in the emissivity of underlying surface on the flux of longwave radiation leaving the atmosphere was analysed. The approach implemented in this paper is based on the functional theory of sensitivity. Using the analytical result obtained in the work on the coefficient of differential sensitivity of the outgoing radiation as well as the computational complex created on the basis of the LBLRTM model, it is shown that the

maximum sensitivity of the outgoing longwave radiation to the variations of emissivity is observed in the ranges of  $780\text{--}1000\text{ cm}^{-1}$  and  $1015\text{--}1200\text{ cm}^{-1}$ .

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