Signal Processing in the Receiving System with Spatially Distributed Receiving Elements

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Abstract. The issues of signal processing and noise suppression in receiving systems with spatially distributed elements are considered. Processing algorithms are based on the theory of Markovian processes. The processing takes into account the broadband nature of the observable spatial-temporal signals. The signal with linear frequency modulation was taken as the useful signal.

Keywords: spatially distributed receiving elements; wideband signal; chirp signal; Markovian process.

1 Introduction

The task of detecting a wideband signal (in terms of space-time) by spatially distributed elements seems to be laborious, since only in some cases can be divided temporal and spatial filtering [1]. Also, applying the Bayesian optimal signal detection criterion requires reversing the correlation noise matrices of order corresponding to the square of the product of the number of receiving spatially distributed elements k_a and the number of time moments k_t. This requires laborious calculations, leads to errors, and at small angles of arrival of the noise, the correlation matrix is close to a degenerate matrix. These problems can be solved by using the Markovian random process model to describe the noise on spatially distributed receiving elements.

2 Spatial Time Process Model

The problem of detecting a broadband signal in the space-time sense against a background of noise is considered for the case of a line of spatially distributed antenna elements. A signal with linear frequency modulation (1) in radar is often used as a useful signal.

$$s_i(t) = S_0 \cos\left[\omega_0 \left(t - (i-1)\tau_0 + b\left(t - (i-1)\tau_0 - \frac{\tau_p}{2}\right)^2 + \varphi\right)\right], \tag{1}$$

where ω_0 is the carrier frequency of the radio signal, b is the rate of change of frequency with linear frequency modulation, τ_p is the duration of the pulse signal, τ_0 is the relative delay between adjacent elements of the spatially distributed receiving system, depending on the angle of arrival of the signal Θ_S and the distance between the antenna elements, $0 \le t \le \tau_p$. The matched filter of the time part of the space-time filter is based on this signal.

The space-time process with fixation t is transformed into a spatial fluctuation with a harmonic character. Figure 1 illustrates a model of this fluctuation. It is agreed that the source of noise is located at a considerable distance from the receiving antenna elements. We have a flat phase wave front and observe the same fluctuation on all linearly distributed antenna elements, which is delayed by τ_0 between adjacent elements.

We believe that the noise in the space-time representation has the form presented below.

$$u_i(t) = \Xi(t - (i-1)\tau_0)\cos[\omega_0 (i-1)\tau_0 + \Psi(t - (i-1)\tau_0)]$$

This noise signal with fixed t and и Θ_n is an oscillatory random function of the antenna element number, and the parameter in equation (2) determines the average normalized frequency of spatial fluctuations:

$$\omega_{\text{н}} = 2\pi \frac{d \sin \Theta_{\Pi}}{c}. \tag{2}$$

Thus, Figure 2 shows a graph of spatial fluctuations on the elements of a linear antenna without taking into account fluctuations caused by modulations $\Xi(t - (i-1)\tau_0)$ and $\Psi(t - (i-1)\tau_0)$.

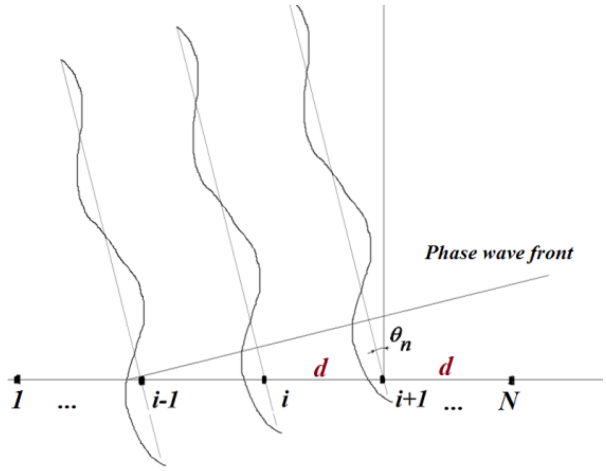

Figure 1. The formation of spatial-temporal fluctuations.

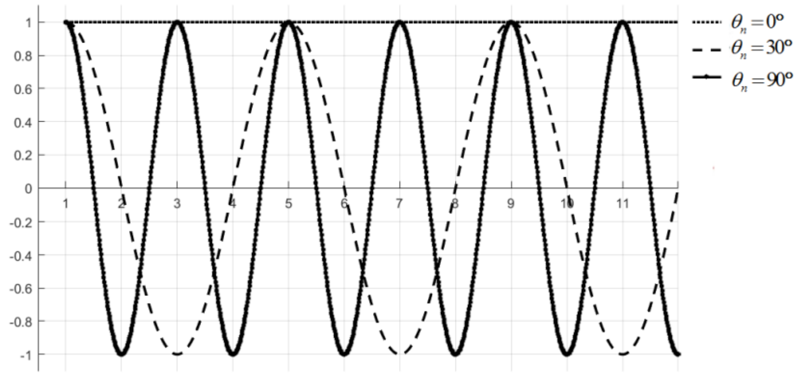

Figure 2. Graph of spatial fluctuations on the line of antenna elements.

In this case, when different sections of the broadband signal are sequentially hit the line of spatially distributed elements, only one frequency is visible in the discrete spectrum of the spatial signal, regardless of the presence of modulation. This property is convenient when there are several noise signals with different directions of arrival. This character is continued when considering the spatial-temporal fluctuations. Figure 3 presents the spatial-temporal spectrum of one interfering fluctuation after it passes through a matched filter with a complex frequency response corresponding to (1).

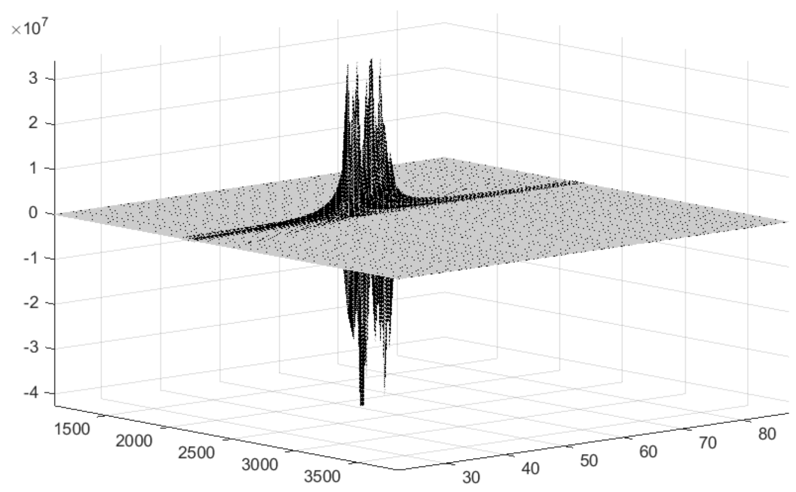

Figure 3. Spatial-temporal spectrum of noise on the line of antenna elements.

It should be noted that the signal wideband in time is narrowband in space, that is, the base of the spatial signal is close to 1.

3 Markovian Model of noise

The model is based on the autoregressive model, that is, the linear prediction model [2]:

$$u_i = \sum_{k=1}^{K} a_k u_{i-k} + \nu_i, \quad (3)$$

Since it is necessary to describe quasiharmonic processes, the minimum order of the Markovian model for the formation of the spatial fluctuation of the noise is the second order.

$$u_i = a_1 u_{i-1} + a_2 u_{i-2} + \nu_i, \quad (4)$$

where ν_i is the prediction noise, which is the information equivalent of the initial interference, a_i are the prediction coefficients. The method of moments is used to determine these prediction coefficients. This method is based on the use of relations connecting the desired parameters with the moments of the observed processes [4].

As an indicator of the correspondence of the noise of the Markovian model, Figure 4 shows the temporary implementation of the noise before and after the decorrelation procedure (4). After applying the processing, it can be seen from Figure 4 that the frequency of the signal crossing the zero level has increased, which can serve as a sign of a decrease in the correlation between neighboring samples.

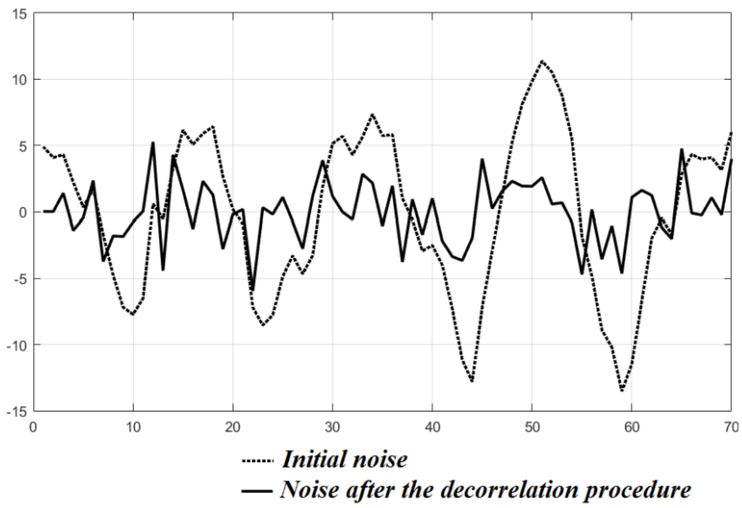

Figure 4. Temporary implementation of noise.

Figure 5 gives the normalized auto correlation functions of the interference signal before and after decorrelation, where it can be seen that the auto correlation function of the resulting information noise is closer to the auto correlation function of white Gaussian noise (tending to a delta-shaped form).

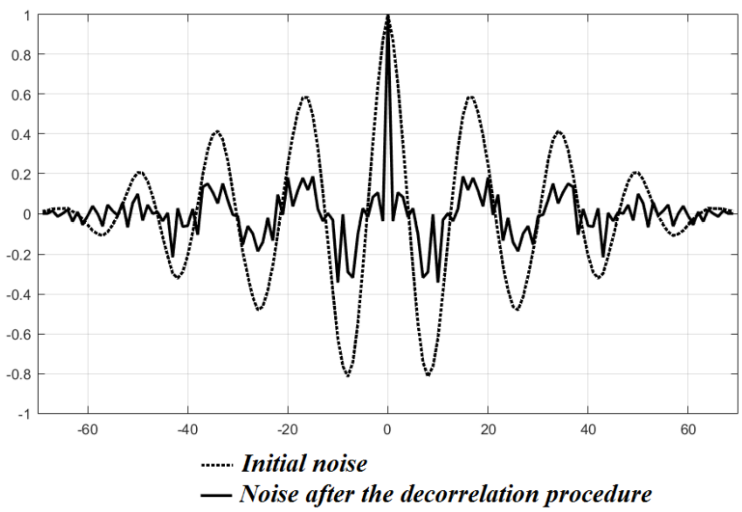

Figure 5. Auto correlation functions of the noise signal.

Figure 6 shows the auto correlation functions of the initial interference, interference after decorrelation according to (4) when we use two prediction coefficients, and interference after decorrelation according to (3) when we use ten prediction coefficients. The graph displays that the processing quality increases with an increase in the number of prediction coefficients and the sample size of the interference.

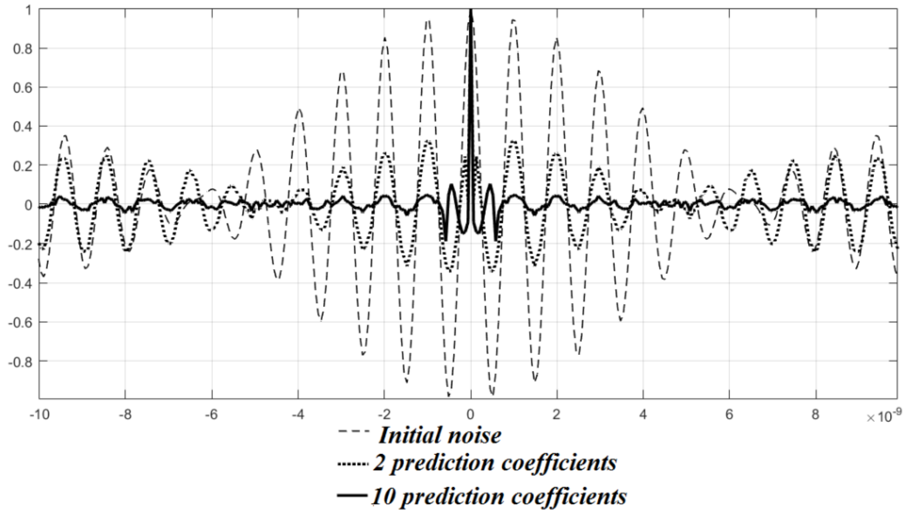

Figure 6. . Auto correlation functions of the noise signal with a different number of prediction coefficients.

If a signal is received from a known direction against a background of white noise, then its processing can always be divided into spatial and temporal [1,5]. Then the joint distribution of the noise samples on all spatially distributed elements for a model with two prediction coefficients has the form:

$$W_0(U) = W_0(u_1, u_2, \dots, u_N) = W_0(u_1, u_2) \prod_{i=3}^{N} \pi(u_i | u_{i-1}, u_{i-2}), \quad (5)$$

The decisive statistics for (4) and (5), taking into account [3], will have the form:

$$Z(U) = {U_{22}}^T {R_2}^{-1} S_{22} + \sum_{i=3}^{N} \left[{U_{3i}}^T {R_3}^{-1} S_{3i} - {U_{2i}}^T {R_2}^{-1} S_{2i} \right], \quad (6)$$

where U_{2i}, U_{3i}, S_{2i}, S_{3i} are the shortened vectors from the samples of the signal and noise oscillations at fixed t and и Θ_n, R_2 and R_3 are the correlation matrices of the shortened vectors. In a theoretical study, the quality criterion of the proposed method was to improve the signal-to-noise ratio (SNR) after spatial processing to the input signal-to-noise ratio. In Figure 7 is shown the improvement of the SNR depending on the direction of arrival of one noise fluctuation and with a fixed direction to the useful signal.

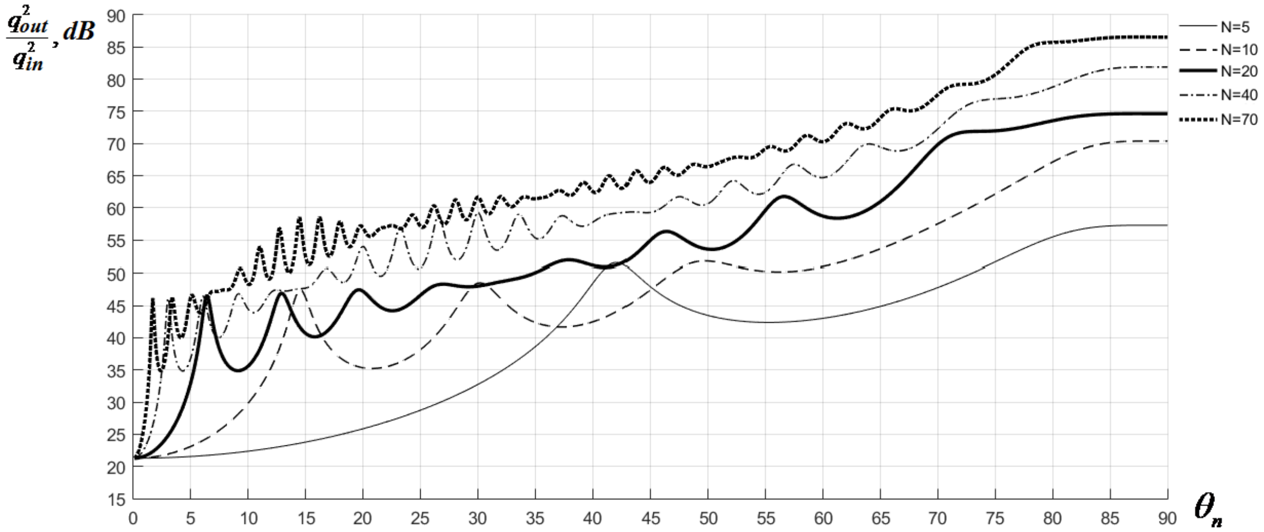

Figure 7. The dependence of the SNR improvement from the direction of the noise arrival.

Figure 8 shows the improvement of the SNR depending on the number of antenna elements in the line of spatially distributed elements in the presence of one noise fluctuation and a fixed direction to the useful signal.

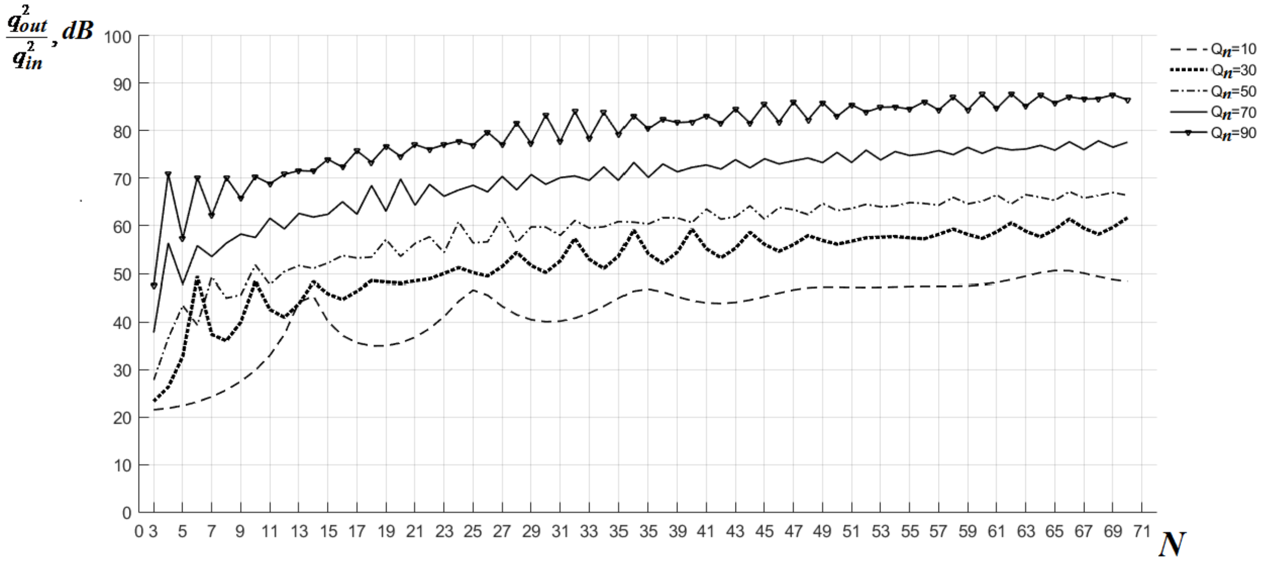

Figure 8. The dependence of the SNR improvement on the number of antenna elements.

We turn to the spatial-temporal detection algorithm, provided that the set of samples at the given time t is independent, that is, we have the vector:

$$U^{\langle t\rangle} = \left\| {u_1}^{\langle t\rangle}, {u_2}^{\langle t\rangle}, \dots, {u_N}^{\langle t\rangle} \right\|^T. \tag{7}$$

The decisive statistics according to (6) and (7) has the form:

$$Z(U) = \sum_{t=1}^{M} Z^{\langle t\rangle}\left(U^{\langle t\rangle}\right),$$

$$Z(U) = \sum_{t=1}^{M} \left\{ {U_{22}^{\langle t\rangle}}^T {R_2}^{-1} {S_{22}}^{\langle t\rangle} + \sum_{i=3}^{N} \left[{U_{3i}^{\langle t\rangle}}^T {R_3}^{-1} {S_{3i}}^{\langle t\rangle} - {U_{2i}^{\langle t\rangle}}^T {R_2}^{-1} {S_{2i}}^{\langle t\rangle} \right] \right\}.$$

Thus, temporary accumulation of the above-described spatial processing occurs.

Signal processing in the presence of several noise requires a larger number of prediction coefficients in (3).

4 Conclusion

The paper considers the issues of signal processing against the background of noise in reception systems with spatially distributed antenna elements. Processing algorithms are built on the model of Markovian random processes, which makes it possible to factorize spatial-temporal processing, and, therefore, leads to a simplification of the implementation of the algorithm. Using the spatial spectrum allows to determine the amount and direction of arrival of active interference.

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