The Use of the Sequential Quadratic Programming Method for Unmixing of Hyperspectral Images

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Abstract. The paper reviews an algorithm for the classification of landscape elements and identification of small objects in hyperspectral images by the method of sequential quadratic programming that determines the fraction of these objects in a pixel. The detection of a projective cover of vineyards is taken here as an example of practical implementation of the algorithm.

Keywords: Hyperspectral data, mixtures of objects, sequential quadratic programming, spectral components

1 Introduction

Due to high spectral resolution, hyperspectral (HS) data is getting increasingly popular in studying the spectral characteristics of Earth objects. It is used both for objects classification and detection and for development of promising multispectral remote sensing tools for the Earth providing selection of the most informative spectral channels in order to solve thematic tasks in various fields of study. However, their low spatial resolution does not always definitely identify a fragment of the image because a pixel contains not only one object, but their mixture. This is particularly observed when the size of the object is smaller than the size of a pixel on the ground, or when the pixel is located on the border of two objects. So in these cases, to separate the mixture of objects in HS images is an important task.

It is often assumed that the spectral mixture of objects is linear:

$$R = M \cdot S + \varepsilon,$$

where $M = \{m_{ij}\}$, $i = \overline{1,l}, j = \overline{1,p}$ is a mixing matrix, each column of which contains a spectral vector of endmembers (objects), l is the number of spectral channels, p is the number of objects ;

$S = \{s_{ij}\}$, $i = \overline{1,p}$ и $j = \overline{1,n}$ is a matrix whose columns are relative abundances of objects with M_j spectral signatures, that is, the elements s_{ij} are the probability of assigning the j-th pixel to spectral signature M_j, n is the number of pixels in the analyzed fragment of HS image;

$R = \{r_{ij}\}$, $i = \overline{1,l}$ и $j = \overline{1,n}$ is a matrix whose columns are spectral vectors of the analyzed fragment of HS image;

ε is a proportion of additive noise.

At the same time there are constraints that are imposed on the coefficients of mixture (the sum of the coefficients is assumed to be equal to 1, each of the coefficients must be non-negative):

$$s_{ij} \geq 0 \text{ и } \sum_{j=1}^{p} S_j = 1.$$

In order to determine fractions in pixel S, linear spectral unmixing methods are used: the method of minimizing standard deviation (linear square), the orthogonal subspace projection method, and others that differ in criteria and optimization methods [1, 2]. At the same time, some of them do not take into account all the constraints imposed on the coefficients of unmixing which brings results that are difficult to interpret.

The arising task of minimizing the value of the standard deviation of a linear mixture of signatures from a true pixel value of HS data with full constraints (Fully Constrained Linear Square, FCLS) belongs to the class of nonlinear optimization problems with constraints. It can be solved using the simplex method, gradient projection method, generalized reduced gradient method, linearization, penalty function method and Lagrangian duality.

In this research it is proposed to solve the problem of calculation of unmixing coefficients S using the method of sequential quadratic programming (SQP). SQP is one of the most modern methods in nonlinear programming taking into account full constraints imposed on the unmixing coefficients [3]. In this case, it is assumed that there is information on the spectral signatures of objects M presented in the image, including a small target object. For the formation of an array of initial a priori spectral information M, various algorithms can be used: the minimum volume simplex analysis MVSA, N-FINDR, independent and dependent component analysis (ICA and DCA), etc. [4, 5, 6]. In the work [7], selection of the spectral components of HS data array is also carried out by the SQP method. For this, preliminary the simplex of minimal volume is optimized to a set of spectral vectors R of a HS image, and the special methods are applied to a shape analysis of signatures.

2 Unmixing Algorithm on the Basis of the SQP Method

In the algorithm, the problem of spectral unmixing is described as the problem of minimization of the standard deviation ε_i between a mixture of fractions and initial HS data. In order to go to the quadratic programming problem, the objective function according to equation (1) was rewritten as follows:

$$f = \sum_{i=1}^{n} {\varepsilon_i}^T \varepsilon_i = \sum_{i=1}^{n} (R_i - MS_i)^T (R_i - MS_i).$$

Taking into consideration the above mentioned restraints on the components of mixture the SQP is proposed as a problem solution method. The main difficulty with it is the need to formalize the constraints in the form of linear equalities and inequalities:

$$\mathrm{AS} \leq \mathrm{b} \text{ и } \mathrm{A}_{eq}\mathrm{S} = b_{eq}.$$

Accordingly, the condition of equality was transformed as follows:

$$\mathrm{A}_{eq} \cdot \mathrm{vec(S)} = b_{eq},$$

where $\mathrm{A}_{eq} = I_n \otimes (1_\mathrm{p})^\mathrm{T}$, $b_{eq} = (1_\mathrm{n})^\mathrm{T}$, vec(S) - vector whose elements are the elements of the columns of matrix S going one after another;

I is a unit matrix;

$\otimes$ is an operator of the Kronecker tensor product for matrices.

The inequality condition was reduced to the form:

$$-\mathrm{A} \cdot \mathrm{vec(S)} \leq b,$$

where $\mathrm{A} = I_{p*n}$ и $\mathrm{b} = (0_{p*n})^T$.

Pixel j was identified as the target object i, if unmixing coefficient s_{ij} exceeded a definite threshold (for example, 0.95).

3 Conclusion

The implementation of this algorithm of unmixing was tested on HS data obtained by Resurs-P spacecraft on the territory of Crimean vineyards. Figure 1 shows the result of processing of this data in the form of map reflecting a projective cover of the vineyards. The spectral signatures of soil and vineyards with different leaf area index LAI (from 1 to 2) in the range of 350 ... 2500 nm, recorded during field measurements by spectroradiometer FieldSpec®4, were taken as the initial data. The field measurements were conducted at the same time as the space shooting. The resulting unmixing coefficients, capturing the soil fraction in the pixel $\mathrm{s_{ij}}$, were used to calculate a projective cover of the vineyards $P_i = 1 - \mathrm{s_{ij}}$. The assessment of the accuracy of the projective cover was carried out in the course of field work, as well as according to ultra-high resolution data (pixel size about 0.5 m) obtained by a digital color camera from the aircraft. On average, the accuracy of estimates for the test sites was about 91 percent.

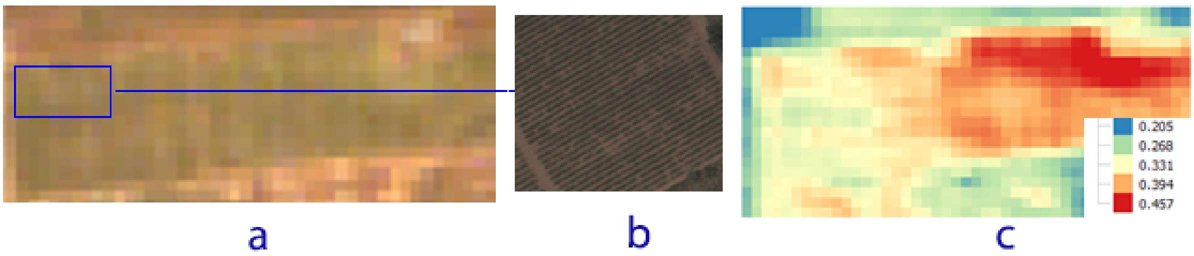

Figure 1. The results of applying the SQP method to assess the projective cover of vineyards (a - Initial HS data from Resurs-P spacecraft, b - Data from a color digital camera, c - Processing result)

Similarly, according to unmixing coefficients extracted from the algorithm the properties of objects whose size is smaller than a pixel size of HS image are detected and restored. The use of the proposed algorithm further allows us to get more reliable maps of landscapes from HS data of low spatial resolution using subpixel mapping, for example, based on a multiagent system [8].

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