

# Model Suites and Multi-model Specifications in IT

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**Abstract:** Model-based systems engineering (MBSE) focuses on creating and exploiting multiple related models that help define, design, and document a system under development. The principal question about this approach concerns the level of association between these models. As our goal is a modeling of a system, we can assume that multiple models will form a tightly coupled system, but in practice, partial models usually form a loosely bounded model kit. In this research we discuss two notions: model suite and model set, give them characterization based on mathematical theory.

**Keywords:** Model-driven development, model suite, model set.

## 1 Introduction

A distinctive feature of Model-based systems engineering (MBSE) is the contemporaneous use of multiple specifications that characterize the object under study from different viewpoints (perspectives) [1]. The principle of multi-modeling stipulates that a single model cannot adequately describe a complex system. The specification of a system constitutes of several singleton models (projections), each adequately reflects individual aspects of original's structure or behavior, and together they form a complete, integrated and coherent representation that is adequate to the modeling goals. However, terminological complexity arises; the term model can designate both a particular singleton perspective as well as a complex specification as a whole. Such substitution of concepts leads to the problem situation when analyst does not go beyond a partial description, and the developer, feeling the incompleteness of the specification, will have to think through and come up with the missing details, but based on his specific technical vision, which usually does not correspond to the analyst's understanding and does not accurately reflect real customer needs. Thus is very important to define a model suite as a multitude of partial problems necessary and sufficient for the development. Until we define the concept of a model suite, a design will remain a craft, while engineering practice is needed. Thus, there is an urgent task of clarifying the concept of a model and the properties of a multi-model specification.

We emphasize two problems of multi-model specifications: incompleteness and inconsistency. The first characteristic means that all individual models in the aggregate should describe all the properties of the original that are important for modeling purposes. Like a technical product drawing, which constitutes of three projections: front, side and top views and in the absence of one the manufacturing of this product becomes impossible; the model of the software system combines several individual perspectives, so the absence of at least one of them makes the model incomplete. However, unlike technical drawing, where the number of projections is determined by the three-dimensionality of the world around us and is fixed by regulatory documents, in software engineering the necessary and sufficient number of projections remains unclear. The choice of perspectives is at the mercy of analysts so that two specialists who solve the same problem can select completely different sets of projections, and in both cases these sets can be incomplete. Since the model, by definition, represent only those properties of the original, which are important for the modeling purposes, and discards other properties that are insignificant, the question arises: what is the mechanism of model reduction? As long as we do not respond to it, we cannot ensure the adequacy of the model to its goals.

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The second characteristic requires that individual projections should be consistent among themselves. They depict one object from different viewpoints, therefore, what is seen in the details on one of the projections should be depicted in a compressed form on another. For example, in a technical drawing, the edge of the body that is represented by a line on one view turns into a point on another. However, in information systems the question of the individual projections consistency remains open. Developers rely more on software modules designed to harmonize projections. However, before programming this links between models, it is necessary to determine the essence of a relationship among corresponding models. To do this, we again need to study the mechanism of model reduction.

Recently, many terms have appeared that designate an assemblage of models: a set, a collection, a kit [2], an integrated model [3], a megamodel [4], etc., denoting a complex representation that combines several partial descriptions. In this case, the question remains open, what is the difference between the corresponding terms? How to describe the connections between individual models? What is the coordination of individual perspectives forming a model?

## 2 State of the art

Modeling is one of the most important development stages; it results in a collection of models designed in a form suitable for further system implementation. The success of the entire development depends on how well these models are elaborated. A typical project involves numerous models in different notations. Often these models are arranged in sets. Models representing a view from a particular standpoint are usually called a perspective. It is obvious that the partial models that form the set should be well-coordinated and well-integrated between themselves, otherwise they can contradict each other. In most cases, interdependencies among singleton models in a set are not given in an explicit form. The term a model can mean either a singleton model as a part of a set, or the entire set as a whole, making understanding ambiguous.

Multi-model specifications are widely used in model-based systems engineering (MBSE) that focuses on creating and exploiting domain models as the primary means of information exchange between engineers, rather than on document-based information exchange [5]. For example, the CIMOSA architecture suggests enterprise modeling using four perspectives [6]. The Zachman framework exploits six projections [7]. ARIS utilizes four perspectives [8]. Process modeling methodology includes four projections [9]. In all examples the perspectives are introduced empirically, are not substantiated theoretically, therefore it is difficult to compare these specifications. All specifications keep silence about providing consistency between projections.

B. Thalheim contradistinguishes a loosely bounded model sets, used for distributed or collaborating IT and tightly coupled model suites with a consistent design and common data [10]. In his opinion, a model suite consists of a coherent collection of models representing different points of view and attention. He portrays a model suite as a kit of models with explicit associations among the models, with explicit controllers for maintenance of coherence of the models, with application schemata for their explicit maintenance and evolution, and tracers for the establishment of their coherence. Changes within one model must be propagated to all dependent models. Each singleton model must have a well-defined semantics as well as a number of representations for the display of model content. The representation and the model must be tightly coupled. B. Thalheim and M. Tropmann-Frick defined model suite essential properties: well-formedness, adequacy, sufficiency. In their opinion a well-formed instrument is adequate for a collection of origins if (i) it is analogous to the origins to be represented according to some analogy criterion, (ii) it is more focused (e.g. simpler, truncated, more abstract or reduced) than the origins being modeled, and (iii) it is sufficient to satisfy its purpose [11].

The object of this study would be multi-model specifications that unify numerous models in different modeling notations, while the subject would be a model suite constituted of tightly coupled submodels with a consistent design and common data. The objective of this study is to characterize the model suite and its essential properties. To achieve the goal we will use a semiotic approach and a mathematical models theory. In the framework of the first approach, the model is considered as semiotic — created using an artificial modeling language, so that linguistic methods of a study are applicable. The second one suggests considering a model as a relational system — a set of abstract objects connected by relations, so that the methods of the mathematical theory of models are applicable to them. We limit this observation to the modeling of material things, leaving ideal world out of concern.

## 3 Problem statement

H. Stachowiak differentiates natural model, having the same physical nature as an original, and semiotic one made of signs [1]. Ch. Pierce proposed to distinguish between textual languages, the alphabet of which consists of letters, combined into meaningful words, and iconic languages where each sign represents a separate concept and provokes the emergence of a sensory image [12]. D. Harel and B. Rumpe attributed graphical modeling languages as iconic, their alphabet consists of a finite number of symbols, each carries its own semantic [13]. Yu.A. Schreider notes that universe of discourse (a set of things of the real world which are subject to modeling) is first simulated by the semiotic model, which is fashioned with another «mathematical» one [14]. The last one does not explore the properties of natural things as such, but only relations between abstract «mathematical» objects. In mathematics, notions of a «structure» and a «model with a given structure» have a similar meaning. But when we investigate a universe of discourse, we have to distinguish between a model as a set of elements of a certain nature, connected to each other in a way that repeats the relations between thing which are subject to modeling, and a structure as an abstract category for which is no matter what the carrier set consists of and what is the real nature of relations. Following Yu.A. Schreider, we distinguish mathematical -  $\mathfrak{M}$  and semiotic -  $\mathfrak{S}$  models [15].

Figure 1 illustrates the proposed approach. A domain is formed by a set of material things. The right side illustrates principle how an analyst perceives a reality. It is based on the Frege's reference triangle, its sides can be interpreted as follows: the conceptualization mapping associates each thing with a certain concept; the semantic mapping relates a concept to a sign denoting it; the representation renders a sign to a thing and defines a consistency between the model and the original. A concept abstracts a notion related to a thing. The totality of all concepts constitutes ontology. A sign is a logical name assigned to the respective concept. A sum of all signs forms an alphabet of a language. Thus a sign of a modeling language denotes a thing if there is a concept associated with both [16]. The left side of this picture demonstrates the correspondence between a set of things and a set of abstract mathematical objects, we call this a formalization mapping. The correspondence between mathematical and semiotic models we call a reduction mapping.

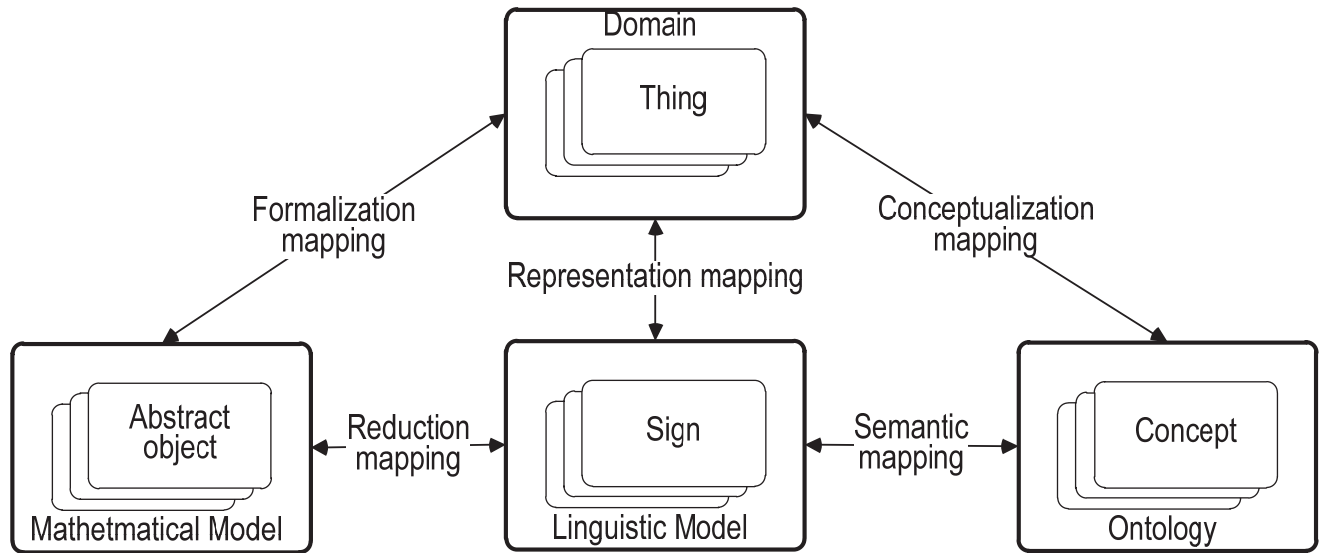


Figure 1 – Correspondence between Linguistic and Mathematic models

This article is structured as follows. First, we explore the basic concepts of the mathematical model theory then analyze semiotic model mappings and reduction properties. Later we investigate a correspondence between mathematical and semiotic models. Finally, we discuss the role of a signature for model comparison, give an interpretation of model suite essential properties, and argue the applicability of a proposed approach.

### Mathematical model

In mathematics a term a model (relational structure) means a set of abstract mathematical objects, together with a collection of relations defined on this set. Each relation is characterized by its arity to determine the number of objects participating in this relationship [17]. We will call a signature the collection of all relations ( $r$ ) of the corresponding arity.

$$\mathfrak{B}_M = \langle \Theta; R \rangle, \text{ where:} \tag{1}$$

$\Theta$  - the carrier set of abstract mathematical objects;

$R = \{ r_1^{(j_1)}, \dots, r_n^{(j_n)} \}$  – a signature, where the upper index in the round parentheses indicates the arity of the corresponding relation.

To neatly judge the similarity of mathematical models we use a concept of isomorphism. Two models  $\mathfrak{B}_{M_1} = \langle \Theta_1; R_1 \rangle$  and  $\mathfrak{B}_{M_2} = \langle \Theta_2; R_2 \rangle$  are isomorphic if there is a one-to-one (bijective) mapping between the elements of their carrier sets  $\Theta_1$  and  $\Theta_2$  and the signatures of both models coincide  $R_1 = R_2$ . Isomorphism is written:  $\varphi: \mathfrak{B}_{M_1} \rightarrow \mathfrak{B}_{M_2}$ . The isomorphism of models means similarity of both structures. In other words, both models are formed by elements of different physical nature but structured in the same way. Two models are homomorphism if this mapping is injective.

Now we consider a case, when two models are similar in some aspects but different in another, following H. Stachowiak we call it a model reduction property [18]. Let us study two main mechanisms of reduction: a decrease of the model signature and a lessening of the model carrier set. In the first case, the carrier sets of both models coincide  $\Theta_1 = \Theta_2$ , and not empty signature of the first model is a subset of the signature of the second model  $R_1 \subseteq R_2$ . One can call the first model a projection of the second. For an elementary projection, the signature includes only one relation, a complex projection consists of several relations. Now we consider two projection of one model. If both signatures don't have common relations, we call them autonomous, but if they intersect we name them interdependent. In the second case, the carrier set of the first model is a subset of the carrier set of the second one  $\Theta_1 \subseteq \Theta_2$ , while both signatures coincide and have the same arity  $R_1 = R_2$ . One can call the first model a submodel of the second one. Let us note that a reduction of the carrier set

depends on the signature. One can reduce only the object that is insignificant in regard to a particular relation in the signature. Consider a pair of models with the same signatures, but having a different level of detailization, for example, two geographical maps. A detailed map depicts pathways while a small scale one leaves them out showing highways only. Both maps (models) depict the same relation, while a domain is organized as an ordered list, to decline unnecessary details. Combining both mechanisms we can state that a finite projection of finite submodel is a local submodel [17].

Now we can make the following conclusions. First, necessary to distinguish relations between abstract objects belonging to one set and mapping of objects that reside in various sets. Second, main mechanisms of model reduction are: (i) decrease of model signature and (ii) a lessening of the model carrier set, both mechanisms are interrelated and must be implemented in concert. Third, a complex model can be represented as set of simple projections having a common carrier set.

#### 4 Semiotic model mapping property

According to Yu. Gastev consistency between a model and an original can be characterized using the algebraic notion of morphism [19]. C. Gurr declares that a syntax of notation can be explained if we manage to restore relations between objects of the subject area [20].

Aiming to evaluate whether a model provides a clear representation of a real-world, Y. Wand and R. Weber «rely on basic notions from the mathematics of mappings» [21]. For this they analyze a mapping from a set of signs forming modeling alphabet to a set of concepts founding ontology. However, they regard ontology just as a thesaurus, do not consider relations between concepts.

D. Guizzardi believes that the semantics, syntax and pragmatics of a modeling notation should be consistent with ontology, but he denies an unambiguous connection between concrete syntax and ontological model [22].

All referenced studies regard diverse mappings and understand morphism differently. We carry out this research in the context of Frege's triangle (see Figure 2), that illustrates a principle how an analyst perceives a subject area. Let distinguish between three sets of different nature: material objects, ideal concepts and signs. Three sides of the triangle can be interpreted as mappings: conceptualization - assigns an object to a concept, semantic - link a concept to a sign, representation - connects a sign of a model to an object of a subject area. We can assert that C. Gurr studies the representation mapping, particularly its algebraic morphism, while Y. Wand and R. Weber analyze the semantic mapping, they understand morphism in the context of the set theory. Here and after we imply a morphism as a relation preserving linear transformation from one mathematical structure to another one, so that relations in the source domain are mapped to equivalent relations in the destination or codomain [23].

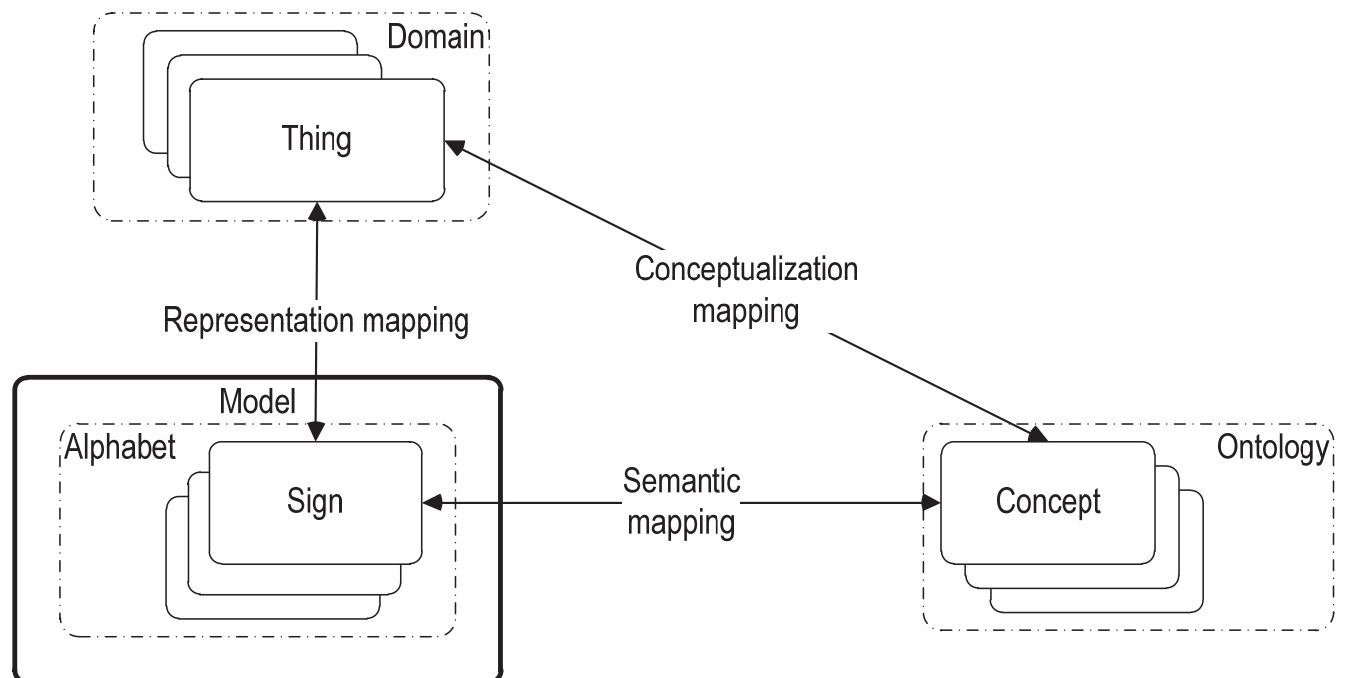


Figure 2 – Frege's triangle

Let evaluate relations between elements of three genera: material objects, concepts and signs. Our classification is not exhaustive, it aims to demonstrate an approach. We limit ourselves to n-ary relations, where  $n \leq 2$ .

## Conceptualization mapping

We will assume that a set of objects forming a certain subject area can be divided into disjoint subsets, called equivalence classes, so that an object belongs to an equivalence class if and only if it possesses a characteristic property. Let state, if  $M_i$  and  $M_j$  are equivalence classes and  $M$  is the entire domain, then:

$$M = \cup M_i \text{ and } M_i \cap M_j = \emptyset, i \neq j, \quad (2)$$

A conceptualization mapping connects each class with an appropriate concept. All objects belonging to one class reference single concept. We evaluate possible mappings (see Figure 3):

- Equipotency (bijection) – each equivalence class is mapped onto the individual concept, so that cardinalities of two sets are equal,
- Indiscernibility (surjection) – one ontology concept map different disjoint equivalence classes, which is unacceptable.
- Uncertainty (injection) – different concepts map to a single equivalence class, which can cause errors.
- Meaningless reality – some things in the universe of discourse have no relevant concepts, can't be mapped into a model.
- Empty notion – some concepts do not have a relevant object, the cardinality of concepts set exceeds the number of equivalence classes.

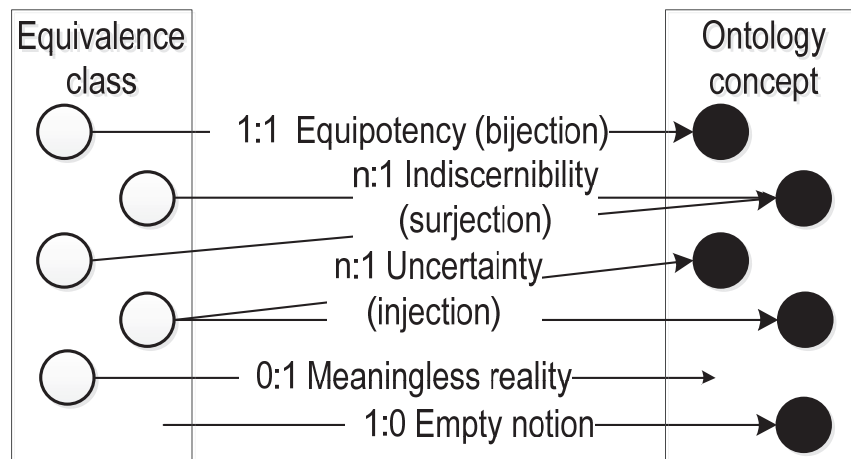


Figure 3 – Conceptualization mapping

Surely, not all of the above combinations are permissible. Ontology by convention is a complete and unambiguous definition of a subject domain, therefore uncertainty and indiscernibility cases should be instantly discarded [24]. Ontology always includes a finite number of notions, so we can speak about the scope or size of ontology as compared to a subject domain. If ontology includes an empty notion, this means that the ontology's scope exceeds the size of a domain. We consider a model erroneous if it contains something that the original does not have. For example, a centaur or mermaid are accepted as concepts in a natural language but are not allowed in the artificial modeling language. If the ontology scope is smaller than the domain size, then meaningless reality appears, some denotatum cannot be mapped into the model. It's quite a common case when a model discards those things, which are not important for modeling objectives. We believe that ontology is not limited to a thesaurus, which means that the conceptualization mapping should preserve relationships between the things and transfer them onto the relations between concepts. In the case of equipotency a conceptualization mapping is isomorphism – all concepts are mapped to appropriate classes and all conceptual relations between classes are defined. In the case of meaningless reality a conceptualization mapping is homomorphism – some equivalence classes do not have the necessary concept and particular conceptual relations remain undefined.

## 5 Semantic mapping

Now let us have a look at the semantic mapping connecting signs of a modeling language alphabet and concepts of the ontology. First we consider a mapping of two sets. The following alternatives can be distinguished (see. Figure 4):

- Unambiguity (monosemy) – a sign has only one meaning; this is an ideal case, when the model is capable of fully reflecting the reality.
- Ambiguity (polysemy) – a sign has several meanings; this is an unacceptable case, since any sign should have only one meaning;

- Equivalence (synonymy) – different language signs have similar or equivalent meanings; though synonymy is not prohibited, it causes confusion in models and should be avoided;
- Vagueness (anasemy) – some language signs have no meaning; we assume this unacceptable;
- Deficiency of representational ability of the language – there is no appropriate language sign for some ontology notions.

In the artificial modeling language ambiguity and equivalence should be excluded. The meaninglessness of a language sign is also unacceptable as it demonstrates the bad design of a modeling language. Synonymy, when different signs have the same semantics, makes the model difficult to understand. Synonymy is not a critical flaw in the language, but should be avoided.

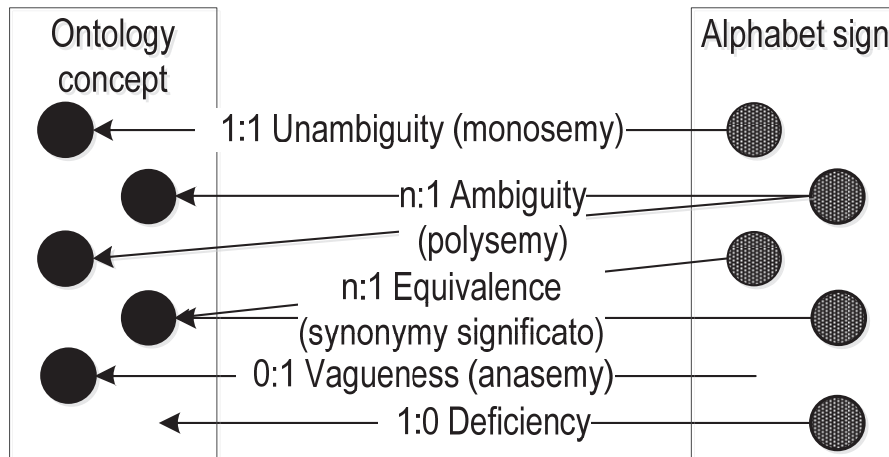


Figure 4 – Semantic mapping

### Representation mapping

The peculiarity of a semiotic model is that it can include several similar signs of the alphabet all of them referencing different objects of reality that belong to one equivalence class (see Figure 5).

- Equivalence - for each domain object there is exactly one sign of the model
- Duality (ambivalence) - there are several signs for one domain object, which is definitely unacceptable.
- Contradiction - one sign corresponds to several domain objects at once, which is inadmissible;
- Empty sign - the model contains a character that does not correspond to any domain object.
- Meaningless reality - some domain objects have no corresponding concepts, they can't be displayed on a model

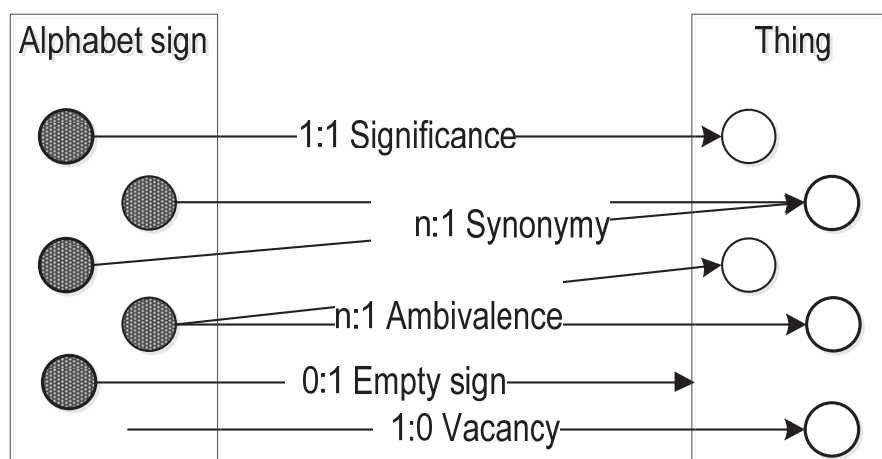


Figure 5 – Representation mapping

The model is created using the alphabet of the modeling notation. Each sign of the model should display exactly one denotate of the prototype. A model can include several identical alphabet characters. All denotate, denoted by one sign of the alphabet, belong to the equivalence class, which map a certain concept.

We make a conclusion that a modeling notation is artificial languages used for special purposes. Its vocabulary and grammar were specifically designed to fulfill certain goals. Compared to a natural language of human communication an

artificial modeling notation sacrifices its beauty and imagery for the sake of accuracy and unambiguity. Therefore, we consider any kind of uncertainty unacceptable. Synonymy is not prohibited, but makes it difficult to interpret a model. Analysts do not always realize that some two signs are synonymous and have the same meaning, understand them as different signs with dissimilar semantics. Modern modeling notations allow synonymy, thus special measures should be taken to streamline synonymy, at least at the level of a modeling agreement. We also noticed that relationships and mappings are often confused. For example, synonymy is a mapping property that is mistakenly called a relation. By separating both, we can do a taxonomy of relations simpler.

## 6 Semiotic model reduction property

We will assume that the mathematical model  $\mathfrak{B}_M$  is designed in such a way that each element of its carrier set corresponds to exactly one thing in the universe of discourse, and the relationship between the elements of the carrier set repeats the relationship between things of the subject domain. In other words, the formalization mapping is an isomorphism.

A semiotic model is usually a contraction of an original [18]. Now we examine how a reduction happens. Let first assume that a semiotic model  $\mathfrak{B}_S$  map all things and preserve all relations in the universe of discourse. We can say that representation mapping is isomorphism. Insofar as both models  $\mathfrak{B}_M$  and  $\mathfrak{B}_S$  are isomorphic to an original, therefore they are isomorphic to each other [17]. First we consider the reduction of relations, we can say, that the semiotic model is isomorphic to a finite projection of the mathematical model  $\mathfrak{B}_M$ . Second we consider a reduction of carrier set, a semiotic model is isomorphic to a finite submodel of the mathematical model  $\mathfrak{B}_M$ . A finite projection of finite submodel is called local submodel. Thus we can postulate that a semiotic model is isomorphic to a local submodel (see Figure 6). Thus, we can make judgments about the semiotic model, by analyzing the corresponding local submodel.

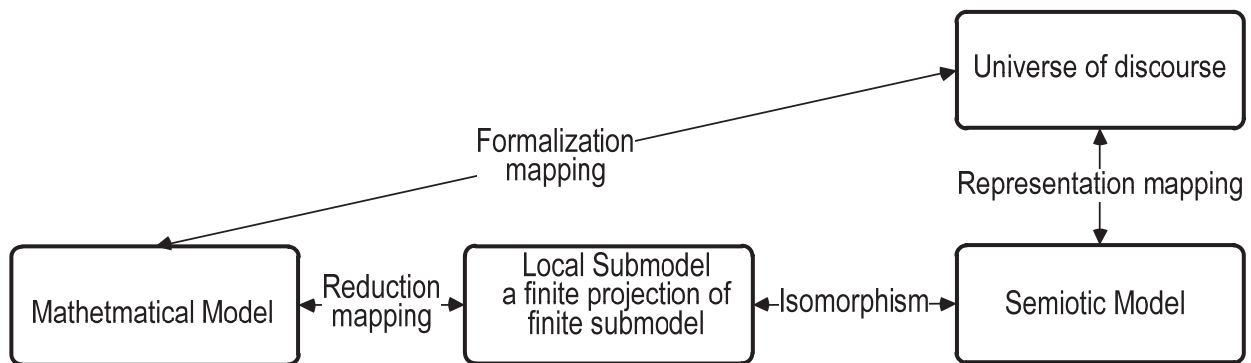


Figure 6 – Model reduction property

## 7 Discussion, a model suite and a model set

Now we give an interpretation of a model suite as a multitude of partial models (projections) having a common carrier set. Each projection highlight specific relations between things constructing a carrier set. The compound of projections should be necessary and sufficient for the development. If different projections of the one model suite do not contain shared relations, they are considered autonomous and can be developed independent to each other. But if they contain common relations, these projections are mutually dependent and should be developed consistently. In the last case there is a need for an additional controller to provide coherence between these projections. We also define a model set as a collection of models that have different carrier sets.

The signature can be considered as an important tool for models qualitative comparison and classification. Suppose that different model are matched. Two models having identical signatures are considered to be of the same type, we can call them eponymous, they can be compared with each other. If the models have completely different signatures, then we conclude that these models are incomparable. In case signatures coincide partially, these models can be compared only in terms of the relations of the same name. Now suppose comparing the modeling notation. One should first match the signatures of those models for the creation of which the studied notations are used. If the signatures coincide, it is necessary additionally compare the formal theories of the respective languages. Models with similar signatures can have the same formal theory and common grammar. Contrary, if formal theories differ, corresponding languages have dissimilar grammars. But if the signatures of models do not match, the notations are not comparable.

A model is analogous to the origins to be represented, if a semiotic mapping is a morphism: (i) there is one-to-one (at least injective) mapping of a set of signs on a set of things; (ii) a mapping preserves relations that exist between the natural things comprising the universe of discourse. A semiotic model is more focused if it correctly utilizes both reduction mechanisms: mitigate unnecessary relations from its signature and decline unimportant things from its carrier set. A semiotic model is considered to be adequate to the goals of modeling when its signature allows the analyst to get answers to the questions he poses.

It is known that the model should be adequate for the purpose of modeling [25]. Let consider how to formulate the goal properly. The SADT methodology states that the purpose of modeling is to obtain answers with a given degree of accuracy on a certain set of questions [26]. These questions are implied in the analysis and govern the creation of the model. If the model does not answer all questions or its answers are inaccurate, then modeling has not reached its goal. Thus, the purpose of modeling is determined by those questions that this model must answer. We can see that the signature defines a set of relationships, each of them answer a question. For example, a model of an enterprise organizational structure displays all employees, their grouping and subordination. It answers four questions - who work in the organization, how are the employees grouped and to whom subordinate, who is authorized to perform a particular unit of work? The signature of this model includes four abovementioned relations. Thus, there is a one-to-one correspondence between the signature of a model and a list of questions it can answer. Therefore, we link the goal of modeling and the signature of the corresponding model. If the signature includes the set of relationships required for the analysis, the corresponding model will be adequate to its goal; otherwise, if the model includes wrong relations it is considered inadequate to the goal. The accuracy of the model can be associated with the reduction of its carrier set. If this type of reduction is absent, so that both carrier sets coincide, then the model has the maximum accuracy. As some elements of the carrier set are discarded, the accuracy of the model decreases. Thus, the degree of reduction of the carrier set characterizes the accuracy of the model.

However, it is necessary to make following comments on the applicability of the proposed approach. In this research we analyze the algebraic model (the relational system) under the assumption that operations on the carrier set of abstract objects are missing. But in reality, operations can exist. If we accept the existence of operations on the carrier set we should consider, whether the named set is closed under each operation? It can be argued that if the operations on the set do not lead to the emergence of new or the destruction of existing objects, the mentioned set is closed, so that the above reasoning is correct. At the same time, there is a narrow class of models, in which, in the result of functional interaction, objects can emerge or destruct. In the last case the reasoning should be clarified.

The motives why this class of models is not included in this research should be explained. We initially limited the subject matter to material things, excluding phenomena and events from the consideration. Phenomena imply a change in the things in the result of functional interaction with other things, and an event is associated with a change in the state of a thing. Thus, in order to be able to correctly analyze phenomena and events, it would be necessary to introduce two new concepts into consideration. First we will need a notion of a state, second it will be necessary to describe a change of the state in the result of functional interaction. But, as we know, an abstract mathematical object neither possesses a state nor interacts with others. We believe that the named contradiction can be resolved as follows. First, it will require conducting an ontological study on the nature of the functional interaction between material things. Secondly, it will entail an investigation of the possibility to present functional interaction in the form of a mathematical relation.

A final comment concerns a notion of a well-formedness. As we presented above in section 4, well-formedness is the quality of a semiotic system that conforms to the grammar of the language which is its part. A way to specify a grammar of a modeling notation is to postulate a set of axioms that belong to a formal theory. Notwithstanding the fact that named approach is well known, the practical outcome is still poor. What is the reason? In order to axiomatize any natural science theory, it is necessary to formulate at least three groups of axioms. First of all logical, then mathematical, finally, the axioms of the given theory [27]. But researchers limit themselves to logical and mathematical axioms, leaving axioms of the natural theory out of consideration. That approach is correct only in case they neglect the functional interaction of material things. But for the class of models, where interaction is essential, all three types of axioms must be utilized. For example, in a model of a type part-whole, we can ignore interaction, contrary, in case of business process modeling an interaction is critical. This confirms the need for the additional research mentioned above.

Finally we discuss some practical results of this study. This analyzes of MBSE approaches is provisional as we compare framework and method, also we consider ARIS as a technique but not as an instrumental tool, offering multiple modeling notations. We start with a well-known Zachman framework, which includes six perspectives. The author does not indicate the relations in each of them, but name every after a question it must answer. The "what" perspective is easy interpreted as the relationship between things that form the domain of material objects being processed. The "how" perspective describes the transformations that take place in things in the result of functional relations. The "who" perspective binds actors to work that induce the transformation. Note that the actors form a separate domain independent of the first one. The "where" perspective geographically locate actors executing a work. The "when" perspective links work to the timeline. Finally, the "why" perspective describes the goals of the work being performed. Note that the goal is usually formulated in financial terms - logical entities having a value in running a business, so we can talk about a third "business" domain. Thus, the Zachman framework uses three different disjoint domains: material things, actors and financial objectives. A similar situation is with CIMOSA and ARIS, which are based on two domains: material things and acting persons. An interesting question is the level of integration between different domains. For example ARIS method postulates that projections are integrated by means of control perspective, whereas Zachman model does not consider the interrelationship of partial models. Thereby we categorize Zachmann as a multi-sort algebraic structure. Until the connections between the three domains are not well defined, it should be classified as a model set. However if one will make some efforts to combine these three domains into one complex carrier set and will accurately describe the dependencies between domains, as well as analyze the relationship in each of projection, to understand the degree of interdependence, the Zachman model will turn into a suite.

Nevertheless, we can see eponymous projections in all specifications above, for example, informational, organizational and functional perspectives are similar. Unfortunately, there is no common enumeration of relationships in each



perspective. That is why analysts interpret these projections individually. Even within the same specification, the models of one projection implemented by different analysts can vary. If one would define a basic set of relations for each projection, it will eliminate the analyst's subjective understanding of the corresponding model, which, of course, will improve the quality of modeling. The specifications above do not define languages used to build the corresponding perspectives, therefore we are not able to analyze the grammars of the matching notations. However, we note that it is possible to set up a unified set of axioms for formal theories for all eponymous projections. This will make possible to define a general grammar for different languages used to describe the projections of the same name.

## Conclusions

A singularity of this paper is in a precise matching the mathematical and semiotic models. Its novelty is in applying algebraic and linguistic methods to study an artificial modeling notation. Within frames of this discussion, we evaluate model mapping and reduction properties; discuss the role of a signature in analyzes of the semantic models. The results obtained in the paper are very important for model-based system engineering. We define a model suite as a multitude of partial models (projections) necessary and sufficient for the development. The main outcome is in the formulation of the model suite essential properties. If the multitude of partial projections is designed in such way that each reflects certain relations between objects of the domain, common to all models, it can be called a suite. Otherwise, if partial models reflect relations between objects belonging to the different domains, these models form a set. If different projections of the one model suite do not contain shared relations, they are considered mutually independent and can be developed autonomously of each other. But if they contain common relations, these projections are mutually dependent and should be developed consistently. In the last case there is a need for an additional controller to provide coherence between these projections. We show, that a model set is a collection of models that have different carrier sets. Thus, well-known modeling methodologies, for example, Zakhman and ARIS utilize several unrelated domains, therefore they form a set and cannot be classified as a model suite. At the same time, we have outlined the methods of domain consolidation, which will allow transforming the collection into a system.

This paper also presents a new approach to the formalization of semiotic models. Its novelty is in the fact that we apply linguistic methods to study an artificial modeling notation. Within frames of this discussion, we prove the hypothesis that semantics and syntax of the modeling language can be justified using an ontology and three mappings connecting real-world objects, concepts of ontology and signs of a modeling notation. Thus, the semantics of the sign is determined through the content of the notion, associated with the corresponding concept of the ontological model, while the syntax of this notation is defined by the relations existing between concepts. The pragmatics of the model is a subject of further research. We suggest:

- 1) Separate relations and mappings. As a result we can see that so-called semantic relations, for example a synonymy, is in fact a type of semantic mapping.
- 2) Analyze mappings inter three sets of different nature: material objects, ideal concepts and signs.
- 3) Consider a mapping as a morphism that carries the relations between the elements of one set onto the relations between the elements of the other.
- 4) Analyze relations between objects of a subject area top down, taking a top-level Bunge-Wand-Weber ontology as a foundation.
- 5) Base a classification of relations on the matching of objects properties. This way a total number of relations types can be decreased.
- 6) Use a signature to differentiate various models types.
- 7) Reduction property of a model is divided into a reduction of relations and a reduction of concepts.

This approach can give the following practical results:

- An analyses of relations inter objects of the subject area can substantiate a syntax of a modeling notation.
- A new taxonomy of models can be based on their signature – ability to represent different relations.
- Analyses of the model reduction property will allow make a better judgment of a modeling quality.

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