# Probability-theoretical model for product assembly parameters assessment

N.V. Ruzanov[1], M.A. Bolotov[1], V.A. Pechenin[1]

[1]*Samara National Research University, 34 Moskovskoe Shosse, 443086, Samara, Russia*

**Abstract**

The proposed work provides a model for estimating the assembly parameters of the products by the example of the cone ring assembly process. There was carried out the simulation of the process of parts mating along the surfaces with form deviation. As a result of Monte Carlo simulation, there were obtained statistical characteristics of the parameters of the assembled units depending on various initial positions of the parts being assembled. The proposed model could be used to assess the accuracy of the products assembled in the aircraft industry.

*Keywords:* assembly parameters; mating of components; Monte Carlo method; probabilistic assessment; probability density

## 1. Introduction

The result of manufactured products assembly depends on the actual shape of the parts used, their initial location and the assembly process. The errors of the actuating mechanisms used during parts manufacture have a significant effect on the accuracy of the manufactured products [1], so that even the parts of the same type do not have actual identical shape. Moreover, even for a single batch of products it is hardly achievable to ensure the exact matching of assembly conditions for various items manufacture.

Assembly parameters assessment will allow determining an achievable accuracy of product manufacturing, which, in turn, will enable to solve a number of problems:

1. Determine the percentage of products meeting process requirements [2].
2. Determine rational tolerances for product parameters [3].
3. Identify critical factors affecting the quality of product assembly (assembly deviation, parts prepositioning or tool selection for parts assembly) [4-7]

An assessment of the assembly parameters of products can be obtained through various approaches. One of them is based on accumulation and analysis of results of the manufactured products inspection phase. Another approach is based on numerical simulation of the product assembly process and subsequent analysis of the obtained results. The use of production statistics requires considerable human and material resources, and therefore is hardly feasible. The first approach being rather difficult to realize is the cause why math modeling methods are widely used to solve the specified problem.

In this paper, the authors present a model for estimation of product assembly parameters, based on assembly process simulation by numerical methods. Moreover, there is given an example of using the proposed model for estimation of assembly parameters of cone rings that are widespread products in the aircraft industry.

## 2. Model description

Product characteristics depend not only on parts comprising it but also on assembly procedures. Assembling of complex products consisting of many parts is a multi-criteria task that takes into account the size of all the components, their mutual arrangement in the finished product, and the alignment procedure [8].

Today, the parts' quality issues are understood deeply, so in order to improve quality characteristics of the product further, the researches aimed at studying the product assembly procedure are becoming increasingly popular. The complexity of implementing multiple product assembly leads to the fact that the majority of such researches is based on the employment of numerical simulation methods in the assembly process.

One of the features of parts assembly in the aircraft industry is the deformation of parts of the product. So as to determine the condition of parts of the product assembled, ANSYS application is used, which enables us to calculate the strength of the parts and assemblies, to solve the problems of gas and hydrodynamics [9]. Such approaches have high computational costs, so in order to simplify the solution of the assembly problem, many researchers consider the mating parts as absolutely rigid bodies.

The authors [10] consider mating two parts along plane surfaces, having form deviation. Researchers proposed a mathematical model simulating the assembly along the planar surfaces; and the result of using of such model is calculation of the clearance between the surfaces in product assembled.

Paper [11] is also devoted to the problem of parts mating along the planar surfaces. The authors of this work suggested a model for describing the deviation of the shape of the specified surfaces and considered the result of the parts mating with various shape deviations of the planar surfaces.

Most of the works focus on the simulation of the assembly process without setting up a formal problem. In paper [12] the authors formalize the product assembly problem and suggest using such concepts as the initial assembly conditions, the assembly quality assessment function, and the assembly sequence function.

The proposed work considers the assessment of the assembly parameters of the product obtained from various assembly process models.

The mutual arrangement of the parts is a fundamental assembly parameter of the product as other geometric characteristics of the finished product can be derived from it (for example, out-of-true running, out-of-flatness, uneven clearance, etc.). The methods for positioning coordinate systems that are bound to these parts are well suited to describe the mutual arrangement of the parts.

Let us consider a product that is assembled from two parts $K_1$ and. The product coordinate system $R$ is compatible with the design coordinate system $R_1$ of the part $K_1$. With such a transformation, the part $K_1$ will be stationary relative to the coordinate system $R$, and the assembly will be carried out by moving the part $K_2$. The assembled state of the product can be described by the position of the design coordinate system $R_2$. The errors in the assembly process will result in the onset of a set of assembled states $\Omega$.

$$\Omega = \left\{ \omega : \ \omega = \begin{pmatrix} dx \\ dy \\ dz \\ \alpha \\ \beta \\ \gamma \end{pmatrix} \right\}, \tag{1}$$

where $dx, dy, dz$ is the displacement of the coordinate origin of system $R_2$ relative to the origin of system $R_1$;

$\alpha, \beta, \gamma$ are the angles of turn of basis vectors of system $R_2$ relative to vectors $R_1$.

The elements of this set describe the mutual arrangement of the parts in the assembled product. To solve the problem of evaluating the assembly parameters of the products, it is necessary to get the parameters for this set. The use of the Monte Carlo method enables us to determine the approximate value of this parameters. According to this method, the first step is to perform a multiple numerical simulation of the assembly of the product and save the simulation results. The second step is to investigate the obtained simulation results and calculate the required parameters.

We will use the following values to estimate the set of assembled states: the mean value, root-mean-square deviation and probability density.

To calculate the average value, let us use the following formula (2):

$$\bar{\omega} = \begin{pmatrix} \overline{dx} \\ \overline{dy} \\ \overline{dz} \\ \bar{\alpha} \\ \bar{\beta} \\ \bar{\gamma} \end{pmatrix}, \ \bar{a} = \frac{1}{n} \sum_{i=1}^{n} a_i, \tag{2}$$

The root-mean-square deviation is calculated with the formula (3):

$$\sigma = \begin{pmatrix} \sigma_{dx} \\ \sigma_{dy} \\ \sigma_{dz} \\ \sigma_{\alpha} \\ \sigma_{\beta} \\ \sigma_{\gamma} \end{pmatrix}, \ \sigma_a = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(a_i - \bar{a}\right)}, \tag{3}$$

Obtaining an analytic expression for a probability density function is a complicated task, so we can use numerical methods to evaluate the state with the approximate values of the function with the required accuracy, based on the distribution histograms and their possible approximation.

Distribution histogram method provides empirical estimates $K_2$ of the density of distribution of the random value [13]. The algorithm for obtaining the distribution density histogram is shown in Fig. 1.

To create a histogram, the observed range of the random variable is divided into several intervals, and then the number of the random value hits in each interval is calculated. The Sturges' rule (4) is used to determine the number of the intervals:

$$n = 1 + \left[\log_2 N\right], \tag{4}$$

The next step is standardization of the received values to meet the condition (5)

$$\int_{\Omega} f(\omega) \, d\omega = 1, \tag{5}$$

The final step is to approximate the probability density function on the basis of the midpoint of the intervals and values calculated in the previous step.

The mean and root-mean-square deviation values, the approximate value of the probability density function describe a set of assembled states and can be used to solve further tasks.

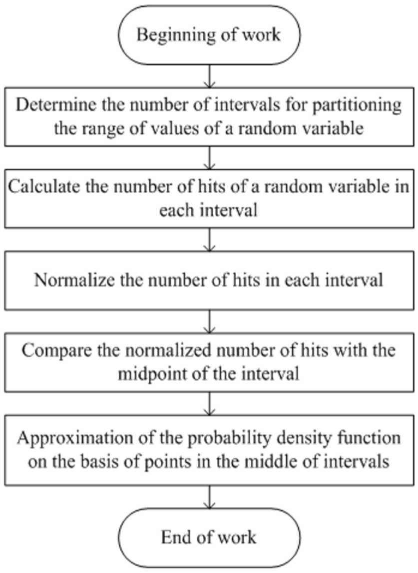


Fig. 1. Algorithm for numerical calculation of probability density. Algorithm for numerical definition of the probability density.

## 3. Simulation results

To test the proposed methodology, there has been carried out the evaluation of the assembly parameters of the parts that have the cone surfaces. Mating of such surfaces is widely used in the aviation industry and the characteristics of the entire product depend on the assembly quality of these parts.

The mathematical model of the cone surface of a part can be represented in a parametric form (6):

$$\begin{cases} \Delta F = \Delta F(u,v) \\ x(u,v) = \left( \dfrac{R_2 - R_1}{H} * \left( \dfrac{R_1 * H}{R_2 - R_1} + H - u \right) + \Delta F(u,v) \right) * \cos(v) \\ y(u,v) = \left( \dfrac{R_2 - R_1}{H} * \left( \dfrac{R_1 * H}{R_2 - R_1} + H - u \right) + \Delta F(u,v) \right) * \sin(v) \\ z(u,v) = u \end{cases}, \tag{6}$$

where $\Delta F$ is the function of deviation of the shape of the actual part from its nominal value;

$R_1, R_2, H$ are the radii of the cone surface and its height;

$u, v$ - surface parameters

$x(u,v), y(u,v), z(u,v)$ - surface point coordinates

A model of a part that has a cone surface is shown in fig. 2.

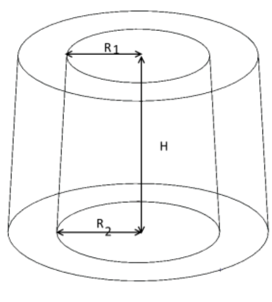


Fig. 2. A model of a part with a cone surface.

Fig. 3 shows a mechanical system model consisting of two parts that have cone surfaces. For parts $K_1$ and $K_2$ the local design coordinate systems $R_1$ and $R_2$ are set. Parts mating is performed along the surfaces $B_1$ and $B_2$. Each surface is set in the local coordinates of the part and is described by the formula (6).

As an assembly procedure, let us consider a translational movement of the second part. This procedure simulates the process of cone rings assembling under press-in technology. The part $K_2$ is lowered onto part $K_1$ until the parts are in contact. For simplicity, let us consider the parts to be absolutely rigid, so that their deformation can be left out of account. In the course of the work, 1000 experiments were carried out on the modeling of the mating of cone surfaces and the amount of data required to carry out the evaluation was collected. Fig. 4 demonstrates the assembled states of the mechanical system.

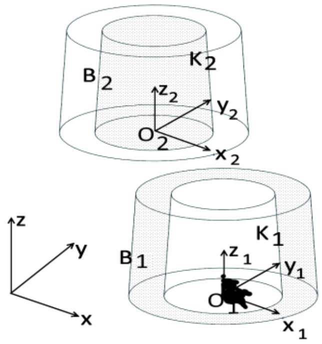


Fig. 3. A mechanical system model for assembling two cone rings.

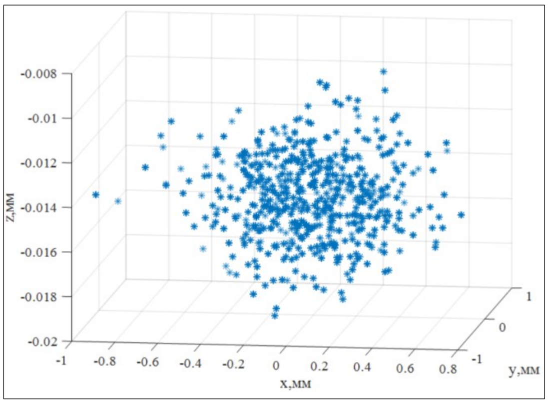


Fig. 4. The assembled states of the mechanical system for the various initial simulation conditions.

The statistical characteristics of the obtained set are specified in Table 1

Table 1. Parameters of the set of assembled states $\Omega$ .

| Parameter | X | Y | Z |
|---|---|---|---|
| Mean value | -0.0066 | -0.0087 | -0.0147 |
| Root-mean-square deviation | 0.2576 | 0.2557 | 0.0018 |

Fig. 5, 6 and 7 demonstrate histograms of the distribution of assembled states along the corresponding coordinates.

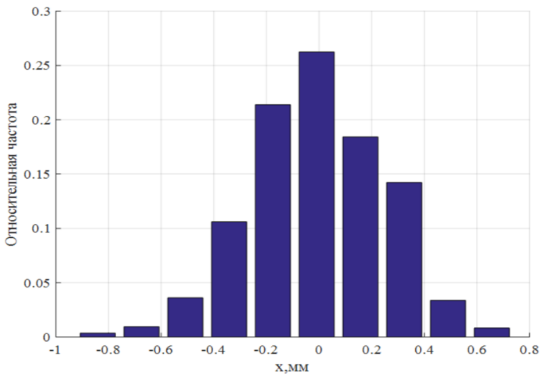


Fig. 5. Distribution density of the x coordinate of the system assembled state.

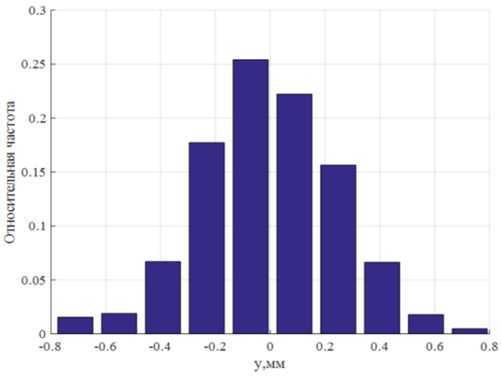


Fig. 6. Distribution density of the y coordinate of the system assembled state.

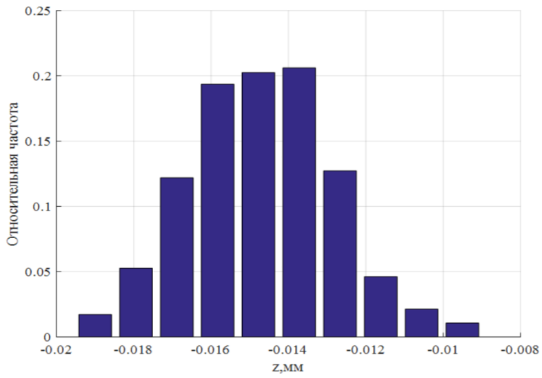


Fig. 7. Distribution density of the z coordinate of the system assembled state.

Based on the simulation results shown in Fig. 4, there was obtained a probability density histogram for the distribution of the assembled states of the system. The obtained chart is an approximate value of the probability density function for the assembled state of the mechanical system. Fig. 8, 9, 10 demonstrate the approximate value of a section of this function for various z heights.

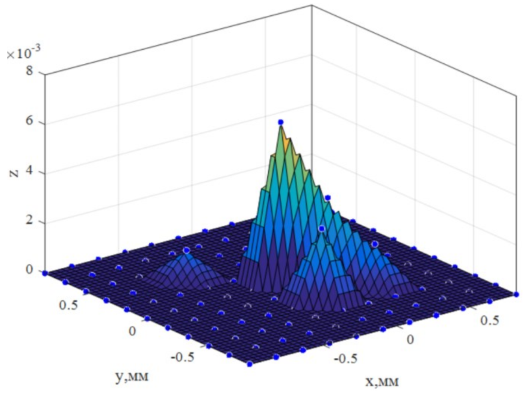


Fig. 8. Section of an approximate probability density value at height z = -0.0189.

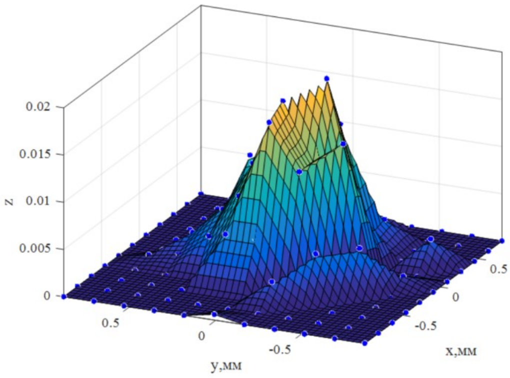


Fig. 9. Section of an approximate probability density value at height z = -0.0154.

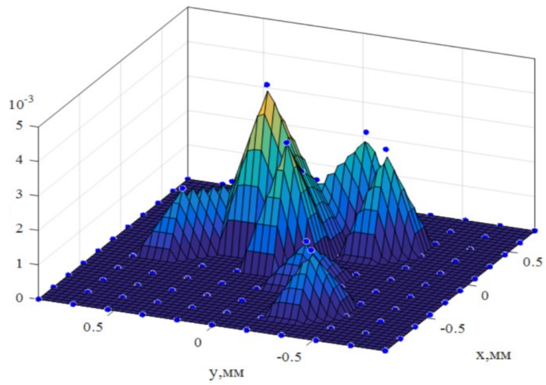


Fig. 10. Section of an approximate probability density value at height z = -0.0108.

The obtained values describe a set of assembled states of the product and can be used to solve further tasks.

## 4. Conclusion

Assessment of the assembly parameters of the products allows to solve a number of important production tasks of the aircraft industry related to the efficiency and the quality of the manufacturing process of the parts. Such an estimation is difficult to implement without processing the product assembly results. One way to get the geometric parameters of an assembled product is numerical simulation of the process of its assembly. The results of multiple simulations can be used to evaluate some assembly parameters.

In the framework of this research, there was proposed a model for estimating the mutual arrangement of parts of the product, which is one of the main assembly parameters. It enables us to obtain many other geometric parameters, such as out-of-true running, out-of-flatness, uneven clearance, etc. The proposed estimation is based on the calculation of the parameters of a set of assembled products: mean and root-mean-square deviation values, approximate probability density. These parameters may be useful for other production tasks. Mean and root-mean-square deviation values for the mutual arrangement of parts can be used to solve the problem of determining rational tolerances for product parameters. The probability density function can be applied to determine the percentage of products that meet the technology requirements. All of these parameters can be used to increase the efficiency of the technological process by identifying the most critical factors influencing the final product quality.

The proposed model was used to assess the mutual arrangement of the parts that have cone surfaces. The next step is to test the results of the numerical simulation in practice.

## Acknowledgments

This work was supported by the Ministry of Education and Science of the Russian Federation in the framework of the implementation of the Program ‘‘Research and development on priority directions of scientific-technological complex of Russia for 2014– 2020”.

## References

[1] Ganchevskaya SV, Skidanov RV. A technique for optimizing the structure of an optical trap to rotate multiple microobjects. Optical Memory and Neural Networks (Information Optics) 2016; 25(3): 160–167.

[2] Grechnikov FV, Khaimovich AI. Development of the requirements template for the information support system in the context of developing new materials involving big data. CEUR Workshop Proceedings 2015; 1490: 364–375. DOI: 10.18287/1613-0073-2015-1490-364-375.

[3] Demin FI, Pronichev ND, Shitarev IL. Technology of turbine engines basic parts manufacture. Samara: Samara State Aerospace University Publishing, 2012; 324 p.

[4] Vdovin RA, Smelov VG, Bolotov MA, Pronichev ND. Paths of Improving the Technological Process of Manufacture of GTE Turbine Blades. IOP Conference Series: Materials Science and Engineering 2016; 142: 1–8. DOI: 10.1088/1757-899X/142/1/012073.

[5] Smelov VG, Sotov AV, Agapovichev AV. Recovery technology features of aerospace parts by layering synthesis. Key Engineering Materials 2016; 684: 316–322. DOI: 10.4028/www.scientific.net/KEM.684.316.

[6] Kolmakova DA, Baturin OV, Popov GM. Effect of manufacturing tolerances on the turbine blades. ASME 2014 Gas Turbine India Conference, GTINDIA, 2014; 1–10.

[7] Kazansky NL, Stepanenko IS, Haimovich AI, Kravchenko SV, Byzov EV, Moiseev MA. Injectional multilens molding parameters optimization. Computer Optics 2016; 40(2): 203–214. DOI: 10.18287/2412-6179-2016-40-2-203-214.

[8] Kirilin A, Shakhmatov E, Soifer V, Akhmetov R, Tkachenko S, Prokofev A. Small satellites "aIST" constellation - design, construction and program of scientific and technological experiments. Procedia Engineering 2015; 104; 43–49. DOI: 10.1016/j.proeng.2015.04.095.

[9] Zubanov V, Shabliy L, Krivcov A. Centrifugal kerosene pump CFD-modeling. Research Journal of Applied Sciences 2014; 9(100: 629–634. DOI: 10.3923/rjasci.2014.629.634.

[10] Pierce RS, Rosen D. Simulation of mating between nonanalytical programing formulation. Journal of Computing and Information Science in Engineering 2007; 7(4): 314–321. DOI: 10.1115/1.2795297.

[11] Samper S, Adragna P-A, Favreliere H, Pillet M. Modeling of 2D and 3D assemblies taking into account form errors of plane surfaces. Journal of Computing and Information Science in Engineering 2009; 9(40): 1–12.

[12] Bolotov MA, Pechenin VA Murzin SP. Method of estimating the indeterminancy of the space mating of precisy optical and mechanical parts. Computer Optics 2016; 40(3): 360–369. DOI: 10.18287/2412-6179-2016-40-3-360-369.

[13] Eliseeva II, Iuzbashev MM. General theory of statistics. M.: Finance and Statistics, 2004; 656 p.