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USING A FRACTIONALIZED INTEGRATOR FOR CONTROL PERFORMANCE ENHANCEMENT

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Abstract. *This paper presents a new approach for robust control against unmodeled system dynamics and additive noises based on the introduction of fractional order functions in the control loop. An intensive research effort is focusing nowadays on fractional order control (FOC). The main reason for this rapidly growing interest is their advantageous properties allowing the improvement of the plant performance and robustness versus external noises and disturbance. The proposed design approach for fractional order systems and their application in control engineering is symmetrically opposed to the classical methodologies which approximate the fractional order operators by a finite set of integer order transfer elements. This new concept consists in approximating integer order transfers by a set of fractional order filters, allowing the system designer to introduce fractional order dynamics and properties to the rational control system under study. The integrator in the PID control algorithm and the classical model reference adaptive control (MRAC) law is fractionalized in order to robustify the control system. The implementation of the fractionalized terms is realized by means of the singularity function numerical approximation method. Illustrative simulation examples show that the disturbance rejection has been significantly improved for both fixed and adaptive controllers. This technique may be easily implemented to a large variety of control schemes.*
Keywords: Fractional order control, Fractional integrator, Robust control, Fractionalization, MRAC, PID control, Noise rejection

1. **Introduction.** Even if the great popularity of fractional calculus is very recent mainly regarding its application in science and engineering, its history goes 300 years back. Particularly, control theory and applications is one of the major fields of application of fractional order systems, with a quickly growing quantity of theoretical and experimental research production [1].

The reason for this success is due to the advantageous properties of fractional order control (FOC) systems and their interesting ability to improve the process robustness against disturbances and noises. A good confirmation is the fact that the first FOC scheme ever proposed in the literature, the so-called "Commande Robuste d'Ordre Non Entier" (CRONE) controller [2], deals with robust control. It uses the constant phase property of the ideal Bode's transfer function $1/s^\alpha$ to obtain a robust feedback control against gain variations.

Another factor is that using fractional order filters in feedback control applications, presents a certain advantageous action on the system dynamical behavior. This is due to the hereditary property of fractional order operators [3] offering an interesting robustness improvement versus external noises [4, 5, 6].

A great research effort is focused nowadays on the design and analysis of new robust fractional order controllers on the basis of the CRONE control approach [2]. Another pioneering contribution was the proposition of combining the classical well-established PID controller with fractional order differentiation, introduced by Podlubny [7]. He developed a generalization of this controller called the $PI^\lambda D^\mu$ controller, involving an integration action of order λ and a differentiation action of order μ. The problem of tuning and performance improvement of fractional order PID controllers was the new challenge towards practical usage of this generalized PID controller in industrial processes (see [8, 9, 10]).

Consequently, the number of robust fractional order control applications is growing exponentially touching various physical processes as can be found in the fractional control literature [11, 12, 13].

In the domain of adaptive control, Model Reference Adaptive Control (MRAC) is a very attractive approach, because it offers a high level of performance while being very easy to implement. This scheme is mainly used to deal with unknown or slowly varying plants. The desired dynamical behavior of this adaptively controlled system is imposed by means of a reference model chosen by the designer. Adjustment of parameters is achieved using the error between the plant's output and the model reference output. Many control researchers are attempting to improve the MRAC robustness in order to deal efficiently in real time with the potential external disturbance that usually affects industrial plants. Control engineering literature counts various robustification techniques that were introduced in elementary adaptive control algorithms (see for instance [14, 15]).

Many efforts have been reported in the literature focusing on new MRAC control concepts design using fractional order differential operators, with important results for temporal behavior and robustness versus external perturbation (see [4, 6, 16, 17, 18] and recently [19]).

This paper proposes a new design approach for fractional order systems and their application in control engineering that is symmetrically opposed to the classical methodologies [2, 20] which approximate the fractional order operators by a finite set of integer order transfer elements (see Figure 1). The new concept consists in approximating integer order transfers in the feedback control loop by a set of fractional order filters. This technique allows the control designer to introduce fractional order dynamics and properties to the rational system under study.

The proposed approach is used for robust control design by introducing fractional order integrators in the classical feedback control loop without changing the overall equivalent closed loop transfer function. The robustification method concerns both of PID and MRAC control schemes.

Considering a transfer function $H(s)$ element of the initial control loop, the idea is to "fractionalize" it as follows [21]:

$$H(s) = H(s)^\alpha \times H(s)^{1-\alpha} \tag{1}$$

where α is a real number such that $0 < \alpha < 1$.

The problem of robust control and disturbance rejection is a permanent challenge and an important issue for feedback control designers (see [22, 23, 24]). Different control techniques have been proposed in order to handle plants with missing information [25,

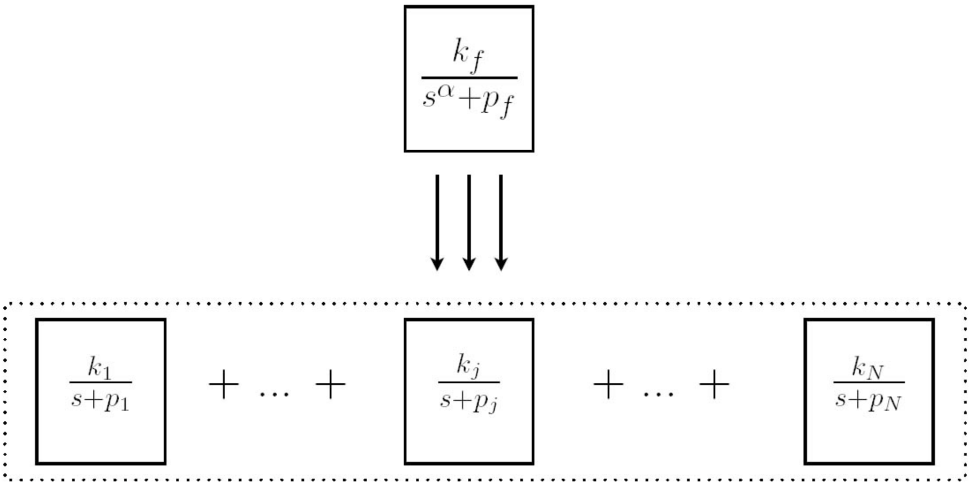

FIGURE 1. Approximation of a fractional order function

26, 27] or in the presence of small disturbances for adaptive controllers [28, 29, 30] and PID controllers [31].

For that concern, our proposed methodology does not change the original global stable control scheme, but may improve its robustness against external noise and perturbation, by taking benefit of the interesting properties of fractional order systems. This new idea even though simple and easy to implement, opens a new maneuvering margin to the design engineers dealing with plants in realistic industrial conditions.

This paper is outlined as follows. Section 2 presents some mathematical background on fractional order integrals and their approximation method in frequency domain together with the Fundamental Predictor-Corrector Algorithm for numerical integration of fractional order differential equations. Section 3 gives a frequential analysis of a fractionalized integrator. In Section 4, a fractionalized PID controller is designed and a numerical simulation example is proposed to illustrate its effectiveness. Then, a fractionalized adaptive controller is introduced in Section 5, with a simulation example given for comparison purpose with the classical control scheme. Section 6 is dedicated to the discussion of the obtained results, and some concluding remarks are then given in Section 7.

2. **Fractional Order Integrators.**

The fractional calculus and fractional order differential equations attracted a great attention these last decades (see [1]). One of the most important reasons for this interest is their ability to model many natural systems and their seducing properties like robustness and dynamical behavior. However, applying fractional-order calculus to dynamic systems control is just a recent focus of interest [2, 7].

2.1. **Mathematical definitions.**

Fractional order differentiation is represented as ${}_aD_t^{\mu}$ where a and t are the bounds and μ ($\mu \in \Re$) the operation order. Many equivalent definitions of this operator have been proposed in the fundamental literature [1]. The most popular definition of the general fractional order differential operator is the Riemann-Liouville (RL) definition:

$$ {}_aD_t^{\mu} f(t) = \frac{1}{\Gamma(1-\mu)} \frac{d^n}{dt^n} \int_a^t (t-\xi)^{-\mu} f(\xi) d(\xi) \tag{2} $$

where $\Gamma(.)$ is the Euler's Gamma function, $(a,t) \in \mathbb{R}^2$ with $a < t$ and n an integer.

The Laplace transform of the RL fractional derivative/integral (2) under zero initial conditions for order μ, $(0 < \mu < 1)$ is given by

$$L\left\{{}_aD_t^{\pm\mu}f(t);s\right\} = s^{\pm\mu}F(s) \tag{3}$$

The following transfer function represents a general form of a single input single output (SISO) fractional order system,

$$F(s) = \frac{b_m s^{\beta_m} + b_{m-1}s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1}s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \tag{4}$$

where α_i and β_j are real numbers such that,

$$\begin{cases} 0 \le \alpha_0 < \alpha_1 < \ldots < \alpha_n \\ 0 \le \beta_0 < \beta_1 < \ldots < \beta_m \end{cases}$$

and s is Laplace operator.

2.2. **Linear approximation of the fractional order integrator.** To practically and easily implement the designed controller has to be of finite dimension whereas the fractional order controllers result in infinite dimension functions.

The approximate implementation of FOC can be classified into

(1) analog approximate implementation method, and
(2) digital approximate implementation method.

In practice, the approximation is preferred in digital form for direct implementation in computer controlled systems [32].

For the purpose of our approach we need to use an integer order model approximation of the fractional order transfer function considered in the original control loop. For this aim, we will make use of the so-called *singularity function method* [20]. This method allows the approximation of a fractional order transfer function by a rational function with a finite number of poles and zeros.

2.3. **The Fundamental Predictor-Corrector Algorithm.** The following paragraph recalls the fractional Adams-Bashforth-Moulton method introduced in [33], that we shall later use it to approximate the fractional order integral operator. In fact it is more practical to use a numerical fractional integration method rather than the transfer function approximation methods presented in Section 2.2 to compute fractional order integration or derivation as the approximating transfer functions are of relatively high orders.

Consider the differential equation

$$D^{\alpha}y(x) = f\left(x, y(x)\right) \tag{5}$$

with initial conditions:

$$y^{(k)}(0) = y_0^{(k)}, \quad k = 0, 1, \ldots, m-1 \tag{6}$$

where $m = [\alpha]$ and the real numbers $y^{(k)}(0)$, $k = 0, 1, \ldots, m-1$, are assumed to be given.

This approach is based on the analytical property that the initial value problem (5), (6) is equivalent to the Volterra integral equation

$$y(x) = \sum_{k=0}^{[\alpha]-1} y^{(k)}(0)\frac{x^k}{k!} + \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f\left(t, y(t)\right)dt \tag{7}$$

Introducing the equispaced nodes $t_j = jh$ with some given $h > 0$ and by using the product trapezoidal quadrature formula with these nodes to replace the integral in (7), the corrector formula is given by

$$y_h(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} \frac{t_{n+1}}{k!} y^{(k)}(0) + \frac{h^\alpha}{\Gamma(\alpha+2)} f\left(t_{n+1}, y_h^P(t_{n+1})\right) + \frac{h^\alpha}{\Gamma(\alpha+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, y_h(t_j)\right) \quad (8)$$

where

$$\begin{aligned} a_{0,n+1} &= n^{\alpha+1} - (n-\alpha)(n+1)^\alpha \\ a_{j,n+1} &= (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1} \quad (1 \le j \le n) \end{aligned} \quad (9)$$

and $y_h^P(t_{n+1})$ is given by,

$$y_h^P(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} \frac{t_{n+1}}{k!} y^{(k)}(0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f\left(t_n, y_h(t_j)\right) \quad (10)$$

where now

$$b_{j,n+1} = \frac{h^\alpha}{\alpha}\left((n+1-j)^\alpha - ((n-j)^\alpha\right), \quad (0 \le j \le n) \quad (11)$$

3. **Frequency Domain Analysis of a Fractionalized Integrator.** In this section, the proposed fractionalization approach is analyzed by considering its application to transfer functions elements of a feedback control system given in (1). In order to show the effectiveness of this technique, let us consider an integrator given by its Laplace transform:

$$G(s) = \frac{1}{s} \quad (12)$$

The fractionalization of the classical integrator (12) as represented in Figure 2 leads to,

$$\frac{1}{s} = \frac{1}{s^\alpha} \cdot \frac{1}{s^{1-\alpha}} \quad (13)$$

where α is a real number such that $0 < \alpha < 1$. Let us take the fractional value $\alpha = 0.4$.

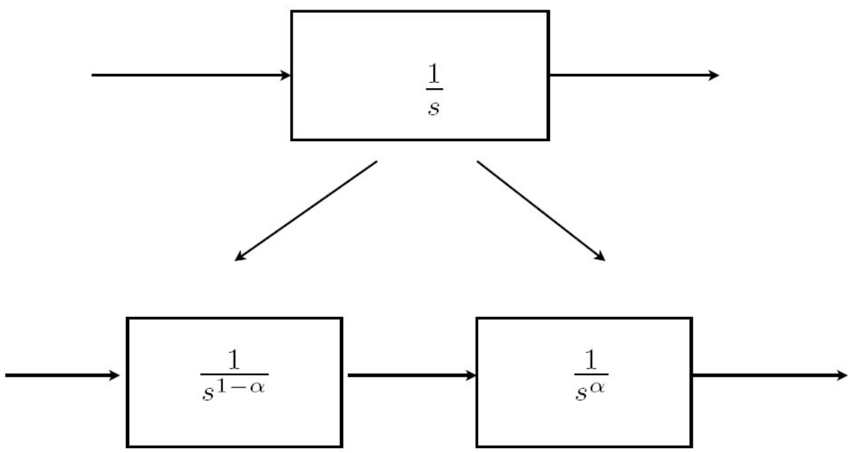

FIGURE 2. Fractionalization of integral operator

Using the singularity approximation method presented in Section 2.2 with the approximation parameters: $\omega_b = 0.1\text{rad/sec}$, $\omega_h = 1000\text{rad/sec}$, $\delta = 1.5d\beta$, we get the approximated functions $h_\alpha(s)$ and $h_{1-\alpha}(s)$ given bellow.

$$h_\alpha(s) = h_{0.4}(s) = \frac{3.862\times10^{-13}s^{10}+6.846\times10^{-9}s^9+2.326\times10^{-5}s^8+0.01793s^7+3.243s^6+138.8s^5+1406s^4+3372s^3+1897s^2+242.1s+5.923}{6.429\times10^{-16}s^{11}+2.703\times10^{-11}s^{10}+2.178\times10^{-7}s^9+0.000398s^8+0.1707s^7+17.33s^6+416.7s^5+2375s^4+3201s^3+1013s^2+72.69s+1}$$

and

$$h_{1-\alpha}(s) = h_{0.6}(s) = \frac{9.045\times10^{-17}s^{11}+5.882\times10^{-12}s^{10}+7.332\times10^{-8}s^9+0.0002073s^8+0.1376s^7+21.6s^6+803.6s^5+7084s^4+1.477\times10^4s^3+7231s^2+802.9s+17.09}{8.002\times10^{-19}s^{12}+9.254\times10^{-14}s^{11}+2.051\times10^{-9}s^{10}+1.031\times10^{-5}s^9+0.01217s^8+3.398s^7+224.8s^6+3527s^5+1.311\times10^4s^4+1.153\times10^4s^3+2380s^2+111.4s+1}$$

Figure 3 shows a comparison in the frequency domain between the integer order integral operator $\frac{1}{s}$ and the product of the approximating filters of the fractional order integral operators $\frac{1}{s^{0.4}}$ and $\frac{1}{s^{0.6}}$, that is $h_{0.4}(s) \times h_{0.6}(s)$. It is clear that this filters product gives a satisfactory approximation of the integral operator in the frequency interval of interest.

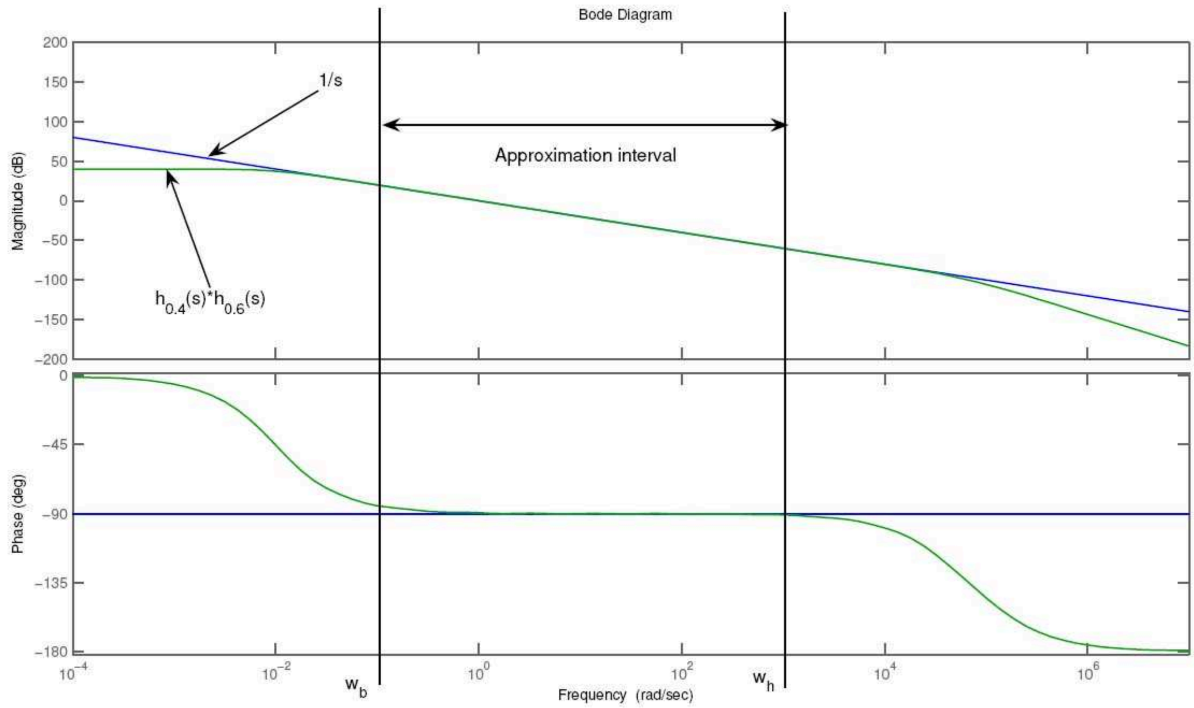

FIGURE 3. Comparison of the integration $\frac{1}{s}$ with the approximation $h_{0.4}(s) \times h_{0.6}(s)$

4. **Fractionalized PID Control.** The PID control scheme is modified here to get more robustness against noise and perturbation. The new PID control law is obtained by using the fractionalization of a control system element, and the integral operator $1/s$ is fractionalized as represented in (13) and Figure 2; that is,

$$\frac{1}{s} = \frac{1}{s^\alpha} \cdot \frac{1}{s^{1-\alpha}}$$

where α is a real number such that $0 < \alpha < 1$.

The feedback control loop of the fractionalized integer order system is shown in Figure 4.

In Figure 4

- C_f is the fractionalized controller transfer function,
- $G(s)$ is the system or plant transfer function,
- y is the output signal,

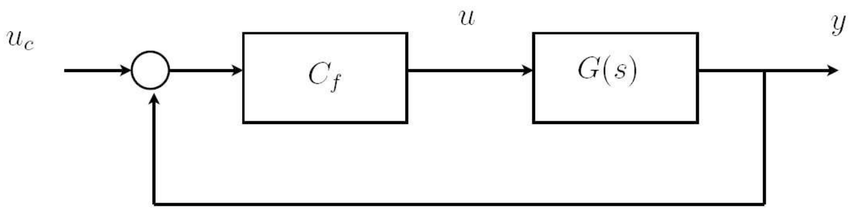

Figure 4. Fractionalized feedback control scheme

- u_c is the reference signal,
- u is the control signal.

The fractionalization of the integer-order PID controller to be designed is given in the following form,

$$\begin{aligned} C_f(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= \frac{1}{s}\left(K_p T_d s^2 + K_p s + \frac{K_p}{T_i}\right) \\ &= \frac{1}{s^\alpha} \cdot \frac{1}{s^{1-\alpha}}\left(K_p T_d s^2 + K_p s + \frac{K_p}{T_i}\right) \end{aligned} \tag{14}$$

where, $0 < \alpha < 1$.

Simulation example. To show the viability of the proposed robustified control design, let us study an illustrative example. The plant model is given by the following transfer function:

$$G(s) = \frac{10(s+1)}{s^2 + 2s + 2} \tag{15}$$

A PID controller is designed using the Ziegler-Nichols rule for the the system model $G(s)$ with the following PID parameters: $K_p = 10$, $K_i = 0.6$, $K_d = 1.5$.

The resulting PID controller is 'fractionalized' as shown in (15) with integrator fractional order $\alpha = 0.3$. Applied to the system $G(s)$ of (15) in the case of ideal conditions and in presence of additive noises, we get the responses of Figure 5 and Figure 6.

To have a more comprehensive idea on the robustness improvement obtained by using the fractionalization method, we will compare the results for different values of the fractional order integral α.

The evaluation of the control system performance will be realized by defining a quadratic error criterion J_α given by,

$$J_\alpha = \int_{t_I}^{t_F} (y(t) - u_c(t))^2 dt \tag{16}$$

where α is the order of integration in the fractionalized PID (in the classical control scheme, $\alpha = 1$). t_F is the time window limit and t_I is the criterion calculus beginning time (chosen to avoid the convergence phase).

Taking $t_I = 0.2$ s and $t_F = 1$ s, we obtain the criterion values of Table 1, for different values of the fractionalizing integration order α.

The comparison of the conventional PID control and the proposed robust fractional order control algorithm is provided in Table 1. With the proposed method, noise behavior can be reduced with about 22% of magnitude and the tuning technique is very simple because there is no requirement of PID parameters' adjustment.

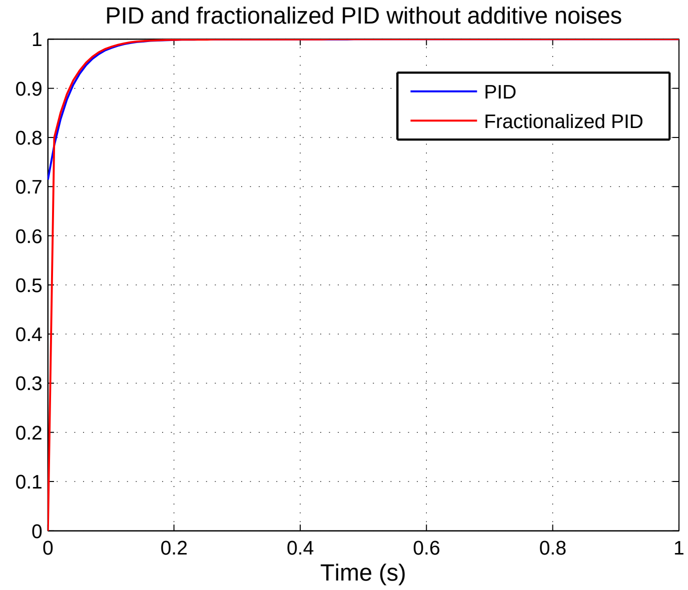

FIGURE 5. System responses to PID and fractionalized PID controllers for $\alpha = 0.3$ (Ideal case)

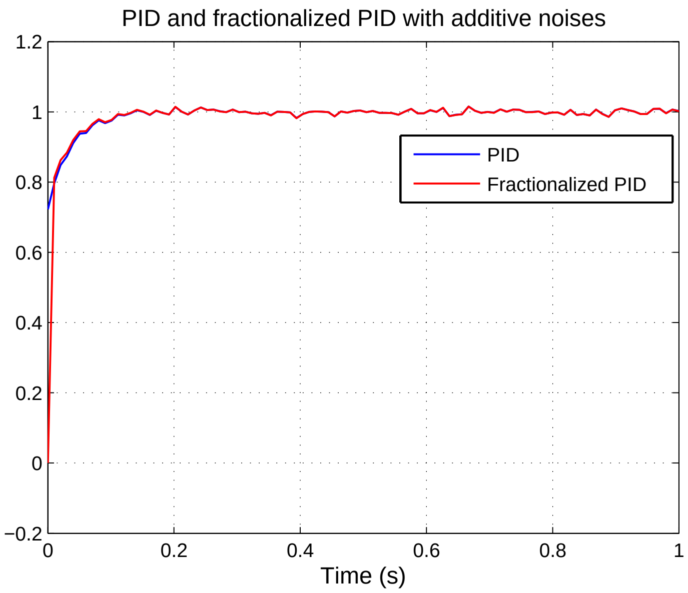

FIGURE 6. System responses to PID and fractionalized PID controllers for $\alpha = 0.3$ (In presence of additive noises)

TABLE 1. PID quadratic error criterion versus fractional integration order α in case of additional input noises

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
J_α	0.0483	0.0468	0.0476	0.0464	0.0455	0.0466	0.0480	0.0472	0.0488	0.0581

5. **Fractionalized Adaptive Control.** In this paper, the Model Reference Adaptive Control (MRAC) strategy, and particularly the M.I.T. Rule algorithm is considered, because of its simplicity and its high level of performance. A new adaptive scheme is proposed, based on the introduction of fractional order operators in order to improve

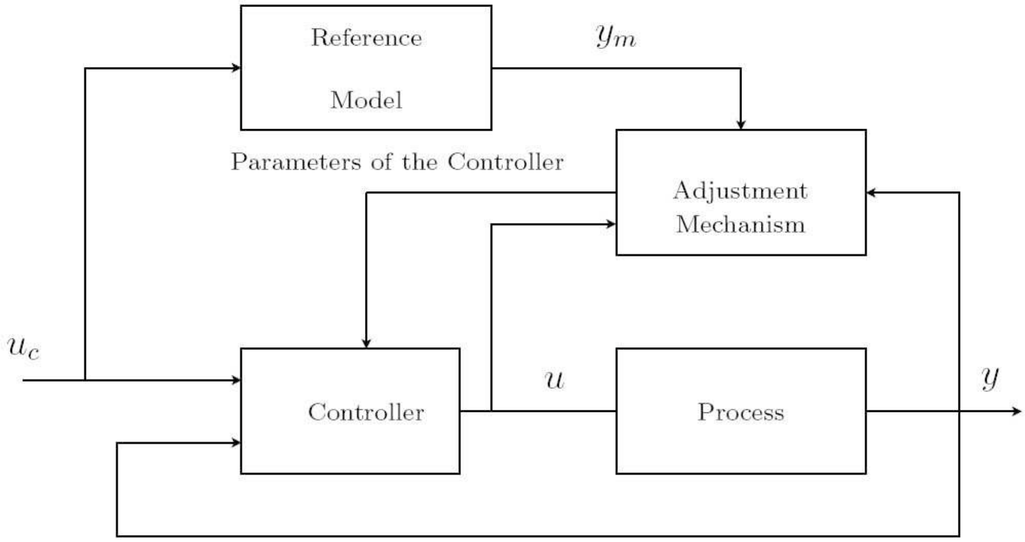

FIGURE 7. Direct Model Reference Adaptive Control

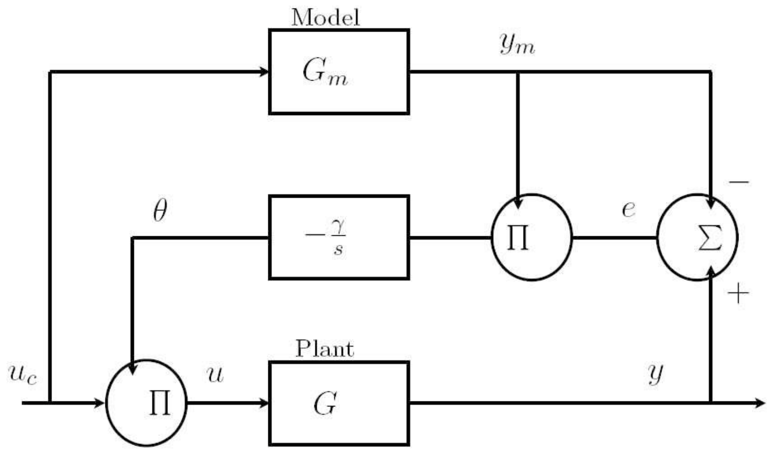

FIGURE 8. Classical adaptation algorithm

the robustness of MRAC against the effect of external noises and perturbations. In this approach the desired performance is specified by the choice of a reference model. Adjustment of parameters is achieved by means of the error between the plant's output and the model reference output. This can be represented in Figure 7 (see [34]).

5.1. **M.I.T. rule.** We consider a closed loop system where the controller has an adjustable parameter vector θ. A model whose output is y_m specifies the desired closed loop response. Let e be the error between the closed loop system output y and the model one y_m, a possibility is to adjust the parameters so that the cost function:

$$J(\theta) = \frac{1}{2}e^2 \tag{17}$$

is minimized. This leads to the normalized control law given by [34],

$$\theta = -\frac{\gamma}{s} \cdot \frac{e\varphi}{\epsilon + \varphi^T \cdot \varphi} \tag{18}$$

where $\epsilon > 0$ is a real number. This last formula indeed improves the stability of the adaptive control loop against high variations of signals.

The control signal is computed using the following relation,

$$u = \varphi^T \theta \tag{19}$$

where φ is the regression vector containing the measured input and output signals u and y and the input reference signal u_c.

This leads to the scheme of Figure 8, where:

- G_m: Reference model transfer function
- G: Plant transfer function
- u_c: Reference signal
- u: Command signal
- y: Plant output
- y_m: Reference model output
- θ: Parameter vector
- γ: Adaptation gain

5.2. **The proposed fractionalized adaptation law.** In order to improve the system's robustness against environmental disturbances we will modify the MRAC control algorithm, by introducing data filtering blocks. The key idea for improving the original control law is fractionalization of the integral action in the MIT rule as presented in (1).

The integral operator $1/s$ in the adaptation law (18) is developed as explained in Figure 2 and Equation (13),

$$\frac{1}{s} = \frac{1}{s^\alpha} \cdot \frac{1}{s^{1-\alpha}}$$

where α is a real number such that $0 < \alpha < 1$.

Replacing in the adaptive control law (18) we get,

$$\theta = -\frac{\gamma}{s^\alpha \cdot s^{1-\alpha}} \cdot \frac{e\varphi}{\epsilon + \varphi^T \cdot \varphi} \tag{20}$$

and by defining the variable ξ we get,

$$\begin{aligned} \xi &= \frac{1}{s^{1-\alpha}} \cdot \frac{e\varphi}{\epsilon + \varphi^T \cdot \varphi} \\ \theta &= -\frac{\gamma}{s^\alpha} \cdot \xi \end{aligned} \tag{21}$$

which is the new fractionalized adaptation law represented in Figure 9. We can write also:

$$\begin{aligned} {}_0D_t^{1-\alpha}\xi(t) &= \frac{e\varphi}{\epsilon + \varphi^T \cdot \varphi} \\ {}_0D_t^{\alpha}\theta(t) &= -\gamma \cdot \xi(t) \end{aligned} \tag{22}$$

The Adams-Bashforth-Moulton method of Equations (8)-(11) presented in Section 2.3 is then used to compute a numerical approximation of $\theta(t)$ from (22).

5.3. **Simulation example.** The resulting fractionalized model reference adaptive control algorithm is now applied in numerical simulation to the control of the following transfer function which represents the experimentally identified model of a DC motor for the online angular velocity control [35],

$$G(s) = \frac{81018}{s^2 + 260.7s + 2394} \tag{23}$$

and the standard second order reference model given by

$$G_m(s) = \frac{100}{s^2 + 9s + 100} \tag{24}$$

5.3.1. *Classical MRAC response.* Applying the classical MRAC scheme of Figure 8 to the plant model (23) with the reference model (24), we obtain the simulation response of Figure 10 with the following initial parameters values: $u(0) = 0$, $\theta_0 = 0.01[1 \ \ -3 \ 2 \ 1]^T$, $\gamma = 0.7$ and $\epsilon = 1$.

5.3.2. *Fractionalized MRAC response.* Now we apply the fractional MRAC of Figure 9 with the new adaptation algorithm of Equation (22) taking the order $\alpha = 0.4$. We obtain

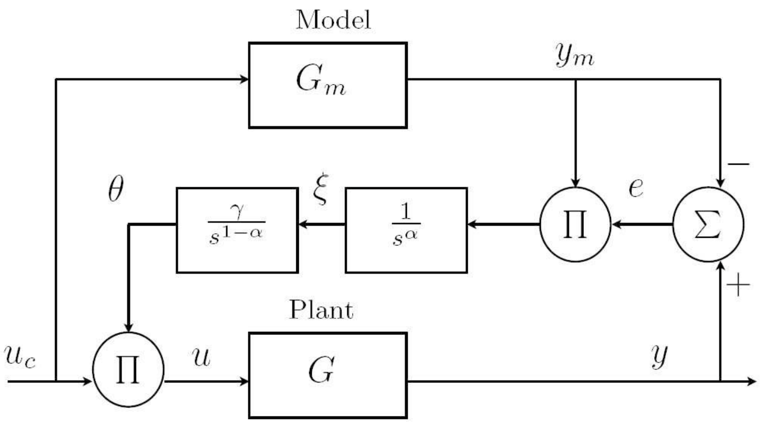

FIGURE 9. Fractionalized adaptation algorithm

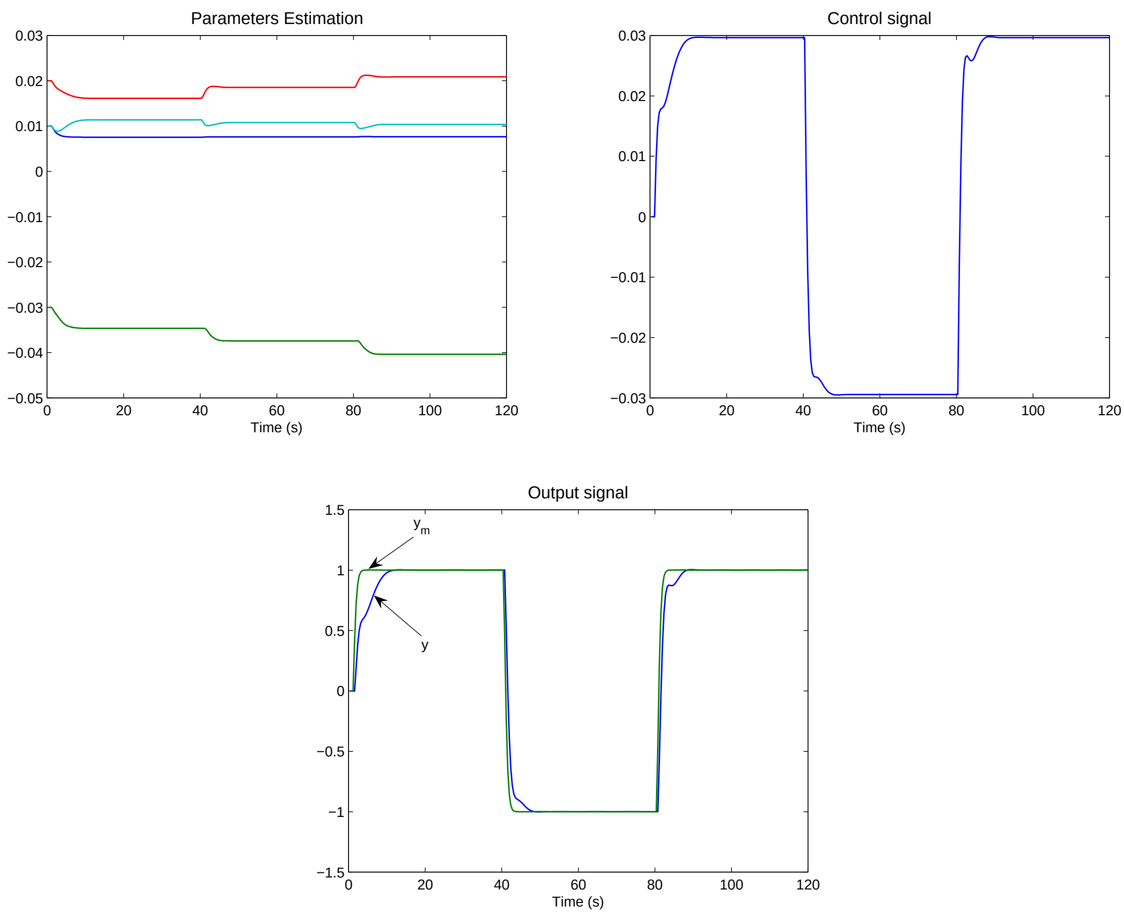

FIGURE 10. Process behavior with classical MRAC

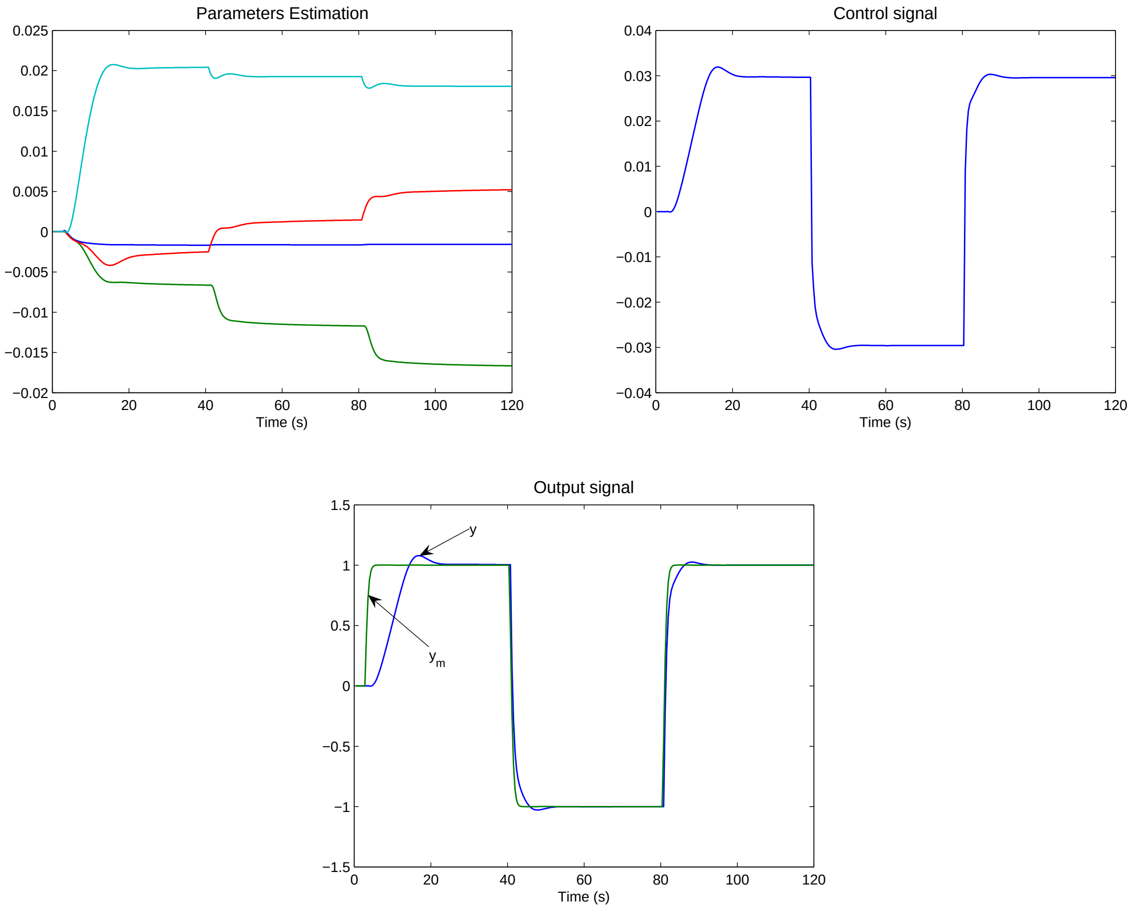

FIGURE 11. Process behavior with fractionalized MRAC for $\alpha = 0.4$

the simulation response of Figure 11 with the following initial parameters values: $u(0) = 0$, $\theta_0 = [0\ 0\ 0\ 0]^T$, $\gamma = 0.0007$ and $\epsilon = 0.001$.

Figure 11 illustrates the good behavior of the controlled plant under the proposed adaptive control scheme. One has to explain the apparent settling time extension (longer parameter convergence time) by the necessary time for additional calculus brought by the fractional order integral approximation involving a growing number of measure data.

5.3.3. *Control robustness in noisy conditions.* In order to validate the proposed fractionalized approach described in the previous sections, regarding the system robustness enhancement, let us perturb the controlled plant by introducing an additional external random noise to the output with an amplitude of 8% of the reference signal one. Applying the classical MRAC control algorithm to the perturbed plant we obtain the simulation results of Figure 12.

The proposed fractionalized adaptive controller gives the response of Figure 13.

From a simple visual comparison it is obvious that the output variations around the reference signal are less important in the case of the fractionalized scheme, which illustrates the effectiveness of this proposed approach to reject additional noises and disturbance.

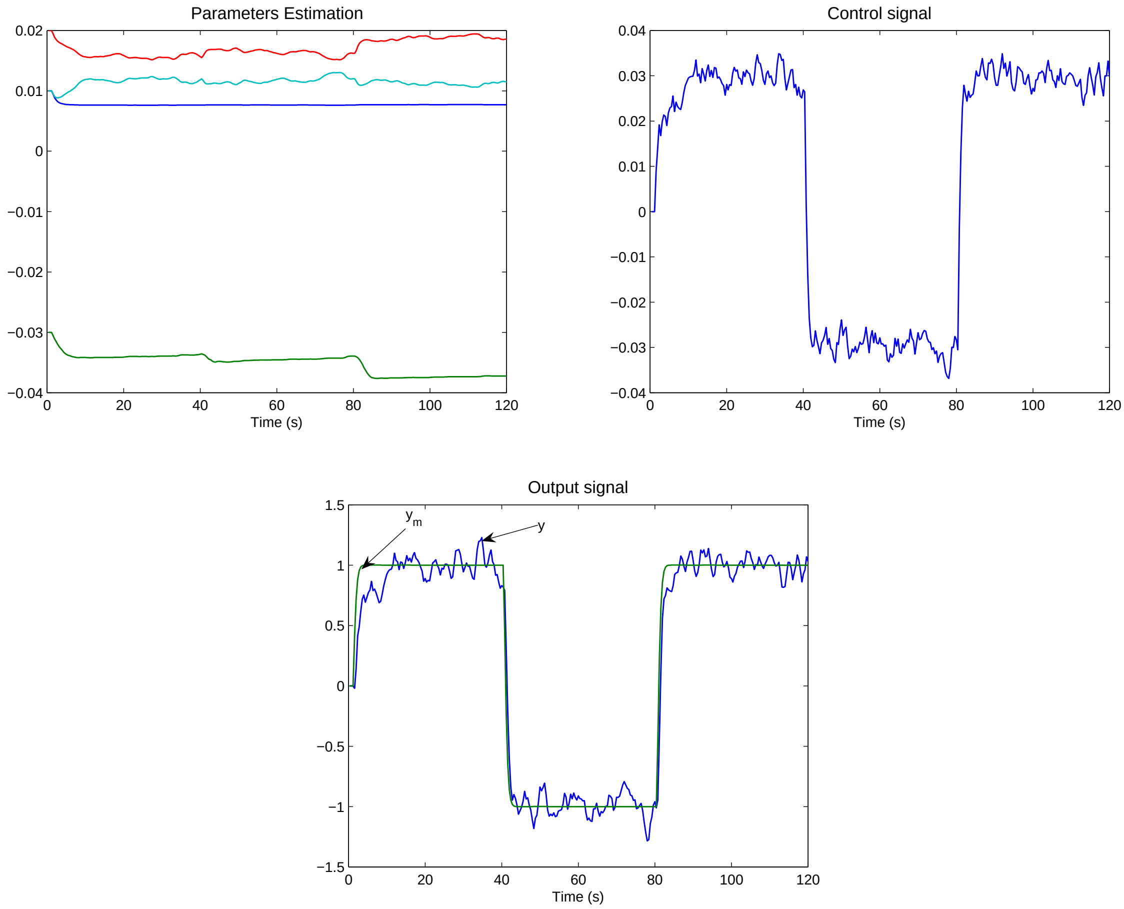

FIGURE 12. Process behavior with classical MRAC with random output noise of 8% of the reference signal amplitude

In order to be able to quantify the control system performance evaluation, let us define a quadratic error objective criterion J_α similar to (16), given by,

$$J_\alpha = \int_{t_I}^{t_F} (y(t) - y_m(t))^2 dt \tag{25}$$

where α is the order of integration in the fractionalized MRAC.

Taking $t_I = 20$ s and $t_F = 120$ s, we obtain the cost function values of Table 2, for different values of the fractionalizing integration order α.

TABLE 2. Fractional MRAC objective function in case of random output noise of 8% of the reference signal magnitude

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
J_α	2.67	2.28	2.24	2.88	2.64	2.97	2.77	2.97	2.82	5.61

One may remark from Table 2 that we obtain a certain diminution of the noise effect of about 50% comparatively to the integer order MRAC results in the scope of this criterion.

6. **Discussion and Remarks.** The proposed fractionalization technique is based on the replacement of rational (integer order) transfer function by a cascaded fractional order elements. The global feedback control system must be equivalent to the original

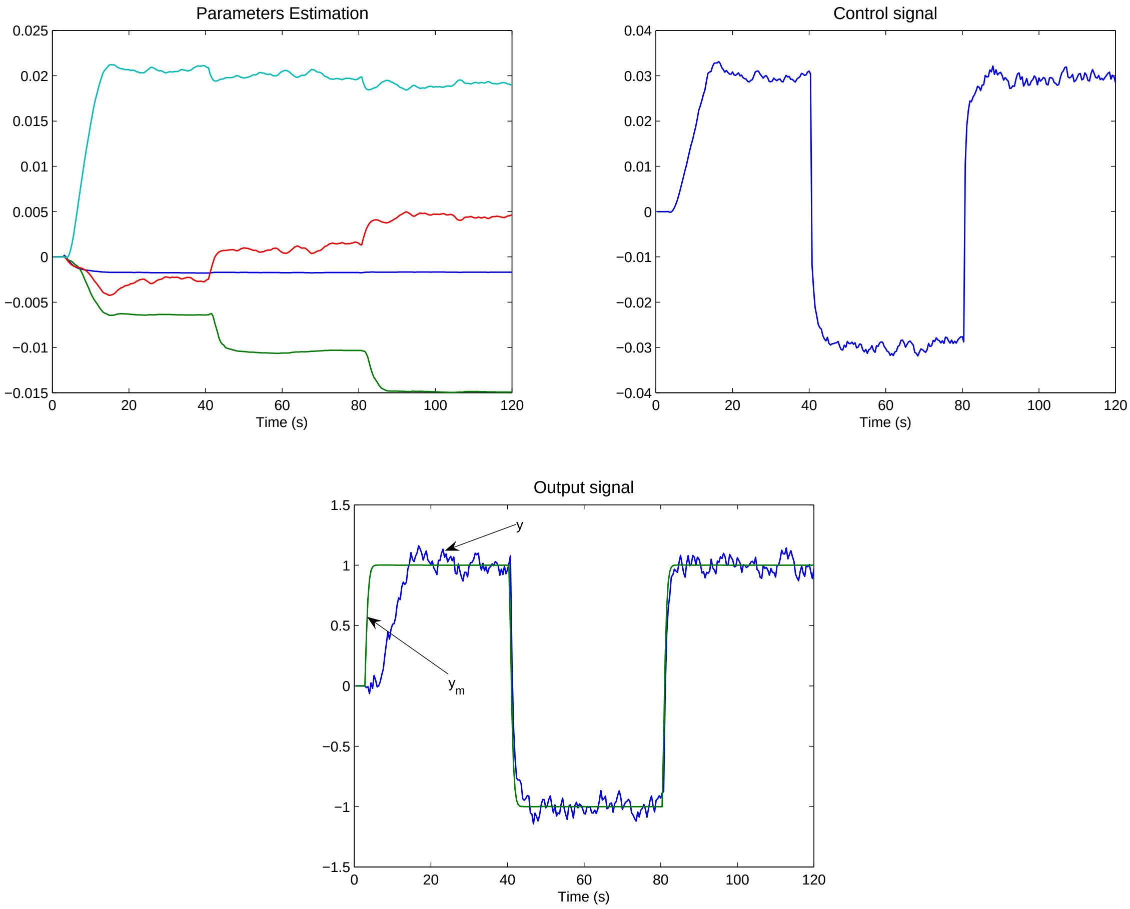

FIGURE 13. Process behavior with Fractionalized MRAC for $\alpha = 0.4$, with random output noise of 8% of the reference signal amplitude

one, for a tolerated approximation error and a working frequency bandwidth. It opens new perspectives for designing more robust and reliable control systems. The following comments can emerge from the examination of the two illustrative applications examples.

- The examination of the numerical example results shows that the use of the fractionalization approach is advantageous for disturbance rejection because as it appears in (2), the calculus of the fractional derivative is dependent on all the history of the signal, which moderates the effect of variations and external random noises [4, 36]. This robustification leaded to an improvement in noise rejection of about 20% for PID control and 50% for MRAC control.
- Notice that the gain parameter γ value is much smaller in the case of fractional integrators, which advantageously augments the stability margin of the overall adaptive control system [4, 17].
- The proposed fractionalized MRAC control is less sensitive to the initial values of the parameter vector θ than for the classical MRAC case. In fact, the initial value θ_0 has been fixed to zero for all the values of the fractional integral order α in Table 2.
- This new robustification approach can be implemented in a larger class of adaptive and non adaptive control systems as MRAC and PID controllers (fractionalization of the integer order integrator).

7. **Conclusions.** A novel approach for robust control has been proposed. This technique consists in introducing fractional order integrators in the classical feedback control loop without changing the overall equivalent closed loop transfer function.

The new control structure is based on the simple idea of using fractional order filters in the control system loop instead of the pure integrator. The fractionalization approach leads to a similar behavior in perfect experimental conditions but brings an important improvement in noises rejection and robustness. Thus, more conservative performances are guaranteed with a very easy implementation procedure.

In particular, the reported numerical examples for both fixed and adaptive control schemes, show that if fractional order double-integrators are introduced, significant reduction in the output-tracking errors can be achieved with the classical integer order integral scheme in presence of disturbances and additive noises.

Further research will concern the extension of this technique to the fractionalization of more general integer order functions, in order to obtain desired fractional dynamics in the closed loop feedback control system.

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