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TIRE'S PARAMETERS IDENTIFICATION BASED ON THE LUGRE FRICTION MODEL

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Abstract. *To study the tire's friction characteristics of the LuGre model, an identification method of the tire's parameters is put forward to obtain the static and dynamic parameters. According to the identified static parameters, a relationship curve between the adhesion coefficient and the slip ratio is plotted, which indicates that the simulation result agrees well with measured results of experiment. In addition, the sensitivity of static parameters* μ_c*,* μ_s *and* σ_2 *to different experimental variables, including the normal force* F_n*, sideslip angle* γ *and slip ratio* s *is analyzed. Experimental results show that the static parameters are sensitive to different experimental conditions.*

Keywords: The LuGre model, Static parameters, Relationship curve, Sensitivity

1. **Introduction.** Establishing a tire model is necessary to describe the structural parameters and mechanical characteristics of tires, which are most associated with vehicle safety, performance, and handling stability [1]. A tire model usually can be divided into a static model and a dynamic model. The static model builds up the relationship between a vehicle's velocity and tire-road friction under a steady state. The typical model is the "Magic Formula" model, which is derived from experimental data to produce a good fit under a steady state (i.e., constant linear and angular velocity) [2-4]. In reality, linear and angular velocity can never be controlled independently and hence, such idealized steady-state conditions are not reached except during the rather particular case of cruising at a constant speed. Furthermore, the Magic Formula model cannot explain the mechanism of a tire's friction dynamically and cannot indicate instantaneous characteristics of tire friction. In addition, most of the existing model-based friction characteristics use a combination of classical friction models, such as the Fiala and Gim models [5,6]. However, in applications with high-precision positioning and low-velocity tracking, the results are not always satisfactory [7,8]. Therefore, to better describe the dynamic friction characteristics and study the parameters identification, the LuGre friction model is used in this paper [9].

According to different experimental conditions, the friction parameters identification can be achieved by statically calculating different experimental data. Also, the LuGre friction model has an advantage that agrees well with experimental results, which means the parameters identification based on this model is feasible and reliable. In contrast to many other static models, this model is appropriate for any vehicle motion situations and for the development of a vehicle dynamics control system relevant to tires. This is especially important during transient phases of vehicle operation, such as braking or acceleration [10].

2. **LuGre Tire Model.** The LuGre tire model is a typical dynamic model. It regards friction as reciprocity between the elastic bristle of the interface with random behavior at the microscopic level. When a tangential force acts on the bristles, the bristle deforms elastically, like a spring, and begins to slip when the distortion is large enough.

The centralized equations based on the LuGre tire model under different road surface conditions are expressed as follows:

$$\frac{dz}{dt} = v_r - \frac{\sigma_0 \left| v_r \right|}{\theta \cdot g(v_r)} \cdot z \tag{1}$$

$$F_x = \left(\sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v_r \right) \cdot F_n \tag{2}$$

$$g(v_r) = \mu_c + (\mu_s - \mu_c) e^{-|v_r/v_s|^{\alpha}} \tag{3}$$

where z denotes the average elastic deflection of the bristles; σ_0 and σ_1 are the longitudinal stiffness and damping coefficients of the bristles, respectively; σ_2 is the relative viscosity damping coefficient; θ is the road surface conditions, which is set as 1 on dry asphalt, 0.65 on wet asphalt, and 0.15 on snow asphalt; F_n is the normal force on the tire; v_r is the relative velocity between the wheel and the vehicle body, and $g(v_r)$ is the positive sliding function that can be used to describe different friction effects. Equation (3) is the typical parameterization of $g(v_r)$ to characterize the Stribeck effect. In Equation (3), v_s is the Stribeck velocity, μ_c is the Coulomb friction coefficient, μ_s is the static friction coefficient, and α is the Stribeck index of friction characteristics, which usually ranges from 0.5-2 under the steady state .

In the LuGre friction model, the parameters that needed to be identified include the static parameters and dynamic parameters, and the static parameters can be identified from the experimental data. However, the dynamic parameters identification is relatively difficult. Table 1 shows the value of the experimental variables, which is used to identify the static parameters.

TABLE 1. Value of experimental variables

Parameter	*Value*
F_n	4000N
v_r	20m/s
r	0.32m
L	0.2m
v_s	5m/s
α	0.5
θ	1/0.65/0.15

3. **Parameter Identification Based on the LuGre Friction Model.** In the centralized LuGre friction model shown in Equations (1) to (3), parameters v_r and F_n can be measured by the experiment, and parameters α and v_s usually are regarded as constants; thus, the parameters that need to be identified are μ_c, μ_s, σ_0, σ_1 and σ_2 [11]. The static friction parameters are μ_c, μ_s and σ_2 and the dynamic parameters are σ_0 and σ_1.

3.1. **Static parameters identification.** Tires are under a steady state when the average deflection of bristles is constant (i.e., $\frac{\partial z}{\partial t} = 0$), and in this case the longitudinal force of steady-state condition is expressed as:

$$F_{ss} = \left(\theta \cdot \left(\mu_c + (\mu_s - \mu_c) \cdot e^{-|v_r/v_s|^{\alpha}} \right) \cdot \mathrm{sgn}(v_r) + \sigma_2 \cdot v_r \right) \cdot F_n \tag{4}$$

The relationship between the longitudinal force F_x and the normal force F_n is given as:

$$F_x = \mu F_n \tag{5}$$

However, the coefficient of road adhesion μ is impossible to define accurately because of its relationship with many factors, including the inherent characteristics of tires, road surface conditions, and relative velocity. The relationship curve μ-s between the coefficient of road adhesion μ and the slip ratio s usually is used to describe the sliding operating mode in a practical application. The LuGre tire friction model based on the relationship curve μ-s is expressed as follows [12]:

$$\mu(s, w) = \theta \cdot g(s) \left[1 - z_s \left(1 - e^{-L/z_s}\right)/L\right] + \sigma_2 \cdot r\omega s/(1 - s) \tag{6}$$

$$z_{ss} = \theta \cdot g(s)(1 - s)/(\sigma_0 \cdot s) \tag{7}$$

$$g(s) = \mu_c + (\mu_s - \mu_c) \cdot e^{-|rws/(v_s(1-s))|^\alpha} \tag{8}$$

where the slip ratio s is given as the range of $s \in [0, 1]$, z_{ss} is the average elastic deflection of the bristles under the steady state, L is the length of rectangular grounding contact area, and ω is the angular velocity of the wheel.

The static parameters are identified through a nonlinear least squares method based on corresponding experimental data by using Equation (4). According the identification method mentioned above, the static parameters identification is simulated based on the LuGre friction model. Table 2 gives out the exact static parameters' values and the identified static parameters' results.

Also, the relationship curve μ-s between the coefficient of road adhesion and the slip ratio can be obtained by using the identified static parameters, as shown in Figure 1. In Equation (6), parameter σ_2 is approaching approximately zero and, hence, the value of the algebraic term $\sigma_2 * r\omega_s/(1 - s)$ is regarded as approximately zero.

As shown in Figure 1, the fitting curve according to the identified static parameters based on the LuGre tire model agrees well with the measured results of experiment, which can dynamically describe the changing laws of tire sliding characteristics under different coefficients of road adhesion.

Table 2. The exact static parameters' and the identified static parameters' results

Parameter (unit)	μ_c	μ_s	σ_2 *(Ns/m)*
Exact parameters	0.8	1.4	0.74×10^{-5}
Identified parameters	0.75	1.45	0.66×10^{-5}
Error	6.25%	3.57%	10.8%

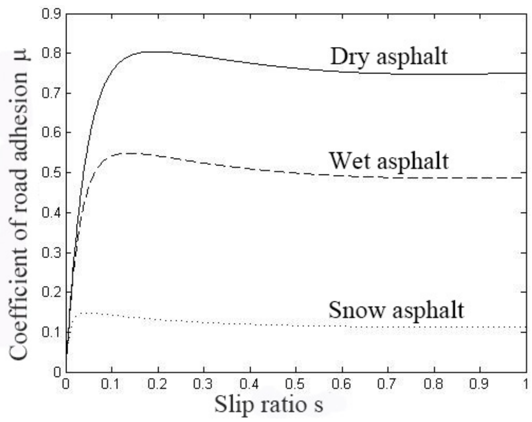

Figure 1. Relationship curve μ-s between coefficient of road adhesion and slip ratio

3.2. **Dynamic parameters identification.** The LuGre model is a typical nonlinear system, the elastic deflection z is immeasurable, and the dynamic parameters are very difficult to identify because of the coupling effect among the static and dynamic parameters. Dynamic parameters usually are identified with a traditional approximation method. However, the identified parameters in this method depend primarily on the selection of the initial parameters, and the identification precision and convergence of the results cannot be guaranteed. In this paper, the dynamic parameters identified by means of a genetic algorithm based on the limit of the system oscillation loop curve can overcome these shortcomings [13].

Based on the identified static parameters mentioned above, and according to Equations (1)-(3), the dynamic parameters can be identified by means of a genetic algorithm method. Supposing that the dynamic parameter vector is identified as $X_d = [\sigma_0, \sigma_1]^T$, taking the wheel angle as identification error, which can be expressed as the following equation.

$$e(t_i) = \theta(t_i) - \hat{\theta}(t_i) \; (i = 1, 2, \ldots, M) \tag{9}$$

where $\theta(t_i)$ is the actual output wheel angle at time t_i, and $\hat{\theta}(t_i)$ is the identified output wheel angle at time t_i.

And a closed-loop PID control scheme was adopted to identify the dynamic parameters. The control law of the PID is expressed as the following equation.

$$u(t) = k_p e + k_d \dot{e} + k_i \int e dt \tag{10}$$

where k_p is the proportional constant; k_d is the differential constant; k_i is the integral constant.

Take the time integral of the absolute error value as the minimum objective function to achieve ideal transient dynamic characteristics. The optimal objective function can be expressed as the following equation.

$$J_m = \int_0^\infty \left(c_1|e(t)| + c_2 u^2(t)\right) dt \tag{11}$$

According to the above steps, the optimal dynamic parameters were iterated out by means of genetic algorithm. And the dynamic parameters are set as follows: σ_0=185N/m, σ_1=1.5Ns/m. Table 3 gives out the exact dynamic parameters' values and the identified dynamic parameters' results.

Table 3. The exact dynamic parameters' and the identified dynamic parameters' results

Parameter (unit)	σ_0 *(N/m)*	σ_1 *(Ns/m)*
Exact parameters	191.6	1.37
Identified parameters	185	1.5
Error	3.44%	9.48%

4. **Parameter Sensitivity Analysis.** To study the sensitivity of static parameters based on the LuGre tire model to experimental variables, studies on the influences of normal force F_n, sideslip angle γ, and slip ratio s to static parameters μ_c, μ_s, and σ_2 were analyzed (The experimental sensitivity of parameter v_s was ignored because it is slightly affected by experimental conditions.). During the analysis process, vehicle speed was a constant and the normal force was equally distributed to each wheel.

It was assumed that the influence of the experimental variables to the other two parameters could be ignored when sensitivity analysis was performed on one of the static parameters. Meanwhile, the other two experimental variables were regarded as constants when sensitivity of one of the experimental variables to the static parameters was analyzed.

4.1. **Influence analysis of normal force to static parameters.** The influence of normal force F_n to the static parameters is shown in Figures 2(a)-2(c).

The influences of normal force F_n to static parameters μ_c, μ_s, and σ_2 present a linear trend, as shown in Figure 2. The coulomb friction coefficient μ_c and the static coefficient μ_s decrease linearly with the increase of the normal force; conversely, the relative viscosity damping coefficient σ_2 increases with it. In addition, parameter μ_s is the most sensitive to the change of normal force F_n (whose absolute value of the straight line gradient is 0.1267); while parameter σ_2 is the least sensitive to normal force F_n (whose absolute value of the straight line gradient is 0.0001). Also, it is important to note that the results are obtained on the premise that there is no coupling effect among the static parameters.

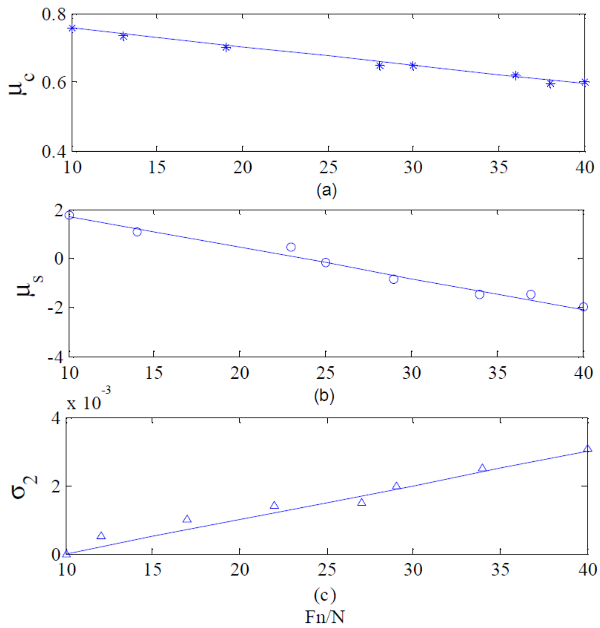

FIGURE 2. Influence of normal force to the identified static parameters

4.2. **Influence analysis of sideslip angle to static parameters.** The influence of sideslip angle γ to the static parameters is shown in Figures 3(a)-3(c).

The influences of sideslip angle γ to coulomb coefficient μ_c and static coefficient μ_s present an obvious nonlinear relationship with monotony; conversely, the influence of sideslip angle γ to the relative viscosity damping coefficient σ_2 presents a linear trend with monotony, as shown in Figure 3. Parameters μ_c, and μ_s decrease with the increase of sideslip angle γ, but the rate of drop tends to be slow. On the contrary, parameter σ_2 goes with a linear increase as the increase of sideslip angle γ. Similarly, experimental results are obtained on the premise that there is no coupling effect among the static parameters.

4.3. **Influence analysis of slip ratio to static parameters.** The influence of slip ratio s to the static parameters is shown in Figures 4(a)-4(c).

It can be seen from Figure 4 that the influences of slip ratio s to coulomb coefficient μ_c, static coefficient μ_s, and relative viscosity damping coefficient σ_2 present obvious nonlinear characteristics with a non-monotonous trend. The influence of slip ratio s to parameters μ_c and μ_s has good agreement: when s grows gradually from zero, parameters μ_c, and μ_s increase sharply close to linear and reach their peaks; when the slip ratio s is about to be

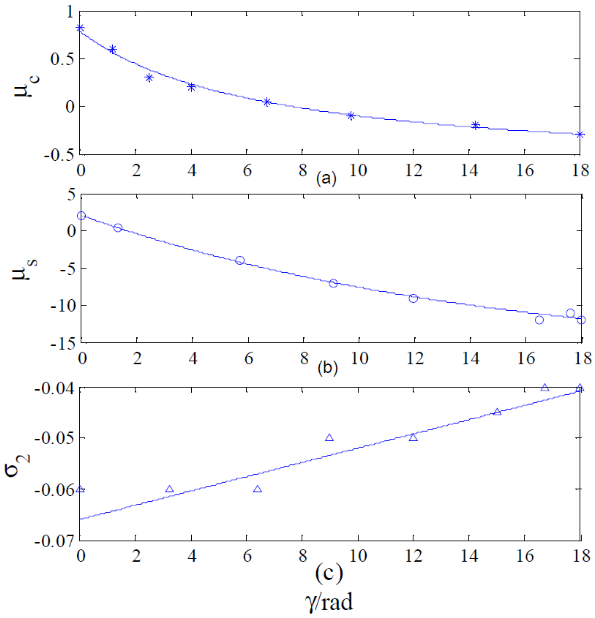

FIGURE 3. Influence of sideslip angle to the identified static parameters

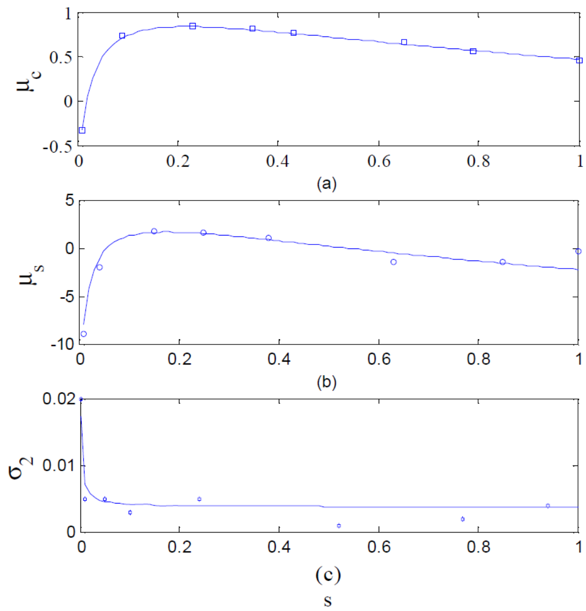

FIGURE 4. Influence of slip ratio to the identified static parameters

0.1, then decline slowly. Compared to the influence of slip ratio s to parameters μ_c, and μ_s, the influence of slip ratio s to the parameter σ_2 is just the opposite: when s grows gradually from zero, parameter σ_2 decreases sharply and reaches its minimum; when the slip ratio s is about to be 0.1, then increases slowly. Similarly, the coupling effect among the static parameters also can be ignored during the influence analysis of sideslip angle to parameters.

Figures 2-4 show that the static parameters μ_c, μ_s, and σ_2 are sensitive to different experimental variables, although they present different changing trends. The simulation results validate the influence of experimental conditions to the static parameters.

5. **Conclusions.** In this paper, the static and the dynamic parameters are identified based on the LuGre friction model. Among the identified parameters, μ_c, μ_s, and σ_2 are

static parameters, whose values can be identified using the least squares method; and σ_0, and σ_1 are dynamic parameters, whose identification process is somehow difficult and complex, which can be identified by means of genetic algorithm. Experimental results show that identification method based on the LuGre friction model is appropriate for studying the static parameters. Study of the sensitivity of different static parameters to different experimental conditions was conducted on the premise that the coupling effect among the static parameters is ignored. Simulation results show that the identification process of the static parameters is sensitive to different experimental variables, including the normal force F_n, the sideslip angle γ, and the slip ratio s.

However, there are still some disadvantages in this paper. For example, the influence of combined analysis of the experimental conditions to the identified parameters has not been presented in this paper; paper mainly completed the parameters identification when the vehicle is in the motion, and realized the sensitivity analysis of the experimental variables to the identification parameters. In the later studies, more research about the combined experimental conditions analysis will be studied.

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