

Throughput and Delay Analysis of Hybrid Wireless Networks with Multi-Hop Uplinks

Devu Manikantan Shila, Yu Cheng and Tricha Anjali

Dept. of Electrical and Computer Engineering,
Illinois Institute of Technology, Chicago, USA

Abstract—How much information can one send through a random ad hoc network of n nodes, if overlaid with a cellular architecture of m base stations? This network model is commonly referred to as hybrid wireless networks and our paper analyzes the above question by characterizing its throughput capacity. Although several research efforts related to throughput capacity exist in the area of hybrid wireless networks, most of these solutions under-explore the capacity analysis. Their results particularly indicate that one can realize only a less than log or no gain on capacity, as compared to pure ad hoc networks, when m scales slower than some threshold. This unsatisfying capacity gain is due to the fact that the base stations were not properly exploited while formulating the capacity analysis. Moreover, these research efforts also assume an one-hop wireless uplink between a node and its associated base station. Nevertheless, with those power-constrained wireless nodes, this assumption clearly indicates an unrealistic scenario. In this paper, we establish the bounds on capacity and delay by resolving the issues in existing efforts and at the heart of our analysis lies a simple routing policy known as same cell routing policy. Our findings particularly stipulate that whether $m = O(\frac{n}{\log n})$ or $\Omega(\frac{n}{\log n})$, each node can realize a throughput that scales, sublinearly or linearly, with m . This is in fact a significant result as opposed to previous efforts which claims that if m grows slower than some threshold, the benefit of augmenting those base stations to the original ad hoc network is insignificant. Our analysis also shows that for a maximum per node throughput $\Lambda(n, m)$, the average end-to-end delay in a hybrid network can be bounded by $\Theta(\Lambda(n, m) \frac{n}{m})$, which has an inverse relationship to m .

I. INTRODUCTION

There has been significant interest in the past on understanding how the capacity of multi-hop wireless networks scales with the number of nodes n in the network [2]-[15]. In their pioneering work [2], Gupta and Kumar studied the capacity of pure ad hoc wireless networks in the limit as the number of nodes grows to a large level. Their results mainly prove that when n nodes are randomly (or arbitrarily) deployed in a planar disk of unit area, the amount of information that can be transmitted by each source-destination (S-D) pair becomes vanishingly small, as $n \rightarrow \infty$. This performance limitation on throughput with increasing n is due to the increasing number of hops between each S-D pair, which in turn implies that those nodes serving as relays essentially spend most of the time relaying traffic from other nodes. Thus, it follows that by decreasing the number of hops between each S-D pair and correspondingly the traffic relayed by the nodes, one can greatly improve the performance of multi-hop wireless networks.

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A significant number of works, [4]-[5],[13] and the references therein, along the tangent of “decreasing hops” have been proposed in the area of multi-hop wireless networks. More specifically, Grossglauser and Tse [4] proved that a constant throughput scaling of $\Theta(1)$ per S-D pair can be achieved, if each packet is constrained to take $O(1)$ hops to destination by utilizing the mobility characteristics of a node. Gamal *et.al.* [5] however showed that this enhanced throughput scaling obtained under node mobility actually comes at the cost of delay and as a result, even a slight dependence on mobility will lead to an abrupt and increased delay. Thus it appears that one should target their efforts on building wireless network models that can furnish each node with a higher throughput, while possibly keeping the delay small.

It has been recently recognized that adding base stations to pure ad hoc wireless networks, commonly referred to as *hybrid wireless networks*, can indeed render larger benefits in terms of both capacity and delay. One can envisage these base stations as a means to carry all the long distance transmissions from a source node, through the wired network, to its intended destination. And since each wireless node is committed to leverage multi-hop transmissions only for short distances, we have smaller number of wireless hops per S-D pair and correspondingly, a larger per node throughput. As delay also increases with the hops, it is obvious that by limiting the number of hops one can realize a smaller delay for hybrid wireless networks, without sacrificing the per node throughput. Several capacity related works exist in the area of hybrid wireless networks. In [6], Liu *et al.* first studied the throughput capacity of hybrid wireless networks under two different routing strategies. Specifically in k -nearest cell routing strategy, it is shown that if m grows asymptotically slower than \sqrt{n} , the maximum per node capacity scales as $\Theta(W \sqrt{\frac{1}{n \log n / m^2}})$. It is not hard to see that the benefits obtained on capacity due to the addition of base stations in this region are insignificant. On the other hand, if m grows asymptotically faster than \sqrt{n} , the maximum per node throughput capacity scales as $\Theta(\frac{Wm}{n})$ which in turn offers a better throughput gain dependent on m . Importantly, note that in the region $m = O(\sqrt{n})$, one can attain only less than log k -fold benefit on capacity as the number of base stations are increased from m to km . We further notice similar capacity limited figures in [8]-[11] i.e., especially when m grows asymptotically slower than some threshold.

The *motivation* for this work is to fundamentally understand

whether the capacity of the hybrid network has been fully explored, especially stimulated by the unsatisfying gain when m scales slower than some threshold. By studying these existing efforts in depth, we identify the following issues in the existing capacity analysis: **ISSUE I:** In [6],[8]-[11], their capacity analysis fails to account for the fact that adding base stations to the original ad hoc wireless networks plays a critical role in decreasing the number of hops between each S-D pair and thereby, the amount of traffic flowing through each relaying node. One may observe that failure to include this aspect in capacity analysis can lead to *inaccurate results*; **ISSUE II:** In [6]-[11], each node in the network is assumed to negotiate with its base station using an one-hop wireless uplink. This implies that those power constrained wireless nodes has to transmit at higher power levels to reach their associated base stations. However, such assumptions are not feasible in practice, especially when wireless nodes are configured to transmit at < 100 mWatts as opposed to the base stations that can transmit at $20 - 60$ Watts [1]; **ISSUE III:** In [6], to formulate the final capacity expression, the parameter n in $W\sqrt{\frac{n}{\log n}}$ (capacity of pure ad hoc network) is simply substituted with nm^{-2} , which is the number of nodes communicating in ad hoc mode. However, those substitutions indirectly imply that nodes are restricted to communicate only with a limited number of nearby neighbors and thus particularly during the breakdown of any infrastructure nodes, several disconnected components will be created in the network. In [7], authors revisit the throughput capacity problem in hybrid wireless networks based on a L -Maximum hop routing strategy. Though their analysis provide better capacity figures in comparison to [6], authors again assume an one-hop wireless uplink between a node and its associated base station [ISSUE II].

Motivated by these shortcomings in existing studies, we particularly address the following two questions in this paper:

- Can we design a better scheme that can resolve the issues in previous efforts, while possibly providing a better asymptotic capacity and delay scaling?
- How much information can one send over a random ad hoc network of n nodes if overlaid with a cellular architecture of m base stations?

This paper attempts to answer these two questions by considering a simple and practical routing policy referred to as *same cell routing policy*. In this policy, a source node route its packets to the destination in ad hoc mode only if both the source and its destination are located in the same cell. Otherwise, the packets are initially transmitted in ad hoc mode to the base station which eventually forwards all the packets to the destination as in a cellular network. Let W bits/sec be the total bandwidth and then for a hybrid network with n nodes and m base stations, we identify the following two regimes:

- $m = O(\frac{n}{\log n})$. In this regime, a per node throughput capacity of $\Theta(W\sqrt{\frac{m}{n \log n}})$ is achieved. It also follows that if the number of base stations are increased from m to km , we can actually obtain a gain of \sqrt{k} on capacity

as opposed to [6], which only provides a less than $\log k$ -fold increase on capacity. The average delay is bounded by $\Theta(\sqrt{\frac{n}{m \log n}})$.

- $m = \Omega(\frac{n}{\log n})$. In this regime, a per node throughput capacity of $\Theta(W\frac{m}{n})$ is achieved. The average delay is bounded by $\Theta(1)$.

It is obvious that independent of the regimes defined by m , the throughput capacity available to each node scales with the number of base stations. We are particularly interested in the region $m = O(\frac{n}{\log n})$, where we observe that the capacity increases sublinearly with the number of base stations. This is an interesting and contrasting result to [6],[9]-[11], which states that if m grows asymptotically slower than some threshold, there is no benefit in augmenting those base stations to the original ad hoc network. More importantly, as compared to previous research efforts [2]-[4],[6]-[7], we clearly demonstrate that our design can guarantee higher benefits on both capacity and delay irrespective of the regimes defined by m . We summarize the distinguishing aspects of our work as follows: (a) **NUMBER OF HOPS:** In contrast to previous efforts, our analysis accurately models the effect of number of hops between each S-D pair on the capacity and delay of a hybrid wireless network; (b) **MULTI-HOP UPLINKS:** We consider a more practical approach in which the power-constrained wireless nodes are allowed to negotiate with their associated base stations using multiple hops; and (c) **FAILURE TOLERANT:** We assume a stand-alone ad hoc network and as a result, even in the failure of any base stations, nodes can still route the packets to its chosen destinations.

Roadmap. The rest of the paper is organized as follows: In section II, we initially present several related research efforts and then provide a detailed description of our hybrid wireless network model. Section III presents the upper and lower bounds on the throughput capacity and delay of a hybrid network under same cell routing policy. In section IV, we investigate the efficiency of our model by comparing with several existing literatures. Section V finally concludes this paper.

II. BACKGROUND AND NETWORK MODEL

A. Related Work

Due to space constraints, we briefly discuss some related works in the area of hybrid wireless networks. In [8], authors study the throughput capacity where both the ad hoc and infrastructure nodes are randomly distributed. They mainly show that each node can achieve a maximum throughput of $\Theta(W/\log n)$ if m scales linearly with n . Following this work, Zemlianov *et. al* [9] investigates the capacity under a different model where the ad hoc nodes are randomly distributed and the base stations are arbitrarily placed. Their results principally show that when $m = O(\sqrt{n/\log n})$, the per node throughput maps to the capacity of pure ad hoc networks and thus, there is no benefit in adding those base stations to the ad hoc network in this region. On the other hand, when $m = \Omega(\sqrt{n/\log n})$, they observe similar results reported in [6]. In [10], authors

investigate the impact of network dimensionality and geometry on the capacity of these networks. Though they observe interesting capacity figures for 1-dimensional strip, their results for 2-dimensional network shows that there is no benefit on capacity from deploying base stations as long as $m = O(\sqrt{n})$. Similar results are also reported in [11]. Importantly, in [8]-[11], authors overlook the effect of number of hops between each S-D pair on the throughput capacity (ISSUE I). These works also assume an one-hop wireless uplink between nodes and their base stations (ISSUE II). One may notice that these aspects can in fact lead to inaccurate as well as impractical solutions. Besides, while comparing with these solutions one can also observe the larger gain obtained in employing our design. In [12], authors study the capacity when access points are regularly placed on the pure ad hoc network. Unfortunately, the results established in that paper are not validated with enough proofs.

B. Hybrid Wireless Network Model

We consider a hybrid wireless network of n wireless nodes, overlaid with a cellular architecture of m base stations on a planar torus¹ of unit area. One may also note that though we mention base stations in this paper, the results are applicable to all infrastructure nodes that are connected by a wired network such as Access Points (AP), Femtocells etc. Fig. 1 depicts the setting of such a hybrid wireless network model in a plane. In particular, a hybrid wireless network consists of two layers, an *ad hoc* layer and a *cellular* layer. In the ad hoc layer, we assume that n wireless nodes are uniformly and independently (randomly) distributed on the surface of a unit area torus, similar to the one proposed by Gupta and Kumar [2], and each node leverages same transmission power to communicate with its neighboring nodes or base stations. As in [4], we also consider that each node is a source of exactly one flow and a destination node for at most $O(1)$ flows. Lastly, we assume a stand-alone wireless network at the ad hoc layer. As a result, even in the absence of any base stations, nodes can still engage in communication with its chosen destinations. This assumption solves ISSUE III in [6].

In the cellular layer, we regularly deploy m base stations, at the top of ad hoc layer, in such a manner that it tessellates the plane into equal-sized squares of area $\frac{1}{m}$. For the sake of clarity of proofs, we assume a square tessellation instead of hexagonal tessellation. One may note that if a hexagonal tessellation is considered, the area of each cell would still be $\frac{k}{m}$, where k is a constant. This in turn implies that all the scaling results derived in this paper will hold true for hexagonal tessellation as well. Besides, this model is also similar to the work in [5], where the authors consider a square tessellation to analyze the performance of employing mobility in pure ad hoc networks. Next, as in a cellular concept, each square is called a **cell** and we place one base station at the center of each cell. Unlike wireless nodes, base stations neither serve as data sources nor as data receivers. Instead, they serve

as relays to forward the traffic for wireless nodes in the ad hoc layer. Moreover, the base stations are also assumed to be connected to each other with a very high bandwidth network so that there are no bottlenecks associated with the base stations. In contrast to wireless nodes, we also assume that there are no power constraints for the base stations. Finally, to ensure that the mutual interference between base stations remains below a threshold, we assume that adjacent cells employ a frequency reuse policy similar to a cellular network [1]; See Section III-C for details.

C. Protocol Interference Model

To study the impact of wireless interference in hybrid wireless networks, we adopt the protocol model in [2]. Suppose that node v_i transmits to another node v_j . In the so-called protocol model, a transmission from node v_i is successfully received by node v_j if the following two conditions are satisfied.

- The distance between node v_i and v_j , $|v_i - v_j|$, is no more than the transmission range of the nodes $r(n)$, i.e., $|v_i - v_j| \leq r(n)$.
- For every other node v_k that is simultaneously transmitting over the same channel, node v_j should lie outside the interference region of v_k , i.e., $|v_k - v_j| \geq (1 + \Delta)r(n)$.

The last condition ensures an exclusion region [1] around the receiving node to prevent a neighboring node from transmitting on the same channel at the same time. The parameter Δ defines the size of the exclusion region and hence, $\Delta > 0$.

D. Routing Policy for Hybrid Wireless Networks

In a hybrid wireless network, each node in the ad hoc layer can communicate with its chosen destination in the corresponding layer using two modes, *purely ad hoc mode* and *hybrid mode*. In a purely ad hoc mode, the source node transmits the data to its intended destination using multiple hops, that is without relying on the base stations in the cellular layer. On the other hand, in a hybrid mode, the source node initially transmits the data to its nearest base station in a multi-hop fashion (i.e., purely ad hoc mode) and eventually, the base station forwards the data through the wired network to the destination node as in a cellular network. One may again note that the approach of utilizing multiple hops by source node to connect to its nearest base station in a hybrid mode is different from the existing solutions [6]-[11], where the authors assume an one-hop wireless uplink between a source node and its base station. Taking into account “multi-hop uplinks” resolves ISSUE II seen in prior schemes and as a result, we call our design as “practical”.

This paper initially considers a simple routing policy called as *same cell routing policy* for hybrid wireless networks. In this policy, a source node transmits the data to its destination in purely ad hoc mode only if both the source and destination are located in the same cell. Otherwise, data is traversed to the destination in hybrid mode. However, one may note that even though the source and destination lie within one-hop

¹The assumption of a torus enables us to avoid technicalities arising out of edge effects, but the scaling results derived here holds for unit square or disk as well.

distance of each other but are located in two different cells, the data will be forwarded in hybrid mode according to the same cell routing policy. This in turn can cause inefficient utilization of the wireless bandwidth, as pointed out in [7]. As a result, besides same cell routing policy, we further analyze another routing policy called as *D length routing policy*; See Appendix A. In this policy, a source node routes the data to its destination in purely ad hoc mode only if the destination can be reached within D (i.e., $\leq D$) distance from the source. Otherwise, the data will be carried to the destination in hybrid mode. Our goal is to find optimal D and interestingly, our results demonstrate that the maximum capacity bounds can be realized at $D = O(1/\sqrt{m})$, similar to what is achieved under same cell routing policy. This clearly shows the effectiveness of our design and analysis.

Each node is assumed to transmit at a maximum data rate of W bits per second over a common wireless channel of bandwidth W . We further partition this wireless channel into three subchannels each of bandwidth, W_a for purely ad hoc transmissions, W_u for uplink transmissions to the base stations and W_d for downlink transmissions from the base stations, respectively. In fact, such a partition will allow us to carry above three transmissions simultaneously in a network without causing interference to each other. Nevertheless, the transmissions occurring in the same subchannel will still cause interference to each other. In addition, since the amount of traffic in the uplink and downlink channels are the same, we can write $W_u = W_d$. As a result, the sum of the transmission rates due to purely ad hoc transmissions as well as base station related transmissions can be expressed as $W = W_a + 2W_u$. One may also note that the bandwidth W_a assigned for purely ad hoc transmissions include not only the traffic from source to destination in the purely ad hoc mode but also the traffic from source to the base station in the hybrid mode. In the sequel, by “purely ad hoc transmissions”, unless stated, we refer to the transmissions occurring in multi-hop fashion in purely ad hoc mode as well as in hybrid mode.

E. Definitions

Throughput. A per-node throughput of $\Lambda(n, m)$ bits per second, for a hybrid wireless network of n nodes and m base stations is said to be achievable, if every node can transmit data to its chosen destination at a rate of $\Lambda(n, m)$ bits per second. In this paper, the throughput capacity of the hybrid wireless network with n nodes and m base stations are expressed by $\Lambda(n, m) = \Lambda_a(n, m) + \Lambda_b(n, m)$, where $\Lambda_a(n, m)$ and $\Lambda_b(n, m)$ denote the throughput capacity contributed by the purely ad hoc mode transmissions and the base station relative transmissions (i.e., uplink and downlink) respectively. Further, since there are total of n source-destination pairs, we define the network capacity to be $n\Lambda(n, m)$.

Average Delay of Hybrid Networks. The delay of a packet is the time it takes for the packet to reach the destination from the source. Thus, the per packet delay is the sum of the times a packet spends at each relay node. As in previous capacity and delay studies [4]-[5], we scale the packet size by

the per node capacity so the transmission delay at each node is constant. Hence, the per packet delay corresponds to the number of hops needed to reach its destination. The average packet delay of a hybrid network $D(n, m)$ is then obtained by averaging over all transmitted packets in the network due to the wireless transmissions. We do not account for the delay caused by the transmissions through the wired network, as a high bandwidth wired backbone network is assumed.

III. CAPACITY AND DELAY OF HYBRID WIRELESS NETWORKS UNDER SAME CELL ROUTING POLICY

This section establishes the upper and lower bounds on the throughput capacity and delay of hybrid wireless networks under same cell routing policy. The related theorems are stated as follows.

Theorem 1. For a hybrid network with n nodes and m base stations, the throughput capacity $\Lambda(n, m)$ furnished to each node under the same cell routing policy is²:

$$\Lambda(n, m) = \Theta\left(\sqrt{\frac{m}{n \log n}} W_a\right) + \Theta\left(\frac{m}{n} W_u\right) \quad (1)$$

where $\Lambda_a(n, m) = \sqrt{\frac{m}{n \log n}} W_a$ and $\Lambda_b(n, m) = \frac{m}{n} W_u$.

Theorem 2. For a hybrid network with n nodes and m base stations, the average delay $D(n, m)$ of each packet under the same cell routing policy is:

$$D(n, m) = \begin{cases} \Theta\left(\sqrt{\frac{n}{m \log n}}\right) & m = O\left(\frac{n}{\log n}\right) \\ \Theta(1) & m = \Omega\left(\frac{n}{\log n}\right) \end{cases} \quad (2)$$

As the purely ad hoc mode and base station relative transmissions are carried in two different subchannels, in the sequel we will derive the bounds on capacity and delay of these transmissions separately. One may also note that the final capacity will be just $\Lambda_a(n, m) + \Lambda_b(n, m)$.

A. Lower Bound on Capacity and Delay for Purely ad hoc transmissions

This section constructs a scheme that achieves $\Lambda_a(n, m)$ bits per second for every node in the network to its chosen destination, with high probability (whp). Consider the ad hoc layer of the hybrid wireless network and tessellate the unit area region by subcells of area $a(n) = \Omega\left(\frac{\log n}{n}\right)$. We then choose the transmission range of each node as $r(n) = 2\sqrt{2a(n)}$ such that a node in a subcell can transmit to some other node lying within its four neighboring subcells. As shown in Fig. 1, we further lay out a virtual layer, cellular layer, formed by m cells each of size $\frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}}$ at the top of this ad hoc layer. To be more precise, such a construction will result in each cell of area $\frac{1}{m}$ to consist of $\frac{1}{ma(n)}$ subcells, each of area $a(n)$. Once the network is constructed, our next step is to present a scheme that allows each node to route the data to its chosen

²The throughput capacity of hybrid wireless networks is said to be of order $\Theta(f(n, m))$ bits per second, if there exist deterministic positive constants c_1 and c_2 such that $\lim_{n \rightarrow \infty} \text{Prob}(\Lambda(n, m) = c_1 f(n, m) \text{ is feasible}) = 1$; and $\lim_{n \rightarrow \infty} \text{Prob}(\Lambda(n, m) = c_2 f(n, m) \text{ is feasible}) < 1$.

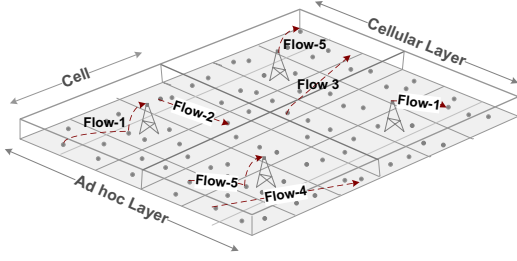


Fig. 1. Same Cell Routing Policy. Flows 2,3 and 4 are forwarded in purely ad hoc mode and Flows 1 and 5 are forwarded in hybrid mode.

destination. For this purpose, draw a straight line, that passes through some subcells, connecting each source-destination (S-D) pair. As mentioned before, if a S-D line lies completely inside (*outside*) a cell the packets are transmitted from source to destination (*base station*) in purely ad hoc mode by hops along the adjacent subcells lying on its S-D line. Therefore to transmit the packet along the S-D line, we need to choose a node from each of these subcells—which in turn requires at least one node per subcell. We now state the Lemma from [5] that bounds the number of nodes present in a subcell of area $a(n) = \Omega(\frac{\log n}{n})$.

Lemma 1. (Ref. [5]) If $a(n)$ is greater than $\frac{50 \log n}{n}$, each subcell has $\Theta(na(n))$ nodes per subcell, whp. \square

From Lemma 1, it follows that each subcell with area $a(n) > \frac{\log n}{n}$ will have at least one node *whp*, thus ensuring successful transmission along each S-D line. Our next step is to schedule transmissions such that each node in a subcell can transmit to nodes in its adjacent subcells, without causing interference to simultaneous senders, at regularly scheduled time slots. We propose a time division multi-access scheme (TDMA) to schedule the transmissions which is in fact based on the following two Lemmas. Lemma 2 shows that there is a schedule for transmitting packets such that once in every $1+c_3$ time slots, each subcell in the tessellation gets one slot to transmit to its adjacent subcells. Once a subcell gets an opportunity to transmit, the nodes within it are required to relay the traffic for each of the S-D routes passing through it. In Lemma 4, we determine the amount of relaying traffic per subcell by bounding the maximum number of routes passing through it.

Before stating Lemma 2, we use the following definition [2],[5]. A subcell X is said to interfere with another subcell Y, if there is a sender in subcell X which is within a distance $(2 + \Delta)r(n)$ of some sender in subcell Y.

Lemma 2. The number of subcells that interfere with any given subcell is bounded by a constant $c_3 = O((2 + \Delta)^2)$, i.e, independent of n , m and $a(n)$.

Proof: According to the definition, any two nodes transmitting simultaneously are separated by a distance of at least $(2 + \Delta)r(n)$ and hence disks of radius $(1 + \frac{\Delta}{2})r(n)$ centered around each transmitter are essentially disjoint. Thus, using

simple geometric arguments we get the number of interfering subcells, c_3 , as at most $\leq 4 \frac{((2+\Delta)\sqrt{2a(n)})^2}{a(n)} = O((2 + \Delta)^2)$, which is a constant independent of n , m and $a(n)$. \blacksquare

This implies that in every $1 + c_3$ slots, each subcell in the tessellation gets one slot to transmit, thus guaranteeing a successful transmission to nodes within a distance of $r(n)$ from their transmitters. Also, note that the fact $1 + c_3$ follows from the vertex coloring of a graph with bounded degree c_3 (see [1] for more details on vertex coloring). Next, we calculate the maximum number of routes passing through any subcell in a cell. To prove Lemma 4, we need Lemma 3 that bounds the number of source-destination (S-D) pairs communicating using purely ad hoc mode and hybrid mode in a given cell.

Lemma 3. For a given cell k of size $\frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}}$, the number of S-D pairs communicating using purely ad hoc mode and hybrid mode are $\frac{n}{m^2}$ and $\frac{n}{m}(1 - \frac{1}{m})$ respectively, under same cell routing policy.

Proof: Consider a tagged cell k . Let X_i^k be an indicator random variable that represents whether the source node i and its destination are in the same cell, k . Thus, we have

$$X_i^k = \begin{cases} 1 & \text{If both source } i \text{ and destination are in cell } k \\ 0 & \text{Otherwise} \end{cases}$$

Recall that in the cellular layer, base stations divide the unit area torus into m cells each with an area of $\frac{1}{m}$, and in the ad hoc layer, we have n nodes randomly distributed. The probability that a source node i is located in cell k is $1/m$; the probability that the destination of node i is also located in cell k is $1/m$. Therefore, $E[X_i^k] = 1/m^2$. Let $X^k = \sum_{i=1}^n X_i^k$ determine the number of source-destination pairs communicating using purely ad hoc mode within cell k . Now applying the linearity of expected value and the fact that all $E[X_i^k]$'s are equal, we have $E[X^k] = \sum_{i=1}^n E[X_i^k] = nE[X_i^k] = \frac{n}{m^2}$. Our next step is to determine the probability that any cell can have at most n/m^2 nodes communicating in purely ad hoc mode. From the application of the Chernoff bound; see Appendix B and setting $\delta = 1$, we obtain that for any cell $\Pr[X^k > \frac{2n}{m^2}] \leq \exp(-\frac{n}{3m^2})$. Since there are total of m cells, by the application of the union bound (see pg.382 [1]), it follows that the above bound holds for all m cells with probability $\leq m \cdot \exp(-\frac{n}{3m^2})$.

Similarly, we will also calculate the number of source-destination pairs $Y^k = \sum_{i=1}^n Y_i^k$ communicating using hybrid mode within cell k , where Y_i^k is defined as follows:

$$Y_i^k = \begin{cases} 1 & \text{If source is in cell } k \text{ and destination in cell } x \setminus k \\ 0 & \text{Otherwise} \end{cases}$$

Again, applying the linearity of expected value and the fact that all $E[Y_i^k]$'s are equal, we have $E[Y^k] = nE[Y_i^k] = \frac{n}{m}(1 - \frac{1}{m})$, where the factor $\frac{1}{m}(1 - \frac{1}{m})$ is the probability that source and destination lie in two different cells. Following the above techniques, we get the $\Pr[\text{any cell has } > \frac{2n}{m}(1 - \frac{1}{m}) \text{ nodes communicating in hybrid mode}] \leq m \cdot \exp(-\frac{n}{3m}(1 - \frac{1}{m}))$. \blacksquare

Lemma 4. The number of S-D routes passing through any subcell in a cell is $O(n\sqrt{\frac{a(n)}{m}})$, whp.

Proof: Consider an arbitrary cell k . From Lemma 3, we have $\frac{n}{m^2}$ S-D pairs communicating using purely ad hoc mode in cell k . Let d_i be the distance between the S-D pair i and h_i be the mean number of hops per packet for each S-D pair i . Then, $h_i = d_i/\sqrt{a(n)}$. Let $H = \sum_{i=1}^{\frac{n}{m^2}} h_i$ be the total number of hops required to send a packet from each sender S to its corresponding destination D in purely ad hoc mode. Consider a tagged subcell in a cell k and define the Bernoulli random variables Z_j^i for S-D pairs $1 \leq i \leq \frac{n}{m^2}$ and hops $1 \leq j \leq h_i$ as follows:

$$Z_j^i = \begin{cases} 1 & \text{If hop } j \text{ of S-D pair } i \text{ originate from a} \\ & \text{node in the subcell of a given cell} \\ 0 & \text{Otherwise} \end{cases} \quad (3)$$

As a result, the total number of S-D lines passing through a subcell due to the purely ad hoc mode is $Z = \sum_{i=1}^{\frac{n}{m^2}} \sum_{j=1}^{h_i} Z_j^i$.

Now consider the random variable $H = \sum_{i=1}^{\frac{n}{m^2}} h_i = \sum_{i=1}^{\frac{n}{m^2}} d_i/\sqrt{a(n)}$. Recall that in same cell routing policy, each node transmits the data to its destination in purely ad hoc mode if both the node and its destination are located in the same cell. Since each cell has a size of $\frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}}$, for all i , we have $d_i \in [0, \frac{1}{\sqrt{m}}]$ and $H = O(\frac{n}{m^{5/2}\sqrt{a(n)}})$. Using this result, we find the bound on $E[Z]$ as follows [5]:

$$\begin{aligned} E[Z] &= E_H[E[Z|H]] = E_H[HE[Z_1^1]] \\ &= \left(\frac{n}{m^{5/2}\sqrt{a(n)}}\right) ma(n) = \frac{n\sqrt{a(n)}}{m^{3/2}} \end{aligned} \quad (4)$$

where the last equality follows from the fact that any hop is equally likely to originate from any subcell of the $1/ma(n)$ subcells. Next, we also need to bound the number of lines passing through a subcell due to the multi-hop traffic (i.e., the traffic from source to the base station) in a hybrid mode. Let Z' be the total number of S-D lines passing through a subcell due to the multi-hop traffic in a hybrid mode. Following the above techniques and replacing $\frac{n}{m^2}$ S-D pairs by $\frac{n}{m}(1 - \frac{1}{m})$ we get $E[Z'] = \frac{n(m-1)\sqrt{a(n)}}{m^{3/2}}$. Since $E[Z]$ and $E[Z']$ are independent, we have the total number of lines passing through a subcell in a cell as $E[Z] + E[Z'] = n\sqrt{\frac{a(n)}{m}}$. Due to space constraints, we omit the proof for computing the probability that any subcell has at most $n\sqrt{\frac{a(n)}{m}}$ lines passing through it, using chernoff bounds. However following the same techniques in [5] and setting $\delta = 2\sqrt{\log n/(E[Z] + E[Z'])}$, we obtain that for any cell the number of lines passing through it are bounded about by $n\sqrt{\frac{a(n)}{m}}$ with probability $\geq 1 - 1/n$. ■

Throughput Capacity: We are now ready to calculate $\Lambda_a(n, m)$, the throughput capacity contributed by purely ad hoc transmissions. It follows from Lemma 2 that each subcell

will receive an opportunity to transmit once in every $1+c_3$ time slots. In other words, in every one second time period each subcell can be active for a constant fraction of time period of length $\Omega(\frac{1}{1+c_3})$ seconds. Lemma 4 suggests that if each time period corresponding to a subcell, that is $\Omega(\frac{1}{1+c_3})$ seconds, is further divided into $\Omega(n\sqrt{\frac{a(n)}{m}})$ time slots, each S-D pair hopping passing through it can use one slot. Equivalently, each node can successfully transmit for $\Omega(\frac{1}{n}\sqrt{\frac{m}{a(n)}})$ fraction of time at the rate of W_a bps. Thus, we have $\Lambda_a(n, m) = \Omega(\frac{W_a}{n}\sqrt{\frac{m}{a(n)}})$, where $a(n) = \frac{\log n}{n}$, as the throughput capacity corresponding to purely ad hoc transmissions.

Average Packet Delay. We will also compute the average packet delay, $D(n, m)$, of all the packets in the network. From Lemma 4, it follows that each S-D pair i , communicating in purely ad hoc mode or hybrid mode, has a length of at most $1/\sqrt{m}$, i.e., $d_i = O(1/\sqrt{m})$. Besides, it also follows from [5], that the delay per packet is the sum of the amount of time spent at each hop. Since each hop covers a distance of $\Theta(\sqrt{a(n)})$, the number of hops per packet for S-D pair i is $\Theta(d_i/\sqrt{a(n)})$. Thus the average number of hops taken by a packet averaged over all S-D pairs cannot be more than $\frac{1}{n}\sum_{i=1}^n d_i/\sqrt{a(n)} = \frac{1}{\sqrt{ma(n)}}$. From the above discussion, we

can conclude the delay $D(n, m)$ as $\Omega(\sqrt{\frac{n}{m \log n}})$. In addition, one may note that as the base stations grows faster than $n/\log n$, the number of hops taken by each packet for S-D pair i is at most $\Theta(1)$. Intuitively as m increases, the size of each cell served by a base station also decreases which in turn leads to decreasing number of hops taken by each packet in purely ad hoc mode. As a result, we can bound the delay by $\Theta(1)$ for $m = \Omega(n/\log n)$ and by $\Omega(\sqrt{\frac{n}{m \log n}})$ for $m = O(n/\log n)$.

B. Upper Bound on Capacity and Delay for Purely ad hoc transmissions

Now, we turn to the upper bounds on the per-node throughput and delay of purely ad hoc transmissions. Before deriving the bound, we state the following Lemma from [2] that determines the number of simultaneous transmissions possible on any particular channel.

Lemma 5. Ref [2]: The number of simultaneous transmissions on any particular channel is no more than $\frac{4}{c_4\pi\Delta^2\tau^2(n)}$ in the Protocol Model. □

Therefore, observing that each purely ad hoc transmission on a given channel is of W_a bits/second, by summing all the transmissions taking place at the same time, we note that they cannot be more than $\frac{4W_a}{c_4\pi\Delta^2\tau^2(n)}$ bits per second in the protocol model. Consider a particular cell of size $\frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}}$ and let \bar{L} be the mean length of the distance taken by a packet in purely ad hoc mode. This also implies that the mean number of hops taken by a packet in a given cell is at least $\frac{\bar{L}}{r(n)}$. From previous discussions we also know that whether in a purely ad hoc mode or a hybrid mode, each source node in a cell needs to transmit

the packet at a distance of at most $\frac{1}{\sqrt{m}}$. Thus we have \bar{L} as at most $\frac{1}{\sqrt{m}}$. [One may note that in [6],[8]-[11], authors do not account for the impact of $\frac{\bar{L}}{r(n)}$ on the capacity analysis which in turn leads to inaccurate results.] Since each source generates $\Lambda_a(n, m)$ bits per second, there are $\frac{n}{m}$ sources in each cell, and each bit needs to be retransmitted on the average by at least $\frac{\bar{L}}{r(n)}$ nodes, it turns out that the total number of bits per second that needs to be served by a given cell as at least $\frac{\bar{L}n\Lambda_a(n, m)}{mr(n)}$. Again noting that there are total of m cells, the total number of bits per second served by the entire network needs to be at least $\frac{\bar{L}n\Lambda_a(n, m)}{r(n)}$. To guarantee that all the purely ad hoc traffic is carried, we need

$$\frac{\bar{L}n\Lambda_a(n, m)}{r(n)} \leq \frac{4W_a}{c_4\pi\Delta^2r^2(n)} \quad (5)$$

Thus we have, $\Lambda_a(n, m) \leq \frac{c_5W_a}{\bar{L}\Delta^2nr(n)}$. From a precursor result in [2], it also follows that $r^2(n) > \frac{\log n}{\pi n}$ is necessary to ensure connectivity in a independently and uniformly distributed ad hoc network. [One may again note that we consider a stand-alone ad hoc network i.e., even in the breakdown of any infrastructure nodes, the network can still function properly. However, in [6], the limited transmission range of nodes may sometimes allow nodes to communicate only with a small number of nearby neighbors and hence, can lead to several disconnected components in the network.] Together with the fact that $\bar{L} = \frac{1}{\sqrt{m}}$, we have the per-node throughput of each node in purely ad hoc mode as $\Lambda_a(n, m) = O(W_a\sqrt{\frac{m}{n\log n}})$ bits per second. Next, we will compute the upper bound on the delay of each packet in the hybrid network. Recall from previous section that the delay per packet is the sum of the amount of time spent at each hop. Noting that the mean number of hops taken by a packet in purely ad hoc mode is about $\frac{\bar{L}}{r(n)}$, we can conclude the delay $D(n, m)$ as $O(\sqrt{\frac{n}{m\log n}})$. This completes the upper bound proof.

C. Capacity of Base Station Relative Transmissions

Our last step is to determine the throughput capacity contributed by the transmissions relative to a base station. One may note that each packet transmitted from a source to destination in hybrid mode will use one uplink transmission and one downlink transmission and as a result each transmission should be counted only once in the computation of the throughput capacity. It follows from Section II-D that the bandwidth allocated for an uplink transmission is W_u bps. Hence, the throughput capacity contributed by per cell cannot be more than W_u which in turn is the upper bound. We will next turn to the lower bound on the capacity. From section II-B, it follows that each cell employs a frequency reuse policy (similar to a cellular network) to avoid mutual interference from adjacent cells. Equivalently, each cell will use a different frequency from its adjacent or interfering cells and if a set of say c_6 different frequencies are used for a group of c_6 adjacent cells, the bandwidth occupied by each cell (transmission rate) will be lower bounded by $\frac{W_u}{c_6}$. Since the upper and lower

bounds are tight, we have the throughput capacity per cell as $\Theta(W_u)$. From earlier discussions we know that each cell has $\frac{1}{ma(n)} = \frac{n}{m\log n}$ subcells, each with $\Theta(na(n))$ nodes. This implies that within each cell there are total of $\Theta(n/m)$ nodes. Therefore, note that if the throughput available to a cell is $\Theta(W_u)$, then each node within a cell gets a throughput of $\Lambda_b(n, m) = \Theta(\frac{mW_u}{n})$.

Since both the upper and lower bounds maps to each other, we have the tight bounds denoted by $\Theta(\dots)$ in Theorems 1 and 2 respectively. This concludes the proof. One may also note that the capacity figures, $\Lambda_a(n, m)$ and $\Lambda_b(n, m)$, in Theorem 1 in fact depends on different channel allocation schemes. Therefore to get the maximum throughput capacity, one has to maximize the throughput over all possible combinations of W_a and W_u . We then have the following cases: (a) when $m = O(\frac{n}{\log n})$, we achieve the maximum throughput capacity by allocating most of the bandwidth for ad hoc mode transmissions and only allocating a minimal amount of bandwidth for the base station relative transmissions in the hybrid mode. In other words, when $W_u/W \rightarrow 0$ or when $W_a = W$, we get the throughput available for each node as $\Lambda(n, m) = \Theta(W\sqrt{\frac{m}{n\log n}})$ (See Appendix B); (b) when $m = \Omega(\frac{n}{\log n})$, we observe that each node realizes the maximum throughput by allocating most of the bandwidth for carrying base station relative transmissions. Alternatively, when $W_a/W \rightarrow 0$ or $W_u = W/2$, we get the throughput available to each node as $\Lambda(n, m) = \Theta(W\frac{m}{n})$. Intuitively, we see that when $m > \frac{n}{\log n}$, the number of nodes communicating in ad hoc mode within a cell decreases and hence, most of the traffic has to be carried through the base station which in turn requires larger bandwidth.

IV. COMPARISON WITH EXISTING SOLUTIONS

In this section, we compare our design with several existing schemes. For the convenience of elucidation, in the sequel we term the pure ad hoc networks in [2], mobile ad hoc networks in [4], k -nearest cell routing policy in [6] and L -Maximum hop routing strategy in [7] as ‘‘PANs’’, ‘‘MOBILITY’’, ‘‘ k -nearest’’ and ‘‘ L -Hop’’ respectively. To study the performance of our design, two parameters are considered, capacity gain and delay gain denoted by G_c and G_d respectively. G_c and G_d are computed as follows: $G_c = \frac{\text{CAPACITY OF OUR DESIGN}}{\text{CAPACITY OF RELATED WORKS}}$ and $G_d = \frac{\text{DELAY OF RELATED WORKS}}{\text{DELAY OF OUR DESIGN}}$.

1) *Comparison with Pure Ad Hoc Wireless Networks:* In [2], Gupta *et.al* studied the capacity of pure ad hoc wireless networks as n grows to a large level. Their results primarily indicate that when a source node randomly chooses its destination node placed at the maximum distance of $O(1)$ apart, the throughput capacity available to each node is of $\Theta(\frac{W}{\sqrt{n\log n}})$. Our paper mainly studied the benefits that can be realized by regularly placing m base stations in these pure ad hoc networks. Specifically, we observe that when $m = O(\frac{n}{\log n})$ and $m = \Omega(\frac{n}{\log n})$, same cell routing policy achieves a gain of \sqrt{m} and $m\sqrt{\frac{\log n}{n}}$ respectively in comparison to PANs. This improvement can be interpreted by looking at the impact

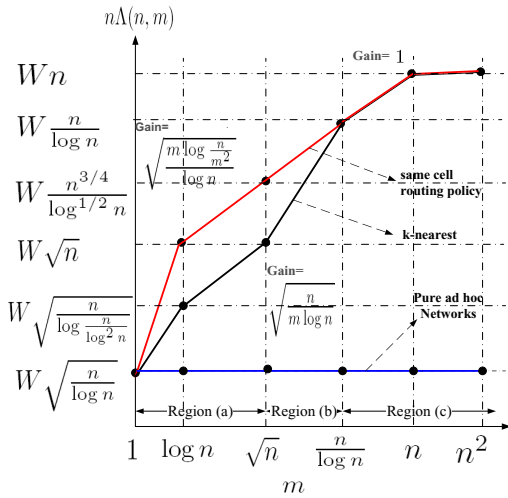


Fig. 2. Network Capacity $n\Lambda(n, m)$ vs. m for same cell routing policy and k -nearest cell [6].

the number of hops on the capacity of each node. It follows from [2] that each source node communicates with a randomly chosen destination at a distance of $O(1)$ apart. Since the average transmission range is of $\sqrt{\log n/n}$, each packet from the source node has to be retransmitted by at least $1/\sqrt{\log n/n} = \sqrt{n/\log n}$ relaying nodes before reaching the final destination. This in turn increases the traffic burden at each relaying node and subsequently leads to a per-node throughput of $1/\sqrt{n \log n}$. However, under same cell routing policy we allow nodes to communicate in purely ad hoc mode only within a distance of $m^{-1/2}$. As a result, the average number of hops between each S-D pair is reduced to $O(\sqrt{n/m \log n})$ ($O(1)$) for $m = O(n/\log n)$ ($\Omega(n/\log n)$). Since it leads to a decreased amount of relaying traffic per node, a higher capacity gain is observed for our design.

2) *Comparison with Hybrid Network Solutions:* In [6], authors propose a k -nearest cell routing policy for hybrid wireless networks. Under this policy, a source node uses ad hoc mode to send data only when the destination is located within its k nearest neighboring cells. Particularly, their results indicate that when $m = O(\sqrt{n})$ the network capacity is $\Theta(W\sqrt{\frac{n}{\log n/m^2}})$, implying that the benefit of adding base stations to pure ad hoc network on capacity is insignificant. However, when $m = \Omega(\sqrt{n})$ the network capacity is $\Theta(mW)$; implying that the capacity increases linearly with the number of base stations. Fig. 2 compares the capacity achieved under same cell routing policy with k -nearest as the number of base stations increases. One can observe a higher gain for our design when compared to k -nearest which is represented by the following three gain regimes (a), (b) and (c). (i) Region a: when $m = O(\sqrt{n})$, $G_c = \sqrt{\frac{m \log \frac{n}{m^2}}{\log n}}$; (ii) Region b: when $m = \Omega(\sqrt{n})$ and $O(\frac{n}{\log n})$, $G_c = \sqrt{\frac{n}{m \log n}}$; and (iii) Region c: when $m = \Omega(\frac{n}{\log n})$, $G_c = 1$. As mentioned earlier, we identified and solved the limiting factors, ISSUE I, ISSUE II and ISSUE III, in [6], which in turn provides a higher gain

for our design. In [7], authors studied the capacity of hybrid

TABLE I
 G_c UNDER THE SAME CELL ROUTING POLICY OVER L-HOP [7]

L	m	G_c
$\Omega(\frac{n^{1/3}}{\log^{2/3} n})$	$(\Omega(\frac{n}{L \log n}), O(\frac{n}{\log n}))$ $\Omega(\frac{n}{\log n})$ $O(\frac{n}{L \log n})$	$\sqrt{\frac{n}{m \log n}}$ 1 $L\sqrt{\frac{m \log n}{n}}$
$O(\frac{n^{1/3}}{\log^{2/3} n})$	$O(L^2 \log n)$ $\Omega(\frac{n}{\log n})$ $\Omega(L^2 \log n), O(\frac{n}{\log n})$	$\frac{\sqrt{nm}}{L^2 \log^{3/2} n}$ 1 $\sqrt{\frac{n}{m \log n}}$

wireless networks under L -Maximum hop routing strategy. In this policy, a node sends the data in ad hoc mode only if the destination can be reached within L hops. Otherwise, the data is transmitted through the base stations. Again, as in [6], authors assume a one-hop wireless uplink between a node and its base station, which seems to be an impractical solution (ISSUE II). Their results mainly indicate that (a) when $L = \Omega(\frac{n^{1/3}}{\log^{2/3} n})$ and m is $\Omega(\frac{n}{L \log n})$, the network capacity is given by $\Theta(mW)$. Otherwise, when $m = O(\frac{n}{L \log n})$, the network capacity is $\Theta(\frac{Wn}{L \log n})$; (b) when $L = O(\frac{n^{1/3}}{\log^{2/3} n})$ and m grows faster than $L^2 \log n$, the network capacity is given by $\Theta(mW)$. Otherwise, when m grows slower than $L^2 \log n$, the network capacity is given by $W(L^2 \log n)$. In Table I, we illustrate the gain of same cell routing policy over L -Hop for several combinations of m and L . For example, we see that when $L = \frac{n^{1/3}}{\log^{2/3} n}$ and $m = \frac{n^{2/3}}{\log^{1/3} n}$, same cell routing policy achieves a gain of $\frac{n^{1/6}}{\log^{1/3} n}$ over L -Hop. That is for a 10,000-node network, our design realizes a gain of ≈ 3 over L -Hop routing policy. This improvement stems from the accuracy of our design.

3) *Delay of Same Cell Routing Policy:* In [5], Gamal *et. al* studied the optimal capacity-delay tradeoff for pure ad hoc networks as well as mobile networks. For mobile networks, they prove that one can achieve a per-node capacity of $\Theta(1)$ with mobility but at the cost of $\Theta(n)$ delay. Whereas for pure ad hoc networks, they show that the average end-to-end packet delay can be bounded by $\Theta(\sqrt{\frac{n}{\log n}})$ which is in turn dependent on the number of nodes n . In both cases, we observe that as n increases, delay also increases largely. Table II illustrates G_d of same cell routing policy over existing schemes in [2],[4]. Again, we observe an improved performance for our design as opposed to those schemes in [2],[4].

TABLE II
 G_d UNDER THE SAME CELL ROUTING POLICY OVER EXISTING SOLUTIONS

Existing Works	$m = O(\frac{n}{\log n})$	$m = \Omega(\frac{n}{\log n})$
PANs MOBILITY	$\frac{\sqrt{m}}{\log n}$	$\frac{\sqrt{nm \log n}}{n}$

V. CONCLUSION

In this paper, we identify three critical factors, *number of hops*, *multi-hop uplinks* and *failure tolerant*, which is overlooked by existing schemes and propose a design that accurately resolves these issues. Specifically, we observe that whether $m = O(\frac{n}{\log n})$ or $\Omega(\frac{n}{\log n})$, our design achieves a per node capacity that scales with m . This is in fact a significant result as opposed to previous efforts which states that if m grows slower than some threshold, the benefit of augmenting those base stations to the original ad hoc network is insignificant. Besides, in comparison to existing works we clearly show the gain one could obtain on delay as well as on capacity in executing our design.

APPENDIX

A. D length Routing Policy

In this section, due to space constraints we only provide the upper bounds on the capacity for D length routing policy. However, the lower bounds can be obtained in a similar manner given by Section III-A. One may also note that only the capacity figures related to the purely ad hoc transmissions differs from the same cell routing policy and the figures related to the base station relative transmissions remain the same. Next to quantify the capacity bounds, we first determine the number of nodes communicating in purely ad hoc mode as well as in hybrid mode. We have the following Lemma.

Lemma 6. The number of S-D pairs communicating using purely ad hoc mode and hybrid mode are $n\pi D^2$ and $n(1 - \pi D^2)$ respectively, under D length routing policy.

Proof: The probability that a node communicates with its destination in purely ad hoc mode i.e., within D length is πD^2 . Likewise, the probability that a node communicates with its destination node in hybrid mode is $1 - \pi D^2$. Now, following the same techniques in Lemma 3 and replacing the corresponding probabilities with πD^2 and $1 - \pi D^2$, we get the number of nodes communicating in ad hoc and hybrid mode as $n\pi D^2$ and $n(1 - \pi D^2)$ respectively. ■

Let \bar{L}_a and \bar{L}_h respectively be the mean length of the distance taken by a packet in purely ad hoc mode and in hybrid mode. Since each node communicates with its chosen destination in ad hoc mode only if it is within D length from the source, we have \bar{L}_a as at most D . From earlier discussions, we also know that in a hybrid mode source communicates with its base station in a multi-hop fashion. Hence in a cell of size $\frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}}$, \bar{L}_h is at most $\frac{1}{\sqrt{m}}$. Replacing the left-hand side of the expression in eqn. (5) in Section III-B by $\frac{\Lambda_a(n, m)}{\tau(n)} \left(n\pi D^3 + n(1 - \pi D^2) \frac{1}{\sqrt{m}} \right)$, we have $\Lambda_a(n, m) \leq \frac{W_a}{\left(\pi D^3 + \frac{1 - \pi D^2}{\sqrt{m}} \right) \sqrt{n \log n}}$. We can thus write the capacity of D length routing policy as follows $\Lambda(n, m) = \Theta\left(\frac{W_a}{\left(\pi D^3 + \frac{1 - \pi D^2}{\sqrt{m}} \right) \sqrt{n \log n}} + \frac{m}{n} W_u \right)$. One may note that when $m = O(n/\log n)$, setting $D = 0$ and 1 maps the capacity of D length routing policy to capacity of same cell routing policy

and pure ad hoc networks respectively. Thus, the maximum capacity realizable under this policy is no more than $W \sqrt{\frac{m}{n \log n}}$ for $m = O(n/\log n)$. Intuitively, as D decreases the probability of nodes communicating in hybrid mode increases; this in turn implies that most of the nodes communicate with the base station in purely ad hoc mode at a distance of $O(m^{-1/2})$. Ultimately, we have $\bar{L}_a = \bar{L}_h = m^{-1/2}$ thus leading to the same expression in eqn. (5).

B. Some useful results

1. **BANDWIDTH ALLOCATION:** From Section II-D, it follows that $W = W_a + 2W_u$. Thus, by replacing W_a in Theorem 1 by $W_a = W - 2W_u$ and rearranging some terms, we get $\Lambda(n, m) = W \sqrt{\frac{m}{n \log n}} + \left(1 - 2\sqrt{\frac{n}{m \log n}} \right) \frac{m}{n} W_u$. Since the factor $\left(1 - 2\sqrt{\frac{n}{m \log n}} \right) < 0$ for $m = O(\frac{n}{\log n})$, we get the maximum capacity by allocating most of the bandwidth for ad hoc mode transmissions i.e., $W_a/W \rightarrow 1$.

2. **CHERNOFF BOUNDS:** Let X_1, \dots, X_n be independent Poisson trails, where $Pr[X_i = 1] = p_i$. Let $X = \sum_{i=1}^n X_i$. Then, for $0 < \delta \leq 1$ we have $Pr[X \geq (1 + \delta)E[X]] \leq \exp\left(-\frac{\delta^2}{3}E[X]\right)$.

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