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Flexible Robots

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Abstract

Mechanical flexibility in robot manipulators is due to compliance at the joints and/or distributed deflection of the links. Dynamic models of the two classes of robots with flexible joints or flexible links are presented, together with control laws addressing the motion tasks of regulation to constant equilibrium states and of asymptotic tracking of output trajectories. Control design for robots with flexible joints takes advantage of the passivity and feedback linearization properties. In robots with flexible links, basic differences arise when controlling the motion at the joint level or at the tip level.

Keywords Joint elasticity • Link flexibility • Regulation by motor feedback • Gravity compensation • Vibration damping • Singular perturbation • Feedback linearization • Noncausal and stable inversion

Introduction

Robot manipulators are usually considered as rigid multi-body mechanical systems. This ideal assumption simplifies dynamic analysis and control design but may lead to performance degradation and even unstable behavior, due to the excitation of vibrational phenomena.

Flexibility is mainly due to the limited stiffness of transmissions at the joints (Sweet and Good 1985) and to the deflection of slender and lightweight links (Cannon and Schmitz 1984). Joint flexibility is common when motion transmission/reduction elements such as belts, long shafts, cables, harmonic drives, or cycloidal gears are used. Link flexibility is present in large articulated structures, such as very long arms needed for accessing hostile environments (deep sea or space) or automated crane devices for building construction. In both situations, static displacements and dynamic oscillations are introduced between the driving actuators and the actual position of the robot end effector. Undesired vibrations are typically confined beyond the closed-loop control bandwidth, but flexibility cannot be neglected when large speed/acceleration and high accuracy are requested by the task.

In the dynamic modeling, flexibility is assumed concentrated at the robot joints or distributed along the robot links (most of the times with some finite-dimensional approximation). In both cases, additional generalized coordinates are introduced beside those used to describe the rigid motion of the arm in a Lagrangian formulation. As a result, the number of available control inputs is strictly less than the number of degrees of freedom of the mechanical system. This type of under-actuation, though counterbalanced by the presence of additional potential energy helping to achieve system controllability, suggests that the design of satisfactory motion control laws is harder than in the rigid case.

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From a control point of view, different design approaches are needed because of structural differences arising between flexible-joint and flexible-link robots. These differences hold for single- or multiple-link robots, in the linear or nonlinear domain, and depend on the physical co-location or not of mechanical flexibility versus control actuation, as well as on the choice of controlled outputs.

In order to measure the state of flexible robots for trajectory tracking control or feedback stabilization purposes, a large variety of sensing devices can be used, including encoders, joint torque sensors, strain gauges, accelerometers, and high-speed cameras. In particular, measuring the full state of the system would require twice the number of sensors than in the rigid case for robots with flexible joints and possibly more for robots with flexible links. The design of controllers that work provably good with a reduced set of measurements is thus particularly attractive.

Robots with Flexible Joints

Dynamic Modeling

A robot with flexible joints is modeled as an open kinematic chain of $n + 1$ rigid bodies, interconnected by n joints undergoing deflection and actuated by n electrical motors. Let $\boldsymbol{\theta}$ be the n-vector of motor (i.e., rotor) positions, as reflected through the reduction gears, and $\boldsymbol{q}$ the n-vector of link positions. The joint deflection is $\boldsymbol{\delta} = \boldsymbol{\theta} - \boldsymbol{q} \not\equiv \mathbf{0}$. The standard assumptions are:

A1 Joint deflections $\boldsymbol{\delta}$ are small, limited to the domain of linear elasticity. The elastic torques due to joint deformations are $\boldsymbol{\tau}_J = \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q})$, where $\boldsymbol{K}$ is the positive definite, diagonal joint stiffness matrix.

A2 The rotors of the electrical motors are modeled as uniform bodies having their center of mass on the rotation axis.

A3 The angular velocity of the rotors is due only to their own spinning.

The last assumption, introduced by Spong (1987), is very reasonable for large reduction ratios and also crucial for simplifying the dynamic model.

From the gravity and elastic potential energy, $\mathcal{U} = \mathcal{U}_g + \mathcal{U}_\delta$, and the kinetic energy $\mathcal{T}$ of the robot, applying the Euler-Lagrange equations to the Lagrangian $\mathcal{L} = \mathcal{T} - \mathcal{U}$ and neglecting all dissipative effects leads to the dynamic model

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{\theta}) = \mathbf{0} \tag{1}$$

$$\boldsymbol{B}\ddot{\boldsymbol{\theta}} + \boldsymbol{K}(\boldsymbol{\theta} - \boldsymbol{q}) = \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{M}(\boldsymbol{q})$ is the positive definite, symmetric inertia matrix of the robot links (including the motor masses); $\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the sum of Coriolis and centrifugal terms $\boldsymbol{c}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ (quadratic in $\dot{\boldsymbol{q}}$) and gravitational terms $\boldsymbol{g}(\boldsymbol{q}) = (\partial\mathcal{U}_g/\partial\boldsymbol{q})^T$; $\boldsymbol{B}$ is the positive definite, diagonal matrix of motor inertias (reflected through the gear ratios); and $\boldsymbol{\tau}$ are the motor torques (performing work on $\boldsymbol{\theta}$). The inertia matrix of the *complete* system is then $\boldsymbol{\mathcal{M}}(q) = \text{block diag}\{M(q), B\}$. The two n-dimensional second-order differential equations (1) and (2) are referred to as the *link* and the *motor* equations, respectively. When the joint stiffness $\boldsymbol{K} \to \infty$, it is $\boldsymbol{\theta} \to \boldsymbol{q}$ and $\boldsymbol{\tau}_J \to \boldsymbol{\tau}$, so that the two equations collapse in the limit into the standard dynamic model of rigid robots with total inertia $\boldsymbol{\mathcal{M}}(\boldsymbol{q}) = \boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{B}$. On the other hand, when the joint stiffness $\boldsymbol{K}$ is relatively large but still

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finite, robots with elastic joints show a *two-time-scale* dynamic behavior. A common large scalar factor $1/\epsilon^2 \gg 1$ can be extracted from the diagonal stiffness matrix as $\boldsymbol{K} = \hat{\boldsymbol{K}}/\epsilon^2$. The *slow* subsystem is associated to the link dynamics

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{\tau}_J, \tag{3}$$

while the *fast* subsystem takes the form

$$\epsilon^2 \ddot{\boldsymbol{\tau}}_J = \hat{\boldsymbol{K}} \left(\boldsymbol{B}^{-1} (\boldsymbol{\tau} - \boldsymbol{\tau}_J) + \boldsymbol{M}^{-1}(\boldsymbol{q}) \left(\boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) - \boldsymbol{\tau}_J \right) \right) \tag{4}$$

For small ϵ, Eqs. (3) and (4) represent a singularly perturbed system. The two separate time scales governing the slow and fast dynamics are t and $\sigma = t/\epsilon$.

Regulation

The basic robotic task of moving between two arbitrary equilibrium configurations is realized by a feedback control law that asymptotically stabilizes the desired robot state.

In the absence of gravity ($\boldsymbol{g} \equiv \boldsymbol{0}$), the equilibrium states are parameterized by the desired reference position $\boldsymbol{q}_d$ of the links and take the form $\boldsymbol{q} = \boldsymbol{q}_d$, $\boldsymbol{\theta} = \boldsymbol{\theta}_d = \boldsymbol{q}_d$ (with no joint deflection at steady state) and $\dot{\boldsymbol{q}} = \dot{\boldsymbol{\theta}} = \boldsymbol{0}$. As a result of passivity of the mapping from $\boldsymbol{\tau}$ to $\dot{\boldsymbol{\theta}}$, global regulation is achieved by a *decentralized PD* law using only feedback from the motor variables,

$$\boldsymbol{\tau} = \boldsymbol{K}_P(\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \boldsymbol{K}_D\dot{\boldsymbol{\theta}}, \tag{5}$$

with diagonal $\boldsymbol{K}_P > 0$ and $\boldsymbol{K}_D > 0$.

In the presence of gravity, the (unique) equilibrium position of the motor associated with a desired link position $\boldsymbol{q}_d$ becomes $\boldsymbol{\theta}_d = \boldsymbol{q}_d + \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q}_d)$. Global regulation is obtained by adding an extra gravity-dependent term $\boldsymbol{\tau}_g$ to the PD control law (5),

$$\boldsymbol{\tau} = \boldsymbol{K}_P(\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \boldsymbol{K}_D\dot{\boldsymbol{\theta}} + \boldsymbol{\tau}_g, \tag{6}$$

with diagonal matrices $\boldsymbol{K}_P > 0$ (at least) and $\boldsymbol{K}_D > 0$. The term $\boldsymbol{\tau}_g$ needs to match the gravity load $\boldsymbol{g}(\boldsymbol{q}_d)$ at steady state. The following choices are of slight increasing control complexity, with progressively better transient performance.

- *Constant* gravity compensation: $\boldsymbol{\tau}_g = \boldsymbol{g}(\boldsymbol{q}_d)$. Global regulation is achieved when the smallest positive gain in the diagonal matrix $\boldsymbol{K}_P$ is large enough (Tomei 1991). This sufficient condition can be enforced only if the joint stiffness $\boldsymbol{K}$ dominates the gradient of gravity terms.
- *Online* compensation: $\boldsymbol{\tau}_g = \boldsymbol{g}(\tilde{\boldsymbol{\theta}})$, $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{K}^{-1}\boldsymbol{g}(\boldsymbol{q}_d)$. Gravity effects on the links are approximately compensated during robot motion. Global regulation is proven under the same conditions above (De Luca et al. 2005).
- *Quasi-static* compensation: $\boldsymbol{\tau}_g = \boldsymbol{g}(\tilde{\boldsymbol{q}}(\boldsymbol{\theta}))$. At any measured motor position $\boldsymbol{\theta}$, the link position $\tilde{\boldsymbol{q}}(\boldsymbol{\theta})$ is computed by solving numerically $\boldsymbol{g}(\boldsymbol{q}) + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{\theta}) = \boldsymbol{0}$. This removes the need of a strictly positive lower bound on $\boldsymbol{K}_P$ (Kugi et al. 2008), but the joint stiffness should still dominate the gradient of gravity terms.

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Additional feedback from the full robot state $(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$, measured or reconstructed through dynamic observers, can provide faster and damped transient responses. This solution is particularly convenient when a joint torque sensor measuring $\boldsymbol{\tau}_J$ is available (*torque-controlled* robots). Using

$$\boldsymbol{\tau} = \boldsymbol{K}_P(\boldsymbol{\theta}_d - \boldsymbol{\theta}) - \boldsymbol{K}_D\dot{\boldsymbol{\theta}} + \boldsymbol{K}_T(\boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{\tau}_J) - \boldsymbol{K}_S\dot{\boldsymbol{\tau}}_J + \boldsymbol{g}(\boldsymbol{q}_d), \tag{7}$$

the four diagonal gain matrices can be given a special structure so that asymptotic stability is automatically guaranteed (Albu-Schäffer and Hirzinger 2001).

Trajectory Tracking

Let a desired sufficiently smooth trajectory $\boldsymbol{q}_d(t)$ be specified for the robot links over a finite or infinite time interval. The control objective is to asymptotically stabilize the trajectory tracking error $\boldsymbol{e} = \boldsymbol{q}_d(t) - \boldsymbol{q}(t)$ to zero, starting from a generic initial robot state. Assuming that $\boldsymbol{q}_d(t)$ is four times continuously differentiable, a torque input profile $\boldsymbol{\tau}_d(t) = \boldsymbol{\tau}_d(\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d, \dddot{\boldsymbol{q}}_d, \ddddot{\boldsymbol{q}}_d)$ can be derived from the dynamic model (1) and (2) so as to reproduce exactly the desired trajectory, when starting from matched initial conditions. A local solution to the trajectory tracking problem is provided by the combination of such feedforward term $\boldsymbol{\tau}_d(t)$ with a stabilizing linear feedback from the partial or full robot state; see Eqs. (6) or (7).

When the joint stiffness is large enough, one can take advantage of the system being *singularly perturbed*. A control law $\boldsymbol{\tau}_s$ designed for the rigid robot will deal with the slow dynamics, while a relatively simple action $\boldsymbol{\tau}_f$ is used to stabilize the fast vibratory dynamics around an invariant manifold associated to the rigid robot control (Spong et al. 1987). This class of composite control laws has the general form

$$\boldsymbol{\tau} = \boldsymbol{\tau}_s(\boldsymbol{q}, \dot{\boldsymbol{q}}, t) + \epsilon\boldsymbol{\tau}_f(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\tau}_J, \dot{\boldsymbol{\tau}}_J). \tag{8}$$

When setting $\epsilon = 0$ in Eqs. (3), (4), and (8), the control setup of the equivalent rigid robot is recovered as

$$(\boldsymbol{M}(\boldsymbol{q}) + \boldsymbol{B})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = \boldsymbol{\tau}_s. \tag{9}$$

Though more complex, the best performing trajectory tracking controller for the general case is based on *feedback linearization*. Spong (1987) has shown that the nonlinear state feedback

$$\boldsymbol{\tau} = \boldsymbol{\alpha}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \dddot{\boldsymbol{q}}) + \boldsymbol{\beta}(\boldsymbol{q})\boldsymbol{v}, \tag{10}$$

with

$$\begin{aligned} \boldsymbol{\alpha} &= \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{B}\boldsymbol{K}^{-1}\left(\left(\ddot{\boldsymbol{M}}(\boldsymbol{q}) + \boldsymbol{K}\right)\ddot{\boldsymbol{q}} + 2\dot{\boldsymbol{M}}(\boldsymbol{q})\dddot{\boldsymbol{q}} + \ddot{\boldsymbol{n}}(\boldsymbol{q}, \dot{\boldsymbol{q}})\right) \\ \boldsymbol{\beta} &= \boldsymbol{B}\boldsymbol{K}^{-1}\boldsymbol{M}(\boldsymbol{q}), \end{aligned}$$

leads globally to the closed-loop linear system

$$\boldsymbol{q}^{[4]} = \boldsymbol{v}, \tag{11}$$

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i.e., to decoupled chains of four input–output integrators from each auxiliary input v_i to each link position output q_i, for $i = 1, \ldots, n$. The control design is then completed on the linear SISO side, by forcing the trajectory tracking error to be *exponentially stable* with an arbitrary decaying rate. The control law (10) is expressed as a function of the *linearizing* coordinates $(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \dddot{\boldsymbol{q}})$ (up to the link jerk), which can be however rewritten in terms of the original state $(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ using the dynamic model equations. This fundamental result is the direct extension of the so-called "computed torque" method for rigid robots.

Robots with Flexible Links

Dynamic Modeling

For the dynamic modeling of a single flexible link, the distributed nature of structural flexibility can be captured, under suitable assumptions, by partial differential equations (PDE) with associated boundary conditions. A common model is the *Euler-Bernoulli* beam. The link is assumed to be a slender beam, with uniform geometric characteristics and homogeneous mass distribution, clamped at the base to the rigid hub of an actuator producing a torque τ and rotating on a horizontal plane. The beam is flexible in the lateral direction only, being stiff with respect to axial forces, torsion, and bending due to gravity. Deformations are small and are in the elastic domain. The physical parameters of interest are the linear density ρ of the beam, its flexural rigidity EI, the beam length ℓ, and the hub inertia I_h (with $I_t = I_h + \rho\ell^3/3$). The equations of motion combine lumped and distributed parameter parts, with the hub rotation $\theta(t)$ and the link deformation $w(x,t)$, being $x \in [0, \ell]$ the position along the link. From Hamilton principle, we obtain

$$I_t\, \ddot{\theta}(t) + \rho \int_0^\ell x\, \ddot{w}(x,t)\, dx = \tau(t) \tag{12}$$

$$EI\, w''''(x,t) + \rho\, \ddot{w}(x,t) + \rho\, x\, \ddot{\theta}(t) = 0 \tag{13}$$

$$w(0,t) = w'(0,t) = 0, \quad w''(\ell,t) = w'''(\ell,t) = 0, \tag{14}$$

where a prime denotes partial derivative w.r.t. to space. Equation (14) are the clamped-free boundary conditions at the two ends of the beam (no payload is present at the tip).

For the analysis of this self-adjoint PDE problem, one proceeds by separation of variables in space and time, defining

$$w(x,t) = \phi(x)\delta(t) \qquad \theta(t) = \alpha(t) + k\delta(t), \tag{15}$$

where $\phi(x)$ is the link spatial deformation, $\delta(t)$ is its time behavior, $\alpha(t)$ describes the angular motion of the instantaneous center of mass of the beam, and k is chosen so as to satisfy (12) for $\tau = 0$. Being system (12)–(14) linear, nonrational transfer functions can be derived in the Laplace transform domain between the input torque and some relevant system output, e.g., the angular position of the hub or of the tip of the beam (Kanoh 1990). The PDE formalism provides also a convenient basis for analyzing distributed sensing, feedback from strain sensors (Luo 1993), or even distributed actuation with piezo-electric devices placed along the link.

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The transcendental characteristic equation associated to the spatial part of the solution to Eqs. (12)–(14) is

$$I_h \gamma^3 \big(1 + \cos(\gamma \ell) \cosh(\gamma \ell)\big) + \rho \big(\sin(\gamma \ell) \cosh(\gamma \ell) - \cos(\gamma \ell) \sinh(\gamma \ell)\big) = 0. \tag{16}$$

When the hub inertia $I_h \to \infty$, the second term can be neglected and the characteristic equation collapses into the so-called *clamped* condition. Equation (16) has an infinite but countable number of positive real roots γ_i, with associated eigenvalues of resonant frequencies $\omega_i = \gamma_i^2 \sqrt{EI/\rho}$ and orthonormal eigenvectors $\phi_i(x)$, which are the natural deformation shapes of the beam (Barbieri and Özgüner 1988). A finite-dimensional dynamic model is obtained by truncation to a finite number m_e of eigenvalues/shapes. From

$$w(x,t) = \sum_{i=1}^{m_e} \phi_i(x) \delta_i(t) \tag{17}$$

we get

$$\begin{aligned} I_t \ddot{\alpha}(t) &= \tau(t) \\ \ddot{\delta}_i(t) + \omega_i^2 \delta_i(t) &= \phi_i'(0) \tau(t), \qquad i = 1, \ldots, m_e, \end{aligned} \tag{18}$$

where the rigid body motion (top equation) appears as decoupled from the flexible dynamics, thanks to the choice of variable α rather than θ. Modal damping can be added on the left-hand sides of the lower equations through terms $2\zeta_i \omega_i \dot{\delta}_i$ with $\zeta_i \in [0,1]$. The angular position of the motor hub at the joint is given by

$$\theta(t) = \alpha(t) + \sum_{i=1}^{m_e} \phi_i'(0) \delta_i(t), \tag{19}$$

while the tip angular position is

$$y(t) = \alpha(t) + \sum_{i=1}^{m_e} \frac{\phi_i(\ell)}{\ell} \delta_i(t). \tag{20}$$

The *joint-level* transfer function $p_{\text{joint}}(s) = \theta(s)/\tau(s)$ will always have relative degree two and only minimum phase zeros. On the other hand, the *tip-level* transfer function $p_{\text{tip}}(s) = y(s)/\tau(s)$ will contain non-minimum phase zeros. This basic difference in the pattern of the transmission zeros is crucial for motion control design.

In a simpler modeling technique, a specified class of spatial functions $\phi_i(x)$ is assumed for describing link deformation. The functions need to satisfy only a reduced set of geometric boundary conditions (e.g., *clamped* modes at the link base), but otherwise no dynamic equations of motion such as (13). The use of finite-dimensional expansions like (17) limits the validity of the resulting model to a maximum frequency. This truncation must be accompanied by suitable filtering of measurements and of control commands, so as to avoid or limit spillover effects (Balas 1978).

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In the dynamic modeling of robots with n flexible links, the resort to *assumed modes* of link deformation becomes unavoidable. In practice, some form of approximation and a finite-dimensional treatment is necessary. Let $\boldsymbol{\theta}$ be the n-vector of joint variables describing the rigid motion, and $\boldsymbol{\delta}$ be the m-vector collecting the deformation variables of all flexible links. Following a Lagrangian formulation, the dynamic model with clamped modes takes the general form (Book 1984)

$$\begin{pmatrix} \boldsymbol{M}_{\theta\theta}(\boldsymbol{\theta},\boldsymbol{\delta}) & \boldsymbol{M}_{\theta\delta}(\boldsymbol{\theta},\boldsymbol{\delta}) \\ \boldsymbol{M}_{\theta\delta}^T(\boldsymbol{\theta},\boldsymbol{\delta}) & \boldsymbol{M}_{\delta\delta}(\boldsymbol{\theta},\boldsymbol{\delta}) \end{pmatrix} \begin{pmatrix} \ddot{\boldsymbol{\theta}} \\ \ddot{\boldsymbol{\delta}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{n}_{\theta}(\boldsymbol{\theta},\boldsymbol{\delta},\dot{\boldsymbol{\theta}},\dot{\boldsymbol{\delta}}) \\ \boldsymbol{n}_{\delta}(\boldsymbol{\theta},\boldsymbol{\delta},\dot{\boldsymbol{\theta}},\dot{\boldsymbol{\delta}}) \end{pmatrix} + \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{D}\dot{\boldsymbol{\delta}} + \boldsymbol{K}\boldsymbol{\delta} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\tau} \\ \boldsymbol{0} \end{pmatrix}, \tag{21}$$

where the positive definite, symmetric inertia matrix $\boldsymbol{\mathcal{M}}$ of the complete robot and the Coriolis, centrifugal, and gravitational terms $\boldsymbol{n}$ have been partitioned in blocks of suitable dimensions, $\boldsymbol{K} > 0$ and $\boldsymbol{D} \geq 0$ are the robot link stiffness and damping matrices, and $\boldsymbol{\tau}$ is the n-vector of actuating torques.

The dynamic model (21) shows the general couplings existing between nonlinear rigid body motion and linear flexible dynamics. In this respect, the linear model (18) of a single flexible link is a remarkable exception.

The choice of specific assumed modes may simplify the blocks of the robot inertia matrix, e.g., orthonormal modes used for each link induce a decoupled structure of the diagonal inertia subblocks of $\boldsymbol{M}_{\delta\delta}$. Quite often the total kinetic energy of the flexible robot is evaluated only in the undeformed configuration $\boldsymbol{\delta} = \boldsymbol{0}$. With this approximation, the inertia matrix becomes independent of $\boldsymbol{\delta}$, and so the velocity terms in the model. Furthermore, due to the hypothesis of small deformation of each link, the dependence of the gravity term in the lower component $\boldsymbol{n}_{\delta}$ is only a function of $\boldsymbol{\theta}$.

The validation of (21) goes through the experimental identification of the relevant dynamic parameters. Besides those inherited from the rigid case (mass, inertia, etc.), also the set of structural resonant frequencies and associated deformation profiles should be identified.

Control of Joint-Level Motion

When the target variables to be controlled are defined at the joint level, the control problem for robots with flexible links is similar to that of robots with flexible joints. As a matter of fact, the models (1), (2), and (21) are both *passive* systems with respect to the output $\boldsymbol{\theta}$; see (19) in the scalar case. For instance, regulation is achieved by a PD action with constant gravity compensation, using a control law of the form (6) without the need of feeding back link deformation variables (De Luca and Siciliano 1993a). Similarly, stable tracking of a joint trajectory $\boldsymbol{\theta}_d(t)$ is obtained by a singular perturbation control approach, with flexible modes dynamics acting at multiple time scales with respect to rigid body motion (Siciliano and Book 1988), or by an inversion-based control (De Luca and Siciliano 1993b), where input–output (rather than full state) exact linearization is realized and the effects of link flexibility are canceled on the motion of the robot joints. While vibrational behavior will still affect the robot at the level of end-effector motion, the closed-loop dynamics of the $\boldsymbol{\delta}$ variables is stable and link deformations converge to a steady-state constant value (zero in the absence of gravity) thanks to the intrinsic damping of the mechanical structure. Improved transients are indeed obtained by active modal damping control (Cannon and Schmitz 1984).

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A control approach specifically developed for the rest-to-rest motion of flexible mechanical systems is *command shaping* (Singer and Seering 1990). The original command designed to achieve a desired motion for a rigid robot is convolved with suitable signals delayed in time, so as to cancel (or reduce to a minimum) the effects of the excited vibration modes at the time of motion completion. For a single slewing link with linear dynamics, as in (18), the rest-to-rest input command is computed in closed form by using impulsive signals and can be made robust via an over-parameterization.

Control of Tip-Level Motion

The design of a control law that allows asymptotic tracking of a desired trajectory for the end effector of a robot with flexible links needs to face the unstable zero dynamics associated to the problem. In the linear case of a single flexible link, this is equivalent to the presence of non-minimum phase zeros in the transfer function to the tip output (20). Direct inversion of the input–output map leads to instability, due to cancellation of non-minimum phase zeros by unstable poles, with link deformation growing unbounded and control saturations.

The solution requires instead to determine the unique reference state trajectory of the flexible structure that is associated to the desired tip trajectory and has *bounded* deformation. Based on regulation theory, the control law will be the superposition of a nominal feedforward action, which keeps the system along the reference state trajectory (and thus the output on the desired trajectory), and of a stabilizing feedback that reduces the error with respect to this state trajectory to zero without resorting to dangerous cancellations.

In general, computing such a control law requires the solution of a set of nonlinear partial differential equations. However, in the case of a single flexible link with linear dynamics, the feedforward profile is simply derived by an inversion defined in the *frequency domain* (Bayo 1987). The desired tip acceleration $\ddot{y}_d(t)$, $t \in [0, T]$, is considered as part of a rest-to-rest periodic signal, with zero mean value and zero integral. The procedure, implemented efficiently using Fast Fourier Transform on discrete-time samples, will automatically generate bounded time signals only. The resulting unique torque profile $\tau_d(t)$ will be a *noncausal* command, anticipating the actual start of the output trajectory at $t = 0$ (so as to *precharge* the link to the correct initial deformation) and ending after $t = T$ (to *discharge* the residual link deformation and recover the final rest configuration).

The same result was recovered by Kwon and Book (1994) in the time domain, by forward integrating in time the stable part of the inverse system dynamics and backward integrating the unstable part. An extension to the multi-link nonlinear case uses an iterative approach on repeated linear approximations of the system along the nominal trajectory (Bayo et al. 1989).

Summary and Future Directions

The presence of mechanical flexibility in the joints and the links of multi-dof robots poses challenging control problems. Control designs take advantage or are limited by some system-level properties. Robots with flexible joints are passive systems at the level of motor outputs, have no zero dynamics associated to the link position outputs, and are always feedback linearizable systems. Robots with flexible links are still passive for joint-level outputs, but cannot be feedback

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linearized in general, and have unstable zero dynamics (non-minimum phase zeros in the linear case) when considering the end-effector position as controlled output.

State-of-the-art control laws address regulation and trajectory tracking tasks in a satisfactory way, at least in nominal conditions and under full-state feedback. Current research directions are aimed at achieving robustness to model uncertainties and external disturbances (with adaptive, learning, or iterative schemes), and further exploit the design of control laws under limited measurements and noisy sensing. Beyond free motion tasks, an accurate treatment of interaction tasks with the environment, requiring force or impedance controllers, is still missing for flexible robots. In this respect, passivity-based control approaches that do not necessarily operate dynamic cancellations may take advantage of the existing compliance, trading off between improved energy efficiency and some reduction in nominal performance.

Often seen as a limiting factor for performance, the presence of joint elasticity is now becoming an explicit advantage for safe physical human-robot interaction and for locomotion. Next generation lightweight robots and humanoids will use flexible joints and also compact actuation with online controlled variable joint stiffness, an area of active research.

Cross-References

- ▸ Feedback Linearization of Nonlinear Systems
- ▸ Hamiltonian Systems Approach to the Control of PDEs
- ▸ Inversion of Nonlinear Systems
- ▸ Modeling of Dynamic System[s]
- ▸ Nonlinear Zero Dynamics
- ▸ PID Control
- ▸ Regulation and Tracking of Nonlinear Systems

Recommended Reading

In addition to the works cited in the body of this article, a detailed treatment of dynamic modeling and control issues for flexible robots can be found in De Luca and Book (2008). This includes also the use of dynamic feedback linearization for a more general model of robots with elastic joints. For the same class of robots, Brogliato et al. (1995) provided a comparison of passivity-based and inversion-based tracking controllers.

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