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A Twenty-Year Review of Time-Delay Feedback Control and Recent Developments

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Abstract—The delayed feedback control (DFC) method has been invented in 1992 (this year is a 20th anniversary). Following the original paper by Pyragas [Phys. Lett. A, 170, 421, 1992], more than 1500 papers devoted or related to the DFC have been published. Many different modifications of the algorithm have been proposed, and significant achievements are attained in the theory of the DFC. Although this theory is non-trivial, currently the mechanism of the DFC action is rather well understood, and the main limitations of the algorithm are established. The DFC has been successfully implemented in a number of experimental systems of different physical nature. The aim of this talk is to present a brief review of important modifications of the DFC algorithm, significant theoretical results and experimental implementations attained during the past twenty years. The recent results concerning adaptive modifications of the DFC and analytical achievements based on phase reduction of time-delay systems will be discussed as well.

1. Brief review of experimental and theoretical results

The DFC algorithm [1] is a simple, robust, and efficient method to stabilize unstable periodic orbits (UPOs) in chaotic systems. Nowadays, it becomes one of the most popular methods in the chaos control research [2]. The method allows a noninvasive stabilization of UPOs of dynamical systems in the sense that the control force vanishes when the target state is reached. The DFC algorithm is reference-free and makes use of a control signal $K\Delta s(t)$ obtained from the difference $\Delta s(t) = s(t) - s(t-\tau)$ between the current state $s(t)$ of the system and the state of the system $s(t-\tau)$ delayed by one-period τ of the target orbit. The UPO may become stable under the appropriate choice of feedback strength K. Note that only the stability properties of the orbit are changed, while the orbit itself and its period remain unaltered. The controlled system can be treated as a black box, since the method does not require any exact knowledge of either the form of the periodic orbit or the system's equations. The method is particularly appealing for experimentalists, since one does not need to know anything about the target orbit beyond its period τ. The DFC algorithm is notably superior to other control methods in fast dynamical systems, since it does not require any real-time computer processing.

Successful implementation of the DFC algorithm has been attained in diverse experimental systems, including electronic chaotic oscillators, mechanical pendulums, lasers, gas discharge systems, a current-driven ion acoustic instability, a chaotic Taylor-Couette flow, chemical systems, high-power ferromagnetic resonance, helicopter rotor blades, and a cardiac system. (cf. [3] for review up to 2006). An important practical application of the DFC algorithm has been recently demonstrated by Yamasue et al. [4]. The authors have successfully implemented the DFC method in an atomic force microscope and managed to stabilize cantilever oscillations. As a result, they remove artifacts on a surface image. Another interesting application of the DFC for the analysis of bifurcations of periodic states in experimental systems has been recently considered in Ref. [5].

A reach variety of modifications of the DFC has been suggested in order to improve its performance (cf. [3]). Here we mention only the most important modification known as an extended DFC (EDFC), which has been introduced in Ref. [6]. The authors improved an original DFC scheme by using an information from many previous states of the system. The EDFC scheme achieves stabilization of UPOs with a greater degree of instability [7, 8].

The theory of DFC is difficult because the delayed feedback induces an infinite number of degrees of freedom. Even linear analysis of such systems is complicated due to the infinite number of Floquet exponents characterizing the stability of controlled orbits. Nevertheless, some analytical approaches have been developed in vicinity to various bifurcations of periodic orbits, such as the period doubling bifurcation [9, 10], the subcritical Hopf bifurcation [11, 12, 13] and the Nejmark-Sacker (discrete Hopf) bifurcation [14].

In 1997 Nakajima [15, 9] proved the so-called odd number limitation, which states that any UPOs with an odd number of real Floquet multipliers greater than unity can never be stabilized by any DFC technique. This limitation has been commonly accepted and intensively discussed in the literature. However, in 2007, Fiedler at al [16] have shown by a simple example that this limitation does not hold in general for autonomous systems (note that for non-autonomous systems it remains valid in general). Recently, a modified (corrected) proof of the limitation for autonomous systems has been presented by Hooton and

Amann [17]. To overcome the odd number limitation a counterintuitive idea based on an unstable controller has been proposed [11, 12, 13].

The group of Pikovsky has proposed to use the DFC for controlling oscillation coherence in noisy systems [18, 19]. This research was further developed by the group of Schöll [20, 33, 22]. The recent achievements in this field are presented in Refs. [23, 24]. The DFC has been also used to control the motion of an overdamped Brownian particle in a washboard potential exerted to a static tilting force [25].

Another interesting proposal of the Pikovsky's group is related to the DFC control of synchronization in globally coupled oscillator networks [26, 28]. This research has a potential application for suppression of pathological rhythms in neural ensembles, which appear due to synchronization of individual neurons and is believed to play the crucial role in Parkinsons disease, essential tremor, and epilepsies. These ideas were further developed by the group of Tass [29, 30].

A number of publications are devoted to the use of DFC for controlling spatio-temporal patterns in reaction-diffusion systems [31, 32, 33, 34].

An interesting idea of implementing a variable or distributed delay time into DFC algorithm is reported in Refs. [35, 36]. It is shown that such a modification may substantially enlarge the domain of stability of controlled orbits or steady-states.

2. Recent theoretical developments

Recent works in the theory of DFC have been mainly devoted to the analysis of the global properties (basins of attraction) of stabilized UPOs [37, 38, 39], adaptive modifications [40, 41], and analytical approaches based on extension of the phase reduction theory to time-delay systems [42, 43].

2.1. Basins of attraction

The standard tool for discussing the control performance of DFC systems consists in linear stability analysis. However, even if such a local analysis predicts stable states, experimental success is not guaranteed because the control performance may strongly depend on initial conditions. To improve the global properties of the linear DFC algorithm several modifications have been proposed. A first heuristic idea has been suggested in the original paper [1]. It has been shown that limiting the size of the control force by a simple cutoff increases a basin of attraction of the stabilized orbit. This idea has proven itself in a number of chaotic systems, and now it is widely used in experimental implementations of DFC method. An alternative two-step DFC algorithm has been proposed in [38]. In [44, 37], the authors have developed a theory which stated that depending on the type of transition at the control boundary there appear basins of attraction of different size; for a continuous transition the basin of attraction is large, while for a discontinuous transition it is small.

Unfortunately, the above-proposed nonlinear DFC schemes are not universal. In recent paper [39], we have elaborated a DFC algorithm with the improved global properties by invoking an egodicity — the universal feature of chaotic systems. The ergodicity means the fact that a trajectory of any chaotic system visits a neighborhood of each periodic orbit with finite probability. We do not perturb the system until it comes in a small neighborhood of the desired orbit and then activate the control. Note that the problem of evaluating the moment when the state of the free system approaches the target orbit has to be considered in an infinite-dimensional phase space, since the DFC force increases the phase space dimension. Using a scalar observable, we have developed a technique based on a linear filter which evaluates the running closeness of the system to the desired orbit in infinite-dimensional phase space.

2.2. Adaptive DFC with a state dependent time delay

Experimental implementation of the DFC method requires a knowledge of delay time, which is equal to the period of actual UPO. However, for autonomous systems, this period is not known a priori. Furthermore, in real experimental situation, the period of actual UPO may evolve due to the evolution of the system parameters under the effects of exogenous, unpredictable factors. In this context, adaptive control techniques with an automatic adjustment of the delay time are desired. The problem of estimating the period of a target UPO from observed experiential data has been considered in several publications. In the original paper [1], it has been shown that the amplitude of DFC perturbation has a resonance-type dependence on the delay time, and the periods of UPOs can be extracted from the minima of this dependence. The first adaptive technique employing online variation of the delay time has been proposed [45]. Here the delay time is adjusted in a discrete way according to the distance between successive maxima of the output signal.

Recently, we have proposed [40] an adaptive DFC algorithm with a state-dependent delay, which is based on resonance dependence of the DFC perturbation on the delay time pointed out in [1]. The state-dependent time delay is varied continuously towards the period of controlled orbit according to a gradient-descent method realized through three simple ordinary differential equations. Another adaptive algorithm has been recently proposed in Ref. [41].

2.3. Phase reduction theory-based treatment of DFC

A sophisticated theoretical foundation for obtaining the UPO period from the observed control signal has been developed in [46]. The control signal in the DFC algorithm vanishes if the delay time is adjusted to be equal to the period of a target UPO. If the delay time differs slightly from the UPO period, a non-vanishing periodic control signal is observed. In Ref. [46] an analytical expression for this period has been derived in the case of the DFC algorithm applied to the systems having a single scalar input. Recently, we have generalized this expression for the multiple-input multiple-output systems controlled by an extended DFC (EDFC) algorithm [43]. Our approach is based on the phase reduction theory of weakly perturbed limit cycle oscillations in systems with time delay, which is developed in our recent publication [42]. This result is important for the experimental implementation of the EDFC algorithm, since it can facilitate the determination of unknown period of control-free UPO. Using an analytical relationship between the period of control signal and the control parameters, the unknown period of the UPO can be determined from only few experimental measurements (c.f. [46]).

3. Conclusions

Based on the above review, we can conclude that the delayed feedback control is still an active area of research. Hopefully, new interesting theoretical and experimental results will appear in the near future.

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