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Effect of measurement noise and excitation on Generalized Response Surface Model Updating

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ABSTRACT

This paper investigates the robustness of a parameter estimation procedure for nonlinear Finite Element (FE) model updating. Through this procedure, polynomial Response Surface (RS) models are constructed to approximate the response of a nonlinear FE model at every time step of the analysis. Subsequently, the optimization problem of model updating is solved iteratively in time which results in histograms of the updating parameters. With the assumption of White Gaussian measurement noise, it is shown that this parameter estimation technique has low sensitivity to the standard deviation of the measurement noise. In order to validate this, a parametric sensitivity study is performed through numerical simulations of nonlinear systems with single and multiple degrees of freedom. The results show the least sensitivity to measurement noise level, selected time window for model updating, and location of the true model parameters in RS regression domain, when vibration frequency of the system is outside the frequency bandwidth of the load. Further application of this method is also presented through a case study of a steel frame with bilinear material model under seismic loading. The results indicate the robustness of this parameter estimation technique for different cases of input excitation, measurement noise level, and true model parameters.

1. Introduction

Structural Health Monitoring (SHM) procedures have been primarily developed to assist with lifetime maintenance of constructed structures through assessment of their condition. With the recent advancement in sensing technology, this goal can be served through instrumentation of the structures and monitoring their global behavior. The main components of vibration-based SHM in interpretation of the monitoring data fall into three categories: identification of dynamic characteristics of the monitored structures [1,2]; detection, localization, and quantification of the damage in the system [3–6]; and updating the Finite Element (FE) simulations of the structures based on their measured responses [7–10]. Among these, FE calibration methods attracted significant attention in the recent decades, mainly because having a FE model calibrated with reference to the actual structure, enables a variety of applications such as futuristic reliability study, assessment of retrofit alternatives, and designing structural control strategies. Moreover, parameter estimation through model calibration serves as the basis for many model-based damage detection algorithms which aim to assess the structural damage in a more objective way than non-parametric damage detection procedures [11–14].

FE model updating is an inverse parameter estimation problem where unknown parameters of an a priori structural FE model are estimated based on measurement data. This parameter estimation problem is solved as a constrained optimization problem with the objective of minimizing an error function representing the discrepancy between certain measured response features and their analytical counterparts. Selection of the reference response features for this optimization problem depends on the behavior of the structure and the future application of the calibrated model. When the FE model is used to study the behavior of the structure in low levels of vibration – in which most of the structures behave linearly – experimentally identified modal quantities (e.g. natural frequencies, and mode shapes) are commonly used for estimation of the model parameters. However, when a system behaves nonlinearly, such features fail to estimate model parameters, and other metrics are required for FE model calibration. While the linear model updating techniques and their applications on full scale structures are well-documented in the literature [15–17], the parameter estimation of nonlinear systems is still under ongoing research due to the individualistic nature of various types of local

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and global nonlinearities which exist in the structural dynamics [18]. Silva et al. [19] presented a comparison of the performance of four time – and frequency-domain measures for use in nonlinear model updating through a numerical two degree of freedom (DOF) system having a spring with cubic stiffness. The parameter estimation based on noise-free simulated data showed that while most of the metrics were effective in cases with local and weak nonlinearities, time-domain measures (proper orthogonal decomposition (POD) and restoring force surface (RFS)) yielded more promising results. However, application of POD and RFS were reported to suffer from sensitivity to the sampling frequency and number of data samples used, and requiring complete knowledge of the system through measuring all model DOFs, respectively. Therefore, developing a generalized time-domain strategy for nonlinear model calibration is of significant value.

Intensive computations and convergence problems are common challenges in many of the proposed model updating techniques, where sensitivity of the structural responses to the model parameters are calculated iteratively by means of the local gradients. As an effort toward decreasing such computational load, application of Response Surface (RS) methodology [20] was introduced in the process of FE model updating. The RS models – commonly used in the form of polynomial functions – approximate the relation between pre-selected inputs and output of the FE model. Then, the optimization problem of model updating is solved using these RS models as surrogates of the full FE model.

Previous studies of this method in updating the uncertain parameters of linear FE models has proved: efficiency of this method over traditional sensitivity-based model updating approaches [21,22]; low computational effort associated with such technique integrated with evolutionary optimization methods [23,24]; reduced computational cost in stochastic model updating [25,26] ; successful application on full scale bridge FE model calibration [27,28]; and success in detection and localization of the structural damage [29]. However, the literature related to application of RS-based model updating for nonlinear systems is scarce. Schultze et al. [30] used RS models in parameter selection and calibration of an FE model of a sandwiched layer of hyper-elastic foam and steel assembled on a drop table based on peak acceleration response and its time of arrival in a series of drop tests. Zhang and Guo [31] proposed a model updating procedure based on principal component decomposition and RS method to update a frame model with thin wall components showing strain-rate-dependence nonlinearity under impact test.

As an effort toward developing a generalized procedure for nonlinear model updating that addresses the above-mentioned issues regarding the nonlinear model updating metrics and computational load, an RS-based time-domain model updating procedure was previously proposed by the authors [32]. This method – called *Generalized Response Surface Model Updating (GRSMU)* – consists of three steps of RS model construction, evaluation, and optimization. Through these three steps, with assumption of known input, and using least square techniques [33], accurate RS models are constructed at every time step of the analysis and minimization problem of parameter estimation is solved iteratively in the length of the time history of the responses of the system. The performance of GRSMU was previously validated through a numerical cases study of a nonlinear frame under sinusoidal loading [32].

As noise contamination is unavoidable in any measurement procedure, which may heavily influence the interpretation of the data, a reliable parameter estimation technique should be robust to measurement noise. Therefore, this paper primarily evaluates the sensitivity of GRSMU estimates to the measurement noise. Moreover, the effect of input excitation frequency content and further application of this method in updating a nonlinear frame under seismic loading are investigated.

The outline of this paper is as follows. In the next section, the algorithm developed to accomplish nonlinear FE model updating using RS models is briefly presented. Afterward, the sensitivity of the proposed algorithm to measurement noise is studied followed by simulated case studies of single – and multi-DOF nonlinear models updated in cases with different assumptions for the frequency of input excitation and noise contamination level. Subsequently, application of this method in estimation of the parameters of a nonlinear steel frame under seismic loading is investigated in different scenarios. Finally, a summary of the paper and conclusions are presented.

2. Generalized Response Surface Model Updating (GRSMU)

In RS-based FE model updating, RS models replace the full FE model in a pre-selected domain of unknown model parameters, here called RS domain. These RS models are constructed using least square techniques [33] by regressing a polynomial function on a set of points sampled from the RS domain. Techniques of designs of experiments [34] can be employed in order to sample these points. However, finding the appropriate model order associated with each parameter and design of model parameters' levels that produce accurate RS models, require a number of trials and errors which may contradict the primary motivation for using the RS models to decrease the computational cost of FE model analyses in model calibration.

GRSMU was previously proposed to systematically design the levels and model order of the RS models, and extend the application of RS modeling for nonlinear model updating in time through RS model construction and optimization iteratively at every time step of the analysis. In order to construct accurate RS models capable of predicting the response of the FE model throughout the RS domain, GRSMU adopts a full factorial design with minimum number of levels and linear RS models. This procedure is subsequently followed by evaluation of the regressed RS models in terms of accuracy and predictability, and increasing the model order or number of levels associated with each model parameter, when required. Fig. 1 presents the flowchart of GRSMU. Eq. (1) formulates the optimization problem of model updating using GRSMU at the l^{th} time step of the nonlinear dynamic analysis.

$$\min_{\theta_j} \quad f_1 = \sqrt{\sum_{i=1}^{s}\left(\frac{RS_{il}(\theta_1,\theta_2,\ldots,\theta_m) - Y_{expil}}{Y_{expil}}\right)^2} \quad i = 1,2,\ldots,s$$
$$s.t. \quad \theta_{jlb} \leqslant \theta_j \leqslant \theta_{jub} \quad j = 1,2,\ldots,m \tag{1}$$

In Eq. (1) $RS_{il}(\theta_1, \theta_2, \ldots, \theta_m)$ denotes the RS model associated with the l^{th} time step of the analysis representing the i^{th} analytical response feature, as a function of the pre-selected uncertain model parameters $(\theta_1, \theta_2, \ldots, \theta_m)$, θ_{jlb} and θ_{jub} represent the lower and upper bounds of the j^{th} model parameter in the RS domain, and y_{expil} is the i^{th} response feature measured at the l^{th} time step of the experiment. Any nonlinear constrained optimization algorithm can be readily adopted to solve this explicitly formulated FE model updating problem.

3. Sensitivity of the GRSMU estimates to measurement noise

This section investigates the effect of measurement noise on the parameter estimation results of GRSMU. This study simulates the measurement error as White Gaussian noise in which the values at any pair of time instances in the noise signal are statistically independent and identically distributed with a zero-mean normal probability distribution.

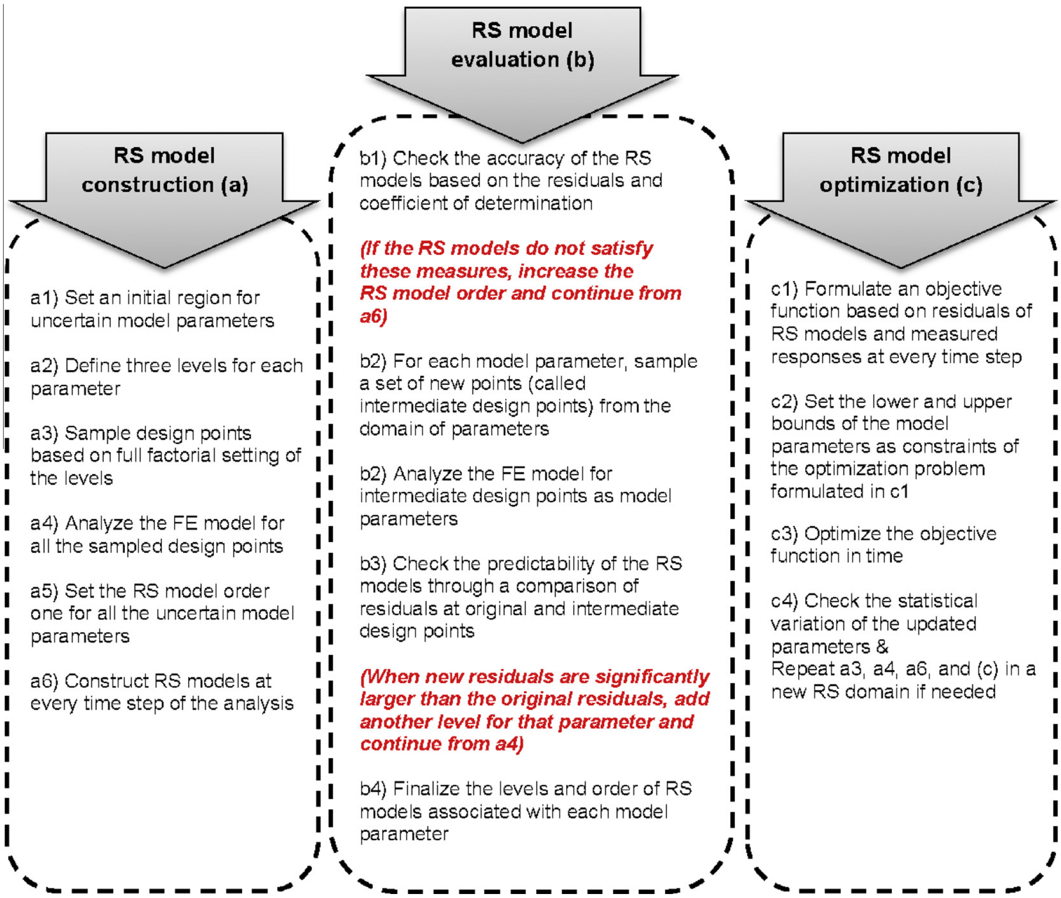

Fig. 1. GRSMU algorithm for nonlinear model updating.

In order to study the sensitivity of GRSMU estimates to noise, assume a single-DOF dynamic system. As Eq. (2) indicates, the measured output of this system (u_m) at any time instance t_i can be considered as a summation of real response (u_m^r) and measurement noise in that time step.

$$u_m(t_i) = u_m^r(t_i) + n(t_i) \tag{2}$$

where $n(t_i)$ is a random variable representing the amplitude of noise in time t_i having a zero-mean normal distribution with standard deviation σ.

With assumption of known mass, the response of an FE model simulating this system is a function of stiffness (k). Over a small domain of k, a linear RS model can approximate the real response of the system at any time step of the analysis. Eq. (3) presents this linear function at time step t_i.

$$\begin{aligned} &\mathrm{RS}(k, t_i) = \hat{\beta}_0(t_i) + \hat{\beta}_1(t_i) \times k \\ &\quad k_{\mathrm{lb}} \leqslant k \leqslant k_{\mathrm{ub}} \end{aligned} \tag{3}$$

In this equation, k_{lb} and k_{ub} denote the lower and upper bounds of domain of k, where the linear RS model (with coefficients $\hat{\beta}_0(t_i$ and $\hat{\beta}_1(t_i)$) replaces the FE model of the system. Eq. (4) formulates the model updating procedure in which parameter estimation is accomplished by minimizing the residual of the predicted and measured responses.

$$\begin{aligned} \min \quad & f(k, t_i) = (\hat{\beta}_0(t_i) + \hat{\beta}_1(t_i) \times k - u_m(t_i))^2 \\ s.t. \quad & K_{lb} \leqslant k \leqslant k_{ub} \end{aligned} \tag{4}$$

Since f is a nonnegative function, its minimum value at every time step (t_i) corresponds to the root of $f(k, t_i)$. This statement holds with the assumption that the domain of the RS model includes the root of $f(k, t_i)$. High amplitudes of noise and/or when model parameters locate outside or on the corners of the RS domain can contradict this assumption. In such cases the solution of this constrained optimization problem is k_{lb} or k_{ub} whichever associates with a smaller f.

Therefore, estimation of k based on the measured response (k_{est}) in time instance t_i is

$$k_{est}(t_i) = \frac{u_m(t_i) - \hat{\beta}_0(t_i)}{\hat{\beta}_1(t_i)} \tag{5}$$

It should be noted that if $f(k, t_i)$ in Eq. (4) has two roots (k_{lb} and k_{ub}) in the domain, the formulation of the problem does not change. Double roots in the domain could occur because (1) k_{lb} and k_{ub} generate the same response at time step t_i, or (2) k_{lb} and k_{ub} generate the same response time history. In case (1), as the time history of the responses are not the same in other time steps, through the parameter estimation in time history of the response, the true k will be estimated. In case (2), by solving Eq. (4) using a global optimization framework which is able to find multiple optima, both k_{lb} and k_{ub} are estimated. Therefore, in both cases, Eq. (5) can be used to demonstrate the estimated stiffness with reference to the measured response of the system.

Since $u_m^r(t_i), \hat{\beta}_0(t_i)$ and $\hat{\beta}_1(t_i)$ are independent of measurement noise, the expected value of $k_{est}(t_i)$ can be written as

$$E[k_{\mathrm{est}}(t_i)] = \frac{u^r{}_m(t_i) - \hat{\beta}_0(t_i)}{\hat{\beta}_1(t_i)} \tag{6}$$

therefore, its sensitivity with respect to the standard deviation of White Gaussian noise is

$$\frac{\partial E[k_{est}(t_i)]}{\partial \sigma} = 0 \quad (7)$$

Eq. (7) shows that the expected value of the estimated stiffness in time is not sensitive to the measurement noise amplitude.

The main assumption in derivation of Eq. (7) is zero-mean assumption for the noise signal. Therefore, for any non-Gaussian or non-stationary noise, it is expected to observe similar estimation performance as long as the zero-mean assumption for the underlying probability density function of the noise signal holds. However, it should also be noted that since this error minimization problem of parameter estimation is completed through constrained optimization techniques, in a finite length time window, and in presence of different levels of measurement noise, it is critical to evaluate the robustness of this technique for different noise structures in separate scenarios. In the following sections, several parametric sensitivity studies are performed to accomplish this goal with assumption of White Gaussian measurement noise.

4. Nonlinear model updating using harmonic loading

This section describes the implementation of the methodology that was developed in Section 3 to study the robustness of GRSMU in a single-DOF and a multi-DOF bilinear system. In each case, the response of the system is simulated under several assumptions of measurement noise level and input excitation. The parameter estimation is then completed in two different time-domain windows, and the estimation error is investigated.

The following subsections describe the sensitivity study carried out for these systems in detail.

4.1. Numerical simulation: single-DOF system

This section studies the sensitivity of GRSMU estimates to the measurement noise level through a numerical case study of a single-DOF nonlinear system under harmonic loading. This single-DOF system is simulated with unit mass (1 lb sec^2/in = 175.09 kg) and bilinear stiffness material model. Stiffness of the system (k) and yielding force are 4 lb/in (0.7 N/mm) and 4 lb (17.79 N), respectively. The natural period of vibration of this system (T_n) is 3.14 sec. Post yielding stiffness ratio of the system (α) is selected as an uncertain model parameter varying between 0.2 and 0.8 to be estimated from the time history of the displacement of the mass.

In order to study the impact of the frequency of the input harmonic loading, in different scenarios period of the applied load (T_{load}) varies so that the ratio of the loading frequency over the natural vibration frequency varies from 0.1 to 10. In these scenarios the amplitude of the load is adjusted so that in all the cases maximum displacement of the system in the longer window used for parameter estimation is 3 in (7.62 cm). A time step of 0.001 sec is used in the time history analysis of this nonlinear system, which satisfies a convergence test with 1.0e−6 lb (4.45e−6 N) tolerance for the norm of the unbalanced force in every time step of the dynamic analysis. This time step is small enough, not to affect the accuracy of the results, as selection of a smaller time step did not change the results of the dynamic analysis.

In every scenario, two time-domain windows are used for the parameter estimation: (1) a T_n-sec long window and (2) a T_{load}-sec long window. The model construction and evaluation steps in the longer window of (1) and (2) in every scenario are completed to obtain the RS models of displacement as functions of α. Subsequently, residuals of simulated measured displacement and regressed RS models are minimized along the selected time window to update α.

The optimization problem of model updating in the T_n-sec long time window is completed with sampling frequency of 100 Hz based on a multi-start optimization framework using interior-point algorithm [35]. Different levels of the measurement noise are assumed in each case. Noise level denotes the ratio of the root mean square of the simulated Gaussian noise signal to the root mean square of the simulated measured signal. Figs. 2 and 3 show the results of the updating procedures where α is set equal to 0.625 and 0.2 to simulate the measured displacement signal.

The results show that, as indicated by Eq. (7), the mean of the updated α is fairly insensitive to the measurement noise level, particularly when it is low or medium. However, when the assumptions made in derivation of Eq. (5) are violated, the constrained optimization problem of RS model updating is likely to result in the bounds of the selected RS domain as the optima. This can cause the mean value of the estimated α to deviate considerably from α_{true}, while the median – having a breakdown point of 50% – robustly estimates the true α. Therefore, in the following cases, the median of the updated model parameters are reported as the point estimate of the true parameters. Figs. 2 and 3 also show that, when sampling frequency in the response measurement is high enough relative to the loading frequency, frequency of the input excitation does not significantly influence the accuracy of the estimated parameters, particularly at low levels of measurement noise.

The updating procedure in the previous scenarios is iterated in a time window equivalent to the period of loading (T_{load}) in each case. The optimization frequency in these cases is adjusted to have the same number of time steps as for the cases with T_n-sec long time window. Fig. 4 displays the error sensitivity of the median of the updated parameters to the measurement noise, when the parameter estimation of this single-DOF system is completed in a T_{load}-sec long time window. This figure shows that when α_{true} = 0.625, the estimation error is less sensitive to the noise level and the length of the time window compared to the cases when α_{true} = 0.2. Furthermore, in the latter cases, the largest estimation error of all of the noise levels is observed when frequency of the loading approaches natural vibration frequency of the system.

It should be noted that, amongst all the cases of the single-DOF model updating, the results of the cases with T_{load}/T_n = 10 ("slow" loading) consistently show robustness to 20% measurement noise level. When the harmonic load is applied "fast" (T_{load}/T_n = 0.1) and T_{load}-sec window is used for parameter estimation, the estimation error is comparable to the results of the "slow" loading; however, when updating is completed in the constant length time window (T_n sec), the estimation error for the "fast" loading case is larger than the "slow" loading case, at 20% measurement noise level. Figs. 5 and 6 illustrate the normalized median deviation of the parameter estimation in all the cases studied here. Since the median is selected as the point estimate of the updated parameters in each scenario, the absolute median deviation with respect to the median of the histograms of updated α is calculated and normalized by the true model parameters in each case.

These figures show that the dispersion of the updated α is roughly insensitive to the selected time window, with the exception of the cases with small ratio of T_{load}/T_n when α_{true} = 0.625. Furthermore, the largest deviation corresponds to the cases with the highest level of noise contamination. When α_{true} = 0.2, the deviation of the updated α increases considerably as the period of the harmonic loading approaches the vibration period of the system. The reason is that in such cases, the response of this nonlinear system in the selected time windows has low sensitivity to permutation of the post yielding stiffness ratio, and thus in the cases with high simulated measurement noise, dispersion of the optimization results increases significantly.

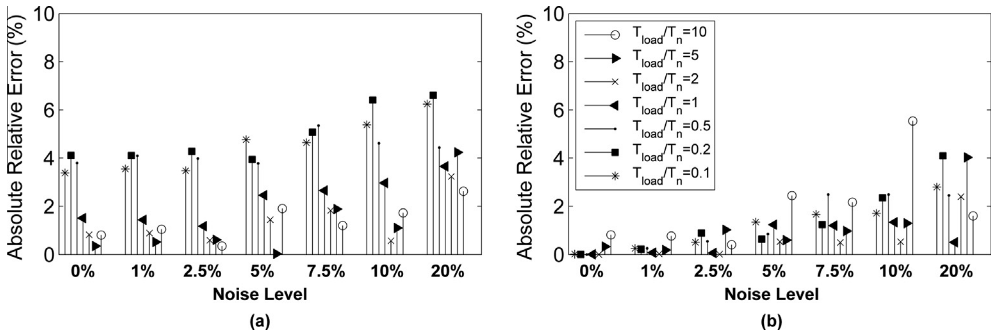

Fig. 2. Error sensitivity in estimated α (Single-DOF system, $\alpha_{true} = 0.625$ and T_n-sec long window): (a) mean and (b) median.

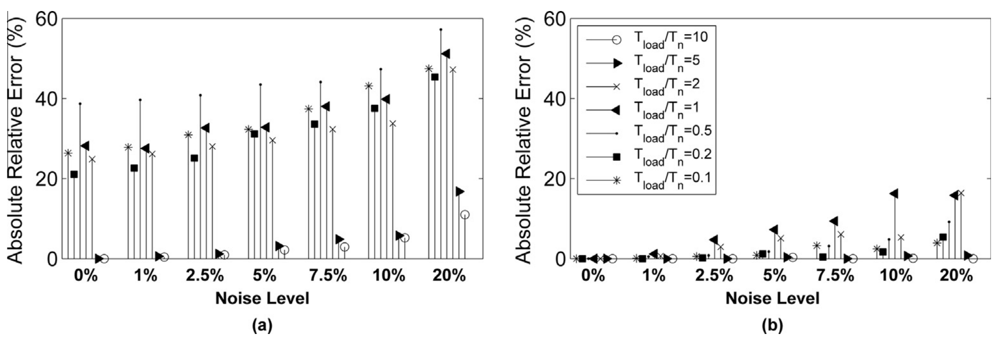

Fig. 3. Error sensitivity in estimated α (Single-DOF system, $\alpha_{true} = 0.2$, and T_n-sec long window): (a) mean and (b) median.

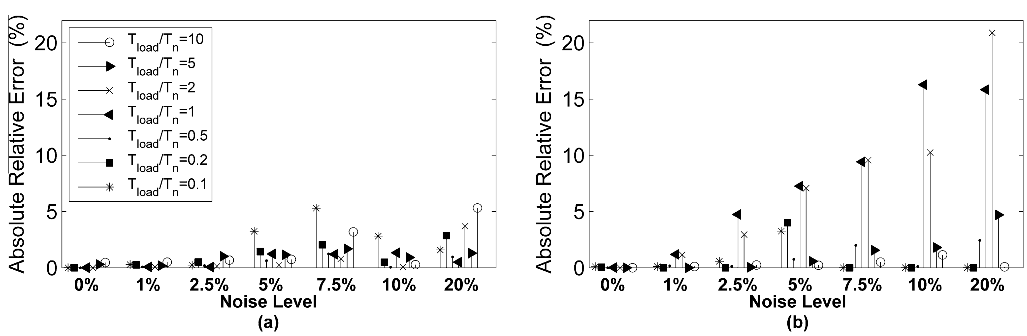

Fig. 4. Error sensitivity of the median estimated α (Single-DOF system, T_{load}-sec long window): (a) $\alpha_{true} = 0.625$ and (b) $\alpha_{true} = 0.2$.

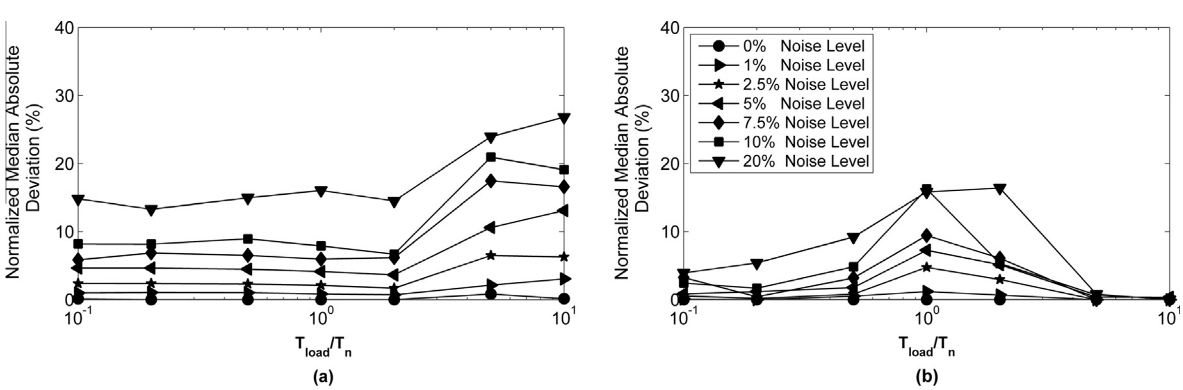

Fig. 5. Normalized median absolute deviation of the estimated α (Single-DOF system, T_n-sec long window): (a) $\alpha_{true} = 0.625$ and (b) $\alpha_{true} = 0.2$.

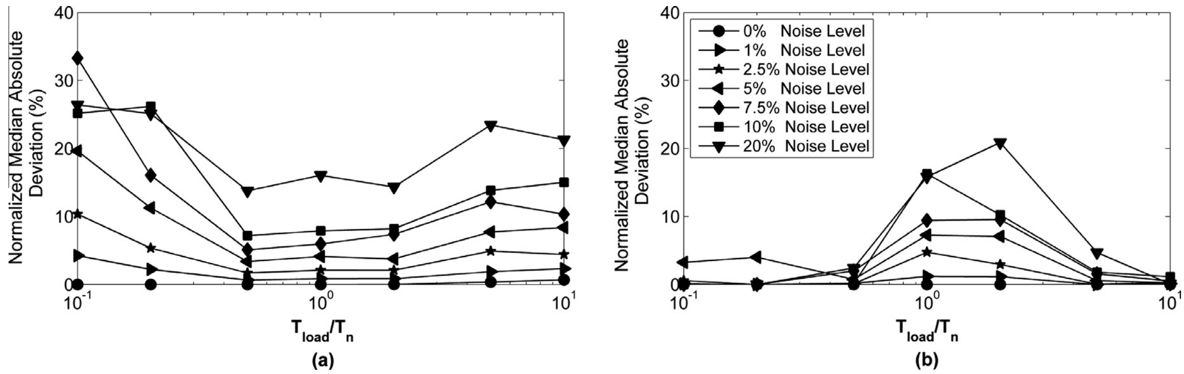

Fig. 6. Normalized median absolute deviation of the estimated α (Single-DOF system, T_{load}-sec long window): (a) α_{true} = 0.625 and (b) α_{true} = 0.2.

4.2. Numerical simulation: multi-DOF system

In order to further investigate the sensitivity of GRSMU to the measurement noise and input excitation, a multi-DOF system is considered in this section. This simulation is for a cantilever steel beam with nonlinear material model under a harmonic load, applying vertically at its tip. Fig. 7 shows the configuration of this simulated beam. This beam, with 30 in (76.2 cm) length, has a 2″ (5.08 cm) square section. The steel behaves bilinearly with modulus of elasticity (E) and yield stress of 29,000 ksi (200 GPa) and 50 ksi (344.8 MPa), respectively. A uniform dead load on the beam is designed so that the fundamental vibration period of this system (T_1) is 1.57 sec. Post yielding stiffness ratio of the material (α) is selected as uncertain model parameter varying between 0.2 and 0.8. Time history of displacement at the tip of the beam ($u(t)$) is used to estimate α in this range in scenarios with different ratios of T_{load}/T_1 varying between 0.2 and 20. In all these cases, maximum displacement in the longer model updating window and the true model parameters are the same as for the single-DOF case discussed previously.

It should be noted that GRSMU framework can be used for parameter estimation in linear and nonlinear systems. For linear systems, in addition to using input–output data for model updating, natural frequencies and mode shapes can be used for parameter estimation through GRSMU which requires no prior knowledge of the input excitation. However, in the cases of nonlinear systems, to use the time domain data for updating the uncertain model parameters, known input excitation is used to run the FE model, generate, and validate the RS surrogate models. Therefore, RS model construction and evaluation in all these single – and multi-DOF cases are completed with assumption of known experimental input excitation, and thus the type of excitation (harmonic, random, etc.) does not bear any effect on the proposed methodology for parametric sensitivity study. In order to study the robustness of GRSMU results to the frequency content of the input excitation, single harmonic loading is chosen in this study which allows controlling one parameter (loading period) at a time and studying the potential effect of dynamic amplification of the system on GRSMU estimates, while in each case several levels of measurement noise contamination is also considered. In applying the input harmonic excitation, the period of loading is set while the amplitude is adjusted in each T_{load}/T_1 case to have equal maximum displacement response in the longer model updating window. This load adjustment is required to establish a fair comparison of the parameter estimation accuracy when loading period is widely changing in different cases.

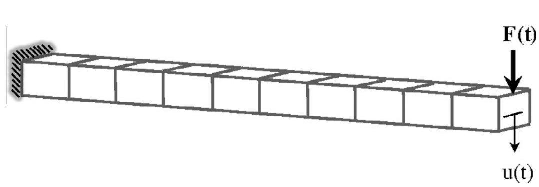

Fig. 7. Configuration of the simulated cantilever beam.

The bilinear material behavior considered in these case studies is plastic, i.e., during the unloading phase the material takes its initial stiffness. Based on this assumption, the instantaneous fundamental period of these single- and multi-DOF systems change between two values; elastic period of vibration and elongated period which is bounded to $\left[1/\sqrt{0.8}\quad 1/\sqrt{0.2}\right]T_n = [1.12\quad 2.34]T_n$ in the single-DOF case, and $(12.34)T_1$ in the case of simulated cantilever beam. In order to compare the results of all the cases considered, fundamental period of vibration (in elastic range) is selected. Since the elongation bound is constant in all the considered scenarios for each case, this would not change the interpretation of the results in terms of the "fast" or "slow" loading.

A 2-dimensional lumped mass FE model is developed in Opensees software [36] using fiber section procedure, Steel01 uniaxialMaterial model, and nonlinearBeamColumn elements. This FE model consists of 10 frame elements, 11 nodes, and overall 30 DOFs. A transient analysis object is used to apply the Newmark method integrated with the Krylov-Newton algorithm [37] to solve the nonlinear equation of motion in each case with a time step of 0.001 sec.

In order to study characteristics of noise signals as samples of a desired Gaussian population, for each case of T_{load}/T_1 ratio, 50 rounds of simulations are conducted for the same noise level. In every scenario, two time windows were used for the parameter estimation: (1) a T_1-sec long window and (2) a T_{load}-sec long window. The steps of RS model construction and validation in each case is carried out in the longer window between (1) and (2). It should be noted that when T_{load}/T_1 = 0.2, due to rapid change of the stiffness of the beam elements under high frequency loading, the response of beam is not predictable so the regressed RS models fail to estimate the response of the FE model over the entire domain of α. Therefore, RS model evaluation is not possible, and thus the optimization step is not completed in the cases corresponding to loading with this period.

Figs. 8 and 9 show the error sensitivity of the median estimated α for all of the 50 simulations when T_{load}/T_1 is 20, 2, and 0.4, and with the assumption of α_{true} = 0.625 and 0.2, respectively. These figures show that the estimated α has larger variation as the noise level increases. When α_{true} = 0.2, the estimation error is sensitive to the length of the selected time window, such that model updating in a longer time window, results in higher estimation error.

Figs. 10 and 11 display the estimation error of the median of the estimated α in all the 50 cases associated with each noise level and T_{load}/T_1 ratio. These figures show that as the noise level increases, the estimation error increases particularly when α_{true} is at the corner of the selected RS domain. Furthermore, the estimation

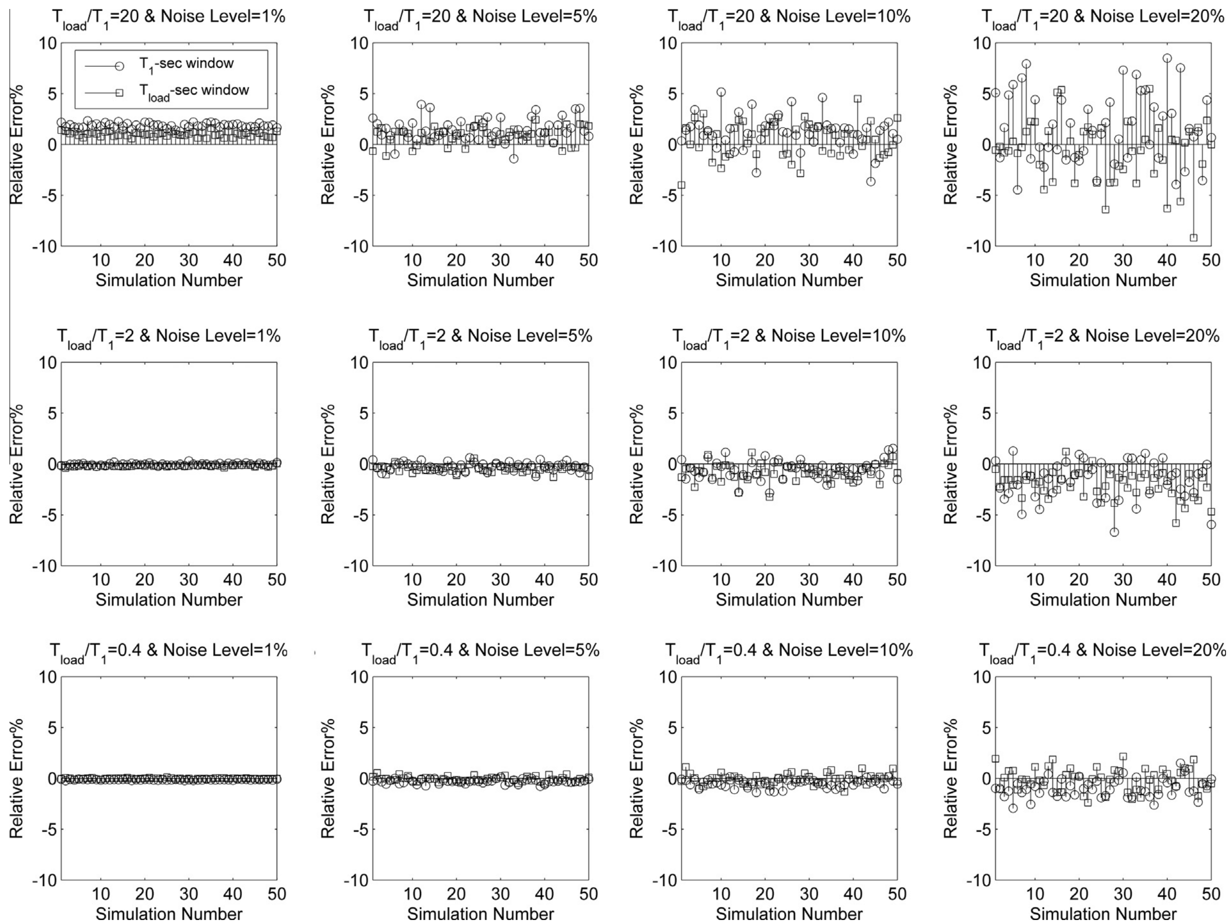

Fig. 8. Error sensitivity of the median estimated α, 50 noise signal simulations: (Multi-DOF system, $\alpha_{true} = 0.625$).

error in the cases with the largest ratio of T_{load}/T_1 appears to have the least sensitivity to the noise level and the selected time window. The reason is that when the vibration frequency of the system is outside of the frequency bandwidth of the load, the response of the model at different levels of the uncertain model parameters has the same frequency content as for the loading (a "steady-state" response). Therefore, the results of the model parameter estimation in time are robust to high measurement noise level and selected time window.

4.2.1. Effect of damping

In order to study the effect of damping on the performance of GRSMU, in this section different levels of damping are considered for the nonlinear cantilever beam. In these simulations, Rayleigh damping is assumed, and the mass – and stiffness-proportional damping coefficients are designed so that 1^{st} and 5^{th} natural modes of vibration of the beam have 0.02, 0.05, and 0.1 damping ratios in different cases. Two levels of loading period (T_{load}/T_1), and four levels of noise contamination are considered. Parameter estimation is carried out in T_1-sec and T_{load}-sec long widows. The results of parameter estimation (shown in Fig. 12) are consistent with the observations in the previous sections; when frequency of loading is high relative to natural frequency of the system, estimation error is sensitive to the length of the optimization window.

5. Nonlinear model updating using seismic data

The previous section demonstrated that GRSMU estimates show robustness to the measurement noise, particularly in the cases where the input excitation has lower frequency content than the fundamental frequency of the system. This implies further application of this method in updating parameters of nonlinear models in time under seismic loading. To validate such application, in this section a steel frame with bilinear material model is considered.

Details of the simulated steel frame, description of the factors considered to study the variability of the results, and the results of the updating procedures are presented in the following subsections.

5.1. Nonlinear frame

The model presented in this section is a steel frame with nonlinear material properties under dynamic loading. The frame consists of one span with overall length of 7′6″ (228.6 cm) supported by columns that are 2′9″ (83.8 cm) long. The cross section of the beam and column members is uniform hollow 2″ (5.08 cm) tube, with 0.083″ (0.21 cm) wall thickness. The column supports are fixed and the frame is considered a "plane frame" which constrains out-of-plane and torsional degrees of freedom. The steel has bilinear behavior with the yield stress of 50 ksi (344.8 MPa). Modulus of elasticity (E) and post yielding stiffness ratio of steel (b) are chosen as the updating parameters. The input excitation in this model is a dynamic load resulting from selected earthquake records applied to the left column-beam joint. To update the pre-selected parameters of the model, simulated time histories of displacement at two locations on the frame are used. Fig. 13 shows the configuration of the frame, loading and responses used for updating the FE model.

5.2. Simulated model

A 2-dimensional massless model is developed in Opensees software [36]. The model consists of 8 nodes and 7 elements dividing

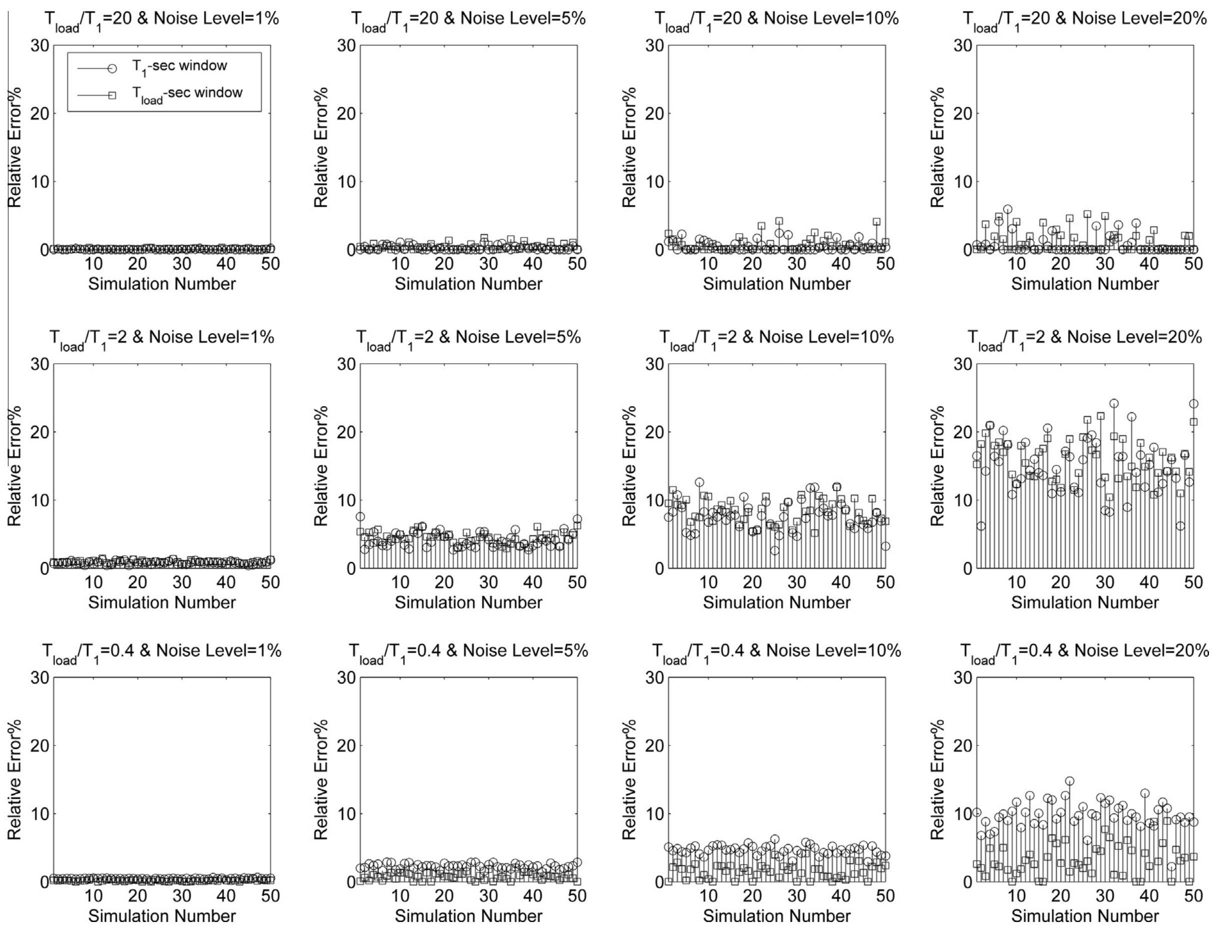

Fig. 9. Error sensitivity of the median estimated α, 50 noise signal simulations: (Multi-DOF system, $\alpha_{\text{true}} = 0.2$).

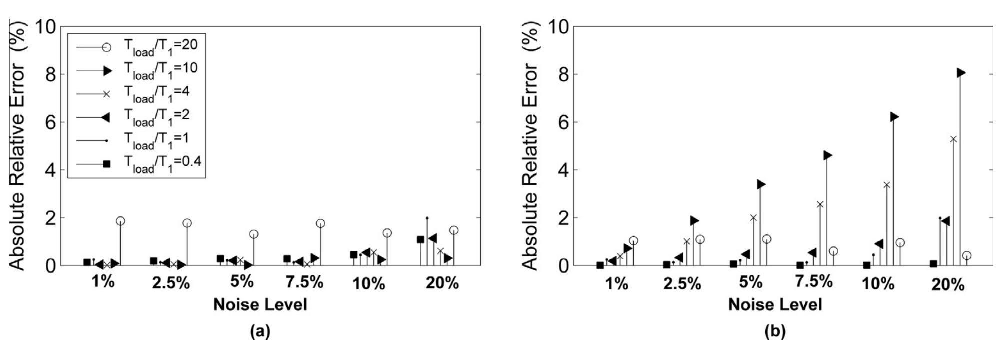

Fig. 10. Error sensitivity of the median estimated α (Multi-DOF system, $\alpha_{\text{true}} = 0.625$): (a) T_1-sec long window and (b) T_{load}-sec long window.

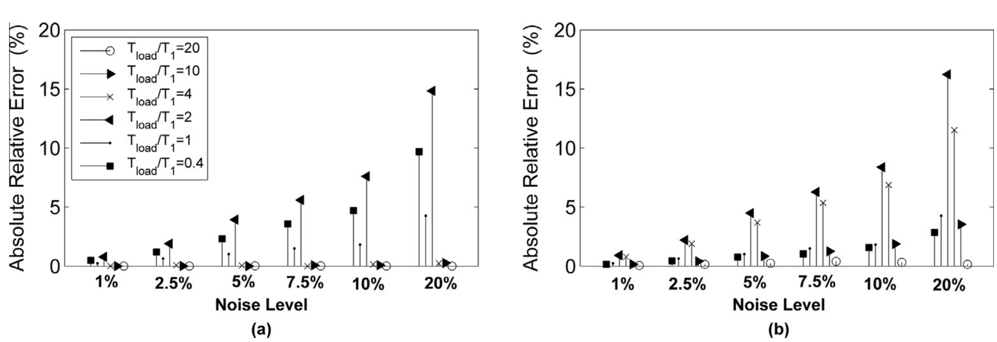

Fig. 11. Error sensitivity of the median estimated α (Multi-DOF system, $\alpha_{true} = 0.2$). (a) T_1-sec long window and (b) T_{load}-sec long window.

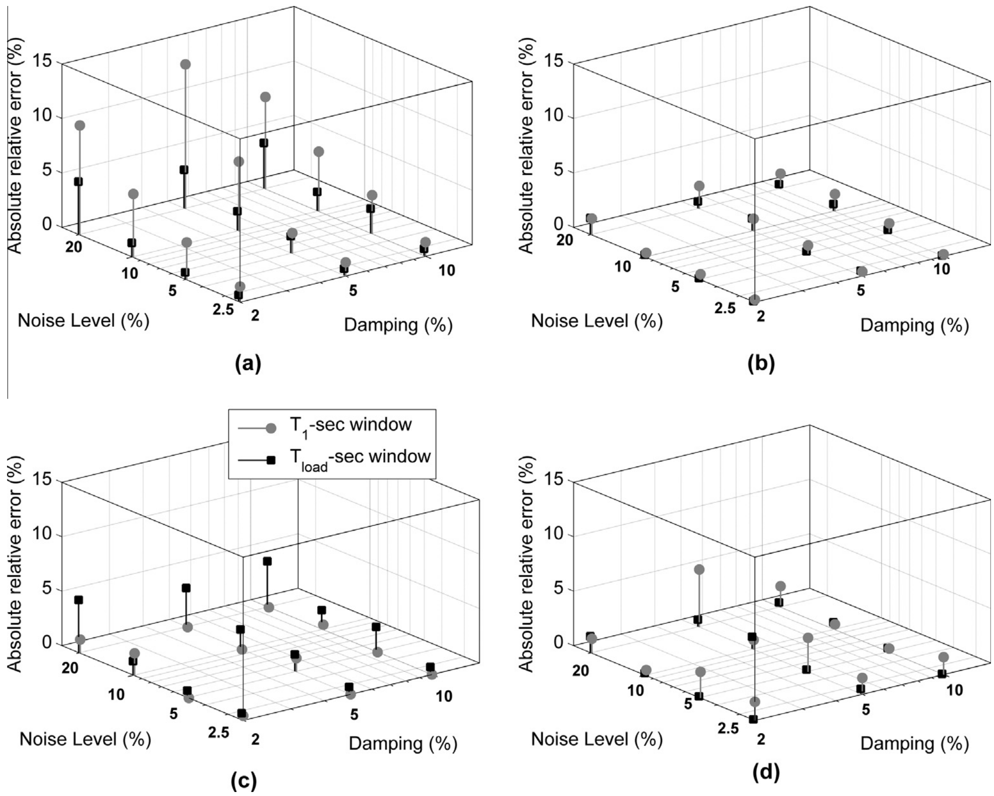

Fig. 12. Error sensitivity in estimated α: (a) $T_{load}/T_1 = 0.4$ and $\alpha_{true} = 0.2$ and (b) $T_{load}/T_1 = 0.4$ and $\alpha_{true} = 0.625$. (c) $T_{load}/T_1 = 20$ and $\alpha_{true} = 0.2$ and (d) $T_{load}/T_1 = 20$ and $\alpha_{true} = 0.625$.

beam and columns members into two and three segments, respectively. Each node has three degrees of freedom, *ux*, *uy* and θz which allow for translation and rotation in *xy* plane. Elements are modeled as nonlinearBeamColumn having Steel01 uniaxialMaterial properties to construct a bilinear steel material object with kinematic strain hardening. Five integration points were assigned along each element to model the distributed plasticity. A fiber section procedure is used to build the tubular steel section from 92 fibers patched together. Due to zero-mass assumption for the steel tube section, the behavior of the system is not dynamic, and thus static or transient analysis objects with appropriate integrators can be used to solve the equation of motion under seismic loading. In this study, a transient analysis object is used to apply the Newmark method integrated with the KrylovNewton algorithm [37].

The main purpose of studying these numerical simulations is to investigate the effect of frequency band limited excitations – at different measurement noise levels – on the GRSMU estimates. As shown in Section 4.2.1, this can be completed regardless of the damping level of the system. Therefore, for this nonlinear frame model damping was not considered.

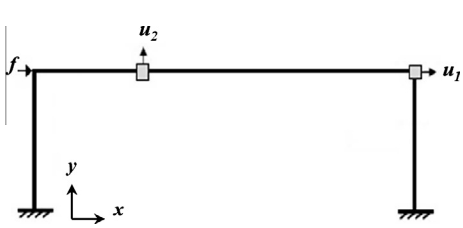

Fig. 13. Configuration of the nonlinear steel frame.

5.3. Parametric study

In order to evaluate the performance of GRSMU algorithm using seismic loading, variability of the model updating results are studied by considering: earthquake loads with different characteristics; various assumptions for true model parameters; and several levels of noise to contaminate the simulated response of the structure.

In this simulation, the RS domain for the updating parameters is set to 27,000–33,000 ksi (186.2–227.5 GPa) for *E* and 0.05–0.25 for *b*. Since the location of true model parameters in the RS domain is always unknown in the inverse problem of model updating, four pairs of model parameters are selected from the RS domain to simulate the measured responses of the nonlinear frame under earthquake loading. Table 1 presents the true model parameters that are used for simulation.

Three earthquake records with different characteristics in terms of duration, fault distance, and frequency content are selected to study the sensitivity of the parameter estimation procedure to seismic input excitation. Fig. 14 shows the time history and Fourier amplitude spectra of these ground motion records.

The selected earthquake records are: (1) Fault-normal component of Kern County earthquake (1952) recorded at LA Hollywood

Table 1
Case studies of model parameters used to simulate the measured signals.

	True model parameters		
	b	*E* ($\times 10^3$ ksi)	*E* (GPa)
Case (1)	0.065	27.5	189.6
Case (2)	0.05	33	227.5
Case (3)	0.18	28	193.1
Case (4)	0.125	31.5	217.2

Stor Pe Lot station [38] which is a long duration far-fault record with a relatively long strong motion portion, (2) Fault-normal component of Northridge earthquake (1994) recorded at Rinaldi Receiving station [38], a near-fault short duration record with a pronounced pulse in its time history, and (3) North–south component of horizontal ground acceleration of the Imperial Valley earthquake (1940) recorded at EL Centro station [39] which has a frequency content more uniform than the first two records and a relatively medium length strong shaking part. These earthquake records are scaled to simulate a dynamic lateral force at floor level which creates 1 in (2.54 cm) maximum $u_1(t)$, when model behaves linearly with E = 33,000 ksi (227.5 GPa).

The effect of measurement noise is also investigated by contamination of the simulated reference responses with Gaussian noise signals with different standard deviations.

5.4. Parameter estimation using GRSMU

The unknown model parameters are estimated based on the measured responses of the frame in 60 simulated scenarios resulting from three different input excitation, 4 different pairs of true model parameters, and 5 different levels of measurement noise. The model construction and evaluation steps of the GRSMU algorithm resulted in a 5×3 design for b and E. The RS models regressed on this design have model order of 4 for b, and 2 for E. In the optimization step, the resulting optimization problem in Eq. (1) is formulated and solved iteratively in a window selected from the response of system to the strong motion segment of each earthquake loading. Table 2 summarizes the information regarding the model updating window associated with each earthquake loading case.

In order to find the global minimum of the formulated objective function at each time step, a multi-start optimization framework is adopted based on interior-point algorithm [35] using four corners of the RS domain as starting point. Figs. 15 and 16 display the histograms of the updating parameters using EQ (1) record to simulate the input seismic loading on the frame. These figures show that GRSMU successfully estimates the model parameters regardless of the location of the true model parameters in the selected RS domain. The parameter estimation procedures are reiterated to capture the variability of the results with respect to the input excitation and noise level in each case. Fig. 17 summarizes the estimation error in all the 60 cases considered in this study. This figure indicates low error sensitivity of GRSMU estimates to measurement noise level in all cases with the exception of case (2) with high level measurement noise. Moreover, it is observed that the results are not sensitive to the choice of the ground motion record used for earthquake loading simulation.

6. Summary and conclusions

GRSMU is a generalized procedure for nonlinear model updating using time-domain data. In GRSMU, the parameter estimation is accomplished through approximation of the input–output relationship of the nonlinear FE model with RS models, and optimization of an objective function based on measured responses and regressed RS models successively through the time history of the measured data. This paper is primarily concerned with the sensitivity of GRSMU estimates to noise, since a reliable parameter estimation technique should be robust to measurement noise which inevitably exists in any monitoring data.

In this study, with the assumption of White Gaussian measurement noise, it is analytically shown that the GRSMU estimates have low sensitivity to the standard deviation of the noise. Numerical simulations of nonlinear systems with several assumptions for measurement noise level, input excitation, true updating parameters, and time-domain window for parameter estimation are used to validate this methodology. The results of the estimation of the post yielding stiffness ratio of the material in these systems through GRSMU show that the estimation error is fairly insensitive to low and medium measurement noise level. Additionally, when the vibration frequency of the system is outside of the frequency bandwidth of the load, the results show the least sensitivity to measurement noise level, selected time window for optimization, and location of the true model parameters in the RS domain.

Further application of GRSMU is also studied through a case study of a steel frame with bilinear material under seismic loading. In this simulation, three earthquake records with different characteristics in terms of duration, fault distance, and frequency content are selected to capture the variability of the parameter estimation

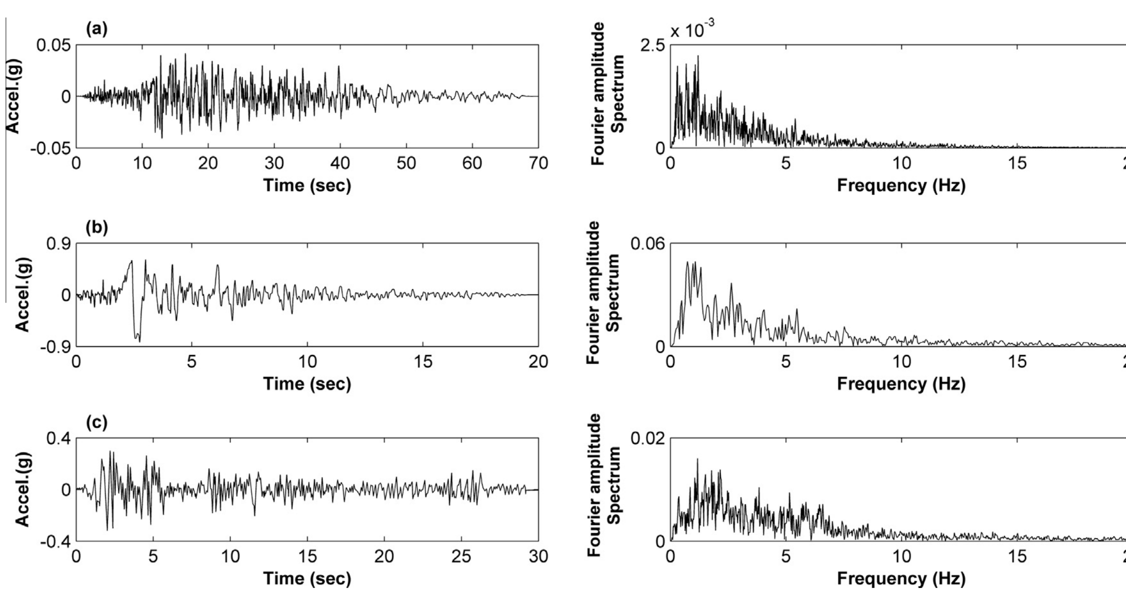

Fig. 14. Acceleration time history and Fourier amplitude spectra of: (a) Kern County earthquake (1952), (b) Northridge earthquake (1994), and (c) Imperial Valley earthquake (1940).

Table 2
Details of the steel frame model calibration using Earthquake records.

	Earthquake record	ts_{opt} (sec)[a]	te_{opt} (sec)[b]	dt_{FEA} (sec)[c]	dt_{opt} (sec)[d]	N_{opt}[e]
EQ (1)	Kern County	11.95	21.5	0.005	0.010	955
EQ (2)	Northridge	2.4	3	0.001	0.001	600
EQ (3)	Imperial Valley	1.66	4.8	0.002	0.004	785

[a] Beginning of the time window used in the model calibration.
[b] End of the time window used in model calibration.
[c] Time step used in Finite Element Analysis (FEA).
[d] Time step used for the parameter estimation in the selected time window.
[e] Number of time steps used in the parameter estimation.

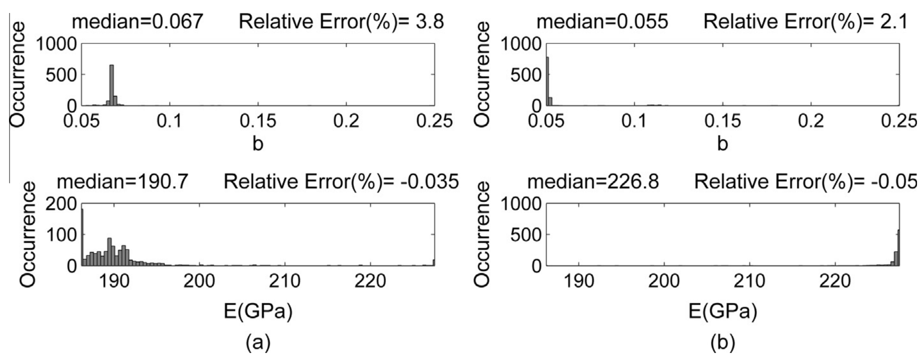

Fig. 15. Histograms of the updated parameters using EQ (1) record (noise-free data): (a) Case (1) and (b) Case (2).

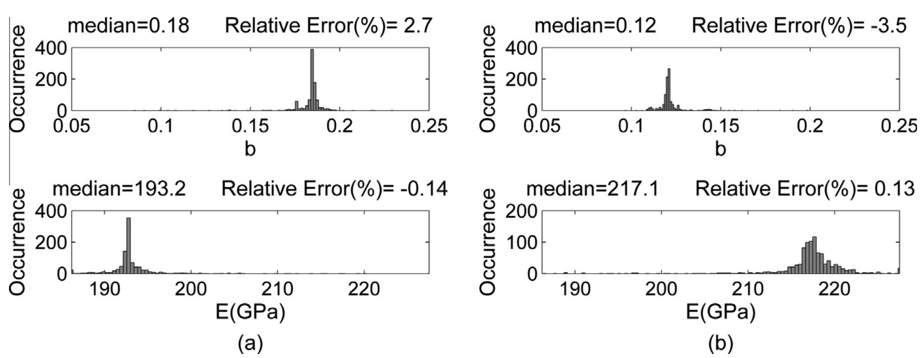

Fig. 16. Histograms of the updated parameters using EQ (1) record (noise-free data): (a) Case (3) and (b) Case (4).

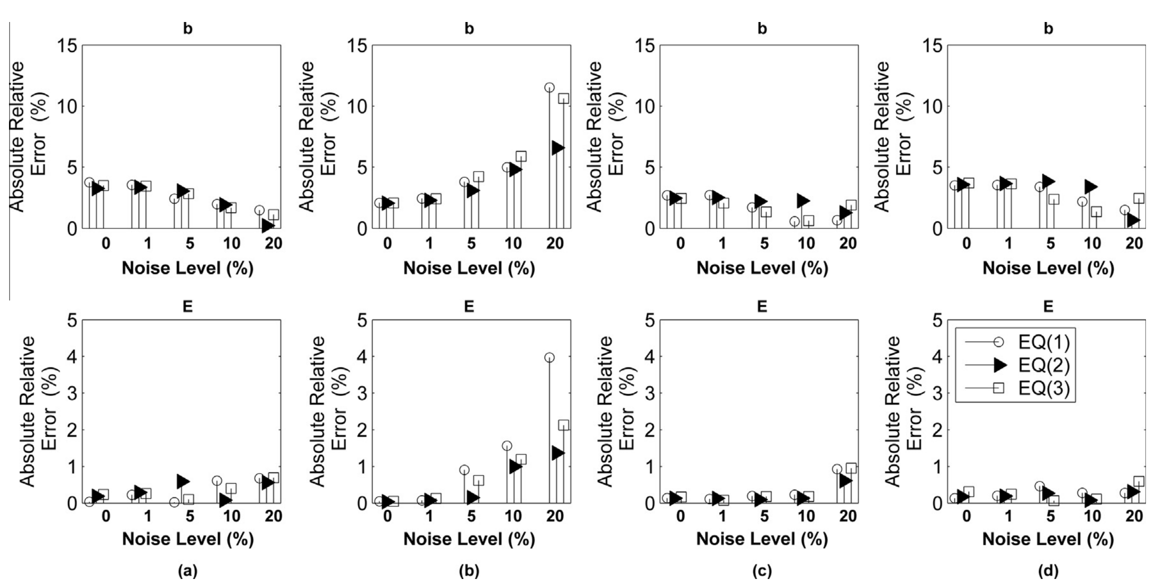

Fig. 17. Error sensitivity in parameter estimation of the steel frame: (a) case (1), (b) case (2), (c) Case (3), and (d) Case (4).

results. The uncertain model parameters are successfully estimated based on the measured responses of the frame in 60 simulated scenarios resulting from 3 different input excitation, 4 pairs of true model parameters, and 5 increasing levels of measurement noise.

It should be noted that as this study is mainly concerned with evaluation of the overall performance of GRSMU algorithm, uniform spatial distribution is assumed for the unknown model parameters. In model-based damage detection scenarios, different spatial distribution could be possibly assumed in order to locate and quantify the structural damage.

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