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Researh Article

Analysis, Adaptive Control and Anti-Synchronization of a Six-Term Novel Jerk Chaotic System with two Exponential Nonlinearities and its Circuit Simulation

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Abstract

This research work proposes a six-term novel 3-D jerk chaotic system with two exponential nonlinearities. This work also analyses system's fundamental properties such as dissipativity, equilibria, Lyapunov exponents and Kaplan-Yorke dimension. The phase portraits of the jerk chaotic system simulated using MATLAB, depict the strange chaotic attractor of the system. For the parameter values and initial conditions chosen in this work, the Lyapunov exponents of the novel jerk chaotic system are obtained as $L_1 = 0.24519$, $L_2 = 0$ and $L_3 = -0.84571$. Also, the Kaplan-Yorke dimension of the novel jerk chaotic system is obtained as $D_{KY} = 2.2899$. Next, an adaptive backstepping controller is designed to stabilize the novel jerk chaotic system having two unknown parameters. Moreover, an adaptive backstepping controller is designed to achieve global chaos anti-synchronization of two identical novel jerk chaotic systems with two unknown system parameters. Finally, an electronic circuit realization of the novel jerk chaotic system is presented using SPICE to confirm the feasibility of the theoretical model.

Keywords: Chaos, chaotic systems, jerk system, Lyapunov exponents, Kaplan-Yorke dimension, circuit simulation.

1. Introduction

Nonlinear dynamics occurs widely in engineering, physics, biology and many other scientific disciplines [1]. There is great interest in the chaos literature in discovery of chaos in nature and physical systems. Poincaré was the first to notice the possibility of chaos according to which a deterministic system exhibits aperiodic behaviour that depends on the initial conditions, thereby rendering long-term prediction impossible [2].

Chaotic systems are nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and also with dense periodic orbits [3]. The sensitivity to initial conditions of a chaotic system is indicated by a positive Lyapunov exponent. A dissipative chaotic system is characterized by the condition that the sum of the Lyapunov exponents of the chaotic system is negative.

Since the discovery of a chaotic system by Lorenz [4] while he was modelling weather patterns with a 3-D model, there is great interest in the literature in the modelling of new chaotic systems. Many paradigms of 3-D chaotic systems have been discovered such as Rössler system [5], Rabinovich system [6], ACT system [7], Sprott systems [8], Chen system [9], Lü system [10], Shaw system [11], Feeny system [12], Shimizu system [13], Liu-Chen system [14], Cai system [15], Tigan system [16], Colpitt's oscillator [17], Zhou system [18],etc.

Recently, many 3-D chaotic systems have been discovered such as Li system [19], Pan system [20], Sundarapandian system [21], Yu-Wang system [22], Sundarapandian-Pehlivan system [23], Zhu system [24], Vaidyanathan systems [25-30], Vaidyanathan-Madhavan system [31], Pehlivan-Moroz-Vaidyanathan system [32], Jafari system [33], Pham system [34], etc.

The study of chaos theory in the last few decades had a big impact on the foundations of Science and Engineering and has found several engineering applications.

Some important applications of chaos theory can be cited as oscillators [35, 36], lasers [37, 38], robotics [39-43], chemical reactors [44,45], biology [46,47], ecology [48,49], neural networks [50-52], secure communications [53-56], cryptosystems [57-60], economics [61-63], etc.

Furthermore, control and synchronization of chaotic systems are important research problems in the chaos literature.

The study of control of a chaotic system investigates methods for finding feedback control laws that globally asymptotically stabilize or regulate the outputs of a chaotic system. Some important methodologies used for this study are active control [64-67], adaptive control [68-74], sliding mode control [75-77], backstepping control [78,79], etc.

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Also, since the pioneering research work by Pecora and Carroll [80], many different methodologies have been developed for synchronization of chaotic systems such as active control [81-91], time-delayed feedback control [92,93], adaptive control [94-105], sampled-data feedback control [106-109], backstepping control [110-116], sliding mode control [117-121], etc.

Especially, the study of anti-synchronization of chaotic systems involves a pair of chaotic systems called master and slave systems, and the design problem is to find an effective feedback control law so that the outputs of the master and slave systems be equal in magnitude and opposite in sign asymptotically. In other words, when anti-synchronization is achieved between the master and slave systems, the sum of the outputs of the two systems converges to zero asymptotically with time.

In the recent decades, there is some special interest in the chaos literature in finding novel chaotic systems, which can be expressed by an explicit third order differential equation describing the time evolution of the single scalar variable x given by

$$\dddot{x} = j(x, \dot{x}, \ddot{x}) \tag{1}$$

The differential equation (1) is described as "jerk system" because the third order time derivative in mechanical systems is called *jerk* [122]. Thus, in order to study different aspects of chaos, the ODE (1) can be considered instead of a 3-D system.

It is well-known that the simplest jerk function that generates chaos is due to Sprott [123] and this jerk function contains just three terms with a quadratic nonlinearity:

$$j(x, \dot{x}, \ddot{x}) = -A\ddot{x} + \dot{x}^2 - x \quad (\text{with } A = 2.017) \tag{2}$$

Sprott showed that the jerk system with the jerk function (2) is a chaotic system with the following Lyapunov exponents $L_1 = 0.0550$, $L_2 = 0$ and $L_3 = -2.0720$. Sprott also calcuated the corresponding Kaplan-Yorke dimension of his jerk chaotic system as $D_{KY} = 2.0265$.

In this research paper, we propose a 3-D novel jerk chaotic system with two exponential nonlinearities. First, we detail the basic qualitative properties of the novel jerk chaotic system. We show that the novel chaotic system is dissipative and we derive the Lyapunov exponents and Kaplan-Yorke dimension of the novel jerk chaotic system.

Next, we derive an adaptive backstepping control law that stabilizes the novel jerk chaotic system when the two system parameters are unknown.

Furthermore, we also derive an adaptive backstepping control law that achieves global chaos anti-synchronization of two identical 3-D novel jerk chaotic systems with unknown parameters.

The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems [124,125].

All the main results in this research paper have been established using adaptive control theory and Lyapunov stability theory. MATLAB simulations are shown to illustrate the phase portraits of the novel jerk chaotic system, dynamics of the Lyapunov exponents, adaptive stabilization and adaptive chaos anti-synchronization for the 3-D novel jerk chaotic system derived in this research paper.

Finally, an electronic circuit realization of the 3-D novel jerk chaotic system using SPICE simulations is presented to confirm the feasibility of the theoretical model.

2. A 3-D Novel Jerk Chaotic System

In the chaos literature, there is some good interest in finding chaotic jerk functions having the special form

$$\dddot{x} + A\ddot{x} + \dot{x} = G(x), \tag{3}$$

where G is a nonlinear function having some special properties [125].

Such systems are called as chaotic memory oscillators in the literature. In [125], Sprott has made an exhaustive study on autonomous conservative and dissipative chaotic systems. Especially, Sprott has listed a set of 16 chaotic memory oscillators (Table 3.3, p. 74, [42]) named as $MO_0, MO_1, \cdots$, $MO_{15}$with details of their Lyapunov exponents.

Sprott's system $MO_{11}$is described by the third order ordinary differential equation

$$\dddot{x} + \ddot{x} + \dot{x} = 5 - \exp(x) \tag{4}$$

It is convenient to express the Sprott differential equation (4) in a system form as:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = x_3 \\ \frac{dx_3}{dt} = 5 - \exp(x_1) - x_2 - x_3 \end{cases} \tag{5}$$

Eq. (5) represents Sprott's 3-D jerk chaotic system having six terms on the R.H.S. with one exponential nonlinearity.

We have chosen the initial conditions for the Sprott system (5) as:

$$x_1(0) = 1.5,\ x_2(0) = 0.6,\ x_3(0) = 1.8 \tag{6}$$

Fig. 1 depicts the strange attractor of the Sprott jerk system (5) for the chosen initial conditions.

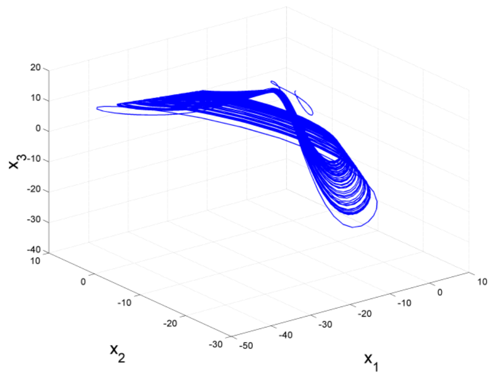

Fig. 1. The strange attractor of the Sprott jerk chaotic system.

The Lyapunov exponents of the Sprott jerk system (5) are calculated as:

$$L_1 = 0.03514, L_2 = 0 \text{ and } L_3 = -1.30479. \tag{7}$$

Thus, the maximal Lyapunov exponent (MLE) of the Sprott jerk system (5) is $L_1(\text{Sprott}) = 0.03514$.

Since the sum of the Lyapunov exponents in (7) is negative, the Sprott jerk system (5) is a dissipative chaotic system.

In this research work, we propose a new jerk system, which is given in system form as:

$$\begin{cases} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= x_3 \\ \frac{dx_3}{dt} &= a - \exp(x_1) - \exp(x_2) - bx_3 \end{cases} \tag{8}$$

In Eq. (8), a and b are assumed to be positive constant parameters.

In this paper, we shall show that the system (8) is chaotic when the parameters a and b take the values:

$$a = 18, b = 0.6 \tag{9}$$

We note that both the Sprott jerk system (5) and the novel jerk system (8) contain the same number of terms on the R.H.S.

However, the two systems are not topologically equivalent since we have replaced the linear term $(-x_2)$ in the third differential equation of the Sprott system (5) with an exponential nonlinearity $-\exp(x_2)$ in the novel system (8). As a consequence, the phase portraits of the two jerk chaotic systems (5) and (8) will be different.

For the parameter values in the chaotic case (9) and the initial conditions given in (6), the Lyapunov exponents of the novel jerk chaotic system (8) are obtained as:

$$L_1 = 0.24519, L_2 = 0 \text{ and } L_3 = -0.84571. \tag{10}$$

Thus, the maximal Lyapunov exponent (MLE) of the novel jerk system (8) is $L_1(\text{Novel System}) = 0.24519$ which is significantly higher than $L_1(\text{Sprott}) = 0.03514$.

Since the sum of the Lyapunov exponents in (10) is negative, the novel jerk chaotic system is dissipative.

For the numerical simulations of the novel jerk chaotic system (8), we have taken the parameter values as in the chaotic case (9) and the initial conditions as (6).

Figure 2 depicts the chaotic attractor of the novel jerk system (8) in 3-D view, while in Figs. 3-5, the 2-D projections of the strange chaotic attractor of the novel jerk chaotic system (8) on (x_1, x_2), (x_2, x_3) and (x_3, x_1) planes, are shown, respectively.

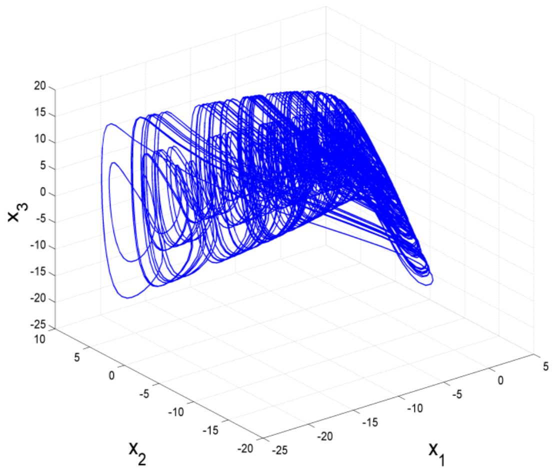

Fig. 2. The strange attractor of the novel jerk chaotic system.

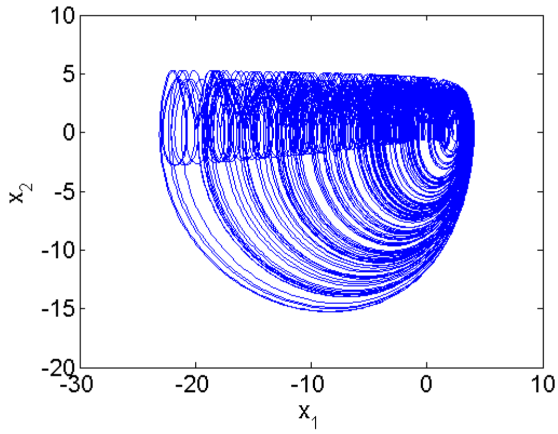

Fig. 3. 2-D projection of the novel chaotic system on (x_1, x_2)-plane.

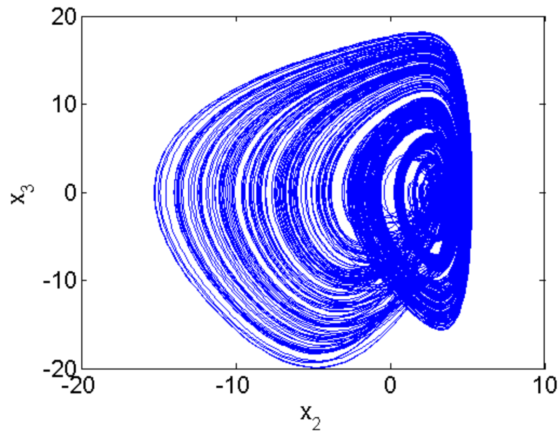

Fig. 4. 2-D projection of the novel chaotic system on (x_2, x_3)-plane.

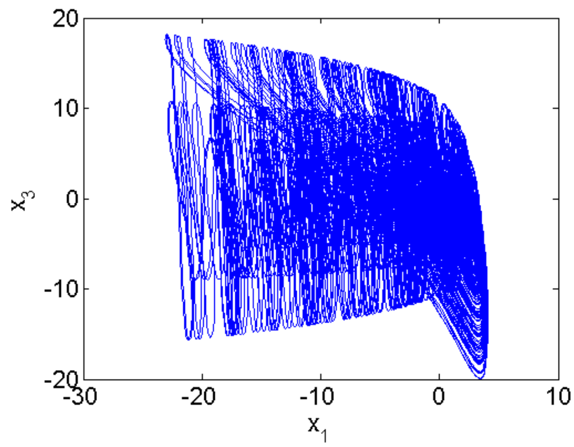

Fig. 5. 2-D projection of the novel chaotic system on (x_1, x_3)-plane.

3. Properties of the 3-D Novel Jerk Chaotic System

In this section, we analyse 3-D novel jerk chaotic system (8) and detail its fundamental properties like dissipativity, symmetry and invariance, equilibria, Lyapunov exponents and Kaplan-Yorke dimension.

3.1. Dissipativity

In vector notation, we may express the system (8) as:

$$\frac{dx}{dt} = f(\boldsymbol{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \quad (11)$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) &= x_2 \\ f_2(x_1, x_2, x_3) &= x_3 \\ f_3(x_1, x_2, x_3) &= a - \exp(x_1) - \exp(x_2) - bx_3 \end{cases} \quad (12)$$

We take the parameter values as in the chaotic case, *viz.* $a = 18$ and $b = 0.6$

Let Ω be any region in $\boldsymbol{R}^3$ with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f.

Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\frac{dV}{dt} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1 dx_2 dx_3 \quad (13)$$

The divergence of the novel jerk chaotic system (8) is easily found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -b = -0.6 \quad (14)$$

Substituting (14) into (13), we obtain the first order ODE

$$\frac{dV}{dt} = -0.6\, V(t) \quad (15)$$

Integrating (15), we obtain the unique solution as:

$$V(t) = \exp(-0.6\, t)\, V(0) \quad (16)$$

It is evident from Eq.(16) that $V(t) \to 0$ exponentially as $t \to \infty$.

This shows that the novel jerk chaotic system (8) is dissipative.

Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel jerk chaotic system (8) settles onto a strange attractor of the system.

3.2. Equilibrium Points

The equilibrium points of the novel chaotic system (8) are obtained by solving the following system of equations (with $a = 18$ and $b = 0.6$)

$$\begin{cases} x_2 &= 0 \\ x_3 &= 0 \\ a - \exp(x_1) - \exp(x_2) - bx_3 &= 0 \end{cases} \quad (17)$$

A simple calculation yields the unique equilibrium point

$$E_0 = \begin{bmatrix} 2.8904 \\ 0 \\ 0 \end{bmatrix} \quad (18)$$

The Jacobian matrix of the system (8) at x is given by

$$J(x) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\exp(x_1) & -\exp(x_2) & -b \end{bmatrix} \quad (19)$$

The Jacobian matrix at the equilibrium E_0 is obtained as:

$$J_0 = J(E_0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -1 & -0.6 \end{bmatrix} \quad (20)$$

Using MATLAB, we find the eigenvalues of J_0 as:

$$\lambda_1 = -2.6995,\ \lambda_{2,3} = 1.0498 \pm 2.3592i \quad (21)$$

Thus, the equilibrium E_0 is a *saddle-focus* point, which is unstable.

3.3. Lyapunov Exponents and Kaplan-Yorke Dimension

For the chosen parameter values (9) and initial conditions (6), the Lyapunov exponents of the novel jerk chaotic system (8) are obtained using MATLAB as:

$$L_1 = 0.24519, L_2 = 0, L_3 = -0.84571 \quad (22)$$

Since the spectrum of Lyapunov exponents (22) has a positive term L_1, it follows that the 3-D novel jerk system (8) is chaotic.

The maximal Lyapunov exponent (MLE) of the novel jerk chaotic system (8) is $L_1 = 0.24519$.

Since the sum of the Lyapunov exponents is negative, it follows that the novel jerk chaotic system (8) is dissipative.

Also, the Kaplan-Yorke dimension of the novel chaotic system (1) is calculated as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.2899 \quad (23)$$

Fig. 6 depicts the dynamics of the Lyapunov exponents of the novel chaotic system (1).

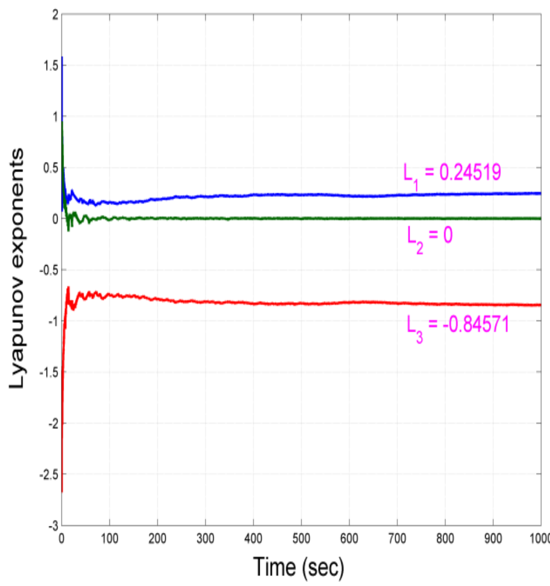

Fig. 6. Dynamics of the Lyapunov Exponents of the Novel System.

4. Adaptive Backstepping Control of the 3-D Novel Jerk Chaotic System

In this section, we use backstepping control to derive an adaptive feedback control law for globally stabilizing the 3-D novel jerk chaotic system with unknown system parameters.

Thus, we consider the 3-D novel jerk system given by

$$\begin{cases} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= x_3 \\ \frac{dx_3}{dt} &= a - \exp(x_1) - \exp(x_2) - bx_3 + u \end{cases} \quad (24)$$

In (24), a and b are unknown constant parameters, and u is a backstepping control law to be determined using estimates $A(t)$ and $B(t)$ for a and b, respectively.

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \end{cases} \quad (25)$$

Differentiating (25) with respect to t, we obtain

$$\begin{cases} \frac{de_a}{dt} &= -\frac{dA}{dt} \\ \frac{de_b}{dt} &= -\frac{dB}{dt} \end{cases} \quad (26)$$

Next, we shall state and prove the main result of this section.

Theorem 1. *The 3-D novel jerk chaotic system (24) with unknown parameters a and b is globally and exponentially stabilized by the adaptive feedback control law*

$$u(t) = -3x_1 - 5x_2 - (3 - B(t))x_3 - A(t) + \\ + \exp(x_1) + \exp(x_2) - kz_3 \quad (27)$$

where $k > 0$ is a gain constant, with

$$z_3 = 2x_1 + 2x_2 + x_3 \quad (28)$$

and the update law for the parameter estimates is given by

$$\begin{cases} \frac{dA}{dt} &= z_3 \\ \frac{dB}{dt} &= x_3 z_3 \end{cases} \quad (29)$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2}z_1^2 \quad (30)$$

where

$$z_1 = x_1 \quad (31)$$

Differentiating V_1 along the dynamics (24), we obtain

$$\frac{dV_1}{dt} = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2) \quad (32)$$

Now, we define

$$z_2 = x_1 + x_2 \quad (33)$$

Using (33), we can simplify (32) as:

$$\frac{dV_1}{dt} = -z_1^2 + z_1 z_2 \quad (34)$$

Next, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2}z_2^2 = \frac{1}{2}(z_1^2 + z_2^2) \quad (35)$$

Differentiating V_2 along the dynamics (24), we obtain

$$\frac{dV_2}{dt} = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3) \quad (36)$$

Now, we define

$$z_3 = 2x_1 + 2x_2 + x_3 \quad (37)$$

Using (37), we can simplify (36) as:

$$\frac{dV_2}{dt} = -z_1^2 - z_2^2 + z_2 z_3 \quad (38)$$

Finally, we define a quadratic Lyapunov function

$$V(\mathbf{z}, e_a, e_b) = V_2(z_1, z_2) + \frac{1}{2}z_3^2 + \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2 \quad (39)$$

From (39), it is clear that V is a positive definite function on R^5.

Differentiating V along the dynamics (24), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a\frac{dA}{dt} - e_b\frac{dB}{dt} \quad (40)$$

where

$$S = z_3 + z_2 + \frac{dz_3}{dt} = z_3 + z_2 + 2\frac{dx_1}{dt} + 2\frac{dx_2}{dt} + \frac{dx_3}{dt} \quad (41)$$

Simplifying the equation (41), we obtain

$$S = 3x_1 + 5x_2 + (3 - b)x_3 + a - \\ - \exp(x_1) - \exp(x_2) + u \quad (42)$$

Substituting the control law (27) into (42), we obtain

$$S = \left(a - A(t)\right) - \left(b - B(t)\right)x_3 - kz_3 \quad (43)$$

Using (25), we can simplify the equation (43) as:

$$S = e_a - e_b x_3 - kz_3 \quad (44)$$

Substituting the value of S from (44) into (40), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - (1 + k)z_3^2 + e_a\left(z_3 - \frac{dA}{dt}\right) + \\ + e_b\left(-x_3 z_3 - \frac{dB}{dt}\right) \quad (45)$$

Substituting (29) into (45), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - (1 + k)z_3^2 \quad (46)$$

Thus, it is clear that $\frac{dV}{dt}$ is a negative semi-definite function on R^5.

From (46), it follows that the vector $\mathbf{z}(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t))$ are globally bounded, *i.e.*

$$[z_1(t) \quad z_2(t) \quad z_3(t) \quad e_a(t) \quad e_b(t)] \in \mathbf{L}_\infty \tag{47}$$

Also, it follows from (46) that

$$\frac{dV}{dt} \le -z_1^2 - z_2^2 - z_3^2 = -\|\mathbf{z}\|^2 \tag{48}$$

That is,

$$\|\mathbf{z}\|^2 \le -\frac{dV}{dt} \tag{49}$$

Integrating the inequality (49) from 0 to t, we get

$$\int_0^t \|\mathbf{z}(\tau)\|^2 d\tau \le V(0) - V(t) \tag{50}$$

From (50), it follows that $\mathbf{z}(t) \in \mathbf{L}_2$, while from (24), it can be deduced that $\frac{d\mathbf{z}}{dt} \in \mathbf{L}_\infty$.

Thus, using Barbalat's lemma [126], we can conclude that $\mathbf{z}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{z}(0) \in \mathbf{R}^3$.

Hence, it is immediate that $\mathbf{x}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{x}(0) \in \mathbf{R}^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (24) and (29), when the adaptive control law (27) is applied.

The parameter values of the novel 3-D jerk chaotic system (24) are chosen as in the chaotic case, *viz.* $a = 18$ and $b = 0.6$. The positive gain constant k is taken as $k = 8$.

Furthermore, as initial conditions of the novel jerk chaotic system (24), we have chosen $x_1(0) = 8.3, x_2(0) = -5.2$ and $x_3(0) = 12.7$.

Also, as initial conditions of the estimates $A(t)$ and $B(t)$, we have taken $A(0) = 15.4$ and $B(0) = 9.2$.

In Fig. 7, the exponential convergence of the controlled states $x_1(t), x_2(t), x_3(t)$ is depicted, when the adaptive control law (27) and parameter update law (29) are implemented.

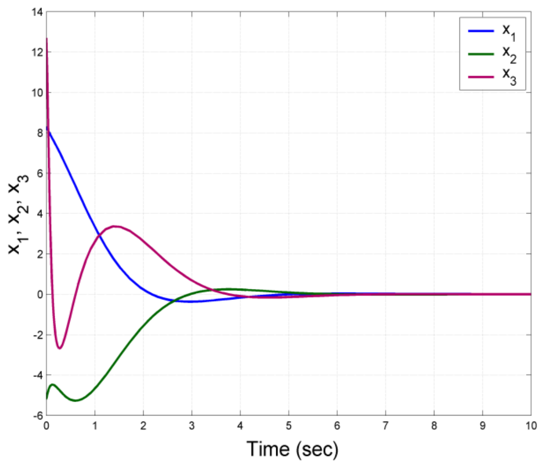

Fig. 7. Time-History of the Controlled States $x_1(t), x_2(t), x_3(t)$.

5. Adaptive Backstepping Anti-Synchronization of the Identical 3-D Novel Jerk Chaotic Systems

In this section, we use backstepping control method to derive an adaptive control law for globally and exponentially anti-synchronizing the identical 3-D novel jerk chaotic systems with unknown system parameters.

Thus, the master system is given by the novel jerk chaotic dynamics

$$\begin{cases} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= x_3 \\ \frac{dx_3}{dt} &= a - \exp(x_1) - \exp(x_2) - bx_3 \end{cases} \tag{51}$$

Also, the slave system is given by the controlled novel jerk chaotic dynamics

$$\begin{cases} \frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= y_3 \\ \frac{dy_3}{dt} &= a - \exp(y_1) - \exp(y_2) - by_3 + u \end{cases} \tag{52}$$

In (51) and (52), the system parameters a and b are unknown and the design goal is to find an adaptive feedback control u that uses estimates $A(t)$ and $B(t)$ for the parameters a and b so as to render the states of the systems (51) and (52) fully anti-synchronized asymptotically.

The anti-synchronization error between the novel jerk chaotic systems (51) and (52) is defined as:

$$\begin{cases} e_1 &= y_1 + x_1 \\ e_2 &= y_2 + x_2 \\ e_3 &= y_3 + x_3 \end{cases} \tag{53}$$

Thus, the anti-synchronization error dynamics is obtained as:

$$\begin{cases} \frac{de_1}{dt} &= e_2 \\ \frac{de_2}{dt} &= e_3 \\ \frac{de_3}{dt} &= 2a - be_3 - \exp(y_1) - \exp(y_2) \\ & \quad - \exp(x_1) - \exp(x_2) + u \end{cases} \tag{54}$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) &= a - A(t) \\ e_b(t) &= b - B(t) \end{cases} \tag{55}$$

Differentiating (55) with respect to t, we obtain

$$\begin{cases} \frac{de_a}{dt} &= -\frac{dA}{dt} \\ \frac{de_b}{dt} &= -\frac{dB}{dt} \end{cases} \tag{56}$$

Next, we shall state and prove the main result of this section.

Theorem 2. *The 3-D novel jerk chaotic systems (51) and (52) with unknown parameters a and b are globally and exponentially anti-synchronized by the adaptive feedback control law*

$$u(t) = -3e_1 - 5e_2 - (3 - B(t))e_3 - 2A(t) + \exp(y_1) + \exp(y_2) + \exp(x_1) + \exp(x_2) - kz_3 \quad (57)$$

where $k > 0$ is a gain constant, with

$$z_3 = 2e_1 + 2e_2 + e_3, \quad (58)$$

and the update law for the parameter estimates is given by

$$\begin{cases} \frac{dA}{dt} = 2z_3 \\ \frac{dB}{dt} = e_3 z_3 \end{cases} \quad (59)$$

Proof. We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \quad (60)$$

where

$$z_1 = e_1 \quad (61)$$

Differentiating V_1 along the dynamics (54), we obtain

$$\frac{dV_1}{dt} = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \quad (62)$$

Now, we define

$$z_2 = e_1 + e_2 \quad (63)$$

Using (63), we can simplify (62) as:

$$\frac{dV_1}{dt} = -z_1^2 + z_1 z_2 \quad (64)$$

Next, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2}(z_1^2 + z_2^2) \quad (65)$$

Differentiating V_2 along the dynamics (54), we obtain

$$\frac{dV_2}{dt} = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \quad (66)$$

Now, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \quad (67)$$

Using (67), we can simplify (66) as:

$$\frac{dV_2}{dt} = -z_1^2 - z_2^2 + z_2 z_3 \quad (68)$$

Finally, we define a quadratic Lyapunov function

$$V(\boldsymbol{z}, e_a, e_b) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2 \quad (69)$$

From (69), it is clear that V is a positive definite function on R^5.

Differentiating V along the dynamics (54), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \frac{dA}{dt} - e_b \frac{dB}{dt} \quad (70)$$

where

$$S = z_3 + z_2 + \frac{dz_3}{dt} = z_3 + z_2 + 2\frac{de_1}{dt} + 2\frac{de_2}{dt} + \frac{de_3}{dt} \quad (71)$$

Simplifying the equation (71), we obtain

$$S = 3e_1 + 5e_2 + (3 - b)e_3 + 2a - \exp(y_1) - \exp(y_2) - \exp(x_1) - \exp(x_2) + u \quad (72)$$

Substituting the control law (57) into (72), we obtain

$$S = 2(a - A(t)) - (b - B(t))e_3 - kz_3 \quad (73)$$

Using (55), we can simplify the equation (73) as:

$$S = 2\, e_a - e_b e_3 - kz_3 \quad (74)$$

Substituting the value of S from (74) into (70), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - (1 + k)z_3^2 + e_a\left(2\, z_3 - \frac{dA}{dt}\right) + e_b\left(-e_3 z_3 - \frac{dB}{dt}\right) \quad (75)$$

Substituting (59) into (75), we obtain

$$\frac{dV}{dt} = -z_1^2 - z_2^2 - (1 + k)z_3^2 \quad (76)$$

Thus, it is clear that $\frac{dV}{dt}$ is a negative semi-definite function on R^5.

From (76), it follows that the vector $\boldsymbol{z}(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t))$ are globally bounded, *i.e.*

$$[z_1(t) \quad z_2(t) \quad z_3(t) \quad e_a(t) \quad e_b(t)] \in \boldsymbol{L}_\infty \quad (77)$$

Also, it follows from (76) that

$$\frac{dV}{dt} \le -z_1^2 - z_2^2 - z_3^2 = -\|\boldsymbol{z}\|^2 \quad (78)$$

That is,

$$\|\boldsymbol{z}\|^2 \le -\frac{dV}{dt} \quad (79)$$

Integrating the inequality (79) from 0 to t, we get

$$\int_0^t \|\boldsymbol{z}(\tau)\|^2 d\tau \le V(0) - V(t) \quad (80)$$

From (50), it follows that $\boldsymbol{z}(t) \in \boldsymbol{L}_2$, while from (54), it can be deduced that $\frac{dz}{dt} \in \boldsymbol{L}_\infty$.

Thus, using Barbalat's lemma [126], we can conclude that $\boldsymbol{z}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\boldsymbol{z}(0) \in \boldsymbol{R}^3$.

Hence, it is immediate that the anti-synchronization error $\boldsymbol{e}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\mathbf{e}(0) \in \mathbf{R}^3$.

Thus, it follows that 3-D novel jerk chaotic systems (51) and (52) are globally and exponentially anti-synchronized for all initial conditions $x(0), y(0) \in \boldsymbol{R}^3$.

This completes the proof. ■

For numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (51), (52) and (59), when the adaptive control law (57) is applied.

The parameter values of the novel 3-D jerk chaotic systems (51) and (52) are taken as in the chaotic case, *viz.* $a = 18$ and $b = 0.6$. The positive gain constant k is taken as $k = 8$.

Furthermore, as initial conditions of the master system (51), we take $x_1(0) = 4.3, x_2(0) = -2.1$ and $x_3(0) = 5.7$. As initial conditions of the slave system (52), we take $y_1(0) = -2.7, y_2(0) = 6.4$ and $y_3(0) = 3.9$.

Also, as initial conditions of the estimates $A(t)$ and $B(t)$, we take $A(0) = 4.5$ and $B(0) = 3.8$.

In Figs. 8-10, the anti-synchronization of the states of the master system (51) and slave system (52) is depicted, when the adaptive control law (57) and parameter update law (59) are implemented. In Fig. 11, the time-history of the anti-synchronization errors $e_1(t), e_2(t), e_3(t)$ is depicted.

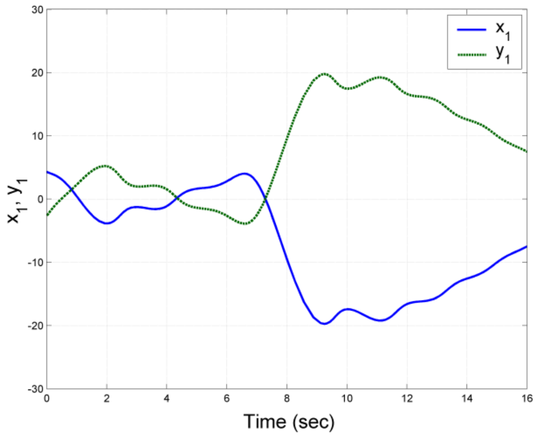

Fig. 8. Anti-Synchronization of the States $x_1(t)$ and $y_1(t)$.

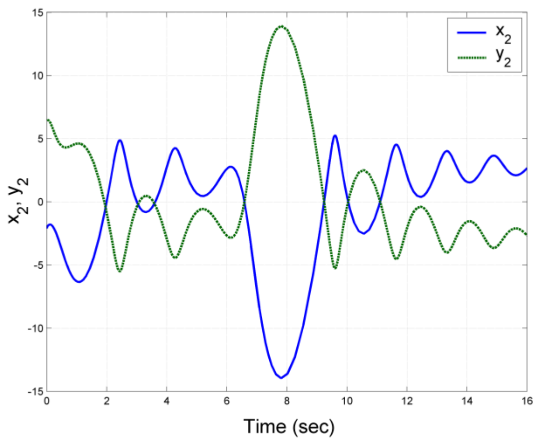

Fig. 9. Anti-Synchronization of the States $x_2(t)$ and $y_2(t)$.

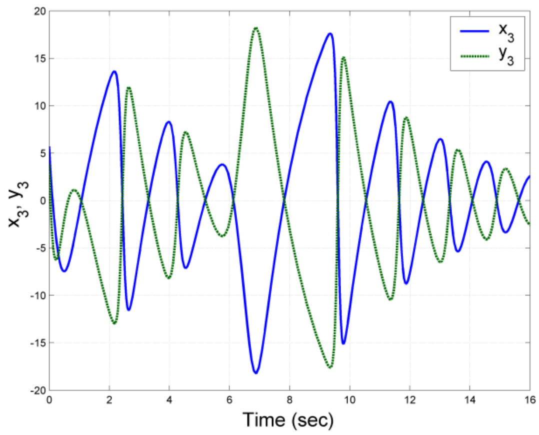

Fig. 10. Anti-Synchronization of the States $x_3(t)$ and $y_3(t)$.

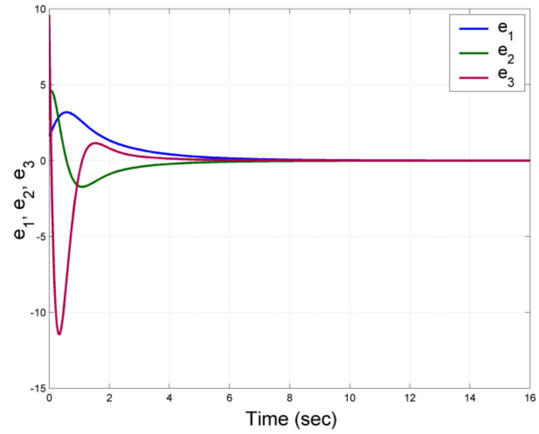

Fig. 11. Time-History of the Errors $e_1(t), e_2(t), e_3(t)$.

6. Circuit Realization of the Novel Jerk System

In this section, we proposed an electronic circuit modelling the new jerk system (8). Because the circuit is designed following an approach based on operational amplifiers [31,33], the state variables of system (8) are scaled down to obtain chaotic attractors in the dynamical range of operational amplifiers. Hence, the new jerk system (8) can be written as:

$$\begin{cases} \frac{dX_1}{dt} & = & X_1 \\ \frac{dX_2}{dt} & = & X_2 \\ \frac{dX_3}{dt} & = & \frac{a}{4} - \frac{1}{4}\exp(4X_1) - \frac{1}{4}\exp(4X_2) - bX_3 \end{cases} \tag{81}$$

in which $X_1 = \frac{x_1}{4}$, $X_2 = \frac{x_2}{4}$ and $X_3 = \frac{x_3}{4}$. The schematic of the designed circuit is shown in Fig. 12.

By applying Kirchhoff's laws to the electronic circuit, its nonlinear equations are derived in the following form:

$$\begin{cases} \frac{dv_{C_1}}{dt} & = & \frac{1}{R_1C_1}v_{C_2} \\ \frac{dv_{C_2}}{dt} & = & \frac{1}{R_2C_2}v_{C_3} \\ \frac{dv_{C_3}}{dt} & = & -\frac{1}{R_3C_3}V_a - \frac{1}{R_4C_3}\exp\left(\left(1+\frac{R_8}{R_7}\right)v_{C_1}\right) \\ & & -\frac{1}{R_5C_3}\exp\left(\left(1+\frac{R_{10}}{R_9}\right)v_{C_2}\right) - \frac{1}{R_6C_3}v_{C_3} \end{cases} \quad (82)$$

where v_{C_1}, v_{C_2}, v_{C_3} are the voltages across the capacitors C_1, C_2, and C_3, respectively.

Here the state variables X_1, X_2, X_3 of system (81) are the voltages v_{C_1}, v_{C_2}, v_{C_3}, respectively.

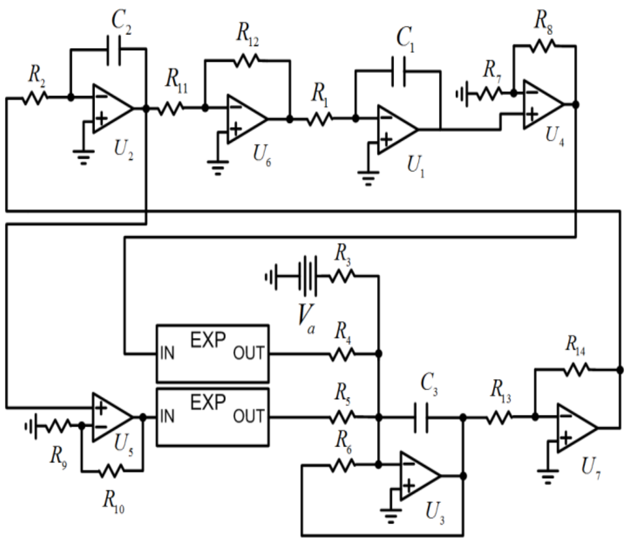

Fig. 12. Circuit diagram for realizing the six-term novel 3-D jerk chaotic system (81).

The values of the electronic components in Fig. 12 are chosen to match known parameters of system (20): $R_1 = R_2 = R_7 = R_9 = R_{11} = R_{12} = 100$ kΩ, $R_3 = 200$ kΩ, $R_4 = R_5 = 400$ kΩ, $R_6 = 166.666$ kΩ, $R_8 = R_{10} = 300$ kΩ, $C_1 = C_2 = C_3 = 1$ nF, and $V_a = -9$ V_{DC}. The power supplies of all active devices are ±15 Volts.

The proposed circuit is implemented by using the electronic simulation package Cadence OrCAD. Figs 13-15 show the obtained phase portraits in (v_{C_1}, v_{C_2})-plane, (v_{C_2}, v_{C_3})-plane, and (v_{C_1}, v_{C_3})-plane, respectively. Clearly, the circuital results agree well with numerical simulation results.

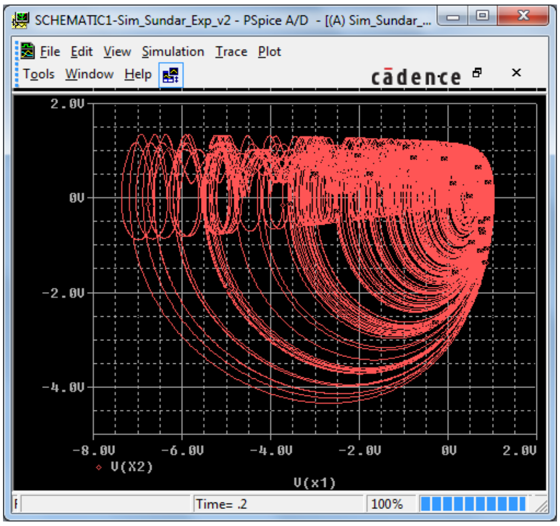

Fig. 13. Chaotic attractor obtained from the circuit in Fig. 12 in (v_{C_1}, v_{C_2})-plane.

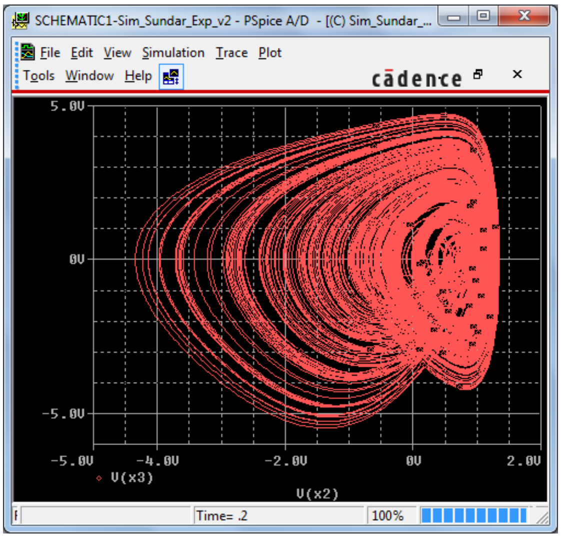

Fig. 14. Chaotic attractor obtained from the circuit in Fig. 12 in (v_{C_2}, v_{C_3})-plane.

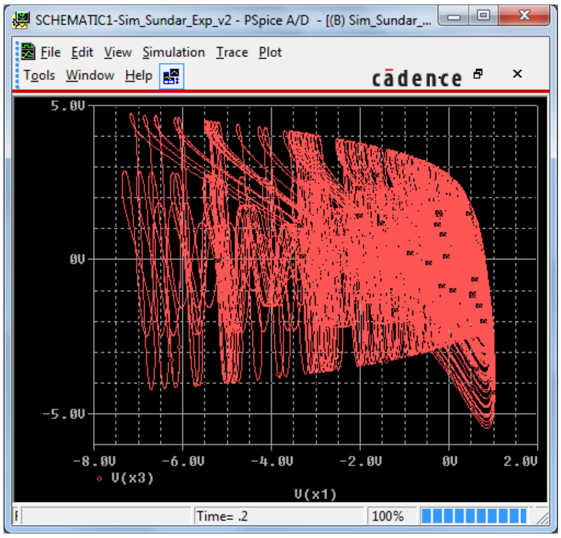

Fig. 15. Chaotic attractor obtained from the circuit in Fig. 12 in (v_{C_1}, v_{C_3})-plane.

7. Conclusion

A six-term novel 3-D jerk chaotic system with two exponential nonlinearities is presented in this work. Fundamental dynamics of new system are investigated through dissipativity, equilibria, Lyapunov exponents and Kaplan-Yorke dimension. In addition, an adaptive backstepping controller is designed not only to stabilize the novel jerk chaotic system with two unknown parameters but also to achieve global chaos anti-synchronization of two identical such systems with two unknown system parameters. Furthermore, an electronic circuit realizing of the novel jerk chaotic system confirms the feasibility of the theoretical model. Hence, it is believed that the new jerk system can be used in diverse chaos-based applications. Complex dynamical behaviors of this system will be further studied in the future researches.

References

1. S.H. Strogatz, Nonlinear dynamics and chaos: With applications to physics, biology, chemistry, and engineering, Perseus Books, Massachusetts, US (1994).
2. J.H. Poincaré, Sur le problème des trois corps et les équations de la dynamique, Divergence des séries de M. Lindstedt, Acta Mathematica, vol. 13, pp. 1-270 (1890).
3. B. Hasselblatt and A. Katok, A first course in dynamics: With a panorama of recent developments, Cambridge University Press (2003).
4. E.N. Lorenz, Deterministic nonperiodic flow, Journal of the Atmospheric Sciences, vol. 20, pp. 130-141 (1963).
5. O.E. Rössler, An equation for continuous chaos, Physics Letters A, vol. 57, pp. 397-398 (1976).
6. M.I. Rabinovich and A.L. Fabrikant, Stochastic self-modulation of waves in nonequilibrium media, Sov. Phys. JETP, vol. 50, pp. 311-317, (1979)
7. A. Arneodo, P. Coullet, and C. Tresser, Possible new strange attractors with spiral structure, Communications in Mathematical Physics, vol. 79, pp. 573-579 (1981).
8. J.C. Sprott, Some simple chaotic flows, Physical Review E, vol. 50, pp. 647-650 (1994).
9. G. Chen and T. Ueta, Yet another chaotic oscillator, International Journal of Bifurcation and Chaos, vol. 9, pp. 1465-1466 (1999).
10. J. Lü and G. Chen, A new chaotic attractor coined, International Journal of Bifurcation and Chaos, vol. 12, pp. 659-661 (2002).
11. R. Shaw, Strange attractors, chaotic behaviour and information flow, Zeitschrift für Naturforschung, vol. 36, pp. 80-112 (1981).
12. B. Feeny and F.C. Moon, Chaos in a forced dry-friction oscillator: Experiments and numerical modeling, Journal of Sound and Vibration, vol. 170, pp. 303-323 (1994).
13. T. Shimizu and N. Moroika, On the bifurcation of a symmetric limit cycle to an asymmetric one in a simple model, Physics Letters A, vol. 76, pp. 201-204 (1980).
14. W. Liu and G. Chen, A new chaotic system and its generation, International Journal of Bifurcation and Chaos, vol. 13, pp. 261-267 (2003).
15. G. Cai and Z. Tan, Chaos synchronization of a new chaotic system via nonlinear control, Journal of Uncertain Systems, vol. 1, pp. 235-240 (2007).
16. G. Tigan and D. Opris, Analysis of a 3D chaotic system, Chaos, Solitons and Fractals, vol. 36, pp. 1315-1319 (2008).
17. G.P. Kennedy, Chaos in the Colpitts oscillator, IEEE Transactions on Circuits and Systems-I, vol. 41, pp. 771-774 (1994).
18. W. Zhou, Y. Xu, H. Lu, and L. Pan, On dynamics analysis of a new chaotic attractor, Physics Letters A, vol. 372, pp. 5773-5777 (2008).
19. D. Li, A three-scroll chaotic attractor, Physics Letters A, vol. 372, pp. 387-393 (2008).
20. L. Pan, D. Xu and W. Zhou, Controlling a novel chaotic attractor using linear feedback, Journal of Information and Computing Science, vol. 5, pp. 117-124 (2010).
21. V. Sundarapandian, Analysis and anti-synchronization of a novel chaotic system via active and adaptive controllers, Journal of Engineering Science and Technology Review, vol. 6, pp. 45-52 (2013).
22. F. Yu, C. Wang, Q. Wan, and Y. Hu, Complete switched modified function projective synchronization of a five-term chaotic system with uncertain parameters and disturbances, Pramana, vol. 80, pp. 223-235 (2013).
23. V. Sundarapandian and I. Pehlivan, Analysis, control, synchronization and circuit design of a novel chaotic system, Mathematical and Computer Modelling, vol. 55, pp. 1904-1915 (2012).
24. C. Zhu, Y. Liu, and Y. Guo, Theoretical and numerical study of a new chaotic system, Intelligent Information Management, vol. 2, pp. 104-109 (2010).
25. S. Vaidyanathan, A new six-term 3-D chaotic system with an exponential nonlinearity, Far East Journal of Mathematical Sciences, vol. 79, pp. 135-143 (2013).
26. S. Vaidyanathan, Analysis and adaptive synchronization of two novel chaotic systems with hyperbolic sinusoidal and cosinusoidal nonlinearity and unknown parameters, Journal of Engineering Science and Technology Review, vol. 6, pp. 53-65 (2013).
27. S. Vaidyanathan, A new eight-term 3-D polynomial chaotic system with three quadratic nonlinearities, Far East Journal of Mathematical Sciences, vol. 84, pp. 219-226 (2014).
28. S. Vaidyanathan, Analysis, control and synchronization of a six-term novel chaotic system with three quadratic nonlinearities, International Journal of Modelling, Identification and Control, vol. 22, pp. 41-53 (2014).
29. S. Vaidyanathan, Analysis and adaptive synchronization of eight-term 3-D polynomial chaotic systems with three quadratic nonlinearities, European Physical Journal: Special Topics, vol. 223, pp. 1519-1529 (2014).
30. S. Vaidyanathan, Ch. Volos, V.T. Pham, K. Madhavan, and B.A. Idowu, Adaptive backstepping control, synchronization and circuit simulation of a 3-D novel jerk chaotic system with two hyperbolic sinusoidal nonlinearities, Archives of Control Sciences, vol. 24(3), pp. 257-285 (2014).
31. S. Vaidyanathan and K. Madhavan, Analysis, adaptive control and synchronization of a seven-term novel 3-D chaotic system, International Journal of Control Theory and Applications, vol. 6, pp. 121-137 (2013).
32. I. Pehlivan, I.M. Moroz, and S. Vaidyanathan, Analysis, synchronization and circuit design of a novel butterfly attractor, Journal of Sound and Vibration, vol. 333, pp. 5077-5096 (2014).
33. S. Jafari and J.C. Sprott, Simple chaotic flows with a line equilibrium, Chaos, Solitons and Fractals, vol. 57, pp. 79-84, 2013.
34. V.T. Pham, C. Volos, S. Jafari, Z. Wei and X. Wang, Constructing a novel no-equilibrium chaotic system, International Journal of Bifurcation and Chaos, vol. 24, 1450073 (2014).
35. H.B. Fotsin and J. Daafouz, Adaptive synchronization of uncertain Colpitts oscillators based on parameter identification, Physics Letters A, vol. 339, pp. 304-315 (2005).
36. A. Sharma, V. Patidar, G. Purohit, and K.K. Sud, Effects on the bifurcation and chaos in forced Duffing oscillator due to nonlinear damping, Communications in Nonlinear Science and Numerical Simulation, vol. 17, pp. 2254-2269 (2012).

37. S. Donati and S.K. Hwang, Chaos and high-level dynamics in coupled lasers and their applications, Progress in Quantum Electronics, vol. 36, pp. 293-341 (2012).
38. N. Li, W. Pan, L. Yan, B. Luo, and X. Zou, Enhanced chaos synchronization and communication in cascade-coupled semiconductor ring lasers, Communications in Nonlinear Science and Numerical Simulation, vol. 19, pp. 1874-1883 (2014).
39. U. Nehmzow and K. Walker, Quantitative description of robot-environment interaction using chaos theory, Robotics and Autonomous Systems, vol. 53, pp. 177-193 (2005).
40. S. Iqbal, X. Zang, Y. Zhu, and J. Zhao, Bifurcations and chaos in passive dynamic walking: A review, Robotics and Autonomous Systems, vol. 62, pp. 889-909 (2014).
41. Ch.K. Volos, I.M. Kyprianidis, and I.N. Stouboulos, A chaotic path planning generator for autonomous mobile robots, Robotics and Autonomous Systems, vol. 60, pp. 651-656, (2012).
42. Ch.K. Volos, I.M. Kyprianidis, and I.N. Stouboulos, Experimental investigation on coverage performance of a chaotic autonomous mobile robot, Robotics and Autonomous Systems, vol. 61(12), pp. 1314-1322 (2013).
43. S. Iqbal, X. Zang, Y. Zhu, and J. Zhao, Bifurcations and chaos in passive dynamic walking: A review, Robotics and Autonomous Systems, vol. 62(6), pp. 889-909 (2014).
44. J.C. Roux, Chaos in experimental chemical systems: two examples, North-Holland Mathematics Studies, vol. 103, pp. 345-352 (1985).
45. Y.N. Li, L. Chen, Z.S. Cai, and X.Z. Zhao, Study on chaos synchronization in the Belousov-Zhabotinsky chemical system, Chaos, Solitons and Fractals, vol. 17, pp. 699-707 (2003).
46. M. Kyriazis, Applications of chaos theory to the molecular biology of aging, Experimental Gerontology, vol. 26, pp. 569-572 (1991).
47. G. Böhm, Protein folding and deterministic chaos: Limits of protein folding simulations and calculations, Chaos, Solitons and Fractals, vol. 1, pp. 375-382 (1991).
48. J.C. Sprott, J.A. Vano, J.C. Wildenberg, M.B. Anderson, and J.K. Noel, Coexistence and chaos in complex ecologies, Physics Letters A, vol. 335, pp. 207-212 (2005).
49. B. Sahoo and S. Poria, The chaos and control of a food chain model supplying additional food to top-predator, Chaos, Solitons and Fractals, vol. 58, pp. 52-64 (2014).
50. G. He, Z. Cao, P. Zhu, and H. Ogura, Controlling chaos in a chaotic neural network, Neural Networks, vol. 16, pp. 1195-1200 (2003).
51. E. Kaslik and S. Sivasundaram, Nonlinear dynamics and chaos in fractional-order neural networks, Neural Networks, vol. 32, pp. 245-256 (2012).
52. I.M. Kyprianidis and A.T. Makri, Complex dynamics of FitzHugh-Nagumo type neurons coupled with gap junction under external voltage stimulation, Journal of Engineering Science and Technology Review, vol. 6(4), pp. 104-114 (2013).
53. K. Suzuki and Y. Imai, Decryption characteristics in message modulation type chaos secure communication system using optical fiber ring resonators, Optics Communications, vol. 259, pp. 88-93 (2006).
54. X.Y. Wang and Y.F. Gao, A switch-modulated method for chaos digital secure communication based on user-defined protocol, Communications in Nonlinear Science and Numerical Simulation, vol. 15, pp. 99-104 (2010).
55. O.I. Moskalenko, A.A. Koronovskii, and A.E. Hramov, Generalized synchronization of chaos for secure communication: Remarkable stability to noise, Physics Letters A, vol. 374, pp. 2925-2931 (2010).
56. A. Abdullah, Synchronization and secure communication of uncertain chaotic systems based on full-order and reduced-order output-affine observers, Applied Mathematics and Computation, vol. 219, pp. 10000-10011 (2013).
57. N. Smaoui and A. Kanso, Cryptography with chaos and shadowing, Chaos, Solitons and Fractals, vol. 42, pp. 2312-2321 (2009).
58. R. Rhouma and S. Belghith, Cryptanalysis of a chaos-based cryptosystem on DSP, Communications in Nonlinear Science and Numerical Simulation, vol. 16, pp. 876-884 (2011).
59. ChK. Volos, I.M. Kyprianidis, and I.N. Stouboulos, Text encryption scheme realized with a chaotic pseudo-random bit generator, Journal of Engineering Science and Technology Review, vol. 6(4), pp. 9-14 (2013).
60. Ch.K. Volos, I.M. Kyprianidis, and I.N. Stouboulos, Image encryption process based on chaotic synchronization phenomena, Signal Processing, vol. 93(5), pp. 1328-1340 (2013).
61. D. Guégan, Chaos in economics and finance, Annual Reviews in Control, vol. 33, pp. 89-93 (2009).
62. Ch.K. Volos, I.M. Kyprianidis, and I.N. Stouboulos, Synchronization phenomena in coupled nonlinear systems applied in economic cycles, WSEAS Trans. Systems, vol. 11(12), pp. 681-690 (2012).
63. P. Caraiani, Testing for nonlinearity and chaos in economic time series with noise titration, Economics Letters, vol. 120, pp. 192-194 (2013).
64. V. Sundarapandian, Output regulation of the Lorenz attractor, Far East Journal of Mathematical Sciences, vol. 42, pp. 289-299 (2010).
65. S. Vaidyanathan, Output regulation of Arneodo-Coullet chaotic system, Communications in Computer and Information Science, vol. 131, pp. 585-593 (2011).
66. S. Vaidyanathan, Output regulation of the unified chaotic system, Communications in Computer and Information Science, vol. 198, pp. 1-9 (2011).
67. S. Vaidyanathan, Output regulation of the Liu chaotic system, Applied Mechanics and Materials, vols. 110-116, pp. 3982-3989 (2012).
68. G. Chen, A simple adaptive feedback control method for chaos and hyper-chaos control, Applied Mathematics and Computation, vol. 217, pp. 7258-7264 (2011).
69. J. Zheng, A simple universal adaptive feedback controller for chaos and hyperchaos control, Computers & Mathematics with Applications, vol. 61, pp. 2000-2004 (2011).
70. S. Vaidyanathan, Adaptive controller and synchronizer design for the Qi-Chen chaotic system, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, vol. 85, pp. 124-133 (2012).
71. V. Sundarapandian, Adaptive control and synchronization design for the Lu-Xiao chaotic system, Lecture Notes in Electrical Engineering, vol. 131, pp. 319-327 (2013).
72. S. Vaidyanathan, A ten-term novel 4-D hyperchaotic system with three quadratic nonlinearities and its control, International Journal of Control Theory and Applications, vol. 6, pp. 97-109 (2013).
73. S. Vaidyanathan, Analysis, control and synchronization of hyperchaotic Zhou system via adaptive control, Advances in Intelligent Systems and Computing, vol. 177, pp. 1-10 (2013).
74. D. Yang and J. Zhou, Connections among several chaos feedback control approaches and chaotic vibration control of mechanical systems, Communications in Nonlinear Science and Numerical Simulation, vol. 19, pp. 3954-3968 (2014).
75. M.T. Yassen, Chaos control of chaotic dynamical systems using backstepping design, Chaos, Solitons and Fractals, vol. 27, pp. 537-548 (2006).
76. J.A. Laoye, U.E. Vincent, and S.O. Kareem, Chaos control of 4D chaotic systems using recursive backstepping nonlinear controller, Chaos, Solitons and Fractals, vol. 39, pp. 356-362 (2009).
77. D. Lin, X. Wang, F. Nian, and Y. Zhang, Dynamic fuzzy neural networks modeling and adaptive backstepping tracking control of uncertain chaotic systems, Neurocomputing, vol. 73, pp. 2873-2881 (2010).
78. S. Vaidyanathan, Sliding mode control based global chaos control of Liu-Liu-Liu-Su chaotic system, International Journal of Control Theory and Applications, vol. 5, pp. 15-20 (2012).
79. S. Vaidyanathan, Global chaos control of hyperchaotic Liu system via sliding mode control, vol. 5, pp. 117-123 (2012).
80. L.M. Pecora and T.L. Carroll, Synchronization in chaotic systems, vol. 64, pp. 821-825 (1990).
81. S. Vaidyanathan and S. Rasappan, New results on the global chaos synchronization for Liu-Chen-Liu and Lü chaotic systems,

Communications in Computer and Information Science, vol. 102, pp. 20-27 (2010).
82. S. Vaidyanathan and S. Rasappan, Hybrid synchronization of hyperchaotic Qi and Lü systems by nonlinear control, Communications in Computer and Information Science, vol. 131, pp. 585-593 (2011).
83. S. Vaidyanathan and K. Rajagopal, Anti-synchronization of Li and T chaotic systems by active nonlinear control, Communications in Computer and Information Science, vol. 198, pp. 175-184 (2011).
84. S. Vaidyanathan and S. Rasappan, Global chaos synchronization of hyperchaotic Bao and Xu systems by active nonlinear control, Communications in Computer and Information Science, vol. 198, pp. 10-17 (2011).
85. S. Vaidyanathan and K. Rajagopal, Global chaos synchronization of hyperchaotic Pang and Wang systems by active nonlinear control, Communications in Computer and Information Science, vol. 204, pp. 84-93 (2011).
86. S. Vaidyanathan, Hybrid chaos synchronization of Liu and Lü systems by active nonlinear control, Communications in Computer and Information Science, vol. 204, pp. 1-10 (2011).
87. P. Sarasu and V. Sundarapandian, Active controller design for generalized projective synchronization of four-scroll chaotic systems, International Journal of Systems Signal Control and Engineering Application, vol. 4, pp. 26-33 (2011).
88. S. Vaidyanathan and K. Rajagopal, Hybrid synchronization of hyperchaotic Wang-Chen and hyperchaotic Lorenz systems by active non-linear control, International Journal of Systems Signal Control and Engineering Application, vol. 4, pp. 55-61 (2011).
89. S. Pakiriswamy and S. Vaidyanathan, Generalized projective synchronization of three-scroll chaotic systems via active control, Lecture Notes of the Institute for Computer Sciences, Social-Informatics and Telecommunications Engineering, vol. 85, pp. 146-155 (2012).
90. V. Sundarapandian and R. Karthikeyan, Hybrid synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems via active control, Journal of Engineering and Applied Sciences, vol. 7, pp. 254-264 (2012).
91. R. Karthikeyan and V. Sundarapandian, Hybrid chaos synchronization of four-scroll systems via active control, Journal of Electrical Engineering, vol. 65, pp. 97-103 (2014).
92. E.M. Shahverdiev and K.A. Shore, Impact of modulated multiple optical feedback time delays on laser diode chaos synchronization, Optics Communications, vol. 282, pp. 3568-3572 (2009).
93. T. Botmart, P. Niamsup, and X. Liu, Synchronization of non-autonomous chaotic systems with time-varying delay via delayed feedback control, Communications in Nonlinear Science and Numerical Simulation, vol. 17, pp. 1894-1907 (2012).
94. S. Bowong, Adaptive synchronization between two different chaotic dynamical systems, Communications in Nonlinear Science and Numerical Simulation, vol. 12, pp. 976-985 (2007).
95. W. Lin, Adaptive chaos control and synchronization in only locally Lipschitz systems, Physics Letters A, vol. 372, pp. 3195-3200 (2008).
96. H. Salarieh and A. Alasty, Adaptive chaos synchronization in Chua's systems with noisy parameters, Mathematics and Computers in Simulation, vol. 79, pp. 233-241 (2008).
97. H. Salarieh and A. Alasty, Adaptive synchronization of two chaotic systems with stochastic unknown parameters, Communications in Nonlinear Science and Numerical Simulation, vol. 14, pp. 508-519 (2009).
98. S. Vaidyanathan and K. Rajagopal, Global chaos synchronization of Lü and Pan systems by adaptive nonlinear control, Communications in Computer and Information Science, vol. 205, pp. 193-202 (2011).
99. V. Sundarapandian and R. Karthikeyan, Anti-synchronization of Lü and Pan chaotic systems by adaptive nonlinear control, European Journal of Scientific Research, vol. 64, pp. 94-106 (2011).
100. V. Sundarapandian and R. Karthikeyan, Anti-synchronization of hyperchaotic Lorenz and hyperchaotic Chen systems by adaptive control, International Journal of Systems Signal Control and Engineering Application, vol. 4, pp. 18-25 (2011).
101. V. Sundarapandian and R. Karthikeyan, Adaptive anti-synchronization of uncertain Tigan and Li systems, Journal of Engineering and Applied Sciences, vol. 7, pp. 45-52 (2012).
102. P. Sarasu and V. Sundarapandian, Generalized projective synchronization of three-scroll chaotic systems via adaptive control, European Journal of Scientific Research, vol. 72, pp. 504-522 (2012).
103. P. Sarasu and V. Sundarapandian, Generalized projective synchronization of two-scroll systems via adaptive control, International Journal of Soft Computing, vol. 7, pp. 146-156 (2012).
104. P. Sarasu and V. Sundarapandian, Adaptive controller design for the generalized projective synchronization of 4-scroll systems, International Journal of Systems Signal Control and Engineering Application, vol. 5, pp. 21-30 (2012).
105. S. Vaidyanathan and K. Rajagopal, Global chaos synchronization of hyperchaotic Pang and hyperchaotic Wang systems via adaptive control, International Journal of Soft Computing, vol. 7, pp. 28-37 (2012).
106. S.H. Lee, V. Kapila, M. Porfiri, and A. Panda, Master-slave synchronization of continuously and intermittently coupled sampled-data chaotic oscillators, Communications in Nonlinear Science and Numerical Simulation, vol. 15, pp. 4100-4113 (2010).
107. X.Z. Jin and J.H. Park, Adaptive synchronization for a class of faulty and sampling coupled networks with its circuit implement, Journal of the Franklin Institute, vol. 351, pp. 4317-4333 (2014).
108. C.K. Zhang, L. Jiang, Y. He, Q.H. Wu and M. Wu, Asymptotical synchronization for chaotic Lur'e systems using sampled-data control, Communications in Nonlinear Science and Numerical Simulation, vol. 18, pp. 2743-2751 (2013).
109. X. Xiao, L. Zhou, and Z. Zhang, Synchronization of chaotic Lur'e systems with quantized sampled-data controller, Communications in Nonlinear Science and Numerical Simulation, vol. 19, pp. 2039-2047 (2014).
110. S. Rasappan and S. Vaidyanathan, Global chaos synchronization of WINDMI and Coullet chaotic systems by backstepping control, Far East Journal of Mathematical Sciences, vol. 67, pp. 265-287 (2012).
111. S. Rasappan and S. Vaidyanathan, Synchronization of hyperchaotic Liu system via backstepping control with recursive feedback, Communications in Computer and Information Science, vol. 305, pp. 212-221 (2012).
112. S. Rasappan and S. Vaidyanathan, Hybrid synchronization of n-scroll Chua and Lur'e chaotic systems via backstepping control with novel feedback, Archives of Control Sciences, vol. 22, pp. 343-365 (2012).
113. R. Suresh and V. Sundarapandian, Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with recursive feedback, Far East Journal of Mathematical Sciences, vol. 73, pp. 73-95 (2013).
114. S. Rasappan and S. Vaidyanathan, Hybrid synchronization of n-scroll chaotic Chua circuits using adaptive backstepping control design with recursive feedback, Malaysian Journal of Mathematical Sciences, vol. 7, pp. 219-246 (2013).
115. S. Vaidyanathan and S. Rasappan, Global chaos synchronization of n-scroll Chua circuit and Lur'e system using backstepping control design with recursive feedback, Arabian Journal for Science and Engineering, vol. 39, pp. 3351-3364 (2014).
116. S. Rasappan and S. Vaidyanathan, Global chaos synchronization of WINDMI and Coullet chaotic systems using adaptive backstepping control design, Kyungpook Mathematical Journal, vol. 54, pp. 293-320 (2014).
117. H.T. Yau, Chaos synchronization of two uncertain chaotic nonlinear gyros using fuzzy sliding mode control, Mechanical Systems and Signal Processing, vol. 22, pp. 408-418 (2008).
118. S. Vaidyanathan and S. Sampath, Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control, Communications in Computer and Information Science, vol. 205, pp. 156-164 (2011).
119. V. Sundarapandian and S. Sivaperumal, Sliding controller design of hybrid synchronization of four-wing chaotic systems, International Journal of Soft Computing, vol. 6, pp. 224-231 (2011).

120. S. Vaidyanathan and S. Sampath, Anti-synchronization of four-wing chaotic systems via sliding mode control, International Journal of Automation and Computing, vol. 9, pp. 274-279 (2012).
121. S. Vaidyanathan, Global chaos synchronization of identical Li-Wu chaotic systems via sliding mode control, Int. J. Modelling, Identification and Control, vol. 22, pp. 170-177, (2014).
122. J.C. Sprott, Some simple chaotic jerk functions, American Journal of Physics, vol. 65, pp. 537-543 (1997).
123. G. Cai and W. Tu, Adaptive backstepping control of the uncertain unified chaotic system, International Journal of Nonlinear Science, vol. 4, pp. 17-24 (2007).
124. M.T. Yassen, Controlling, synchronization and tracking chaotic Liu system using active backstepping design, Physics Letters A, vol. 360, pp. 582-587 (2007).
125. J.C. Sprott, A new class of chaotic circuit, Physics Letters A, vol. 266, pp. 19-23 (2000).
126. H.K. Khalil, Nonlinear System, 3rd ed., Prentice Hall, New Jersey, USA (2002)