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Research Article

# A 3-D Novel Highly Chaotic System with Four Quadratic Nonlinearities, its Adaptive Control and Anti-Synchronization with Unknown Parameters

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***Abstract***

This research work proposes a seven-term 3-D novel dissipative chaotic system with four quadratic nonlinearities. The Lyapunov exponents of the 3-D novel chaotic system are obtained as $L_1 = 11.36204$, $L_2 = 0$ and $L_3 = -47.80208$. Since the sum of the Lyapunov exponents is negative, the 3-D novel chaotic system is dissipative. Also, the Kaplan-Yorke dimension of the 3-D novel chaotic system is obtained as $D_{KY} = 2.23769$. The maximal Lyapunov exponent (MLE) of the novel chaotic system is $L_1 = 11.36204$, which is a large value for a polynomial chaotic system. Thus, the proposed 3-D novel chaotic system is highly chaotic. The phase portraits of the novel chaotic system simulated using MATLAB depict the highly chaotic attractor of the novel system. This research work also discusses other qualitative properties of the system. Next, an adaptive controller is designed to stabilize the 3-D novel chaotic system with unknown parameters. Also, an adaptive synchronizer is designed to achieve anti-synchronization of the identical 3-D novel chaotic systems with unknown parameters. The adaptive results derived in this work are established using Lyapunov stability theory. MATLAB simulations have been shown to illustrate and validate all the main results derived in this work.

*Keywords:* Chaos, chaotic systems, dissipative systems, adaptive control, anti-synchronization.

## 1. Introduction

Chaotic systems are defined as nonlinear dynamical systems which are very sensitive to initial conditions, topologically mixing and also with dense periodic orbits [1].

The sensitivity to initial conditions of a chaotic system is indicated by a positive Lyapunov exponent. A dissipative chaotic system is characterized by the condition that the sum of the Lyapunov exponents of the chaotic system is negative.

Since Lorenz discovered a 3-D chaotic system of a weather model [2], great interest has been shown in the chaos literature in the analysis and modelling of many 3-D chaotic systems such as Rössler system [3], Rabinovich system [4], ACT system [5], Sprott systems [6], Chen system [7], Lü system [8], Shaw system [9], Feeny system [10], Shimizu system [11], Liu-Chen system [12], Cai system [13], Tigan system [14], Colpitt's oscillator [15], WINDMI system [16], Zhou system [17], etc.

Recently, many 3-D chaotic systems have been discovered such as Li system [18], Elhadj system [19], Pan system [20], Sundarapandian system [21], Yu-Wang system [22], Sundarapandian-Pehlivan system [23], Zhu system [24], Vaidyanathan systems [25-30], Vaidyanathan-Madhavan system [31], Pehlivan-Moroz-Vaidyanathan system [32], Jafari system [33], Pham system [34], etc.

We note that the chaotic systems [2-34] are dissipative systems, in which the system limit sets are ultimately confined into a specific limit set of zero volume and the asymptotic motion of the chaotic system settles onto a strange attractor of the system.

Chaos theory has many important applications in science and engineering such as vibration control [35-37], oscillators [38-40], lasers [41-43], robotics [44-47], chemical reactors [48-50], biology [51,52], ecology [53,54], cardiology [55], memristors [56-59], neural networks [60-62], secure communications [63-66], cryptosystems [67-70], network design [71, 72], economics [73-76], market forecasting [77], etc.

Chaos control and chaos synchronization are important research problems in the chaos theory. In the last three decades, many mathematical methods have been developed successfully to address these research problems.

The study of control of a chaotic system investigates methods for designing feedback control laws that globally or locally asymptotically stabilize or regulate the outputs of a chaotic system.

Many methods have been developed for the control and tracking of chaotic systems such as active control [78-82],

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adaptive control [83-89], backstepping control [90-92], sliding mode control [93, 94], etc.

Chaos synchronization problem deals with the synchronization of a couple of systems called the *master* or *drive* system and the *slave* or *response* system. To solve this problem, control laws are designed so that the output of the slave system tracks the output of the master system asymptotically with time.

In the chaos anti-synchronization problem, control laws are designed so that the sum of the outputs of the master and slave systems is driven to zero asymptotically, i.e. the outputs of the two systems are asymptotically equal in magnitude but opposite in phase.

Because of the butterfly effect, both synchronization and anti-synchronization of chaotic systems are challenging problems even when the initial conditions of the master and slave systems are nearly identical because of the exponential divergence of the outputs of the two systems in the absence of any control. The synchronization of chaotic systems has applications in secure communications [95-97], cryptosystems [98, 99], encryption [100-104], etc.

In the chaos literature, many different methodologies have been also proposed for the synchronization and anti-synchronization of chaotic systems such as PC method [103], active control [104-114], time-delayed feedback control [115,116], adaptive control [117-126], sampled-data feedback control [127-130], backstepping control [131-137], sliding mode control [138-143], etc.

In this research work, a seven-term 3-D novel dissipative chaotic system with four quadratic nonlinearities is proposed. The Lyapunov exponents of the 3-D novel chaotic system are found as $L_1$ = 11.36204, $L_2$ = 0 and $L_3$ = –47.80208. The Kaplan-Yorke of the 3-D novel chaotic system is found as $D_{KY}$ = 2.23769. Since the maximal Lyapunov exponent (MLE) of the novel chaotic system has a large value, viz. $L_1$ = 11.36204, it is noted that the 3-D novel chaotic system is highly chaotic.

In Section 2, we describe the equations and phase portraits of the novel chaotic system. In Section 3, we derive the qualitative properties of the novel chaotic system. In Section 4, we derive an adaptive controller for the stabilization of 3-D novel chaotic system with unknown parameters. In Section 5, we derive an adaptive synchronizer for the anti-synchronization of identical 3-D novel chaotic systems with unknown parameters. Section 6 concludes this research work with a summary of main results.

## 2. A Seven-Term 3-D Novel Chaotic System with Four Quadratic Nonlinearities

The dynamics of the seven-term 3-D novel chaotic system is described by

$$\begin{cases} \dfrac{dx_1}{dt} = a(x_2 - x_1) + x_2 x_3 \\ \dfrac{dx_2}{dt} = -c x_1 x_3 + p x_2^2 \\ \dfrac{dx_3}{dt} = -b + x_1 x_2 \end{cases} \tag{1}$$

where $x_1$, $x_2$, $x_3$ are the states and $a$, $b$, $c$, $p$ are positive parameters.

The nonlinear system (1) depicts a chaotic attractor when the parameter values are taken as:

$$a = 40,\ b = 26,\ c = 160,\ p = 0.3 \tag{2}$$

We take the initial conditions as:

$$x_1(0) = 0.8,\ x_2(0) = 0.5,\ x_3(0) = 0.7 \tag{3}$$

The 3-D portrait of the strange chaotic attractor (1) for the parameter values (2) and the initial conditions (3) is depicted in Fig. 1, and the 2-D portraits (projections on the three coordinate planes) are depicted in Figs. 2-4.

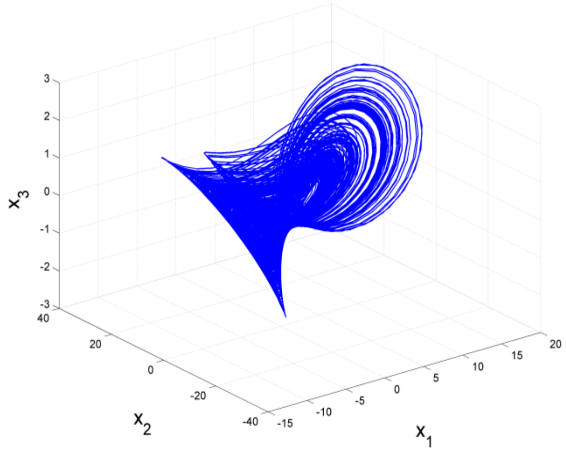


**Fig. 1.** The chaotic attractor of the novel chaotic system in $R^3$.

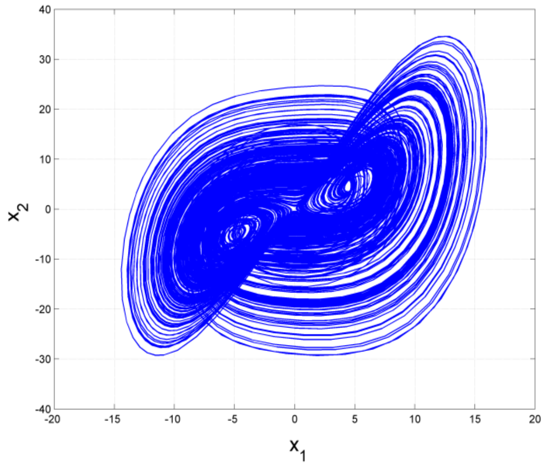


**Fig. 2.** The 2-D projection of the chaotic attractor on the ($x_1$, $x_2$) plane.

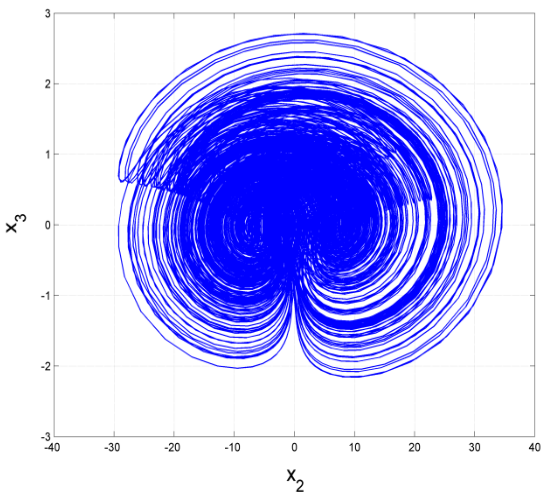


**Fig. 3.** The 2-D projection of the chaotic attractor on the ($x_2$, $x_3$) plane.

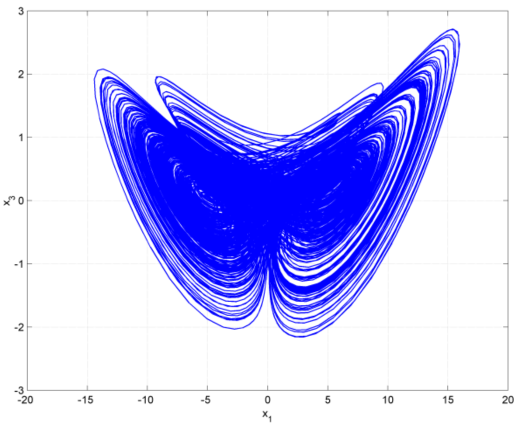


**Fig. 4.** The 2-D projection of the chaotic attractor on the $(x_1, x_3)$ plane.

## 3. Analysis of the 3-D Novel Chaotic System

In this section, qualitative properties of the 3-D novel chaotic system are detailed.

### 3.1. Dissipativity

In vector notation, we may express the system (1) as:

$$\frac{dx}{dt} = f(\boldsymbol{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{4}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) & = & a(x_2 - x_1) + x_2 x_3 \\ f_2(x_1, x_2, x_3) & = & -c x_1 x_3 + p x_2^2 \\ f_3(x_1, x_2, x_3) & = & -b + x_1 x_2 \end{cases} \tag{5}$$

We take the parameter values as in the chaotic case (2).

Let $\Omega$ be any region in $\boldsymbol{R}^3$ with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where $\Phi_t$ is the flow of $f$.

Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\frac{dV}{dt} = \int_{\Omega(t)} (\nabla \cdot f) dx_1 dx_2 dx_3 \tag{6}$$

The divergence of the novel chaotic system (1) is easily found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a < 0 \tag{7}$$

Substituting (7) into (6), we obtain the first order ODE.

$$\frac{dV}{dt} = \int_{\Omega(t)} (-a) dx_1 dx_2 dx_3 = -aV \tag{8}$$

Integrating (8), we obtain the unique solution as:

$$V(t) = \exp(-at) V(0) \qquad \text{for all } t \geq 0 \tag{9}$$

From (9), we find that $V(t)$ shrinks to zero exponentially as $t \to \infty$.

Hence, the 3-D system (1) is dissipative and the asymptotic motion of the 3-D system (1) settles exponentially onto a set of measure zero, i.e. a strange attractor.

### 3.2. Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the following system of equations with the parameter values as in the chaotic case (2):

$$\begin{cases} a(x_2 - x_1) + x_2 x_3 & = & 0 \\ -c x_1 x_3 + p x_2^2 & = & 0 \\ -b + x_1 x_2 & = & 0 \end{cases} \tag{10}$$

Solving (10), we obtain two equilibrium points of the system (1), viz.

$$E_1 = \begin{bmatrix} 5.0996 \\ 5.0984 \\ 0.0096 \end{bmatrix}, \ E_2 = \begin{bmatrix} -5.0984 \\ -5.0996 \\ -0.0096 \end{bmatrix} \tag{11}$$

The Jacobian matrix of the system (1) is obtained as

$$J(x) = \begin{bmatrix} -a & a + x_3 & x_2 \\ -c x_3 & 2p x_2 & -c x_1 \\ x_2 & x_1 & 0 \end{bmatrix} \tag{12}$$

Thus, the Jacobian matrix at $E_1$ is obtained as:

$$J(E_1) = \begin{bmatrix} -40.0000 & 40.0096 & 5.0984 \\ -1.5360 & 3.0590 & -815.9360 \\ 5.0984 & 5.0996 & 0 \end{bmatrix} \tag{13}$$

which has the eigenvalues

$$\lambda_1 = -60.5268, \ \lambda_{2,3} = 11.7929 \pm 73.2294i \tag{14}$$

This shows that the equilibrium $E_1$ is a saddle-focus.

Also, the Jacobian matrix at $E_2$ is obtained as:

$$J(E_2) = \begin{bmatrix} -40.0000 & 39.9904 & -5.0996 \\ 1.5360 & -3.0598 & 815.7440 \\ -5.0996 & -5.0984 & 0 \end{bmatrix} \tag{15}$$

which has the eigenvalues

$$\lambda_1 = -61.9776, \ \lambda_{2,3} = 9.4589 \pm 72.6427i \tag{16}$$

This shows that the equilibrium $E_2$ is also a saddle-focus.

Hence, both equilibrium points $E_1$ and $E_2$ are unstable.

### 3.4. Lyapunov Exponents and Kaplan-Yorke Dimension

For the chosen parameter values (2), the Lyapunov exponents of the novel chaotic system (1) are obtained using MATLAB as:

$$L_1 = 11.36204, L_2 = 0, L_3 = -47.80208 \tag{17}$$

Since the spectrum of Lyapunov exponents (17) has a positive term $L_1$, the system (1) is chaotic.

Since the sum of the Lyapunov exponents is zero, the novel chaotic system (1) is dissipative.

The maximal Lyapunov exponent (MLE) of the novel chaotic system (1) is $L_1$ =11.36204, which is a large value.

This shows that the 3-D novel system (1) is a highly chaotic system.

Also, the Kaplan-Yorke dimension of the novel chaotic system (1) is calculated as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.23769 \tag{18}$$

Fig. 5 depicts the dynamics of the Lyapunov exponents of the novel chaotic system (1).

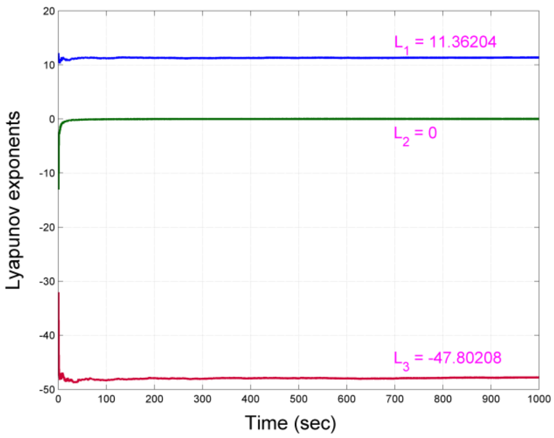


**Fig. 5.** Dynamics of the Lyapunov exponents of the novel system.

## 4. Adaptive Control of the 3-D Novel Chaotic System

In this section, we construct an adaptive controller for globally stabilizing the unstable 3-D novel chaotic system with unknown parameters. The adaptive controller design is carried out using Lyapunov stability theory.

We consider the controlled chaotic system

$$\begin{cases} \frac{dx_1}{dt} &= a(x_2 - x_1) + x_2 x_3 + u_1 \\ \frac{dx_2}{dt} &= -c x_1 x_3 + p x_2^2 + u_2 \\ \frac{dx_3}{dt} &= -b + x_1 x_2 + u_3 \end{cases} \tag{19}$$

where $x_1, x_2, x_3$ are state variables and $a, b, c, p$ are unknown, constant, parameters and $u_1, u_2, u_3$ are adaptive controls to be designed using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ of the unknown parameters $a, b, c, p$, respectively.

We consider the adaptive controller defined by

$$\begin{cases} u_1 &= -\hat{a}(t)(x_2 - x_1) - x_2 x_3 - k_1 x_1 \\ u_2 &= \hat{c}(t) x_1 x_3 - \hat{p}(t) x_2^2 - k_2 x_2 \\ u_3 &= \hat{b}(t) - x_1 x_2 - k_3 x_3 \end{cases} \tag{20}$$

where $k_1, k_2, k_3$ are positive gain constants, and $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ are estimates of the unknown parameters $a, b, c, p$, respectively.

Substituting (20) into (19), we get the closed-loop system as:

$$\begin{cases} \frac{dx_1}{dt} &= \left(a - \hat{a}(t)\right)(x_2 - x_1) - k_1 x_1 \\ \frac{dx_2}{dt} &= -\left(c - \hat{c}(t)\right) x_1 x_3 + \left(p - \hat{p}(t)\right) x_2^2 \\ & \quad -k_2 x_2 \\ \frac{dx_3}{dt} &= -\left(b - \hat{b}(t)\right) - k_3 x_3 \end{cases} \tag{21}$$

The parameter estimation errors are defined by

$$\begin{cases} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \\ e_c(t) &= c - \hat{c}(t) \\ e_p(t) &= p - \hat{p}(t) \end{cases} \tag{22}$$

Substituting (22) into the state dynamics (21), we get

$$\begin{cases} \frac{dx_1}{dt} &= e_a (x_2 - x_1) - k_1 x_1 \\ \frac{dx_2}{dt} &= -e_c x_1 x_3 + e_p x_2^2 - k_2 x_2 \\ \frac{dx_3}{dt} &= -e_b - k_3 x_3 \end{cases} \tag{23}$$

Differentiating (22) with respect to $t$, we get

$$\begin{cases} \frac{de_a}{dt} &= -\frac{d\hat{a}}{dt} \\ \frac{de_b}{dt} &= -\frac{d\hat{b}}{dt} \\ \frac{de_c}{dt} &= -\frac{d\hat{c}}{dt} \\ \frac{de_p}{dt} &= -\frac{d\hat{p}}{dt} \end{cases} \tag{24}$$

Next, we use Lyapunov stability theory for finding an update law for the parameter estimates.

Consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right), \tag{25}$$

which is positive definite on $R^7$.

Differentiating $V$ along the trajectories of (23) and (24), we get

$$\begin{aligned} \frac{dV}{dt} &= -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 \\ &\quad + e_a \left[x_1(x_2 - x_1) - \frac{d\hat{a}}{dt}\right] \\ &\quad + e_b \left[-x_3 - \frac{d\hat{b}}{dt}\right] + e_c \left[-x_1 x_2 x_3 - \frac{d\hat{c}}{dt}\right] \\ &\quad + e_p \left[x_2^3 - \frac{d\hat{p}}{dt}\right] \end{aligned} \tag{26}$$

In view of (26), we take the parameter update law as:

$$\begin{cases} \frac{d\hat{a}}{dt} &= x_1(x_2 - x_1) \\ \frac{d\hat{b}}{dt} &= -x_3 \\ \frac{d\hat{c}}{dt} &= -x_1 x_2 x_3 \\ \frac{d\hat{p}}{dt} &= x_2^3 \end{cases} \tag{27}$$

Next, we state and prove the main result of this section.

**Theorem 1.** *The novel chaotic system (19) is globally and exponentially stabilized by the adaptive control law (20) and the parameter update law (27) for all initial conditions, where $k_1$, $k_2$, $k_3$ are positive constants.*

***Proof.*** We prove this result using Lyapunov stability theory.

For this purpose, we consider the quadratic Lyapunov function $V$ defined by (25), which is positive definite on $R^7$.

Substituting the parameter update law (27) into (26), we obtain the time derivative of $V$ as:

$$\frac{dV}{dt} = -k_1x_1^2 - k_2x_2^2 - k_3x_3^2 \quad (28)$$

which is a negative semi-definite function on $R^7$.

Thus, we can conclude that the state vector $x(t)$ and the parameter estimation error are globally bounded.

We define $k = \min\{k_1, k_2, k_3\}$. Then we get

$$\frac{dV}{dt} \le -k\|x\|^2 \ \text{ or } \ k\|x\|^2 \le -\frac{dV}{dt} \quad (29)$$

Integrating the inequality (29) from 0 to $t$, we get

$$k\int_0^t \|x(\tau)\|^2 d\tau \le V(0) - V(t) \quad (30)$$

From (30), it follows that $x(t) \in L_2$. Using (23), we can conclude that $\dot{x} \in L_\infty$.

Thus, using Barbalat's lemma [145], we conclude that $x(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in R^3$.

This completes the proof. ■

For numerical simulations, the parameter values of the novel chaotic system (19) are taken as in the chaotic case, *viz.* $a = 40$, $b = 26$, $c = 160$ and $p = 0.3$. We take the gain constants as $k_1 = 6$, $k_2 = 6$ and $k_3 = 30$.

The initial conditions of the chaotic system (19) are taken as $x_1(0) = 1.3$, $x_2(0) = 2.7$ and $x_3(0) = -3.5$.

The initial conditions of the parameter estimates are taken as $\hat{a}(0) = 21$, $\hat{b}(0) = 30$, $\hat{c}(0) = 25$ and $\hat{p}(0) = 3$.

Fig. 6 describes the time-history of the state $x(t)$ .

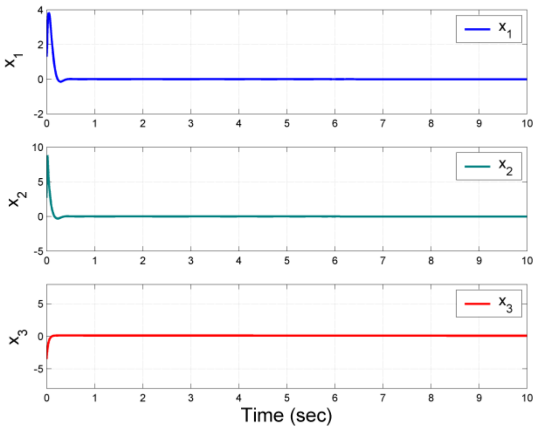


**Fig. 6.** Time-history of the controlled states $x_1$, $x_2$, $x_3$ of the chaotic system

## 5. Adaptive Anti-Synchronization of Identical 3-D Novel Chaotic Systems

In this section, we construct an adaptive synchronizer for global anti-synchronization of identical 3-D novel chaotic systems. The adaptive synchronizer design is carried out using Lyapunov stability theory.

As the master system, we take the novel chaotic system

$$\begin{cases} \frac{dx_1}{dt} = a(x_2 - x_1) + x_2x_3 \\ \frac{dx_2}{dt} = -cx_1x_3 + px_2^2 \\ \frac{dx_3}{dt} = -b + x_1x_2 \end{cases} \quad (31)$$

where $x_1$, $x_2$, $x_3$ are state variables and $a$, $b$, $c$, $p$ are unknown, constant, parameters.

As the slave system, we take the novel chaotic system

$$\begin{cases} \frac{dy_1}{dt} = a(y_2 - y_1) + y_2y_3 + u_1 \\ \frac{dy_2}{dt} = -cy_1y_3 + py_2^2 + u_2 \\ \frac{dy_3}{dt} = -b + y_1y_2 + u_3 \end{cases} \quad (32)$$

where $y_1$, $y_2$, $y_3$ are state variables and $u_1$, $u_2$, $u_3$ are adaptive controls to be designed using estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ of the unknown parameters $a$, $b$, $c$, $p$, respectively.

The anti-synchronization error between the systems (31) and (32) is defined as:

$$\begin{cases} e_1 = y_1 + x_1 \\ e_2 = y_2 + x_2 \\ e_3 = y_3 + x_3 \end{cases} \quad (33)$$

The error dynamics is easily obtained as:

$$\begin{cases} \frac{de_1}{dt} = a(e_2 - e_1) + y_2y_3 + x_2x_3 + u_1 \\ \frac{de_2}{dt} = -c(y_1y_3 + x_1x_3) + p(y_2^2 + x_2^2) + u_2 \\ \frac{de_3}{dt} = -2b + y_1y_2 + x_1x_2 + u_3 \end{cases} \quad (34)$$

We consider the adaptive controller defined by

$$\begin{cases} u_1 = -\hat{a}(t)(e_2 - e_1) - y_2y_3 - x_2x_3 - k_1e_1 \\ u_2 = \hat{c}(t)(y_1y_3 + x_1x_3) - \hat{p}(t)(y_2^2 + x_2^2) \\ \qquad -k_2e_2 \\ u_3 = 2\hat{b}(t) - y_1y_2 - x_1x_2 - k_3e_3 \end{cases} \quad (35)$$

where $k_1$, $k_2$, $k_3$ are positive gain constants and $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ are estimates of the unknown parameters $a$, $b$, $c$, $p$, respectively.

Substituting (35) into (34), we get the closed-loop error dynamics as:

$$\begin{cases} \frac{de_1}{dt} = (a - \hat{a}(t))(e_2 - e_1) - k_1e_1 \\ \frac{de_2}{dt} = -(c - \hat{c}(t))(y_1y_3 + x_1x_3) \\ \qquad +(p - \hat{p}(t))(y_2^2 + x_2^2) - k_2e_2 \\ \frac{de_3}{dt} = -2\left(b - \hat{b}(t)\right) - k_3e_3 \end{cases} \quad (36)$$

The parameter estimation errors are defined by

$$\begin{cases} e_a(t) & = & a - \hat{a}(t) \\ e_b(t) & = & b - \hat{b}(t) \\ e_c(t) & = & c - \hat{c}(t) \\ e_p(t) & = & p - \hat{p}(t) \end{cases} \qquad (37)$$

Substituting (37) into the error dynamics (36), we get

$$\begin{cases} \frac{de_1}{dt} & = & e_a(e_2 - e_1) - k_1 e_1 \\ \frac{de_2}{dt} & = & -e_c(y_1 y_3 + x_1 x_3) + e_p(y_2^2 + x_2^2) - k_2 e_2 \\ \frac{de_3}{dt} & = & -2e_b - k_3 e_3 \end{cases} \qquad (38)$$

Differentiating (37) with respect to $t$, we get

$$\begin{cases} \frac{de_a}{dt} & = & -\frac{d\hat{a}}{dt} \\ \frac{de_b}{dt} & = & -\frac{d\hat{b}}{dt} \\ \frac{de_c}{dt} & = & -\frac{d\hat{c}}{dt} \\ \frac{de_p}{dt} & = & -\frac{d\hat{p}}{dt} \end{cases} \qquad (39)$$

Consider the quadratic Lyapunov function defined by

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_p^2\right), \qquad (40)$$

which is positive definite on $R^7$.

Differentiating $V$ along the trajectories of (38) and (39), we get

$$\begin{aligned} \frac{dV}{dt} = & -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \\ & + e_a\left[e_1(e_2 - e_1) - \frac{d\hat{a}}{dt}\right] \\ & + e_b\left[-2e_3 - \frac{d\hat{b}}{dt}\right] \\ & + e_c\left[-e_2(y_1 y_3 + x_1 x_3) - \frac{d\hat{c}}{dt}\right] \\ & + e_p\left[e_2(y_2^2 + x_2^2) - \frac{d\hat{p}}{dt}\right] \end{aligned} \qquad (41)$$

In view of (41), we take the parameter update law as:

$$\begin{cases} \frac{d\hat{a}}{dt} & = & e_1(e_2 - e_1) \\ \frac{d\hat{b}}{dt} & = & -2e_3 \\ \frac{d\hat{c}}{dt} & = & -e_2(y_1 y_3 + x_1 x_3) \\ \frac{d\hat{p}}{dt} & = & e_2(y_2^2 + x_2^2) \end{cases} \qquad (42)$$

**Theorem 2.** *The novel chaotic systems (31) and (32) are globally and exponentially anti-synchronized by the adaptive control law (35) and the parameter update law (42) for all initial conditions, where $k_1$, $k_2$, $k_3$ are positive constants.*

***Proof.*** We prove this result using Lyapunov stability theory. For this purpose, we consider the quadratic Lyapunov function $V$ defined by (40), which is positive definite on $R^7$. Substituting the parameter update law (42) into (41), we obtain the time derivative of $V$ as:

$$\frac{dV}{dt} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \qquad (43)$$

Since $\frac{dV}{dt}$ is a negative semi-definite function on $R^7$, we can conclude that the anti-synchronization vector $e(t)$ and the parameter estimation error are globally bounded.

We define $k = \min\{k_1, k_2, k_3\}$. Then we get

$$\frac{dV}{dt} \le -k\|e\|^2 \quad \text{or} \quad k\|e\|^2 \le -\frac{dV}{dt} \qquad (44)$$

Integrating the inequality (44) from 0 to $t$, we get

$$k\int_0^t \|e(\tau)\|^2 d\tau \le V(0) - V(t) \qquad (45)$$

From (45), it follows that $e(t) \in L_2$. Using (38), we can conclude that $\dot{e} \in L_\infty$.

Thus, using Barbalat's lemma [145], we conclude that $e(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $e(0) \in R^3$.

This completes the proof. ■

For numerical simulations, the parameter values of the novel chaotic systems are taken as in the chaotic case, *viz.* $a = 40$, $b = 26$, $c = 160$ and $p = 0.3$. We take the gain constants as as $k_1 = 20$, $k_2 = 20$ and $k_3 = 30$.

The initial conditions of the master system (31) are taken as $x_1(0) = 0.4$, $x_2(0) = 2.3$ and $x_3(0) = -0.5$

The initial conditions of the slave system (32) are taken as $y_1(0) = 1.7$, $y_2(0) = 1.2$ and $y_3(0) = -2.8$.

The initial conditions of the parameter estimates are taken as $\hat{a}(0) = 15$, $\hat{b}(0) = 22$, $\hat{c}(0) = 11$ and $\hat{p}(0) = 4$.

Figure 7 describes the time-history of the anti-synchronization error $e(t)$.

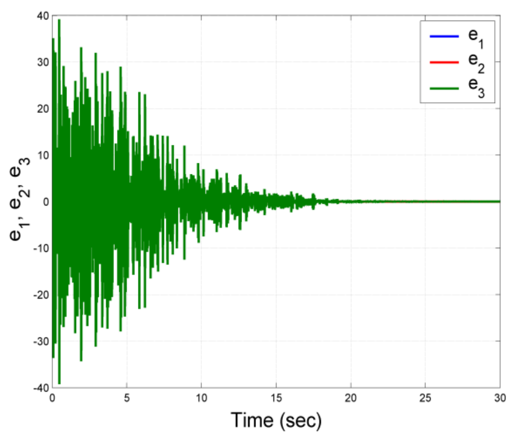


**Fig. 7.** Time-history of the anti-synchronization error.

## 6. Conclusion

In this research work, we have proposed a seven-term 3-D novel chaotic system with four quadratic nonlinearities. The Lyapunov exponents of the 3-D novel chaotic system have been found as $L_1 = 11.36204$, $L_2 = 0$ and $L_3 = -47.80208$. The Kaplan-Yorke of the 3-D novel chaotic system has been found as $D_{KY} = 2.23769$. Since the maximal Lyapunov exponent (MLE) of the novel chaotic system has a large value, viz. $L_1 = 11.36204$, the 3-D novel chaotic system is a highly chaotic system and it has potential applications in encryption and secure communication systems. We have also designed control laws for adaptive stabilization and adaptive anti-synchronization of the 3-D novel chaotic system with unknown system parameters. The main adaptive results derived in this work were established using Lyapunov stability theory. MATLAB simulations have been shown to illustrate the phase portraits of the highly chaotic system and also the adaptive stabilization and anti-synchronization results derived in this work.

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