PAPER

Performance Evaluation of OFDM Amplify-and-Forward Relay System with Subcarrier Permutation

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SUMMARY We perform error probability analysis of the uncoded OFDM fixed gain Amplify-and-Forward (AF) relaying system with subcarrier permutation (SCP). Two SCP schemes, named: the best-to-best SCP (BTB SCP) and the best-to-worst SCP (BTW SCP) are considered. Closed-form expressions for the bit error rate (BER) of the above SCP methods are derived. Numerical results manifest that these SCP schemes may outperform one another, depending on the average channel conditions of the links involved. That is, BTB SCP is better at low signal-to-noise ratio (SNR) values, while BTW SCP prevails in the medium and high SNR regime. Thus, it could be concluded that OFDM AF relaying systems may switch from the BTB SCP to BTW SCP in order to achieve optimum BER performance. Moreover, using the derived end-to-end SNR probability density functions (PDF), tight upper bounds for the ergodic capacities of both SCP schemes are obtained.

key words: *amplify-and-forward, OFDM, relay, subcarrier permutation*

1. Introduction

Relaying systems have attracted a great research interest in recent years, due to their ability to increase coverage area and capacity of wireless systems. The basic realization assumes dual-hop relaying system, where a source terminal communicates with a destination one through a relay terminal. The relay terminal usually performs one of the two main relaying methods: Amplify-and-Forward (AF) or Decode-and-Forward (DF). In the AF relaying mode, the relay amplifies the received signal by employing fixed gain or variable gain, depending on its capability to estimate the source-relay channel. The DF mode assumes that the relay terminal performs decoding of the received signal, and re-encoding before further transmission.

The performance of single carrier relaying systems in different channel conditions, as well as for different relaying strategies, are widely examined in the literature [1]–[3]. Lately, multicarrier relaying systems using Orthogonal Frequency Division Multiplexing (OFDM) as transmission technique have attracted extensive attention. Due to a large number of proven good characterstics, OFDM is already accepted in many standardized wireless communication systems, and it is a candidate for the next generation of WLAN and WWAN systems [4]. Implemented as a transmission technique in relay systems, OFDM brings about additional freedom of making decisions on a subcarrier basis at the relay station, according to the channel conditions on the source-relay and relay-destination links. In this paper we focus on AF relaying mode, as recent research has shown that, in OFDM based AF relay system, the BER performance improvement may be achieved if the appropriate subcarrier permutation (SCP) scheme is performed at the relay station, depending on the average signal-to noise ratio (SNR) on the source-to-relay and relay-to-destination links [5].

The idea of subcarrier permutation in OFDM AF relaying was first introduced in [6], and a little bit later was independently discussed in [7] and [8]. The author of [8] proves that, in special cases, when the signal received by a relay is noise-free, the system achieves maximum capacity if the subcarrier with the highest SNR from the first hop is mapped to the subcarrier with the highest SNR on the second hop, second best - to second best, etc. As the subcarrier-based permutation significantly increase necessary signaling overhead, the author in [8] proposed to group adjacent subcarriers in chunks, and then the relay should perform chunk permutation according to the average chunk's SNRs. General proof that this kind of SCP maximizes received SNR and achievable capacity in OFDM AF relaying systems was first presented in [9]. The capacity analysis, for the case of fixed gain AF relaying, is performed in [10] numerically, using the derived SNR probability density function (PDF). However, when the BER performance is taken into account, it is proven in [5] that the described SCP presents the best solution for OFDM AF relay system only in the low SNR region. By employing majorization theory the authors in [5] proved that the BER performance of the dual-hop OFDM variable gain AF relay system in the medium and high SNR regions can be improved by using the opposite SCP scheme, where the subcarrier with the highest SNR from the first hop is mapped to the subcarrier with the lowest SNR on the second hop, etc. Nonetheless, no analytical BER performance analysis for OFDM relay system employing SCP has been reported to date.

In this paper, the BER performance of the best-to-worst (BTW) and the best-to-best (BTB) SCP schemes, for the uncoded dual-hop OFDM AF relaying system, is inves-

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tigated. Assuming that semi-blind fixed gain relaying is employed, closed form BER expressions are obtained for DPSK-modulated OFDM AF relaying in both BTW SCP and BTB SCP scenarios, while for the case of BPSK modulation tight approximations of the exact BERs for both SCP schemes are derived. It is worth mentioning that the same BER evaluation procedure used for BPSK modulation can also be used for any M-ary quadrature amplitude modulation (M-QAM). The analytical BER expressions are verified by computer simulations. Moreover, we derive and compare the upper bounds of the total ergodic capacities of these two SCP schemes, in order to have an insight into the potential trade off between the BER and capacity performance, when the relaying system switches from one SCP scheme to another.

2. System Model

We consider an OFDM dual-hop relaying system with a source terminal S, a half-duplex relay terminal R, and a destination one D, all equipped with a single antenna. The destination terminal is assumed to be out of reach of the source, thus, all the communication is realized through R, which is placed between S and D. The relay terminal has FFT (Fast Fourier Transformation) and IFFT (Inverse Fast Fourier Transformation) blocks for OFDM demodulation and OFDM modulation, respectively. Furthermore, R has a block that performs subcarrier permutation, mapping the subccarriers from the first hop to subcarriers on the second hop according to their transfer functions. The relay structure is illustrated in Fig. 1. The scenario with perfect time and frequency synchronization between all the communication terminals is considered. It is also assumed that R has ideal channel knowledge of both S-R and R-D links, and D knows the permutation function performed at R.

The post-FFT signal on the i-th subcarrier, received at the relay station, is given by

$$Y_{R,i} = H_{1,i}X_i + \mathcal{N}_{1,i}, \ \ 1 \leq i \leq N \tag{1}$$

where N is total number of subcarriers, $H_{1,i}$ is i-th subcarrier transfer function, and X_i is data symbol sent by source on the i-th subcarrier. $\mathcal{N}_{1,i}$ represents additive white Gaussian noise for the i-th subcarrier with variance $\mathbf{E}(|\mathcal{N}_{1,i}|^2) = \mathcal{N}_{0_1}$, where $\mathbf{E}(\cdot)$ denotes the expectation operator. The relay operates in

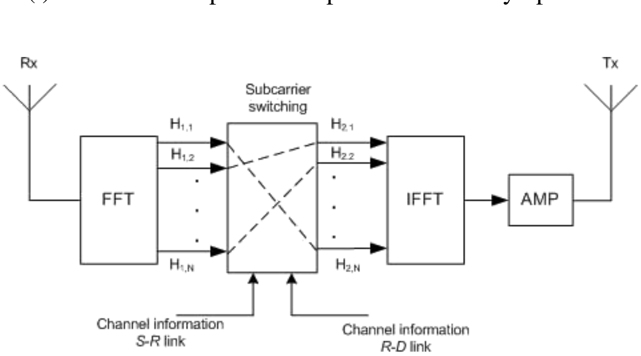

Fig. 1 Block scheme of the relay station.

the fixed-gain AF mode, where the signal that reaches the relay is amplified by a fixed gain, G. Assuming that the SCP function $\nu(i)$ at the relay station maps the i-th subcarrier from the first hop to the k-th subcarrier of the second hop, the signal at the destination can be presented in the frequency domain as

$$\begin{aligned} Y_{D,k} &= GH_{2,k}Y_{R,\nu(i)} + \mathcal{N}_{2,k} \\ &= GH_{2,k}H_{1,i}X_i + GH_{2,k}\mathcal{N}_{1,i} + \mathcal{N}_{2,k}, \end{aligned} \tag{2}$$

where $1 \leq k \leq N$ and $H_{2,k}$ denotes the k-th subcarrier transfer function on the second hop. $\mathcal{N}_{2,k}$ is the additive white Gaussian noise at the destination on the k-th subcarrier, with variance $\mathbf{E}(|\mathcal{N}_{2,k}|^2) = \mathcal{N}_{0_2}$.

Fadings in the S-R and R-D channels are assumed to be independent and identically distributed (i.i.d.) among the subcarriers. Moreover, we assume Rayleigh fading in each subcarrier, so that the PDF and the cumulative distribution function (CDF) of the SNR in each of the S-R subchannels is given by $f_{SR}(x) = \lambda_{SR}e^{-\lambda_{SR}x}$ and $F_{SR}(x) = 1 - e^{-\lambda_{SR}x}$, while the corresponding PDF and CDF of the SNR in each of the R-D subchannels are given by $f_{RD}(x) = \lambda_{RD}e^{-\lambda_{RD}x}$ and $F_{RD}(x) = 1 - e^{-\lambda_{RD}x}$, respectively. $\lambda_{SR} = 1/\overline{\gamma}_{SR}$ and $\lambda_{RD} = 1/\overline{\gamma}_{RD}$ denote the inverse of the average SNR on the S-R and R-D link, where $\overline{\gamma}_{SR} = \mathcal{E}_S\mathbf{E}\left(|H_{1,k}|^2\right)/\mathcal{N}_{0_1}$ and $\overline{\gamma}_{RD} = \mathcal{E}_R\mathbf{E}\left(|H_{2,k}|^2\right)/\mathcal{N}_{0_2}$, with $\mathcal{E}_S$ and $\mathcal{E}_R$ representing average symbol power per subcarrier transmitted by S and R, respectively. From (2) the end-to-end SNR on the k-th subcarrier can be written as [2]

$$\gamma_{k,end} = \frac{\frac{\mathcal{E}_S|H_{1,i}|^2}{\mathcal{N}_{0_1}}\frac{|H_{2,k}|^2}{\mathcal{N}_{0_2}}}{\frac{|H_{2,k}|^2}{\mathcal{N}_{0_2}} + \frac{1}{G^2\mathcal{N}_{0_1}}} = \frac{\gamma_{i,SR}\gamma_{k,RD}}{\gamma_{k,RD} + C} \tag{3}$$

where C is a constant that depends on the relay gain G through $C = \mathcal{E}_R/\left(G^2\mathcal{N}_{0_1}\right)$.

3. Evaluation of the End-to-End SNR PDF

3.1 Statistics of the k-th Subcarrier Permutation

It is obvious from (3) that for the evaluation of the end-to-end SNR density function in BTW SCP and BTB SCP scenarios, the ordered statistics of N i.i.d. random variables need to be known. Let $f^w_{k,SR}(\cdot)$ denotes the PDF of the SNR of the k-th weakest subcarrier out of the N total ones in the S-R link. Using the example for ordered statistics of random variables, given in [11], $f^w_{k,SR}(\cdot)$ can be presented as:

$$\begin{aligned} f^w_{k,SR}(x) &= N\binom{N-1}{k-1}f_{SR}(x) \\ &\quad \times (F_{SR}(x))^{k-1}(1 - F_{SR}(x))^{N-k} \end{aligned} \tag{4}$$

where $\binom{\cdot}{\cdot}$ represents the binomial coefficient. For the analyzed scenario, having the i.i.d. subcarriers with Rayleigh fading, $f^w_{k,SR}(x)$ can be written as

$$f^w_{k,SR}(x) = N\binom{N-1}{k-1}\lambda_{SR}e^{-\lambda_{SR}x}$$

$$\times\left(1-e^{-\lambda_{SR}x}\right)^{k-1}e^{-\lambda_{SR}x(N-k)} \quad (5)$$

After using the binomial expansion

$$\left(1-e^{-\lambda_{SR}x}\right)^{k-1}=\sum\nolimits_{i=0}^{k-1}(-1)^i\binom{k-1}{i}e^{-\lambda_{SR}xi} \quad (6)$$

(5) reduces to

$$f_{k,SR}^{w}(x)=\sum_{i=0}^{k-1}\lambda_{SR}\alpha_i e^{-\beta_i\lambda_{SR}x} \quad (7)$$

where α_i and β_i are given as

$$\alpha_i=(-1)^i N\binom{N-1}{k-1}\binom{k-1}{i} \quad (8)$$

$$\beta_i=i+N-k+1. \quad (9)$$

Using a similar approach as that given in [11], the PDF for decreasing ordered random variables can be derived. As a result, the PDF of the k-th strongest subcarrier out of the N total ones in the R-D link can be represented as

$$f_{k,RD}^{s}(x)=N\binom{N-1}{k-1}f_{RD}(x)\times\left(F_{RD}(x)^{N-k}\right)\left[1-F_{RD}(x)\right]^{k-1} \quad (10)$$

After substituting the PDF and CDF functions corresponding to Rayleigh fading channels, $f_{k,RD}^{s}(x)$ can be expressed as

$$f_{k,RD}^{s}(x)=\sum_{i=0}^{N-k}\lambda_{RD}\delta_i e^{-\varepsilon_i\lambda_{RD}x} \quad (11)$$

where δ_i and ε_i are given as

$$\delta_i=(-1)^i N\binom{N-1}{k-1}\binom{N-k}{i} \quad (12)$$

$$\varepsilon_i=i+k. \quad (13)$$

3.2 End-to-End SNR PDF for BTW SCP

In BTW SCP scheme, the order of the channel gains associated with the subcarriers employed in the S-R channel determines the order of the subcarriers used in the R-D channel, so that the signal reaching the relay via the strongest subcarrier in the S-R channel is forwarded to the destination through the weakest subcarrier in the R-D channel etc. Assuming that the subcarriers in the S-R link are increasingly ordered at the R station, according to their transfer functions, and then mapped to the R-D link subcarriers, which are decreasingly ordered, the end-to-end SNR of the k-th subcarrier pair is given as [2]

$$\gamma_{k,end}=\frac{\gamma_{k,SR}\gamma_{k,RD}}{C+\gamma_{k,RD}}=\frac{\gamma_{k,SR}}{1+Cz_k} \quad (14)$$

where $z_k=1/\gamma_{k,RD}$. The PDF of the new variable z_k is given as $f_{z_k}(z_k)=(1/z_k^2)f_{RD}^{s}(1/z_k)$. Using the transformation $y_k=1+Cz_k$, the PDF of the denominator in (14) is derived as

$$f_{y_k}(y_k)=C\frac{\mathcal{U}_{\{y_k-1\}}}{(y_k-1)^2}\sum_{i=0}^{N-k}\lambda_{RD}\delta_i e^{-\varepsilon_i\lambda_{RD}\frac{C}{y_k-1}} \quad (15)$$

where $\mathcal{U}_{\{y_k-1\}}$ is the unitary step function, i.e., $\mathcal{U}_{\{y_k-1\}}=1$ if $y_k\geq 1$ and zero otherwise. The PDF of $\gamma_{k,end}$ is then calculated as in [11]

$$f_{\gamma_{k,end}}^{BTW}(x)=\int_0^\infty y_k f_{\gamma_{SR},y}(xy_k,y_k)\,dy_k \quad (16)$$

with $f_{\gamma_{SR},y}(xy_k,y_k)=f_{k,SR}^{w}(xy_k)f_{y_k}(y_k)$ representing the joint PDF of the independent random variables γ_{SR} and y_k, and $f_{k,SR}^{w}(xy_k)$ is given in (7). It follows that

$$f_{\gamma_{k,end}}^{BTW}(x)=\int_0^\infty y_k f_{k,SR}^{w}(xy_k)f_{y_k}(y_k)\,dy_k \quad (17)$$
$$=\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\lambda_{SR}\lambda_{RD}\alpha_j\delta_i C\times\int_1^\infty\frac{y_k\exp\left(-\lambda_{SR}\beta_j xy_k-\lambda_{RD}\varepsilon_i\frac{C}{y_k-1}\right)}{(y_k-1)^2}dy_k$$

Using the transformation $m_k=1/(y_k-1)$ and [12, Eq. (3.471.12)], (17) reduces to

$$f_{\gamma_{k,end}}^{BTW}(x)=\frac{2}{\overline{\gamma}_{SR}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\alpha_j\delta_i e^{-\beta_j\frac{x}{\overline{\gamma}_{SR}}}\times\left[\sqrt{\frac{C\beta_j x}{\varepsilon_i\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{C\beta_j\varepsilon_i x}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)+\frac{C}{\overline{\gamma}_{RD}}K_0\left(2\sqrt{\frac{C\beta_j\varepsilon_i x}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)\right] \quad (18)$$

where $K_0(\cdot)$ and $K_1(\cdot)$ are zero and first order modified Bessel functions of the second kind defined in [13, Eqs. (9.6.21), (9.6.22)].

3.3 End-to-End SNR PDF for BTB SCP

In the BTB SCP scheme the strongest subcarrier from the first hop is mapped to the strongest subcarrier in the second hop, etc. Considering that the subcarriers in the S-R and R-D links are increasing ordered according to their transfer functions, which implies that their SNR PDFs are given as (7), the PDF of SNR for the k-th subcarrier pair in BTB SCP scenario is obtained as

$$f_{\gamma_{k,end}}^{BTB}(x)=\frac{2}{\overline{\gamma}_{SR}}\sum_{j=0}^{k-1}\sum_{i=0}^{k-1}\alpha_j\alpha_i e^{-\beta_j\frac{x}{\overline{\gamma}_{SR}}}\times\left[\sqrt{\frac{C\beta_j x}{\beta_i\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{C\beta_j\beta_i x}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)+\frac{C}{\overline{\gamma}_{RD}}K_0\left(2\sqrt{\frac{C\beta_j\beta_i x}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}}\right)\right] \quad (19)$$

4. Performance Analysis

The subsequent presented BER and ergodic capacity analysis is performed assuming perfectly time and frequency synchronized OFDM AF relay system, with implemented SCP.

4.1 Average BER

4.1.1 BTW SCP

Having the PDF of the SNR for the k-th subcarrier pair in BTW SCP (18), the moment generating function (MGF) of $\gamma_{k,end}$ can be derived as

$$\mathcal{M}_{\gamma_{k,end}}(s) = \mathbf{E}(e^{-s\gamma}) = \int_0^\infty e^{-s\gamma} f^{BTW}_{\gamma_{k,end}}(\gamma)\, d\gamma \tag{20}$$

After some basic transformations, a closed-form solution for the integral in (20) can be found in terms of the Whittaker function, by using [12, Eqs. (6.614.4), (6.631.3)]. The obtained MGF can be expressed using the more common exponential integral function, with the help of the identities [13, Eqs. (13.6.28), (13.6.30), (6.5.19)]

$$\begin{aligned}\mathcal{M}_{\gamma_{k,end}}(s) = &\frac{1}{\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{N-k} \frac{\alpha_j \delta_i}{T_j(s)} \left[\frac{1}{\varepsilon_i} + e^{\frac{CB_{j,i}}{T_j(s)}} \right. \\ &\left. \times E_1\left(\frac{CB_{j,i}}{T_j(s)}\right)\left(\frac{C}{\overline{\gamma}_{RD}} - \frac{CB_{j,i}}{\varepsilon_i T_j(s)}\right)\right]\end{aligned} \tag{21}$$

where $T_j(s) = s + \beta_j/\overline{\gamma}_{SR}$, $B_{j,i} = \beta_j \varepsilon_i / \overline{\gamma}_{SR}\overline{\gamma}_{RD}$ and $E_1(\cdot)$ is the exponential integral function defined in [13, Eq. (5.1.1)].

Using the MGF expression in closed-form as shown in (21), the BER performance evaluation for different digital modulations over fading channels can be conducted via the MGF-based approach [14]. For example, the k-th subcarrier pair BER for the differential phase shift-keying (DPSK) is $P_{b,k} = 0.5\mathcal{M}_{\gamma_k,end}(1)$, and for the OFDM system the total average BER is obtained through

$$P_b = \frac{1}{N} \sum\nolimits_{k=1}^{N} P_{b,k} \tag{22}$$

However, for coherent modulations, such as M-QAM or M-ary phase shift keying (M-PSK), the derivation of the exact closed form BER expression is more complex, as the integrand includes the exponential integral function. Therefore, in order to obtain a closed-form approximate BER expression for BPSK we use a PDF-based approach, and the approximation of the complementary error function erfc($\cdot$) given in [15].

The average BER for the k-th subcarrier pair in the BTW SCP scheme is given as

$$P_{b,k} = \int_0^\infty P_{b|\gamma_k} f^{BTW}_{\gamma_{k,end}}(\gamma) d\gamma \tag{23}$$

with $P_{b|\gamma_k}$ denoting modulation dependent conditional BER. For BPSK signalling $P_{b|\gamma_k} = Q(\sqrt{2\gamma})$, where $Q(\cdot)$ is the Gaussian Q function which can be expressed in terms of erfc($\cdot$) as $Q(x) = 0.5\text{erfc}(x/\sqrt{2})$ [11]. Using the approximation of erfc($\cdot$) [15]

$$\text{erfc}(x) \simeq \frac{1}{6} e^{-x^2} + \frac{1}{2} e^{-4x^2/3} \tag{24}$$

the integral in (23) becomes

$$P_{b,k} = \frac{1}{2} \int_0^\infty \left(\frac{1}{6} e^{-\gamma} + \frac{1}{2} e^{-\frac{4\gamma}{3}} \right) f^{BTW}_{\gamma_{k,end}}(\gamma)\, d\gamma. \tag{25}$$

Substituting (18) in (25) and using the integrals [12, Eqs. (6.614.4), (6.631.3)], as well as the identities [13, Eqs. (13.1.33), (13.6.28), (13.6.30), (6.5.19)], a closed-form expression is obtained in terms of $E_1(\cdot)$

$$\begin{aligned}P_{b,k} = &\frac{1}{2\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{N-k} \alpha_j \delta_i \left[\frac{1}{2\varepsilon_i} \left(\frac{1}{3T_j(1)} + \frac{1}{T_j(4/3)} \right) \right. \\ &+ \frac{1}{6} \frac{e^{\frac{CB_{j,i}}{T_j(1)}}}{T_j(1)} E_1\left(\frac{CB_{j,i}}{T_j(1)}\right)\left(\frac{C}{\overline{\gamma}_{RD}} - \frac{CB_{j,i}}{2\varepsilon_i T_j(1)}\right) \\ &\left. + \frac{e^{\frac{CB_{j,i}}{T_j(4/3)}}}{2T_j(4/3)} E_1\left(\frac{CB_{j,i}}{T_j(4/3)}\right)\left(\frac{C}{\overline{\gamma}_{RD}} - \frac{CB_{j,i}}{2\varepsilon_i T_j(4/3)}\right)\right]\end{aligned} \tag{26}$$

where $T_j(1) = 1 + \frac{\beta_j}{\overline{\gamma}_{SR}}$ and $T_j(4/3) = \frac{4}{3} + \frac{\beta_j}{\overline{\gamma}_{SR}}$. The average BER is then calculated as $P_b = \frac{1}{N} \sum_{k=1}^{N} P_{b,k}$.

The same BER derivation approach can be implemented for obtaining BER expressions for M-QAM modulations, having that the conditional BER for M-QAM modulation is generally given as $P_{b|\gamma} = \sum_{i=1}^{M} \mathcal{A}_i Q(\sqrt{\mathcal{B}_i \gamma})$, where $\mathcal{A}_i$ and $\mathcal{B}_i$ are modulation dependent constants [16].

4.1.2 BTB SCP

Using (19) and the same procedure implemented in the case of the BTW SCP, the MGF of $\gamma_{k,end}$ for the BTB SCP is obtained as

$$\begin{aligned}\mathcal{M}_{\gamma_{k,end}}(s) = &\frac{1}{\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{k-1} \frac{\alpha_j \alpha_i}{T_j(s)} \left[\frac{1}{\beta_i} + e^{\frac{CA_{j,i}}{T_j(s)}} \right. \\ &\left. \times E_1\left(\frac{CA_{j,i}}{T_j(s)}\right)\left(\frac{C}{\overline{\gamma}_{RD}} - \frac{CA_{j,i}}{\beta_i T_j(s)}\right)\right]\end{aligned} \tag{27}$$

where $A_{j,i} = \beta_j \beta_i / \overline{\gamma}_{SR}\overline{\gamma}_{RD}$.

Following the same steps as the ones used for the BER derivation for the BTW SCP, a BER expression for the k-th subcarrier pair in the BTB SCP scenario and BPSK modulation is evaluated as

$$\begin{aligned}P_{b,k} = &\frac{1}{2\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{k-1} \alpha_j \beta_i \left[\frac{1}{2\beta_i} \left(\frac{1}{3T_j(1)} + \frac{1}{T_j(4/3)} \right) \right. \\ &+ \frac{1}{6} \frac{e^{\frac{CA_{j,i}}{T_j(1)}}}{T_j(1)} E_1\left(\frac{CA_{j,i}}{T_j(1)}\right)\left(\frac{C}{\overline{\gamma}_{RD}} - \frac{CA_{j,i}}{2\beta_i T_j(1)}\right)\end{aligned} \tag{28}$$

$$+ \frac{e^{\frac{CA_{j,i}}{T_j(4/3)}}}{2T_j(4/3)} E_1\left(\frac{CA_{j,i}}{T_j(4/3)}\right)\left(\frac{C}{\overline{\gamma}_{RD}} - \frac{CA_{j,i}}{2\beta_i T_j(4/3)}\right)\Bigg]$$

4.2 Ergodic Capacity

The ergodic capacity for the k-th subcarrier pair in the analyzed system can be calculated using the obtained end-to-end SNR density function as

$$\boldsymbol{C}_k = \frac{1}{2}\mathbf{E}(\log_2(1+\gamma)) = \frac{1}{2}\int_0^{\infty} \log_2(1+\gamma) f_{\gamma_{k,end}}(\gamma)\, d\gamma \quad (29)$$

where $f_{\gamma_{k,end}} \in \{f^{BTW}_{\gamma_{k,end}}, f^{BTB}_{\gamma_{k,end}}\}$, and the factor 1/2 comes due to the transmission over two time-slots [1]. Having $f_{\gamma_{k,end}}$ as (18) or (19), precludes finding closed-form solution for the integral in (29). However, the ergodic capacity in (29) can be upper bounded using Jensen's inequality [17] and noting that $\log(\cdot)$ is a concave function, yielding

$$\boldsymbol{C}_k \leq \frac{1}{2}\log_2(1+\mathbf{E}(\gamma)). \quad (30)$$

Using the derived PDF expression (18) and the integral given in [12, Eqs. (6.631.3)], the expectation $\mathbf{E}(\gamma)$ for the k-th subcarrier pair in the case of BTW SCP is derived as

$$\mathbf{E}(\gamma) = \sum_{j=0}^{k-1}\sum_{i=0}^{N-k} \frac{\alpha_j \delta_i}{\beta_j^2} \frac{1}{\sqrt{CB_{j,i}}} e^{\frac{1}{2}\frac{CB_{j,i}\overline{\gamma}_{SR}}{\beta_j}} \times \left[2\sqrt{\frac{\beta_j}{\varepsilon_i}\frac{\overline{\gamma}_{SR}}{\overline{\gamma}_{RD}}} C W_{-2,\frac{1}{2}}\left(\frac{CB_{j,i}\overline{\gamma}_{SR}}{\beta_j}\right) + \frac{C}{\overline{\gamma}_{RD}}\sqrt{\beta_j \overline{\gamma}_{RD}} W_{-\frac{3}{2},0}\left(\frac{CB_{j,i}\overline{\gamma}_{SR}}{\beta_j}\right)\right] \quad (31)$$

where $W_{a,b}(\cdot)$ denotes the Whittaker function defined in [13, Eq. (13.1.32)]. Substituting relation (31) in (30), the upper bound for the k-th subcarrier pair ergodic capacity can be obtained.

For the BTB SCP scenario, the expectation $\mathbf{E}(\gamma)$ for the k-th subcarrier pair is given as

$$\mathbf{E}(\gamma) = \sum_{j=0}^{k-1}\sum_{i=0}^{k-1} \frac{\alpha_j \alpha_i}{\beta_j^2} \frac{1}{\sqrt{CA_{j,i}}} e^{\frac{1}{2}\frac{CA_{j,i}\overline{\gamma}_{SR}}{\beta_j}} \times \left[2\sqrt{\frac{\beta_j}{\beta_i}\frac{\overline{\gamma}_{SR}}{\overline{\gamma}_{RD}}} C W_{-2,\frac{1}{2}}\left(\frac{CA_{j,i}\overline{\gamma}_{SR}}{\beta_j}\right) + \frac{C}{\overline{\gamma}_{RD}}\sqrt{\beta_j \overline{\gamma}_{RD}} W_{-\frac{3}{2},0}\left(\frac{CA_{j,i}\overline{\gamma}_{SR}}{\beta_j}\right)\right] \quad (32)$$

The total upper bound for the ergodic capacity of OFDM AF relay system with SCP is evaluated as $\boldsymbol{C} = \sum_{k=1}^{N} \boldsymbol{C}_k$.

5. Numerical Results

The following presented analytical and simulation results assume perfectly synchronized OFDM AF relay system with implemented SCP. The OFDM system has N=16 subcarriers, which in a real scenario can be considered as 16 chunks with uncorrelated transfer functions from chunk to chunk. It is also assumed that $\mathcal{E}_R = \mathcal{E}_S$, $\mathcal{N}_{0_1} = \mathcal{N}_{0_2}$, and that R is equally spaced between S and D, so that $\overline{\gamma}_{SR} = \overline{\gamma}_{RD}$. The relay gain G is calculated assuming that the average subcarrier symbol power transmitted by the relay is $\mathcal{E}_R$ and using the relay's knowledge of the average fading power on the S-R link

$$G^2 = \mathbf{E}\left[\frac{\mathcal{E}_R}{\mathcal{E}_S |H_{1,k}|^2 + \mathcal{N}_{0_1}}\right] \quad (33)$$

yielding [2]

$$G^2 = \frac{\mathcal{E}_R}{\mathcal{E}_S \Omega_1} e^{1/\overline{\gamma}_{SR}} E_1\left(\frac{1}{\overline{\gamma}_{SR}}\right) \quad (34)$$

where $\Omega_1 = \mathbf{E}[|H_{1,k}|^2]$. Using (34), the constant C becomes

$$C = \frac{\overline{\gamma}_{SR}}{e^{1/\overline{\gamma}_{SR}} E_1(1/\overline{\gamma}_{SR})} \quad (35)$$

Simulation results are obtained through Monte Carlo simulations, where only the frequency domain part of the analyzed system is taken in consideration, as it is assumed to be perfectly synchronized. The subcarrier transfer functions on the first and second hop are generated as independent complex Gaussian random variables with zero mean and variance 1/2, meaning that the average subcarrier power is equal to 1. Ten OFDM symbols are transmitted through each channel realization.

5.1 BER Performance

Figure 2 presents the BER performance of DPSK modulated OFDM AF relay system for the BTW SCP and the BTB SCP. For the sake of comparison, the BER performance of the OFDM AF relay system without (w/o) SCP is presented. BER for the AF system w/o SCP is analytically obtained using the MGF derived in [2]. The obtained analytical results

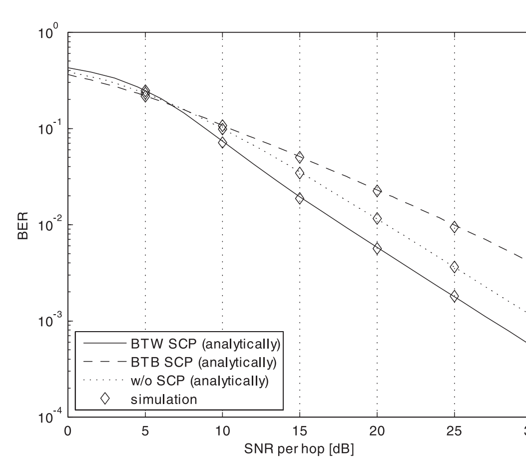

Fig. 2 BER of DPSK modulated OFDM AF system with SCP.

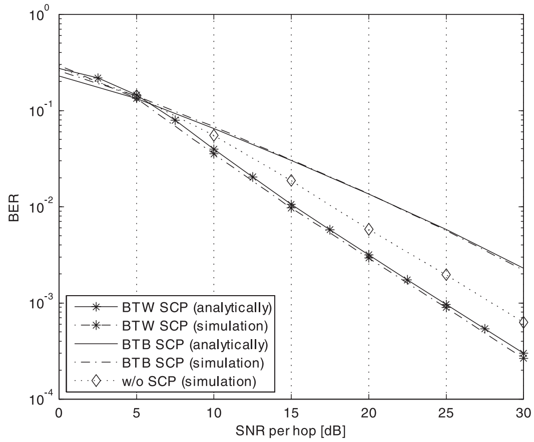

Fig. 3 BER of BPSK modulated OFDM AF system with SCP.

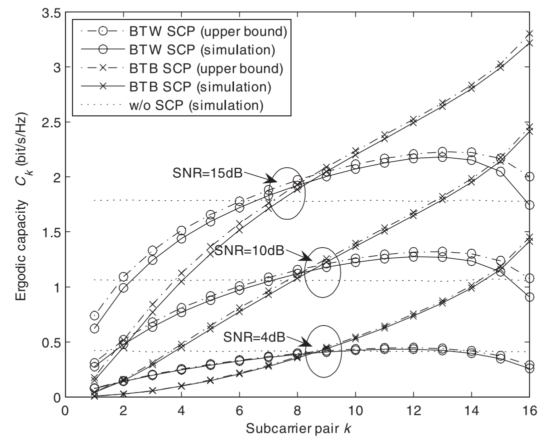

Fig. 4 Ergodic capacity on a subcarrier basis.

are completely verified by simulations. As expected, for the low SNR values BTB SCP achieves the best BER performance. It outperforms BTW SCP system up to the SNR of 6.5 dB approximately, and the system w/o SCP up to the SNR of 7.5 dB. For SNR values above 6.5 dB, the BTW SCP scheme has the lowest BER and its advantage in BER performance increases very fast as the SNR values increase. It already achieves almost 3 dB SNR gain over the system w/o SCP and more than 7 dB SNR gain over the BTB SCP scheme, for the BER value of 10^{-2}.

BER results of the BPSK modulated OFDM AF system with BTW SCP, BTB SCP and w/o SCP schemes are given in Fig. 3. The analytically obtained BER results for both SCP schemes present a tight approximation of the exact BER values. The small difference compared to the exact results is expected, as the approximation of erfc(·) is used in averaging the BER over the region of all SNR values. Considering that simulation results are the exact ones, it can be seen that the intersection point for the BER graphs of BTW SCP and BTB SCP is now approximately 4 dB, meaning that after that point the BTW SCP scheme achieves significantly lower BER. For example, the SNR gain is approximately 7 dB for BER of 10^{-2}. Note that for SNR=30 dB, the BTB SCP scheme attains BER of $2.2{\cdot}10^{-3}$, whereas the BER for the BTW SCP scheme is $2.6{\cdot}10^{-4}$, i.e., more than eight times lower.

Having in mind that the fadings in S-R and R-D links are Rayleigh distributed, it is clear that mapping of the weakest subcarriers from the first hop to the corresponding weakest subcarriers on the second hop, in BTB SCP scheme, enables only minor end-to-end SNR improvement for these worst subcarrier pairs (k=1, k=2,…) with increase of the average SNR in the S-R and R-D links. Those worst subcarrier pairs achieve high BER values even in the region of medium and high average SNRs, thus making enough difference in average BER performance to enable BTW SCP scheme to outperform BTB SCP. Namely, in BTW SCP scheme, the deviation between average SNR and end-to-end SNR of any subcarrier pair is less than the corresponding deviation in BTB SCP scheme. However, this feature of BTW SCP scheme turns out to be disadvantage in the region of small average SNRs, as it means that all subcarrier pairs will achieve high BER values. On the other side, in BTB SCP scheme, the strongest subcarrier pairs (in the analyzed scenario for k=16, k=15,…) have end-to-end SNR much above the average SNR, which is very important at low average SNR values. Those best subcarrier pairs bring advantage in average BER performance in the region of low SNRs over the BTW SCP scheme.

From the presented BER results, it is obvious that OFDM AF relaying systems may switch from the BTB SCP to BTW SCP scheme depending on the average SNR in S-R and R-D links. Apparently, this hybrid SCP scheme is expected to achieve optimum BER performance.

5.2 Ergodic Capacity

Figure 4 presents the calculated upper bounds, as well as ergodic capacities obtained by simulation of the SCP schemes considered above, for each subcarrier pair. The ergodic capacity of the system w/o SCP is also presented, for the sake of comparison. The presented results assume average SNRs per hop of 4 dB, 10 dB and 15 dB. In the simulations, independent subcarrier transfer functions with Rayleigh distribution are generated in the S-R link ($H_{1,k}^{(n)}$), and the R-D link ($H_{2,k}^{(n)}$), for each channel realization, where $1 \leq n \leq n_{tot}$ denotes the n-th channel realization. The instantaneous end-to-end capacity for the k-th subcarrier pair is then found as

$$\boldsymbol{C}_k^{(n)} = \frac{1}{2}\log_2\left(1 + \frac{(G^{(n)})^2|H_{1,k}^{(n)}H_{2,k}^{(n)}X_k^{(n)}|^2}{(G^{(n)})^2|H_{2,k}^{(n)}|^2\mathcal{N}_{0_1} + \mathcal{N}_{0_2}}\right) \tag{36}$$

Depending on the implemented subcarriers reordering in the first and the second hop, BTW or BTB k-th subcarrier pair instantaneous capacity is simulated. The ergodic k-th subcarrier capacity is then determined as $\boldsymbol{C}_k = \frac{1}{n_{tot}}\sum_{n=1}^{n_{tot}}\boldsymbol{C}_k^{(n)}$, and the total system capacity as $\boldsymbol{C} = \sum_{k=1}^{N}\boldsymbol{C}_k$.

From Fig. 4, it is obvious that the calculated upper bounds present tight bounds of the exact ergodic capacities

Table 1 Total ergodic capacity.

SNR per hop	4 dB	10 dB	15 dB
BTW SCP [$\frac{b/s}{Hz}$] - simul.	5.149	15.666	27.857
BTW SCP [$\frac{b/s}{Hz}$] - bound	5.360	16.505	29.115
BTB SCP [$\frac{b/s}{Hz}$] - simul.	7.930	18.302	29.574
BTB SCP [$\frac{b/s}{Hz}$] - bound	8.197	18.825	30.369
AF w/o SCP [$\frac{b/s}{Hz}$] - simul.	6.657	16.957	28.562

for both SCP schemes. In the given figure, the ergodic capacity in BTB SCP scheme is enhanced with increasing subcarrier pair k, as the end-to-end SNR enhances along with subcarrier pair k. k=1 in BTW SCP scenario denotes subcarrier pair with the weakest subcarrier from the S-R link and the strongest subcarrier from the R-D link, while k=16 assumes opposite situation. Thus, due to the Rayleigh distributed subcarrier transfer functions, the ergodic capacity is improved along with the end-to-end SNR, from k=1 to k=13 (k=12 for SNR=4 dB), and then both, the end-to-end SNR and the ergodic capacity goes down with increasing subcarrier pair k. For the system w/o SCP, the ergodic capacity is equal for all subcarrier pairs, as their average SNRs are the same.

Table 1 gives the total system ergodic capacities for both SCP schemes, as well as for the ordinary AF system w/o SCP. As it can be seen, the BTB SCP scheme achieves the highest total ergodic capacity, while the BTW SCP scheme has the lowest total ergodic capacity. Comparing the total ergodic capacities for the BTB SCP scheme obtained through simulations and the corresponding upper bound capacities, one may conclude that the upper bound results give up to 3.3% higher capacities. For the BTW SCP scheme, the upper bound ergodic capacity results give up to 5% higher ergodic capacities than simulation results.

Considering only ergodic capacities obtained by simulation, for SNR=4 dB, which presents the BPSK-modulated BER intersection point for the BTW SCP and BTB SCP schemes, the capacity loss due to the implementation of the BTW SCP scheme is 35.3%. Comparing to the scheme w/o SCP, the BTW SCP scheme has a capacity loss equal to 22.9% for the same SNR value. As the average SNR per hop increases, the capacity loss of BTW SCP decreases. For instance, for the SNR values of 10 dB and 15 dB, the BTW SCP scheme has a capacity loss equal to 14.4% and 5.9%, respectively, comparing to the BTB SCP scheme. For the same SNR values, compared to the scheme w/o SCP, the BTW SCP scheme has a capacity loss equal to 7.6% and 2.4%, respectively.

6. Conclusions

We performed an error probability analysis for the two OFDM fixed gain amplify-and-forward (AF) relaying schemes, namely best-to-worst (BTW) subcarrier permutation (SCP) and best-to-best (BTB) SCP. Closed-form expressions for the bit error rate (BER) were derived, which is, to the best of author's knowledge, the first time that this problem is analytically treated up to date. In terms of BER, it has been shown that the BTB SCP scheme outperforms BTW SCP in the low SNR regime, while in the medium and high SNR region, the relative performance of BTB SCP and BTW SCP are reversed, in the sense that BTW SCP yields lower BER. It has also been pointed out that, in order to optimize the BER performance, the OFDM AF relaying system may switch from one SCP scheme to another, depending on the average S-R and R-D channel conditions. Additionally, the total ergodic capacities of the above schemes were compared, so as to have an insight into the possible trade off between these two performance measures when the relaying system switches from the one analyzed SCP scheme to another.

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