New Solution for BER Performance Improvement of OFDM AF Relay Systems

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Abstract — Dual-hop OFDM amplify-and-forward (AF) variable gain (VG) relay systems may improve bit error rate (BER) performance if ordered subcarrier mapping (SCM) is applied at the relay station (R). In previous works we showed that in the region of small and medium values of average signal-to-noise ratios (SNRs) on hops best-to-best SCM (BTB SCM) scheme should be applied, while in the region of high values of average SNRs on hops very small BER performance improvement is achieved by implementing best-to-worst SCM (BTW SCM) scheme. In this paper we propose new solution, where in the assumed relay system with BTB SCM the subcarriers having the lowest SNR on each hop are omitted. Analytical and simulation result comparison, for the scenario with DPSK modulation implemented and Rayleigh fading statistics on both hops, have confirmed that the proposed solution achieves the best BER performance for all the SNR values.

***Key words* — BER, OFDM AF relay system, variable gain, subcarrier mapping.**

I. INTRODUCTION

OFDM (*Orthogonal Frequency Division Multiplexing*) based relay systems as a solution for performance improvement and coverage extension of mobile cellular systems have already found its place in both accepted standards for IMT-Advanced systems, [1]. The adopted solutions assume that in dual-hop relay system, a relay station (R) will perform decode-and-forward (DF) based relaying, where the signal received from the source of information (S) will be fully decoded, and then again re-encoded before forwarding toward destination (D). This type of R stations will be placed at the edge of the cells primarily for coverage extension, and will behave as base stations with its own cell IDs. On the other side, many researches were conducted analyzing the possibility for implementation of another type of R stations that should be placed inside the cells as a solution for overall performance improvement, [2]-[8]. In this type of R stations amplify-and forward (AF) relaying, where the signal received by R is only amplified and then forwarded to D, has shown few advantages over the DF systems, like are less delay introduced, simpler realization, etc. Thus, it is expected that some of the future specifications of IMT-Advanced systems will adopt OFDM AF based relaying for the R stations inside the cells, [1]. This is the reason why are OFDM based AF relay systems the topic of intensive interest of research community, seeking for the new solutions for their optimization, performance improvement, better energy efficiency, etc. AF relay systems may implement fixed gain (FG) at the R station, or variable (VG) if the R station can estimate S-R channel.

As an interesting solution for performance improvement of dual-hop OFDM based AF relay systems, the ordered subcarrier mapping (SCM) at R station was proposed, [2]. In this way ergodic capacity may be enhanced, [2]-[6], and/or bit error rate (BER) performance may be improved, [5]-[8]. The proposed solution for capacity enhancement assume that R station, knowing channel state information from both S-R and R-D links, orders subcarriers from the first hop in accordance to their instantaneous signal-to-noise ratios (SNRs) and than maps them to subcarriers on the second hop, which are also increasingly ordered with respect to their instantaneous SNRs. This SCM scheme is denoted as best-to-best SCM (BTB SCM). However, in [7] it is shown that for BER performance improvement in AF relaying systems, for the high value of SNRs on both hops, the so called best-to-worst SCM (BTW SCM) should be implemented. It assumes that the subcarriers from the first hop are increasingly ordered in accordance to their instantaneous SNRs at the R station and than are mapped to subcarriers which are decreasingly ordered with respect to their instantaneous SNRs. The analytically derived BER results for differentially phase shift keying (DPSK) modulated OFDM AF FG relay system with SCM, [8] and OFDM AF VG relay systems with SCM, [6] have confirmed this. However, we showed in [6] that OFDM AF VG relay system with BTW SCM mapping attains very small, almost neglectful BER performance improvement in high SNR region, compared to the OFDM AF VG system without SCM. This raises a question on the justification of this solution.

Thus, in this paper we propose a solution with BTB SCM, but where the subcarriers having lowest SNRs on both hops are not used. We examined analytically and through simulations BER performance of DPSK modulated proposed system and compare it with the BER performances of other OFDM AF VG relay systems' solutions. Additionally, in order to get an insight in

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efficiency of the proposed solution, we compared its BER performance with BER performance of the other OFDM AF VG relay systems, as well as with BER performance of OFDM direct transmission between S and D, in the case of same transmitted power per subcarrier.

The paper is organized as follows. Section II describes the analyzed OFDM AF VG relay system with SCM and the considered scenario. Section III contains BER performance analysis of this system. Section IV presents the analytically and simulation obtained BER and results and comments on this results. Section V concludes the paper.

II. System model

A dual-hop OFDM VG non-regenerative relay system, with three communication terminals, S, R, and D, each equipped with single antenna is considered. The scenario without direct link between S and D is assumed, where R operates in half-duplex mode. R station has FFT (Fast Fourier Transformation) block, performing OFDM demodulation. After being demodulated, the signals in parallel branches are amplified in such a way that the influence of the subcarrier channels on the S-R link is compensated (Fig.1). This is achieved through implementation of a gain, which is at the i-th subcarrier equal to $G_i=1/H_{1,i}$, $i=1,..,M$, where $H_{1,i}$ is the transfer function of the i-th subcarrier channel on the first hop. In the model considered, we assume that the R station has a perfect knowledge of each subcarrier channels' transfer function at both hops. Subcarrier mapping block follows after the amplification of each subcarrier, and then the signal is again OFDM modulated through IFFT (Inverse Fast Fourier Transformation) block. In order to perform signal demodulation at the system receiving end, it is necessary that *D* knows the permutation scheme performed at *R*.

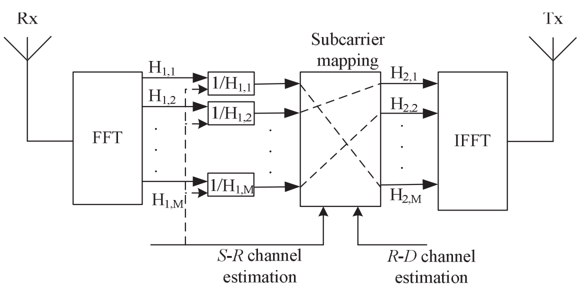

Figure 1. OFDM AF VG relay station with SCM

The signal received at the R station on the i-th subcarrier, after OFDM demodulation can be presented as:

$$Y_{R,i} = X_i H_{1,i} + N_{1,i}, \quad 1 \le i \le M, \tag{1}$$

where M is total number of subcarriers and X_i is data symbol sent by source on the i-th subcarrier. $N_{1,i}$ represents an additive white Gausssian noise on the i-th subcarrier with variance $\mathbf{E}(|N_{1,i}|^2)=N_{01}$. $\mathbf{E}(\cdot)$ denotes the expectation operator. Assuming that the SCM function $v(i)$ performs mapping of the i-th subcarrier from the first hop to the k-th subcarrier on the second hop, the frequency domain signal at *D* can be written as:

Frequency domain signal at R station, on i-th subcarrier, may be represented as:

$$\begin{aligned} Y_{D,k} &= G_i H_{2,k} Y_{R,v(i)} + N_{2,k} \\ &= G_i H_{2,k} H_{1,i} X_i + G_i H_{2,k} N_{1,i} + N_{2,k}, \quad 1 \le k \le M, \end{aligned} \tag{2}$$

with $H_{2,k}$ denoting the k-th subcarrier channel transfer function on the second hop. $N_{2,k}$ is an additive white Gaussian noise on the k-th subcarrier at the destination, having variance $\mathbf{E}(|N_{2,k}|^2)=N_{02}$. Fadings in the S-R and R-D channels are assumed to be independent and identically distributed (i.i.d.) with the Rayleigh distribution, resulting in the exponential form of PDF of SNR on both channels. Using (2) and the described gain G_i applied at the subcarrier level at R, the instantaneous SNR on the k-th subcarrier at D can be presented as:

$$\gamma_{k,end} = \frac{\gamma_{i,SR}\gamma_{k,RD}}{\gamma_{i,SR}+\gamma_{k,RD}}, \tag{3}$$

where $\gamma_{i,SR}$ and $\gamma_{k,RD}$ denote instantaneous SNR on the i-th subcarrier of the S-R link and on the k-th subcarrier of the R-D link, respectively.

III. Performance analysis

We have conducted BER performance analysis for the dual-hop DPSK modulated OFDM AF VG relay system with SCM, in the case of Rayleigh fading channels on both hops in paper [6]. We first derived PDF of SNR for the k-th subcarrier at D for the systems implementing BTB SCM and BTW SCM. However, for the case of DPSK modulation implemented, it is more convenient to use MGF based for BER performance analysis, as in that case BER for the k-th subcarrier at D is obtained through, [9]:

$$P_{b,k} = 0.5 \cdot \mathcal{M}_{\gamma_{k,end}}(1), \tag{4}$$

with $\mathcal{M}_{\gamma_{k,end}}(\cdot)$ denoting MGF of SNR function. BER of the analyzed OFDM AF VG relay system with SCM is derived through averaging (4) over all M subcarriers:

$$P_b = \frac{1}{M}\sum_{k=1}^{M} P_{b,k}. \tag{5}$$

Thus, we derived MGF of SNR at the k-th subcarrier at D for the relay systems implementing both analyzed SCM schemes. In the scenario with BTB SCM scheme at the R station, MGF of SNR for the k-th subcarrier at D is obtained as:

$$\begin{aligned} \mathcal{M}_{\gamma_{k,end}}(s) &= \frac{16}{3}\sum_{j=0}^{k-1}\sum_{i=0}^{k-1}\frac{\alpha_j\alpha_i}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\frac{1}{\left(s+I_{j,i}+2\sqrt{A_{j,i}}\right)^2} \\ &\times\left[\frac{4I_{j,i}}{s+I_{j,i}+2\sqrt{A_{j,i}}}\,{}_2F_1\!\left(3,\frac{3}{2};\frac{5}{2};\frac{s+I_{j,i}-2\sqrt{A_{j,i}}}{s+I_{j,i}+2\sqrt{A_{j,i}}}\right)\right. \\ &\left.+\,{}_2F_1\!\left(3,\frac{3}{2};\frac{5}{2};\frac{s+I_{j,i}-2\sqrt{A_{j,i}}}{s+I_{j,i}+2\sqrt{A_{j,i}}}\right)\right] \end{aligned} \tag{6}$$

In (6) ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ represents the Gaussian hypergeometric function defined in [10, eq. (9.100)], while the introduced coefficients are equal to $I_{j,i}=\beta_j/\bar{\gamma}_{SR}+\beta_i/\bar{\gamma}_{RD}$ and $A_{j,i}=\beta_j\beta_i/\bar{\gamma}_{SR}\bar{\gamma}_{RD}$. $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$ denote average SNRs on S-R link and R-D link, respectively. The introduced coefficients are equal to:

$$\alpha_i=(-1)^i M\binom{M-1}{k-1}\binom{k-1}{i};\quad \beta_i=i+M-k+1, \tag{7}$$

with (:) representing the binomial coefficients.

MGF of SNR for the k-th subcarrier at D in case of BTW SCM implementation is equal to:

$$\begin{aligned}\mathcal{M}_{\gamma_{k,end}}(s)=\mathbf{E}\left(e^{-\gamma s}\right)=\frac{16}{3}\sum_{j=0}^{k-1}\sum_{i=0}^{M-k}\frac{\alpha_j\delta_i}{\overline{\gamma}_{SR}\overline{\gamma}_{RD}}\frac{1}{\left(s+L_{j,i}+2\sqrt{B_{j,i}}\right)^2}\\ \times\left[\frac{4L_{j,i}}{s+L_{j,i}+2\sqrt{B_{j,i}}}\ {}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{s+L_{j,i}-2\sqrt{B_{j,i}}}{s+L_{j,i}+2\sqrt{B_{j,i}}}\right)\right.\\ \left.+\ {}_2F_1\left(3,\frac{3}{2};\frac{5}{2};\frac{s+L_{j,i}-2\sqrt{B_{j,i}}}{s+L_{j,i}+2\sqrt{B_{j,i}}}\right)\right].\end{aligned} \tag{8}$$

In (8) $L_{j,i}=\beta_j/\overline{\gamma}_{SR}+\varepsilon_i/\overline{\gamma}_{RD}$ and $B_{j,i}=\beta_j\varepsilon_i/\overline{\gamma}_{SR}\overline{\gamma}_{RD}$.The coefficients δ_i and ε_i are obtained through:

$$\delta_i=(-1)^i M\binom{M-1}{k-1}\binom{M-k}{i};\quad \varepsilon_i=i+k. \tag{9}$$

Substituting (7) or (8) in (4) and then in (5), BER for the DPSK modulated OFDM AF VG relay system with BTB SCM or BTW SCM is obtained. If we exclude the component k=1 from summation in BER expression for the system with BTB SCM, then BER value of the proposed system without worst subcarrier pair is obtained.

In order to examine weather the additional signal processing in systems with SCM can be justified considering the issue of power consumption, the obtained BER results should be presented as a function of total transmitted power per subcarrier P_T. We assumed equal power allocation among the S and R station, and among all the subcarriers, i.e. $P_S=P_R=P_T/2$. Now, the average SNRs on S-R and R-D links can be written as $\overline{\gamma}_{SR}=A_1P_S$ and $\overline{\gamma}_{RD}=A_2P_R$, respectively, where A_1 and A_2 include parameters as the antenna gains, path loss, noise power and similar. For example, if using Friis propagation model, A_i, i=1, 2, can be written in the form:

$$A_i=\frac{G_{t,i}G_{r,i}\lambda^2}{(4\pi)^2 d_i^{\alpha} LN_{0i}}, \tag{10}$$

where $G_{t,i}$ is the transmitter antenna gain on the i-th hop, $G_{r,i}$ is the receiver antenna gain, λ is the wavelength, d_i is the distance between the transmitter and receiver on th i-th hop, L is the system loss factor, α=2 for free space and 3<α<4 in urban environment, while N_{0i} is the noise variance at the i-th hop. Without loss of the generality, we took that the transmitter antenna gains at S and R are equal, $G_{t,1}=G_{t,2}$, and the receiver antenna gains at the R and D are also equal, $G_{r,1}=G_{r,2}$, as well as that the noise variances at the R and D are the same, $N_{01}=N_{02}$. Moreover, we assumed that in the case of relayed transmission, S, R and D are placed on a straight line, and that all the links are affected by the same shadowing environment. The average SNR at D in the case of direct transmission can be written as $\overline{\gamma}_{SD}=A_{eq}P_T$, where for this simplified propagation model, by taking α=3, A_{eq} is related to A_1 and A_2 through:

$$A_{eq}=\frac{A_2}{\left(1+\left(A_2/A_1\right)^{1/3}\right)^3}. \tag{11}$$

For the case of direct OFDM transmission in Rayleigh fading environment, MGF of SNR for the k-th subcarrier has the form:

$$\mathcal{M}_{\gamma_k,end}(s)=1\big/\left(1+s\overline{\gamma}_{SD}\right). \tag{12}$$

IV. Results

We analyzed the scenario with perfectly time and frequency synchronization among the three communicating terminals in dual-hop OFDM VG AF relay system with SCM at R. The OFDM system with M=64 subcarriers is modeled, if not otherwise stated. It is assumed that the noise variances at R and D are the same ($\mathcal{N}_{01}=\mathcal{N}_{02}$). Simulation results are obtained through Monte Carlo simulations, where only the part of the analyzed relay system which belongs to frequency domain is modeled. This is accurate approach as the system is assumed to be perfectly synchronized. The subcarrier transfer functions on the first and second hop are generated as independent complex Gaussian random variables with zero mean and variance 1/2, meaning that the average subcarrier power is equal to 1.

BER results for DPSK modulated OFDM VG AF relay system with and without SCM obtained analytically, as well as the simulation obtained results are given in Fig. 2. Moreover, BER performances of this system in which the worst subcarrier pair is not used (w/o k=1), for the cases of OFDM AF VG relay system having 32, 64 and 128 subcarriers are given. The scenario with equal average SNRs on both hops is assumed.

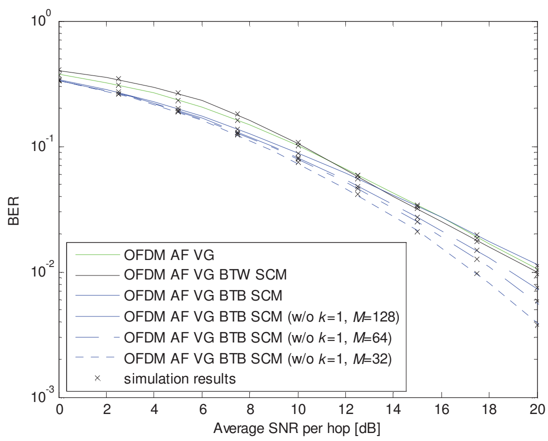

Figure 2. BER of DPSK OFDM VG AF relay system

It can be seen from the Fig. 2, that the analytically obtained results are completely verified with the simulation results, as they perfectly match for all the analyzed systems and for all the given SNR values. The presented results confirm that in the region of small and medium values of average SNRs per hop (up to 13.5dB) the system implementing BTB SCM achieves the best BER performances, which is very important, as it improves BER in the cases when the channel conditions on both hops may be very bad. Above 13.5dB of average SNR per hop, the system with BTW SCM has the lowest BER. However, BER performance improvement attained with BTW SCM scheme in this region of average SNRs per hop is very small, and it can be almost neglected in comparison to OFDM AF VG system not employing SCM. This result brings into question if the implementation of BTW SCM scheme in this kind of OFDM relay systems is justified. Thus, we analyzed the BER performances of OFDM AF VG relay system with BTB SCM, but in the case where the subcarriers with the lowest SNR values on each hop (mapped into "worst subcarrier pair") are not used. The BER results given in Fig. 2 show that this solution improves BER performance of OFDM AF VG relay system for all the SNR values on both hops in each analyzed system (M=128, M=64 and M=32). Compared to the system without SCM, for the BER value of 10^{-2}, this solution brings SNR gain of approximately 1.5dB when M=128, about 2dB when M=64 and 3dB of SNR gain when M=32. In the OFDM relay systems with higher number of subcarriers (e.g. M=512, M=1024,..) more than one subcarrier pair may be omitted for further BER performance improvement.

BER performances of the proposed solutions and of DPSK modulated OFDM AF VG relay systems with SCM, as a function of power transmitted per subcarrier P_T, are given in Fig. 3. In order to get an insight in power balance of OFDM AF VG relay system with SCM, comparison of BER performances with BER performances of OFDM AF VG relay system as well as with the direct transmission scenario are also provided. We have chosen parameters A_1=2 and A_2=10, which for downlink communication corresponds to the scenario where the distance between the R and D terminals is shorter than the one between the S and R terminals.

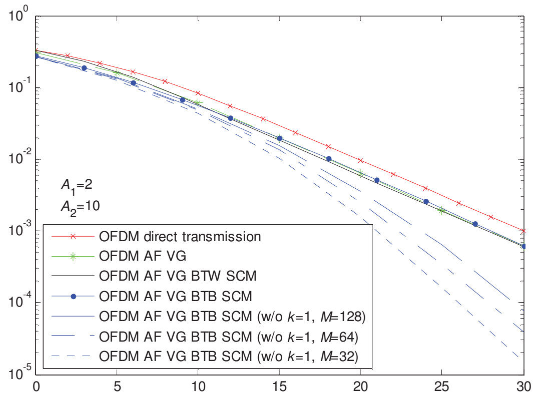

Figure 3. BER of DPSK OFDM VG AF relay system as a function of P_T

From the given BER results, the advantage of using relay transmission over the direct transmission is clear, as the power saving is more than 2.5dB for all he BER values lower than 10^{-2}. Moreover, relay transmission decreases interference induced to other communicating terminals. It is again shown that system with BTB SCM achieves the best BER performances in the regime of low and medium transmitted power. In the assumed scenario, for P_T values above 15dB, the system with BTW SCM scheme slightly outperforms the system not implementing SCM and the system with BTW SCM. However, like in previous BER result analysis conducted, it is confirmed that omitting the worst subcarrier pair in the case when BTB SCM is implemented presents the optimal solution for BER performance improvement. Thus for example, for BER value of 10^{-3}, SNR gain of the system employing BTB SCM without k=1, compared to the OFDM AF VG relay system, is equal to 5dB, 6.5dB and 7.5dB when M=128, M=64 and M=32 subcarriers, respectively.

V. Conclusions

We propose new solution based on ordered SCM for BER performance improvement in OFDM AF VG relay systems. It assumes that the worst subcarrier pair in OFDM AF VG relay system with BTB SCM mapping is not used. Analytical and simulation results for DPSK modulated assumed system have shown that it outperforms all the other OFDM AF VG relay systems in terms of BER. Thus, it can be recognized as the optimal solution for BER performance improvement of the analyzed OFDM AF VG relay system.

Additionally, we compared BER performances of the proposed solution with the BER performances other OFDM AF VG relay systems, as well as with the case of OFDM direct transmission between S and D, as the function of total transmitted power. It is again confirmed that the proposed system implementing BTB SCM, but not using the worst subcarrier pair, achieves the best BER performances among the all analyzed systems having the same total transmitted power.

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