BER Performance of OFDM Amplify-and-Forward Relaying System with Subcarrier Permutation

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Abstract— **We perform error probability analysis of the uncoded OFDM fixed gain Amplify-and-Forward (AF) relaying system with subcarrier permutation (SCP). Two SCP schemes, named: the best-to-best SCP (BTB SCP) and the best-to-worst SCP(BTW SCP) are considered. Closed-form expressions for the bit error rate (BER) of the above SCP methods are derived. Numerical results manifest that these SCP schemes may outperform one another, depending on the average channel conditions of the links involved. That is, BTB SCP is better in the low signal-to-noise ratio (SNR), while BTW SCP prevails in the medium and high SNR regime. Thus, it could be concluded that OFDM AF relaying systems may switch from the BTB SCP to BTW SCP in order to achieve optimum BER performance.**

I. Introduction

Relaying systems have attracted a great research interest in recent years, due to their ability to increase coverage area and capacity of wireless systems. The basic realization assumes dual-hop relaying system, where a source terminal communicates with a destination one through a relay terminal. The relay terminal usually performs one of the two main relaying methods: Amplify-and-Forward (AF) or Decode-and-Forward (DF). In the AF relaying mode, the relay amplifies the received signal by employing fixed gain or variable gain, depending on its capability to estimate the source-relay channel. The DF mode assumes that the relay terminal performs decoding of the received signal, and re-encoding before further transmission.

The performance of single carrier relaying systems in different channel conditions, as well as for different relaying strategies, are widely examined in the literature [1]-[3]. Lately, multicarrier relaying systems using Orthogonal Frequency Division Multiplexing (OFDM) as transmission technique have attracted extensive attention. Due to a large number of proven good characteristics, OFDM is already accepted in many standardized wireless communication systems, and it is a candidate for the next generation of WLAN and WWAN systems [4].

The idea of subcarrier permutation (SCP) in OFDM AF relaying was first introduced in [5], and a little bit later was independently discussed in [6] and [7]. The author of [7] proves that, in special cases, when the signal received by a relay is noise-free, the system achieves maximum capacity if the subcarrier with the highest signal-to-noise (SNR) from the first hop is mapped to the subcarrier with the highest SNR on the second hop, second best - to second best, etc. As the subcarrier-based permutation significantly increase necessary signaling overhead, the author in [7] proposed to group adjacent subcarriers in chunks, and then the relay should perform chunk permutation according to the average chunk's SNRs. General proof that this kind of SCP maximizes achievable capacity and receive SNR in OFDM AF relaying was first presented in [8]. The capacity analysis, for the case of fixed gain AF relaying, is performed in [9] numerically, using the derived SNR probability density function (PDF). However, when the BER performance is taken into account, it is proven in [10] that the described SCP presents the best solution for OFDM AF relay system only in the low SNR region. By employing majorization theory the authors in [10] proved that the BER performance of the dual-hop OFDM variable gain AF relay system in the medium and high SNR regions can be improved by using the opposite SCP scheme, where the subcarrier with the highest SNR from the first hop is mapped to the subcarrier with the lowest SNR on the second hop, etc. Nonetheless, any analytical BER performance analysis for these two SCP schemes has not been reported to date.

In this paper, the BER performance of the best-to-worst (BTW) SCP and the best-to-best (BTB) SCP schemes, for the uncoded dual-hop OFDM AF relaying system, is investigated. Assuming that semi-blind fixed gain relaying is employed, closed form BER expressions are obtained for DPSK-modulated OFDM AF relaying in both BTW SCP and BTB SCP scenarios, while for the case of BPSK modulation tight approximations of the exact BERs for both SCP schemes are derived. It is worth mentioning that the same BER evaluation procedure used for BPSK modulation can also be used for any M-ary quadrature amplitude modulation (M-QAM). The analytical BER expressions are verified by computer simulations.

II. System Model

We consider an OFDM dual-hop relaying system with a source terminal S, a half-duplex relay terminal R, and a destination one D, all equipped with a single antenna. The relay terminal has FFT (Fast Fourier Transformation) and IFFT (Inverse Fast Fourier Transformation) blocks for OFDM demodulation and OFDM modulation, respectively. Furthermore, R has a block that performs subcarrier permutation, mapping the subcarriers from the first hop to subcarriers on the second

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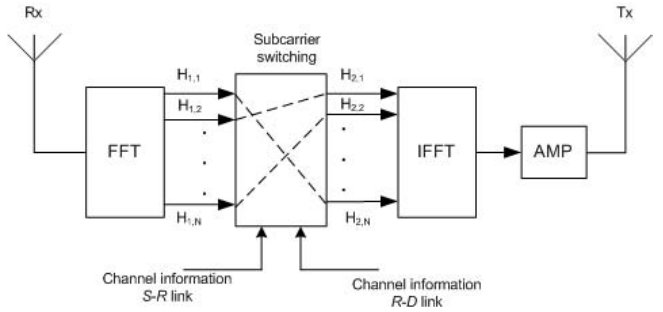

Fig. 1. Block diagram of the relay terminal

hop according to their transfer functions. The relay structure is illustrated in Fig. 1. It is assumed that R has a perfect channel knowledge of both S-R and R-D links, and D knows the permutation function performed at R. The post-FFT signal on the i-th subcarrier, received at the relay station, is given by

$$Y_{R,i} = H_{1,i}X_i + \mathcal{N}_{1,i}, \ \ 1 \le i \le N \tag{1}$$

where N is total number of subcarriers, $H_{1,i}$ is i-th subcarrier transfer function, and X_i is data symbol sent by source on the i-th subcarrier. $\mathcal{N}_{1,i}$ represents additive white Gaussian noise for the i-th subcarrier with variance $\mathbf{E}(|\mathcal{N}_{1,i}|^2) = \mathcal{N}_{0_1}$, where $\mathbf{E}(\cdot)$ denotes the expectation operator. The relay operates in the fixed-gain AF mode, where the signal that reaches the relay is amplified by a fixed gain, G. Assuming that the SCP function $\nu(i)$ at the relay station maps the i-th subcarrier from the first hop to the k-th subcarrier of the second hop, the signal at the destination can be presented in the frequency domain as

$$\begin{aligned} Y_{D,k} &= GH_{2,k}Y_{R,\nu(i)} + \mathcal{N}_{2,k} \\ &= GH_{2,k}H_{1,i}X_i + GH_{2,k}\mathcal{N}_{1,i} + \mathcal{N}_{2,k}, \end{aligned} \tag{2}$$

where $1 \le k \le N$ and $H_{2,k}$ denotes the k-th subcarrier transfer function on the second hop. $\mathcal{N}_{2,k}$ is the additive white Gaussian noise at the destination on the k-th subcarrier, with variance $\mathbf{E}(|\mathcal{N}_{2,k}|^2) = \mathcal{N}_{0_2}$.

Fadings in the S-R and R-D channels are assumed to be independent and identically distributed (i.i.d.) among the subcarriers. Moreover, we assume Rayleigh fading in each subcarrier, so that the PDF and the cumulative distribution function (CDF) of the SNR in each of the S-R subchannels is given by $f_{SR}(x) = \lambda_{SR}e^{-\lambda_{SR}x}$ and $F_{SR}(x) = 1 - e^{-\lambda_{SR}x}$, while the corresponding PDF and CDF of the SNR in each of the R-D subchannels are given by $f_{RD}(x) = \lambda_{RD}e^{-\lambda_{RD}x}$ and $F_{RD}(x) = 1 - e^{-\lambda_{RD}x}$, respectively. $\lambda_{SR} = 1/\overline{\gamma}_{SR}$ and $\lambda_{RD} = 1/\overline{\gamma}_{RD}$ denote the inverse of the average SNR on the S-R and R-D link, where $\overline{\gamma}_{SR} = \mathcal{E}_S\mathbf{E}\left(|H_{1,k}|^2\right)/\mathcal{N}_{0_1}$ and $\overline{\gamma}_{RD} = \mathcal{E}_R\mathbf{E}\left(|H_{2,k}|^2\right)/\mathcal{N}_{0_2}$, with $\mathcal{E}_S$ and $\mathcal{E}_R$ representing average symbol power per subcarrier transmitted by S and R, respectively. From (2) the end-to-end SNR on the k-th subcarrier can be written as [2]

$$\gamma_{k,end} = \frac{\frac{\mathcal{E}_S|H_{1,i}|^2}{\mathcal{N}_{0_1}}\frac{|H_{2,k}|^2}{\mathcal{N}_{0_2}}}{\frac{|H_{2,k}|^2}{\mathcal{N}_{0_2}} + \frac{1}{G^2\mathcal{N}_{0_1}}} = \frac{\gamma_{i,SR}\gamma_{k,RD}}{\gamma_{k,RD} + \mathcal{C}} \tag{3}$$

where $\mathcal{C}$ is a constant that depends on the relay gain G through $\mathcal{C} = \mathcal{E}_R/\left(G^2N_{01}\right)$.

III. Evaluation of the end-to-end SNR PDF

A. Statistics of the k-th subcarrier permutation

It is obvious from (3) that for the evaluation of the end-to-end SNR density function in BTW SCP and BTB SCP scenarios, the ordered statistics of N i.i.d. random variables need to be known. Let $f^w_{k,SR}(\cdot)$ denotes the PDF of the SNR of the k-th weakest subcarrier out of the N total ones in the S-R link. Using the example for ordered statistics of random variables, given in [11], $f^w_{k,SR}(\cdot)$ can be presented as:

$$\begin{aligned} f^w_{k,SR}(x) &= N\binom{N-1}{k-1}f_{SR}(x) \\ &\quad \times \left(F_{SR}(x)\right)^{k-1}\left(1 - F_{SR}(x)\right)^{N-k} \end{aligned} \tag{4}$$

where $\binom{\cdot}{\cdot}$ represents the binomial coefficient. For the analyzed scenario, having the i.i.d. subcarriers with Rayleigh fading, $f^w_{k,SR}(x)$ can be written as

$$\begin{aligned} f^w_{k,SR}(x) &= N\binom{N-1}{k-1}\lambda_{SR}e^{-\lambda_{SR}x} \\ &\quad \times \left(1 - e^{-\lambda_{SR}x}\right)^{k-1}e^{-\lambda_{SR}x(N-k)} \end{aligned} \tag{5}$$

After using the binomial expansion

$$\left(1 - e^{-\lambda_{SR}x}\right)^{k-1} = \sum_{i=0}^{k-1}(-1)^i\binom{k-1}{i}e^{-\lambda_{SR}xi} \tag{6}$$

(5) reduces to

$$f^w_{k,SR}(x) = \sum_{i=0}^{k-1}\lambda_{SR}\alpha_i e^{-\beta_i\lambda_{SR}x} \tag{7}$$

where α_i and β_i are given as

$$\alpha_i = (-1)^i N\binom{N-1}{k-1}\binom{k-1}{i} \tag{8}$$

$$\beta_i = i + N - k + 1. \tag{9}$$

Using a similar approach as that given in [11], the PDF for decreasing ordered random variables can be derived. As a result, the PDF of the k-th strongest subcarrier out of the N total ones in the R-D link can be represented as

$$\begin{aligned} f^s_{k,RD}(x) &= N\binom{N-1}{k-1}f_{RD}(x) \\ &\quad \times \left(F_{RD}(x)^{N-k}\right)\left[1 - F_{RD}(x)\right]^{k-1} \end{aligned} \tag{10}$$

After substituting the PDF and CDF functions corresponding to Rayleigh fading channels, $f^s_{k,RD}(x)$ can be expressed as

$$f^s_{k,RD}(x) = \sum_{i=0}^{N-k}\lambda_{RD}\delta_i e^{-\varepsilon_i\lambda_{RD}x} \tag{11}$$

where δ_i and ε_i are given as

$$\delta_i = (-1)^i N \binom{N-1}{k-1} \binom{N-k}{i} \quad (12)$$

$$\varepsilon_i = i + k. \quad (13)$$

B. End-to-end SNR PDF for BTW SCP

In the BTW SCP, the order of the channel gains associated with the subcarriers employed in the S-R channel determines the order of the subcarriers used in the R-D channel, so that the signal reaching the relay via the strongest subcarrier in the S-R channel is forwarded to the destination through the weakest subcarrier in the R-D channel etc. Assuming that the subcarriers in the S-R link are increasing ordered according to their transfer functions, and the subcarriers in the R-D link are decreasing ordered, the end-to-end SNR of the k-th subcarrier permutation is given as

$$\gamma_{k,end} = \frac{\gamma_{k,SR}\gamma_{k,RD}}{\mathcal{C} + \gamma_{k,RD}} = \frac{\gamma_{k,SR}}{1 + \mathcal{C} z_k} \quad (14)$$

where $z_k = 1/\gamma_{k,RD}$. The PDF of the new variable z_k is given as $f_{z_k}(z_k) = (1/z_k^2) f_{RD}^s (1/z_k)$. Using the transformation $y_k = 1 + \mathcal{C} z_k$, the PDF of the denominator in (14) is derived as

$$f_{y_k}(y_k) = \mathcal{C} \frac{\mathcal{U}_{\{y_k - 1\}}}{(y_k - 1)^2} \sum_{i=0}^{N-k} \lambda_{RD} \delta_i e^{-\varepsilon_i \lambda_{RD} \frac{\mathcal{C}}{y_k - 1}} \quad (15)$$

where $\mathcal{U}_{\{y_k-1\}}$ is the unitary step function, i.e., $\mathcal{U}_{\{y_k-1\}} = 1$ if $y_k \geq 1$ and zero otherwise. The PDF of $\gamma_{k,end}$ is then calculated as in [11]

$$f_{\gamma_{k,end}}^{BTW}(x) = \int_0^\infty y_k f_{\gamma_{SR},y}(x y_k, y_k)\, dy_k \quad (16)$$

with $f_{\gamma_{SR},y}(x y_k, y_k) = f_{k,SR}^w (x y_k) f_{y_k}(y_k)$ representing the joint PDF of the independent random variables γ_{SR} and y_k, and $f_{k,SR}^w(x y_k)$ is given in (7). It follows that

$$f_{\gamma_{k,end}}^{BTW}(x) = \int_0^\infty y_k f_{k,SR}^w (x y_k) f_{y_k}(y_k)\, dy_k \quad (17)$$
$$= \sum_{j=0}^{k-1} \sum_{i=0}^{N-k} \lambda_{SR} \lambda_{RD} \alpha_j \delta_i \mathcal{C}$$
$$\times \int_1^\infty \frac{y_k \exp\left(-\lambda_{SR} \beta_j x y_k - \lambda_{RD} \varepsilon_i \frac{\mathcal{C}}{y_k - 1}\right)}{(y_k - 1)^2} dy_k$$

Using the transformation $m_k = 1/(y_k - 1)$ and [12, eq. (3.471.12)], (17) reduces to

$$f_{\gamma_{k,end}}^{BTW}(x) = \frac{2}{\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{N-k} \alpha_j \delta_i e^{-\beta_j \frac{x}{\overline{\gamma}_{SR}}} \quad (18)$$
$$\times \left[\sqrt{\frac{\mathcal{C} \beta_j x}{\varepsilon_i \overline{\gamma}_{SR} \overline{\gamma}_{RD}}} K_1 \left(2 \sqrt{\frac{\mathcal{C} \beta_j \varepsilon_i x}{\overline{\gamma}_{SR} \overline{\gamma}_{RD}}} \right) + \frac{\mathcal{C}}{\overline{\gamma}_{RD}} K_0 \left(2 \sqrt{\frac{\mathcal{C} \beta_j \varepsilon_i x}{\overline{\gamma}_{SR} \overline{\gamma}_{RD}}} \right) \right]$$

where $K_0(\cdot)$ and $K_1(\cdot)$ are zero and first order modified Bessel functions of the second kind defined in [13, eqs. (9.6.21), (9.6.22)].

C. End-to-end SNR PDF for BTB SCP

In the BTB SCP scheme the strongest subcarrier from the first hop is mapped to the strongest subcarrier in the second hop, etc. Considering that the subcarriers in the S-R and R-D links are increasing ordered, which implies that their SNR PDFs are given as (7), the PDF of SNR for the BTB SCP is obtained as

$$f_{\gamma_{k,end}}^{BTB}(x) = \frac{2}{\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{k-1} \alpha_j \alpha_i e^{-\beta_j \frac{x}{\overline{\gamma}_{SR}}}$$
$$\times \left[\sqrt{\frac{\mathcal{C} \beta_j x}{\beta_i \overline{\gamma}_{SR} \overline{\gamma}_{RD}}} K_1 \left(2 \sqrt{\frac{\mathcal{C} \beta_j \beta_i x}{\overline{\gamma}_{SR} \overline{\gamma}_{RD}}} \right) + \frac{\mathcal{C}}{\overline{\gamma}_{RD}} K_0 \left(2 \sqrt{\frac{\mathcal{C} \beta_j \beta_i x}{\overline{\gamma}_{SR} \overline{\gamma}_{RD}}} \right) \right] \quad (19)$$

IV. Performance analysis

A. Average BER for BTW SCP

Having the PDF of the SNR for the k-th subcarrier pair in BTW SCP (18), the moment generating function (MGF) of $\gamma_{k,end}$ can be derived as

$$\mathcal{M}_{\gamma_{k,end}}(s) = \mathbf{E}(e^{-s\gamma}) = \int_0^\infty e^{-s\gamma} f_{\gamma_{k,end}}^{BTW}(\gamma)\, d\gamma \quad (20)$$

After some basic transformations, a closed-form solution for the integral in (20) can be found in terms of the Whittaker function, by using [12, eqs. (6.614.4), (6.631.3)]. The obtained MGF can be expressed using the more common exponential integral function, with the help of the identities [13, eqs. (13.6.28), (13.6.30), (6.5.19)]

$$\mathcal{M}_{\gamma_{k,end}}(s) = \frac{1}{\overline{\gamma}_{SR}} \sum_{j=0}^{k-1} \sum_{i=0}^{N-k} \frac{\alpha_j \delta_i}{T_j(s)} \left[\frac{1}{\varepsilon_i} + e^{\frac{\mathcal{C} B_{j.i}}{T_j(s)}} \times E_1\left(\frac{\mathcal{C} B_{j,i}}{T_j(s)} \right) \left(\frac{\mathcal{C}}{\overline{\gamma}_{RD}} - \frac{\mathcal{C} B_{j,i}}{\varepsilon_i T_j(s)} \right) \right] \quad (21)$$

where $T_j(s) = s + \beta_j / \overline{\gamma}_{SR}$, $B_{j,i} = \beta_j \varepsilon_i / \overline{\gamma}_{SR} \overline{\gamma}_{RD}$ and $E_1(\cdot)$ is the exponential integral function defined in [13, eq. (5.1.1)].

Using the MGF expression in closed-form as shown in (21), the BER performance evaluation for different digital modulations over fading channels can be conducted via the MGF-based approach [14]. For example, the k-th subcarrier pair BER for the differential phase shift-keying (DPSK) is $P_{b,k} = 0.5 \mathcal{M}_{\gamma_k, end}(1)$, and for the OFDM system the total average BER is obtained through

$$P_b = \frac{1}{N} \sum\nolimits_{k=1}^{N} P_{b,k} \quad (22)$$

However, for coherent modulations, such as M-QAM or M-ary phase shift keying (M-PSK), the derivation of the exact closed form BER expression is more complex, as the integrand includes the exponential integral function. Therefore, in order to obtain a closed-form approximate BER expression for BPSK we use a PDF-based approach, and the approximation of the complementary error function erfc$(\cdot)$ given in [15].

The average BER for the k-th subcarrier permutation in the BTW SCP scheme is given as

$$P_{b,k} = \int_0^\infty P_{b|\gamma_k} f^{BTW}_{\gamma_{k,end}}(\gamma) d\gamma \tag{23}$$

with $P_{b|\gamma_k}$ denoting modulation dependent conditional BER. For BPSK signalling $P_{b|\gamma_k} = Q(\sqrt{2\gamma})$, where $Q(\cdot)$ is the Gaussian Q function which can be expressed in terms of erfc$(\cdot)$ as $Q(x) = 0.5\text{erfc}(x/\sqrt{2})$ [11]. Using the approximation of erfc$(\cdot)$ [15]

$$\text{erfc}(x) \simeq \frac{1}{6}e^{-x^2} + \frac{1}{2}e^{-4x^2/3} \tag{24}$$

the integral in (23) becomes

$$P_{b,k} = \frac{1}{2}\int_0^\infty \left(\frac{1}{6}e^{-\gamma} + \frac{1}{2}e^{-\frac{4\gamma}{3}}\right) f^{BTW}_{\gamma_{k,end}}(\gamma)\, d\gamma. \tag{25}$$

Substituting (18) in (25) and using the integrals [12, eqs. (6.614.4), (6.631.3)], as well as the identities [13, eqs. (13.1.33), (13.6.28), (13.6.30), (6.5.19)], a closed-form expression is obtained in terms of $E_1(\cdot)$

$$\begin{aligned} P_{b,k} = \frac{1}{2\overline{\gamma}_{SR}} \sum_{j=0}^{k-1}\sum_{i=0}^{N-k} \alpha_j \delta_i \Bigg[\frac{1}{2\varepsilon_i}\left(\frac{1}{3T_j(1)} + \frac{1}{T_j(4/3)}\right) \\ + \frac{1}{6}\frac{e^{\frac{\mathcal{C}B_{j,i}}{T_j(1)}}}{T_j(1)} E_1\left(\frac{\mathcal{C}B_{j,i}}{T_j(1)}\right)\left(\frac{\mathcal{C}}{\overline{\gamma}_{RD}} - \frac{\mathcal{C}B_{j,i}}{2\varepsilon_i T_j(1)}\right) \\ + \frac{e^{\frac{\mathcal{C}B_{j,i}}{T_j(4/3)}}}{2T_j(4/3)} E_1\left(\frac{\mathcal{C}B_{j,i}}{T_j(4/3)}\right)\left(\frac{\mathcal{C}}{\overline{\gamma}_{RD}} - \frac{\mathcal{C}B_{j,i}}{2\varepsilon_i T_j(4/3)}\right)\Bigg] \end{aligned} \tag{26}$$

where $T_j(1) = 1 + \frac{\beta_j}{\overline{\gamma}_{SR}}$ and $T_j(4/3) = \frac{4}{3} + \frac{\beta_j}{\overline{\gamma}_{SR}}$. The average BER is then calculated as $P_b = \frac{1}{N}\sum_{k=1}^{N} P_{b,k}$.

The same BER derivation approach can be implemented for obtaining BER expressions for M-QAM modulations, having that the conditional BER for M-QAM modulations is generally given as $P_{b|\gamma} = \sum_{i=1}^{M} \mathcal{A}_i Q(\sqrt{\mathcal{B}_i\gamma})$, where $\mathcal{A}_i$ and $\mathcal{B}_i$ are modulation dependent constants [16].

B. Average BER for the BTB SCP

Using (19) and the same procedure implemented for the MGF derivation in the case of the BTW SCP, the MGF for the BTB SCP is obtained as

$$\begin{aligned} \mathcal{M}_{\gamma_{k,end}}(s) = \frac{1}{\overline{\gamma}_{SR}} \sum_{j=0}^{k-1}\sum_{i=0}^{k-1} \frac{\alpha_j\alpha_i}{T_j(s)} \Bigg[\frac{1}{\beta_i} + e^{\frac{\mathcal{C}A_{j,i}}{T_j(s)}} \\ \times E_1\left(\frac{\mathcal{C}A_{j,i}}{T_j(s)}\right)\left(\frac{\mathcal{C}}{\overline{\gamma}_{RD}} - \frac{\mathcal{C}A_{j,i}}{\beta_i T_j(s)}\right)\Bigg] \end{aligned} \tag{27}$$

where $A_{j,i} = \beta_j\beta_i/\overline{\gamma}_{SR}\overline{\gamma}_{RD}$.

Following the same steps as the ones used for the BER derivation for BTW SCP, a BER expression for the k-th subcarrier permutation in the BTB SCP scenario and BPSK modulation is evaluated as

$$\begin{aligned} P_{b,k} = \frac{1}{2\overline{\gamma}_{SR}} \sum_{j=0}^{k-1}\sum_{i=0}^{k-1} \alpha_j \beta_i \Bigg[\frac{1}{2\beta_i}\left(\frac{1}{3T_j(1)} + \frac{1}{T_j(4/3)}\right) \\ + \frac{1}{6}\frac{e^{\frac{\mathcal{C}A_{j,i}}{T_j(1)}}}{T_j(1)} E_1\left(\frac{\mathcal{C}A_{j,i}}{T_j(1)}\right)\left(\frac{\mathcal{C}}{\overline{\gamma}_{RD}} - \frac{\mathcal{C}A_{j,i}}{2\beta_i T_j(1)}\right) \\ + \frac{e^{\frac{\mathcal{C}A_{j,i}}{T_j(4/3)}}}{2T_j(4/3)} E_1\left(\frac{\mathcal{C}A_{j,i}}{T_j(4/3)}\right)\left(\frac{\mathcal{C}}{\overline{\gamma}_{RD}} - \frac{\mathcal{C}A_{j,i}}{2\beta_i T_j(4/3)}\right)\Bigg] \end{aligned} \tag{28}$$

V. Numerical Results

The subsequent presented analytical and simulation results assume perfectly synchronized OFDM AF relaying system with implemented SCP. The OFDM system has N=16 subcarriers, which in a real scenario can be considered as 16 chunks with uncorrelated transfer functions from chunk to chunk. It is also assumed that $\mathcal{E}_R = \mathcal{E}_S$ and $\mathcal{N}_{0_1} = \mathcal{N}_{0_2}$, so that $\overline{\gamma}_{SR} = \overline{\gamma}_{RD}$. The relay gain G is calculated assuming that the average subcarrier symbol power transmitted by the relay is $\mathcal{E}_R$ and using the relay's knowledge of the average fading power on the S-R link

$$G^2 = \mathbf{E}\left[\frac{\mathcal{E}_R}{\mathcal{E}_S|H_{1,k}|^2 + \mathcal{N}_{0_1}}\right] \tag{29}$$

yielding [2]

$$G^2 = \frac{\mathcal{E}_R}{\mathcal{E}_S\Omega_1} e^{1/\overline{\gamma}_{SR}} E_1\left(\frac{1}{\overline{\gamma}_{SR}}\right) \tag{30}$$

where $\Omega_1 = \mathbf{E}[|H_{1,k}|^2]$.

Simulation results are obtained through Monte Carlo simulations, where only the frequency domain part of the analyzed system is taken in consideration, as it is assumed to be perfectly synchronized. The subcarrier transfer functions on the first and second hop are generated as independent complex Gaussian random variables with zero mean and variance 1/2, meaning that the average subcarrier power is equal to 1. Ten OFDM symbols are transmitted through each channel realization.

Fig. 2 presents the BER performance of DPSK modulated OFDM AF relay system for the BTW SCP and the BTB SCP. For the sake of comparison, the BER performance of the OFDM AF relay system without (w/o) SCP is presented. The BER for the AF system w/o SCP is analytically obtained using the MGF derived in [2]. The obtained analytical results are completely verified by simulations. As expected, for the low SNR values BTB SCP achieves the best BER performance. It outperforms BTW SCP system up to the SNR value of 6.5dB approximately, and the system w/o SCP up to the SNR value of 7.5dB. For SNR values above 6.5 dB, the BTW SCP scheme has the lowest BER and its advantage in BER performance increases very fast as the SNR values increase. It already achieves more than 1dB SNR gain over the system w/o SCP and almost 2dB SNR gain over the BTB SCP scheme, for the BER value of 10^{-1}.

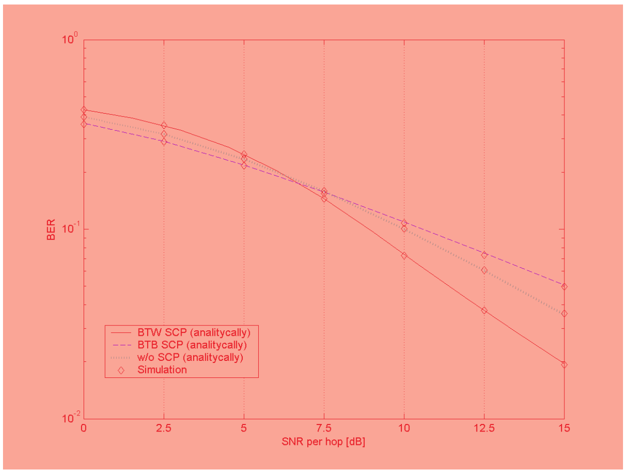

Fig. 2. BER of DPSK modulated OFDM AF system with SCP

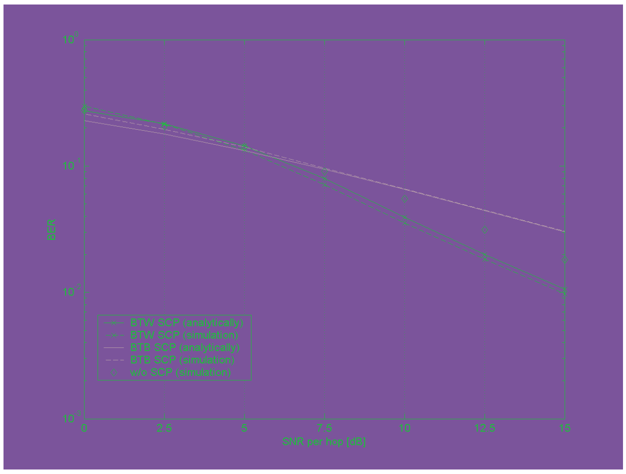

Fig. 3. BER of BPSK modulated OFDM AF system with SCP

BER results of the BPSK modulated OFDM AF system with BTW SCP, BTB SCP and w/o SCP schemes are given in Fig. 3. The analytically obtained BER results for both SCP schemes present a tight approximation of the exact BER values. The small difference compared to the exact results is expected, as the approximation of $\mathrm{erfc}(\cdot)$ is used in averaging the BER over the region of all SNR values. Considering that simulation results are the exact ones, it can be seen that the intersection point for the BER graphs of BTW SCP and BTB SCP is now approximately 4dB, meaning that after that point the BTW SCP scheme achieves significantly lower BER. For example, the SNR gain is more than 1dB for a BER of 10^{-1}. Note that for SNR=15dB, the BTB SCP scheme attains a BER of $3 \cdot 10^{-2}$, whereas the BER for the BTW SCP scheme is 10^{-2}, i.e., three times lower.

Having the presented BER results in mind, it is obvious that OFDM AF relaying systems may switch from the BTB SCP to BTW SCP scheme depending on the average SNR in S-R and R-D links. Apparently, this hybrid SCP scheme is expected to achieve optimum BER performance.

VI. Conclusions

We performed an error probability analysis for the two OFDM fixed gain amplify-and-forward (AF) relaying schemes, namely best-to-worst (BTW) subcarrier permutation (SCP) and best-to-best (BTB) SCP. Closed-form expressions for the bit error rate (BER) were derived, which is, to the best of author's knowledge, the first time that this problem is analytically treated up to date. In terms of BER, it has been shown that the BTB SCP scheme outperforms BTW SCP in the low SNR regime, while in the medium and high SNR region, the relative performance of BTB SCP and BTW SCP are reversed, in the sense that BTW SCP yields lower BER. It has also been pointed out that, in order to optimize the BER performance, the OFDM AF relaying system may switch from one SCP scheme to another, depending on the average S-R and R-D channel conditions.

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