# Minimum Error Probability Cooperative Relay Design

Bin Liu, Biao Chen and Rick S. Blum

***Abstract*— In wireless networks, user cooperation has been proposed to mitigate the effect of multipath fading channels. Recognizing the connection between cooperative relay with *finite alphabet* sources and the distributed detection problem, we design relay signaling via channel aware distributed detection theory. Focusing on a wireless relay network composed of a single source-destination pair with $L$ relay nodes, we derive the necessary conditions for optimal relay signaling that minimizes the error probability at the destination node. The derived conditions are person-by-person optimal: each local relay rule is optimized by assuming fixed relay rules at all other relay nodes and fixed decoding rule at the destination node. An iterative algorithm is proposed for finding a set of relay signaling approaches that are simultaneously person-by-person optimal. Numerical examples indicate that the proposed scheme provides performance improvement over the two existing cooperative relay strategies, namely amplify-forward and decode-forward.**

***Index Terms*— Wireless relay network, cooperative relay, finite alphabet, decentralized detection.**

## I. INTRODUCTION

In wireless networks, a severe limiting factor is multipath induced channel fading. One of the most effective methods in mitigating fading is to exploit diversity. Examples include spatial diversity when multiple antennas are used at the transceivers, multipath diversity in frequency selective channels, and temporal diversity in time selective fading channels through the use of coding/interleaving. More recently, a new diversity resource has attracted considerable attention, especially in the context of wireless *ad hoc* networks [1]–[3]. There, multiple nodes collaborate in transmitting their information, thus providing diversity by exploiting the independence of the fading channels of different users. This is generally referred to as the cooperative diversity, and the collection of cooperating nodes, including the source and the destination nodes, are referred to as a relay network.

Historically, study of relay networks has focused on the capacity issue, e.g., achievable rates. The classical three-node relay network was first introduced by van der Meulen [4] and its capacity was extensively studied by Cover and El Gamal [5]. Gastpar and Vetterli [6] considered the capacity of wireless networks with multiple relay nodes and showed that the lower and upper bounds became the same asymptotically as the number of nodes in the network goes to infinity. Sendonaris *et al* [1], [2] were the first to introduce the concept of *user cooperation diversity* where the mobile users shared their antennas and other resources to obtain diversity gain through distributed transmission. Focusing on a two user case, it was shown that user cooperation results in an increase in capacity for both users. In addition, the achievable rates are less susceptible to channel variations, making the cooperative network a more robust system. Kramer *et al* considered several coding strategies for various relay networks in [7] and showed that a strategy that mixes decode-forward and compress-forward achieves capacity if the terminals form two closely-spaced clusters.

The performance of wireless relay networks has also been evaluated by diversity gain and outage probability. By constraining the nodes to half-duplex mode, Laneman *et al* [3] developed various cooperative transmission protocols and showed that most of the protocols achieve full diversity order (equal to the number of cooperative nodes). Space-time code-based cooperative transmission protocols were developed in [8] and were also shown to achieve full diversity. In [9], [10], symbol error probabilities were derived in the high signal-to-noise ratio (SNR) regime for the general multi-hop, multi-branch wireless relay model using the amplify-forward (AF) scheme; the result provides insight on the optimum placement of relay nodes. Chen and Laneman [11] focused on the decode-forward (DF) scheme and developed a general framework for maximum likelihood (ML) demodulation in cooperative wireless communication systems.

In this paper, we focus on a relay network consisting of a single source-destination pair and $L$ relay nodes. As illustrated in Fig. 1, each relay node receives the signal from the source node and generates a processed signal based on its received signal. The processed signals from all the relay nodes are sent to the destination node using orthogonal channels. The destination node uses the relay signals along with the signal sent directly from the source node to determine the source signal. Novel in the current work is the attempt to find channel aware processing that *minimizes the error probability at the destination node*. The proposed design approach exploits the finite-alphabet (FA) property of the source message, thereby enabling us to pose the cooperative relay design as a distributed multiple hypotheses testing problem. Notice that this FA property is ubiquitous in almost all wireless systems. A similar idea has been explored in [12] to study a diversity combining scheme using the quantized outputs from multiple antennas with independently faded binary frequency shift keying (BFSK) signals. Distinctive in the current work, in addition to considering a general FA source instead of BFSK, is that the relay outputs are assumed to also go through general non-ideal channels. Our approach is to generalize the channel aware distributed signaling design for binary hypothesis testing problem [13], [14] to this cooperative relay problem and derive

Bin Liu and Biao Chen are with Syracuse University, Department of EECS, 335 Link Hall, Syracuse, NY 13244. Email: biliu{bichen}@ecs.syr.edu. Rick S. Blum is with Lehigh University, Department of ECE, 304 Packard Lab, 19 Memorial Drive West, Bethlehem, PA 18015. Email: rblum@eecs.lehigh.edu.
This work was presented in part at the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Philadelphia, PA, March 2005. This work was supported in part by the National Science Foundation under Grants ECS-0501534 and CCR-0112501, and by the Air Force Research Laboratory under aggreement No. F49620-03-1-0214 and FA8750-05-2-0120.

a numerical procedure to compute the optimal local relay rules for minimum error probability at the fusion center.

While DF also utilizes the FA property, the proposed approach is based optimum detection theory and thus provides superior error probability performance. To motivate our proposed idea, we consider a simple relay network with one source-destination pair and two relay nodes. We also assume a parallel relay scheme where there is no direct transmission between the source node and the destination node. The source is binary with repetition coding; i.e., one transmits "$+1+1+1+1$" or "$-1-1-1-1$" where the redundancy is used to combat channel impairment. We also restrict each relay node to send a four-bit sequence to the destination node. If we adopt a DF idea, each relay node attempts to recover the original binary source and resends it to the destination node. However, for this simple example, it will be seen that the optimum relay rule amounts to quantizing the local likelihood ratio; and better performance may result if one uses all possible output alphabet at the relay for the quantization. Contrasting this to the DF approach, one can consider our approach as using 'soft' information from the relays as opposed to hard decisions for DF. As such, applying the distributed detection theory allows us to fully exploit the redundancy in the FA sources for improved detection performance.

Even without the redundancy in the FA sources, jointly designing the relay and destination signaling can still result in improved performance compared with DF. Consider, for example, a simple case that the source signals are either "$+1$" or "$-1$". The relay nodes are also restricted to transmit a binary ("$+1$" or "$-1$") signal to the destination node. Assume that the channels between the source and the two relay nodes have identical channel SNRs, while SNRs of the channels between the two relays and the destination differ significantly from each other. One natural question is: how do we jointly determine the relay and destination processing/signaling that may minimize the error probability at the destination node? Clearly, if one resorts to the DF idea, each relay will try to recover the original signal and retransmit it to the destination node. As such, one can immediately conclude that this idea leads to identical relay rules at the two relay nodes. On the other hand, as the channels between the relays and the destination have different SNRs, should one design the processing/signaling differently for better performance. As demonstrated in Section IV, the optimum relaying for minimum error probability indeed uses different signaling at the two relays. Our goal is to come up with a mechanism to find out the optimal relay signaling.

The proposed cooperative relay signaling design assumes a clairvoyant case, i.e., the designer knows the global channel state information (CSI). While this is unrealistic, it provides important benchmark performance and reveals a significant gap in terms of error probability performance between what is achievable with the existing schemes and what is achievable theoretically. More importantly, the insight one draws from this clairvoyant case study may prove critical in devising cooperative signaling scheme under a more realistic setting with only distributed CSI knowledge (i.e., each relay node knows only its own CSI).

The rest of the paper is organized as follows: Section II describes the system model and the problem formulation. The problem setting allows us to derive, in Section III, the necessary conditions for optimal cooperative relay strategies at relay nodes to minimize the error probability at the destination node. In the same section, we also consider several special models and including the three-node relay network, the parallel relay model, and the singular relay network. Numerical examples are presented in Section IV to show the substantial performance gain of our approach over two existing relay strategies. We conclude in Section V.

## II. Statement of the Problem

Consider a wireless relay network which includes one source node, $L$ relay nodes and one destination node (Fig. 1). The data transmission is divided into two steps. In the first step, the source node broadcasts a signal $S$ to all the relay nodes as well as the destination node. In the second step, the relay nodes then transmit the relay signals to the destination node in orthogonal channels. We assume that $S$ is drawn from an FA set $\mathcal{S}=\{s_0,\cdots,s_{M-1}\}$ with prior probabilities $\{\pi_0,\cdots,\pi_{M-1}\}$. Further, the received signals $X_1,\cdots,X_L$ at the relays and the received signal $Z$ at the destination, which describe the broadcast channel during the first step, are characterized by

$$p(X_1,\cdots,X_L,Z|S)=p(Z|S)\prod_{l=1}^{L}p(X_l|S), \qquad (1)$$

i.e., $X_l$ and $Z$ are conditionally independent given $S$. Here the transmitted signal $S$ can be a vector, and the received signal $X_l$ and $Z$ would have a similar structure. The $l^{th}$ relay node sends a relay signal $U_l$ to the destination node based on its received signal $X_l$,

$$U_l=\gamma_l(X_l) \qquad l=1,\cdots,L. \qquad (2)$$

We assume that, without loss of generality, $U_l$ belongs to a FA set $\mathcal{T}=\{u_0,u_1,\cdots,u_{N-1}\}$. While it may appear natural to require $N=M$, as in the case of DF, we can accommodate $N\neq M$ in the proposed scheme. Indeed, as to be seen later, allowing $N\neq M$ is advantageous as it provides flexibility in the relay signaling design. We note here that the condition $N\neq M$ need not necessarily mean that the source sequence and the relay message have different lengths. Redundancy is typically built into the source sequence (e.g., channel coding), while the relay node may exploit all possible alphabets, as illustrated in the example in Section I. The relay outputs $U_1,\cdots,U_L$ are also sent through parallel transmission channels characterized by

$$p(Y_1,\cdots,Y_L|U_1,\cdots,U_L)=\prod_{l=1}^{L}p(Y_l|U_l) \qquad (3)$$

Note that all the signals, including $S$, $Z$, $X_l$, $Y_l$ and $U_l$, are assumed to be vectors.

Upon collecting the channel outputs from the relay nodes, $\mathbf{y}=\{Y_1,\cdots,Y_L\}$, and from the source node, $Z$, the destination node makes a final decision

$$U_0=\gamma_0(\mathbf{y},Z) \qquad (4)$$

where $U_0 \in \{s_0, \cdots, s_{M-1}\}$ indicates which signal was sent from the source node.

An error happens if $U_0 \neq S$. The goal is, therefore, to jointly design the local relay schemes $\gamma_l(\cdot), l = 1, \cdots, L$ and the decoding rule $\gamma_0(\cdot)$ such that the overall error probability at the destination node, $P(U_0 \neq S)$, is minimized. From the distributed detection point of view, this relay system can be regarded as an $M$-ary hypotheses testing system with each hypothesis corresponding to one of the input alphabet symbols; i.e., $H_i : S = s_i$. Given independence among the transmission channels, the signals received at relay nodes are independent conditioned on the input source, or equivalently, a given hypothesis. Thus, the joint probability density function (*pdf*) of the signals received at the relays becomes

$$p(X_1, \cdots, X_L|H_i) = \prod_{l=1}^{L} p(X_l|H_i), \qquad i = 0, \cdots, M-1. \tag{5}$$

Similarly, for the signals received at the destination node, the joint *pdf* conditioned on the decision made at the relays is

$$p(Y_1, \cdots, Y_L, Z|U_1, \cdots, U_L, H_i) = p(Z|H_i) \prod_{l=1}^{L} p(Y_l|U_l), \qquad i = 0, \cdots, M-1 \tag{6}$$

We point out here that integrating the transmission channels into the decoding rule design has been investigated before in the context of decision fusion in fading channels for wireless sensor networks (WSN) [15]–[17]. The optimal decoding rule in the Bayesian sense amounts to the maximum *a posteriori* probability (MAP) decision; i.e.,

$$U_0 = \gamma_0(\mathbf{y}, Z) = \arg \max_{s_i : i \in \{0,1,\cdots,M-1\}} \pi_i p(\mathbf{y}, Z|H_i). \tag{7}$$

Given a specified set of local relay strategies and the channel characteristics, this MAP decision rule can be obtained in a straightforward manner. As such, in the next section, we will focus on the local relay signaling design.

We close this section with a summary of the cooperative relay design problem.

**Problem statement**

In a wireless relay network as described in Fig. 1, given

- a FA source $\mathcal{S} = \{s_0, \cdots, s_{M-1}\}$ with prior probabilities $\{\pi_0, \cdots, \pi_{M-1}\}$,
- the channels from the source to relay nodes described by $p(X_l|S)$ for $l = 1, \cdots, L$,
- the channels from the relay nodes to the destination node described by $p(Y_l|U_l)$ for $l = 1, \cdots, L$,
- the channel from the source to destination node described by $p(Z|S)$,
- and a decoding rule $\gamma_0(\cdot)$ at the destination node,

design the local relay rules $\gamma_l(\cdot)$ for $l = 1, \cdots, L$ that minimize the overall error probability at the destination node $Pr(U_0 \neq S)$.

## III. Optimal Local Relay Strategies

This is a joint optimization problem. In order to obtain a globally optimal scheme, we should simultaneously optimize the local relay schemes at all the relay nodes. This joint optimization, however, is not feasible due to the distributed nature of the problem [18]. In this paper, we adopt a person-by-person optimal (PBPO) approach, i.e., we optimize the local relay rule $\gamma_l(\cdot)$ for the $l^{th}$ relay node given fixed relay rules at all other relay nodes and a fixed decoding rule $\gamma_0(\cdot)$ at the destination node. As such, the conditions obtained are necessary, but not sufficient, for optimality. This PBPO approach has been widely adopted in various distributed inference problems (see, e.g., [19], [20]).

Define

$$\begin{aligned} \mathbf{u} &= [U_1, U_2 \cdots, U_L], \\ \mathbf{x} &= [X_1, X_2, \cdots, X_L], \end{aligned}$$

so that the error probability at the destination node can be written as

$$P_e \triangleq 1 - P_D = 1 - \int_{X_l} \sum_{j=0}^{N-1} P(U_l = u_j|X_l) D_{lj} dX_l \tag{8}$$

where, for $l = 1, \cdots, L$, $j = 0, \cdots, N-1$

$$D_{lj} = \sum_{i=0}^{M-1} \pi_i P(U_0 = s_i|U_l = u_j, H_i) p(X_l|H_i) \tag{9}$$

and

$$P(U_0 = s_i|U_l = u_j, H_i) = \int_Z \int_{\mathbf{y}} P(U_0 = s_i|\mathbf{y}, Z) p(Z|H_i) p(\mathbf{y}|U_l = u_j, \ldots \tag{10}$$

Equations (8) - (10) can be obtained by expanding the error probability with respect to the $l^{th}$ relay rule $\gamma_l(\cdot)$. The derivation is straightforward and follows the same spirit as that in [13], hence we skip the details.

Thus, to minimize $P_e$, or equivalently maximize $P_D$, we set $P(U_l = u_{j^*}|X_l) = 1$ where $j^*$ is the index that maximizes $D_{lj}(X_l)$. Hence we have,

*Theorem 1:* The optimal relay rule for the $l^{th}$ relay node must satisfy

$$U_l = \gamma_l(X_l) = \arg \max_{u_j : j \in \{0,1,\cdots,N-1\}} D_{lj}(X_l) \tag{11}$$

for $D_{lj}(\cdot)$ defined in (9).

The major issue of Theorem 1 is to evaluate $D_{lj}(\cdot)$. While it is possible to evaluate it analytically for some special cases, in general it requires numerical evaluation which is fairly straightforward.

As expressed in (9) and (10), given the fixed local relay rules of the other relay nodes, $p(\mathbf{y}|U_l = u_j, H_i)$, and the decoding rule at the destination node, $P(U_0 = s_i|\mathbf{y}, Z)$, $D_{lj}(\cdot)$ only depends on the local observations at the $l^{th}$ relay node and is a linear combination of the likelihood function of the local observations. Following the definition of likelihood ratio quantizer (LRQ) for multiple hypotheses testing [21], the optimal local relay rule as described in Theorem 1 is a LRQ.

An important distinction between the current work and that of [22] is that we are considering an $M$-ary hypotheses testing problem with general input (e.g., vector input such as a packet). As such, one does not have the luxury of equating the local relay rule to a scalar quantization problem; instead, one needs to quantize a $(M-1)$-dimensional sufficient statistic [23]. Thus convergence checking by comparing relay rules is generally not viable.

The fact that we use the PBPO criterion implies that the derived conditions are only necessary but not sufficient conditions for optimality. Recognizing that the necessary conditions for the relay function $\gamma_l(\cdot)$ is coupled with the decoding rule, we propose the following iterative algorithm to find the relay and decoding rules that are at least locally optimum.

Iterative algorithm

1) Initialize the local relay strategies for each relay node $\gamma_l^{(0)}, l = 1, \cdots, L$ and set the iteration index $r = 1$;
2) Obtain the optimal decoding rule $\gamma_0^{(r)}$ using (7) for fixed local relay rules $\gamma_l^{(r-1)}$, $l = 1, \cdots, L$;
3) For each $l$, obtain the PBPO local relay rule $\gamma_l^{(r)}$ of $l^{th}$ relay node using (11) given the fixed local relay rules for the other relay nodes and fixed decoding rule;
4) Evaluate the error probability $P_e^{(r)}$ at the destination node given the relay rules $\boldsymbol{\gamma}^{(r)} = \{\gamma_1^{(r)}, \cdots, \gamma_L^{(r)}\}$ and decoding rule $\gamma_0^{(r)}$, and compare it with $P_e^{(r-1)}$. If the difference is less than a prescribed value, stop. Otherwise, set $r = r + 1$ and go to Step 2.

For each iteration, we optimize one rule given that the other rules are fixed. Therefore, the error probability is guaranteed to be non-increasing after each step. Thus the algorithm always converges as the error probability is lower bounded by zero.

### A. *Special cases*

The relay network described in Fig. 1 is rather general; it encompasses many special cases. For example, setting $L = 1$ reduces it to the classical three-node relay network; and the corresponding optimum decoding rule and optimal local relay rule can be obtained by letting $L = 1$ in (7) and (11). While this three-node network is not materially different from the general case, it does significantly reduces the computational complexity. Since there is a single relay node, there is no iteration among the relay rules. Instead, one only needs to iterate between the decoding rule and the relay rule.

Another interesting case is the parallel relay network where there is no direct transmission from the source node to the destination node. Following the same spirit of the derivation in Section III, we can easily get the optimal decoding rule and optimal relay rule which are similar to (7) and (11) except that $Z$ is omitted from the expression.

We now consider the simplest possible relay system: there is only a single ($L = 1$) relay node and there is no direct link between the source and the destination node. Notice that this simple model can be considered as a special case of either the three-node relay model or the parallel network. We term this as a *singular relay network*. In the context of channel optimized quantizer design for WSN, we have shown in [14], [22] that for $M = 2$ (i.e., a binary source), the optimum relay rule for a singular relay network is channel-blind; i.e., the local relay rule will remain unchanged when the relay-destination channel characteristics change. For this special case, the local relay rule is the same as that in the case with ideal relay-destination channel as this ideal channel can be treated as a limiting case of the fading channel. We show in the following that it is not true for the general case of $M > 2$; that is, for a singular relay network with a general FA source, the relay signaling should always be channel aware.

By setting $L = 1$ in (7) and (11) and omitting $Z$, we can easily obtain the decoding rule

$$U_0 = \gamma_0(Y) = \arg \max_{s_i : i \in \{0,1,\cdots,M-1\}} \pi_i p(Y|H_i) \tag{12}$$

and local relay rule

$$u = \gamma(X) = \arg \max_{u_j : j \in \{0,1,\cdots,N-1\}} D_j(X) \tag{13}$$

where

$$D_j(X) = \sum_{i=0}^{M-1} \pi_i P(U_0 = s_i | U = u_j) p(X|H_i). \tag{14}$$

Define

$$Z_{jl}(X) = \{X : D_j(X) < D_l(X)\}$$

which specifies a set such that a lower probability of error will result when the members of the set are assigned to index $j$ instead of $l$. Define

$$P_{ijl} = P(U_0 = s_i | U = u_j) - P(U_0 = s_i | U = u_l) \tag{15}$$

and

$$L_i(X) = \frac{p(X|H_i)}{p(X|H_0)}.$$

Since

$$\sum_{i=0}^{M-1} P_{ijl} = 0$$

we have

$$\begin{aligned} D_j - D_l &= \sum_{i=0}^{M-1} \pi_i p(X|H_i) P_{ijl} \\ &= \sum_{i=1}^{M-1} \pi_i p(X|H_i) P_{ijl} - \sum_{i=1}^{M-1} \pi_0 p(X|H_0) P_{ijl} \\ &= \sum_{i=1}^{M-1} \pi_i p(X|H_0) P_{ijl} \left( L_i(X) - \frac{\pi_0}{\pi_i} \right) \end{aligned}$$

From (15), the change of channel characteristics may alter the value of $P_{ijl}$, which will result in a different region for deciding index $j$ instead of $l$. In other words, the optimum relay rule for the singular relay network needs to be channel aware when $M > 2$.

## IV. Performance Evaluation

In this section, through a number of numerical examples, we demonstrate the performance advantage of our approach over some existing relay strategies, namely DF and AF, for the relay network defined in Fig 1. For DF, each relay node makes its own decision using an MAP rule:

$$U_l = \arg \max_{s_i : i \in \{0,\cdots,M-1\}} \pi_i p(X_l|H_i) \qquad l = 1, \cdots, L \tag{16}$$

and re-encodes it and sends it to the destination node. This is different from the relay signaling specified in Theorem 1, i.e., Eqs. (11) and (9), where all the relay rules are coupled with each other. We remark here that the DF approach considered in this paper is the vanilla version discussed in [8], [11]. We assume that the relay node always forwards its best estimate to a destination node.

For AF, the output of the relay node is simply a scaled version of the received signal, i.e.,

$$U_l = c_l X_l \quad l = 1, \cdots, L$$

where the scaling factor $c_l$ is determined so that all schemes have the same average power constraint. For fading channels, we have

$$c_l^2 = \frac{P_s}{P_s|\alpha_{1l}|^2 + \sigma_{1l}^2}$$

where $P_s$ is the power constraint which is assumed to be the same for all the relay nodes as well as the source node, $\alpha_{1l}$ is the channel coefficient and $\sigma_{1l}^2$ is the variance of channel noise. At the destination node, all the schemes implement the MAP rule to obtain the final decision.

Throughout our simulations, we assume that the channels between the source and the relay nodes are identically and independently distributed (i.i.d.) Rayleigh fading channels with average SNR denoted by $SNR_{sr}$. Similarly, the channels between the relay and destination node are also assumed to be i.i.d. Rayleigh fading channels with average SNR denoted by $SNR_{rd}$ (except for the first example where both relay nodes experience different $SNR_{rd}$). Notice that this is a somewhat simplifying assumption: In a homogeneous environment where the path loss exponent is a constant, the above assumption amounts to requiring that the relay nodes are equidistant to the source node as well as to the destination node. We will vary one of these two SNR with the other fixed; this captures the change in the placement of the relay nodes in terms of their distances to the source and to the destination nodes. The SNR for the direct link between the source and the destination node is denoted as $SNR_{sd}$. Further, all the channels are assumed to be slow fading channels so that the channel coefficients remain unchanged during the transmission of one symbol or a packet.

The signal sent from the source node is assumed to be a $K$-bit codeword drawn from a $M$-ary codebook with equal probability. Hence $M \leq 2^K$. Each bit is assumed to use BPSK modulation. We also assume that the local decision at each relay node is $K$ bits, thus the relay output has a maximum alphabet size of $N = 2^K$.

### *A. Parallel Relay Network*

We first consider an example that we discussed in Section I, the parallel relay network with $K = 1$, $M = N = 2$ and $L = 2$, i.e., a single BPSK symbol is sent from the source and is to be relayed to the destination node using two relay nodes. We assume that the BPSK signal has equal prior probability, i.e.,

$$P(S = -1) = P(S = +1) = 0.5.$$

We also assume that $SNR_{sr}$ is identical for both relay nodes but $SNR_{rd}$ may be different. In this case, the relay rule used by DF for the $l^{th}$ relay node can be easily obtained from (16),

$$SST \triangleq Re\{\alpha_l^* X\} \underset{-1}{\overset{+1}{\lessgtr}} 0$$

where $\alpha_l$ is the channel coefficient for the channel between the source node and the $l^{th}$ relay node and $Re\{\cdot\}$ means real part. Application of Theorem 1 and our iterative algorithm show that our approach also compares $SST$ to a threshold but our threshold is obtained by jointly designing the relay rules and the decoding rule, which leads to performance gains. In table I, with identical $SNR_{sr}$ for both relay nodes and different $SNR_{rd}$ for each relay node, we compare the thresholds of $SST$ and overall error probability between DF and the proposed approach. As one can see, the proposed approach has better performance than DF and the thresholds of $SST$ are different for the different relays for our approach.

We then consider a little different case where $SNR_{rd}$ is identical for both relay nodes. Fig. 2 and Fig. 3 plot the error probability at the destination node as a function of $SNR_{sr}$ and $SNR_{rd}$, respectively. From Fig. 2, where $SNR_{rd}$ is fixed at 5dB, the proposed approach provides the best performance among all three relay schemes. In Fig. 3 where $SNR_{sr}$ is fixed at 5dB, the AF outperforms the proposed method at high $SNR_{rd}$ values. This is not surprising since the optimum performance is achieved with centralized processing, i.e., when all local observations are accessible by the decoder. With high $SNR_{rd}$, the analog signal can be received at the destination almost noiselessly, hence it amounts to the centralized processing. The proposed scheme attempts to find the optimum relay scheme among all possible $K$-bit quantizers to minimizes the error probability at the destination node. The AF apparently does not belong to the class of the $K$-bit quantizers.

We next consider a special case that we also discussed in Section I, the repetition coded binary source. This is equivalent to a binary hypotheses testing with soft (multi-bit) output. To alleviate the computational burden, one can approximate the fading channel using a binary symmetric channel (BSC) where the crossover probability can be properly calculated using the channel SNR. The BSC provides a reasonable, *albeit* coarse, approximation of the fading channel; more ever, one can apply directly the distributed detection algorithm developed in [22] to find the optimal relay rules. We thus compare the BSC approximation with our approach using the actual fading channel model and the two existing relay strategies (i.e., AF and DF). Consider the system with $L = 2$ relay nodes and $K = 4$ bit source input. We generate the error probability plots as a function of $SNR_{sr}$ and $SNR_{rd}$, respectively. From Fig. 4 where we vary $SNR_{sr}$ but fix $SNR_{rd} = 0dB$, one can see that the proposed approach provides uniformly better performance compared with the other alternatives. Notice that all the error probabilities level off as $SNR_{sr}$ increase. This is not unexpected: with large $SNR_{sr}$, the channels between the source and the relay nodes can be considered as ideal. Thus the error probability performance is limited by the finite and fixed $SNR_{rd}$. We also notice that the BSC approximation provides a reasonable performance compared with the proposed approach.

Fig. 5 is the error probability plot as a function of $SNR_{rd}$ with fixed $SNR_{sr} = 0dB$. Again, one observes error probability floor as $SNR_{rd}$ increases due to the fact that $SNR_{sr}$ is fixed. Furthermore, the AF eventually outperforms all other schemes as $SNR_{rd}$ gets large – this is again because at very high channel SNR between the relays and the destination, AF essentially amounts to a centralized processing. On the other hand, the DF is the first to level off in the error probability performance. This is because the DF uses a hard decision relaying – this is clearly not optimal at high SNR for the channel between the relays and the destination.

We also consider a more practical scenario where the packet is coded with a $(7, 4)$ Hamming code [24] with $L = 2$ relay nodes and the generator matrix we use is

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

As shown in Fig. 6 and Fig. 7, the proposed approach again has the best performance.

### B. Three-node Relay Network

We compare the performance of the proposed scheme with two existing relay schemes for the classical three-node model. In generating the error probability plots, we vary one channel SNR and fix the other two. As shown in Figs. 8 - 10, the proposed approach still has the best performance. When we vary $SNR_{sr}$ or $SNR_{rd}$, the plots we obtain are similar to previous examples: the proposed scheme is uniformly better than others for varying $SNR_{sr}$ and the advantage of the proposed scheme over DF diminishes at low SNR for varying $SNR_{rd}$. Since we have a direct transmission from source to destination node, when we vary $SNR_{sd}$ and fix the other two, the performance gain of the proposed scheme diminishes to zero at high SNR, as shown in Fig. 10.

## V. Conclusion

In this work, a novel cooperative relay signaling that applies channel aware decentralized detection theory was proposed to fully exploit the FA property of the source message. Aimed at minimizing the error probability at the destination node, we derived the necessary conditions for an optimal distributed signaling scheme for a FA source. An iterative algorithm was presented to find distributed relay schemes that are at least locally optimum. We further examined some special cases, including the classical three-node relay network and the parallel relay network. For the special case of a single relay node with no direct link between the source and the destination node, i.e., the singular relay network, we pointed out the significant difference between a binary source and a general $M$-ary source ($M > 2$), that is, while the optimal relay rule is channel blind for the singular relay network with a binary source, it is channel aware when $M > 2$. Performance comparison with two existing relay strategies, namely AF and DF, was conducted numerically. In almost all cases of practical interest, the proposed approach exhibits notable advantages over existing relay schemes that do not exploit the redundancy in FA sources.

One drawback of the proposed scheme is that the optimal signaling design requires global channel information. Distributed signaling design that only uses local channel information is more practical and will be investigated in the future. Similar work has been carried in the context of distributed detection for sensor networks [25] and can be extended to the cooperative relay signaling design. Another drawback is that the relay rule design of all relay nodes are *coupled* in the proposed design approach. This significantly increases the complexity of the design algorithm which typically scales exponentially in the number of nodes. One remedy is to resort to the large system regime to optimize the error exponent instead of the error probability, thereby circumventing the iterative algorithm that is needed to achieve the person-by-person optimality in error probability performance.

## References


[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity: Part I System description," *IEEE Trans. Comm.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity: Part I Implementation aspects and performance analysis," *IEEE Trans. Comm.*, vol. 51, pp. 1939–1948, Nov. 2003.

[3] J.N. Laneman, D.N.C. Tse, and G.W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[4] E.C. van der Meulen, "A survey of multi-way channels in Inf. Theory: 1961-1977," *IEEE Trans. Inf. Theory*, vol. 23, pp. 1–37, Jan. 1977.

[5] T.M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, pp. 572–584, 1979.

[6] M. Gastpar and M. Vetterli, "On the capacity of wireless networks: the relay case," in *Proc. IEEE INFOCOM 2002*, New York, NY, June 2002, pp. 1577–1586.

[7] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. on Inf. Theory*, vol. 51, pp. 3037–3063, Sept 2005.

[8] J.N. Laneman and G.W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2415–2425, 2003.

[9] A. Ribeiro, X. Cai, and G. B. Giannakis, "Symbol error probabilities for general cooperative link," *to appear in the IEEE Trans. Wireless Communications*, 2005.

[10] W. Mo and Z. Wang, "Average symbol error probability and outage probability analysis for general cooperative diversity system at high signal to noise ratio," in *Proc. CISS*, Princeton, NJ, Mar. 2004.

[11] D. Chen and J.N. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Comm.*, to appear.

[12] R.S. Blum, "Distributed detection for diversity reception of fading signals in noise," *IEEE Trans. Inf. Theory*, vol. 45, pp. 158–164, Jan. 1999.

[13] B. Chen and P.K. Willett, "On the optimality of likelihood ratio test for local sensor decisions in the presence of non-ideal channels," *IEEE Trans. Inf. Theory*, vol. 51, pp. 693–699, Feb. 2005.

[14] B. Liu and B. Chen, "Channel-optimized quantizers for decentralized detection in sensor networks," *to appear in the IEEE Trans. Inf. Theory*, vol. 52, July 2006.

[15] B. Chen, R. Jiang, T. Kasetkasem, and P.K. Varshney, "Fusion of decisions transmitted over fading channels in wireless sensor networks," in *Proc. of the 36th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2002, pp. 1184–1188.

[16] R. Niu, B. Chen, and P.K. Varshney, "Decision fusion rules in wireless sensor networks using fading statistics," in *Proc. 37th Annual Conference on Information Sciences and Systems*, Baltimore, MD, Mar. 2003.

[17] B. Chen, R. Jiang, T. Kasetkasem, and P.K. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Processing*, vol. 52, pp. 3454–3458, Dec. 2004.

[18] R. Radner, "Team decision problems," *Annals of Mathematical Statistics*, vol. 33, pp. 857–881, 1962.

[19] I.Y. Hoballah and P.K. Varshney, "Distributed Bayesian signal detection," *IEEE Trans. Inf. Theory*, vol. 35, pp. 995–1000, Sept. 1989.

[20] Z.-B. Tang, K.R. Pattipati, and D.L. Kleinman, "A distributed M-ary hypothesis testing problem with correlated observations," *IEEE Trans. Automatic Control*, vol. 37, pp. 1042–1046, July 1992.

[21] J.N. Tsitsiklis, "Extremal properties of likelihood-ratio quantizers," *IEEE Trans. Commun.*, vol. 41, pp. 550–558, 1993.

[22] B. Liu and B. Chen, "Joint source-channel coding for distributed sensor networks," in *Proc. 2004 Annual Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2004.

[23] J.N. Tsitsiklis, "Decentralized detection," in *Advances in Statistical Signal Processing*, H.V. Poor and J.B. Thomas, Eds. JAI Press, Greenwich, CT, 1993.

[24] J. Proakis, *Digital Communications*, McGraw Hill, 1995.

[25] B. Liu and B. Chen, "Distributed detection in sensor networks with channel fading statistics," in *Proc. 2006 International Conference on Acoustic Speech and Signal Processing (ICASSP'06)*, Toulouse, France, May 2006.


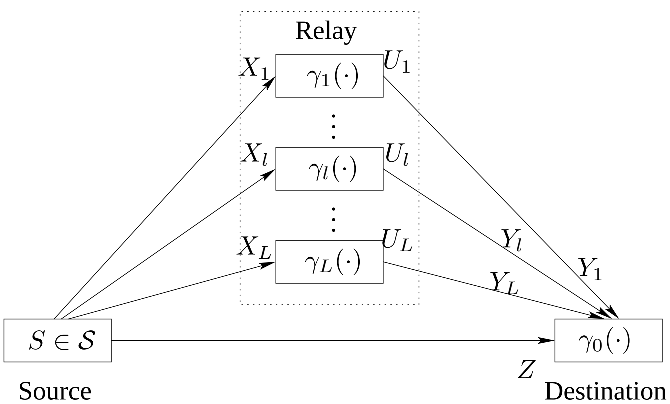


Fig. 1. A wireless relay network with $L$ relay nodes and a direct link connecting the source and the destination nodes.

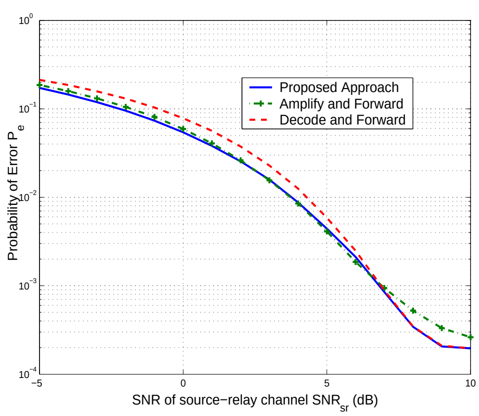


Fig. 2. Error probability versus SNR of source-relay channel for $L = 2, M = 2, K = 1$ ($SNR_{rd} = 5dB$).

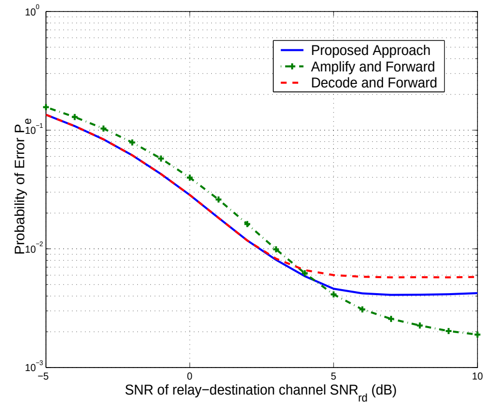


Fig. 3. Error probability versus SNR of relay-destination channel for $L = 2, M = 2, K = 1$ ($SNR_{sr} = 5dB$).

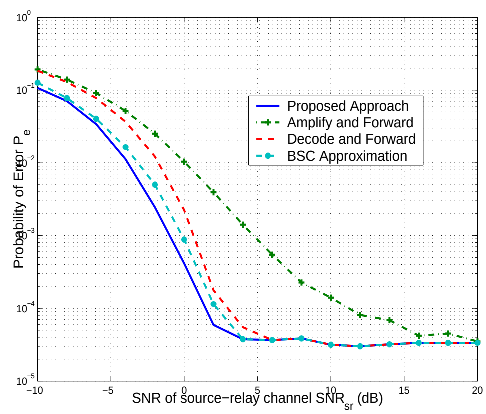


Fig. 4. Error probability versus SNR of source-relay channel for $L = 2, M = 2, K = 4$ ($SNR_{rd} = 0dB$).

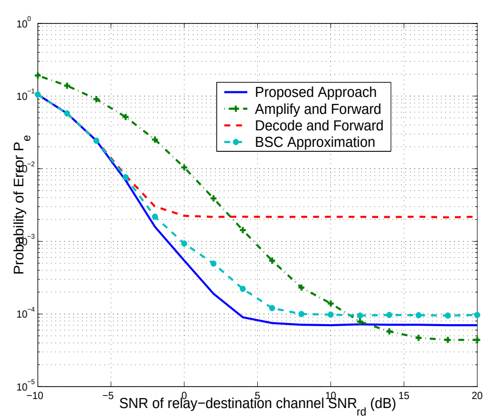


Fig. 5. Error probability versus SNR of relay-destination channel for $L = 2, M = 2, K = 4$ ($SNR_{sr} = 0dB$).

TABLE I

THE COMPARISON OF THRESHOLDS OF $SST$ AND ERROR PROBABILITY BETWEEN DF AND PROPOSED APPROACH ($SNR_{sr} = 5dB$)

| $SNR_{rd}$ **for the first relay node** | $SNR_{rd}$ **for the second relay node** | **Relay scheme** | **threshold at the first relay node** | **threshold at the second relay node** | **Pe** |
|---|---|---|---|---|---|
| 5dB | 0dB | DF | 0 | 0 | 0.0080 |
| | | Proposed approach | 0.0005 | -0.0063 | 0.0073 |
| 5dB | 5dB | DF | 0 | 0 | 0.0063 |
| | | Proposed approach | -0.0599 | -0.0599 | 0.0045 |
| 5dB | 10dB | DF | 0 | 0 | 0.0060 |
| | | Proposed approach | -0.0786 | -0.0512 | 0.0038 |

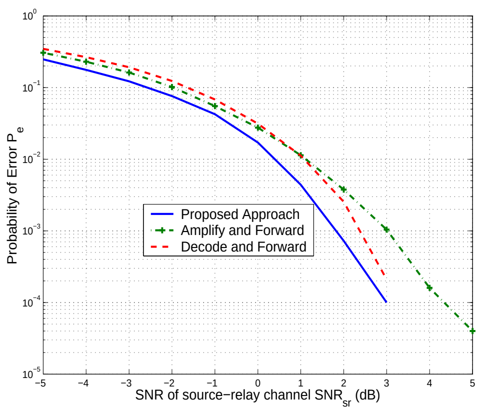


Fig. 6. Error probability versus SNR of source-relay channel for the case using $L = 2$ and $(7, 4)$ code as source input ($SNR_{rd} = 5dB$).

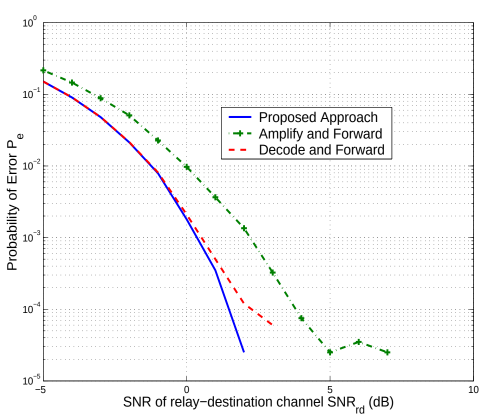


Fig. 7. Error probability versus SNR of relay-destination channel for the case using $L = 2$ and $(7, 4)$ code as source input ($SNR_{sr} = 5dB$).

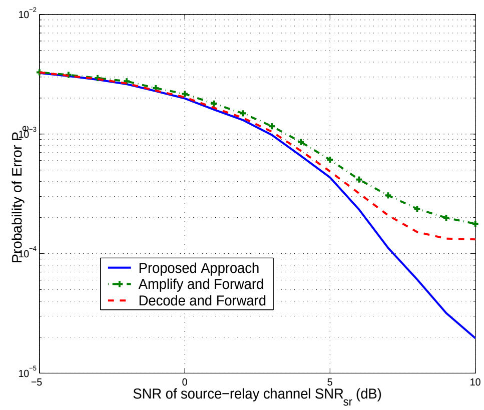


Fig. 8. Error probability versus SNR of source-relay channel for classical model with $M = 3, K = 3$ ($SNR_{rd} = 5dB$, $SNR_{sd} = 5dB$).

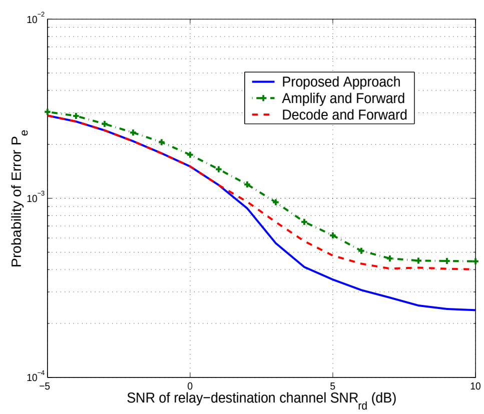


Fig. 9. Error probability versus SNR of relay-destination channel for classical model with $M = 3, K = 3$ ($SNR_{sr} = 5dB$, $SNR_{sd} = 5dB$).

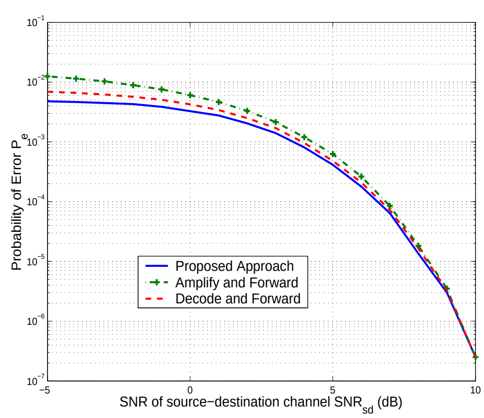


Fig. 10. Error probability versus SNR of source-destination channel for classical model with $M = 3, K = 3$ ($SNR_{sr} = 5dB$, $SNR_{rd} = 5dB$).